Noise-induced rupture process: Phase boundary and scaling of waiting time distribution

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A bundle of fibers has been considered here as a model for composite materials, where breaking of the fibers occur due to a combined influence of applied load (stress) and external noise. Through numerical simulation and a mean-field calculation we show that there exists a robust phase boundary between continuous (no waiting time) and intermittent fracturing regimes. In the intermittent regime, throughout the entire rupture process avalanches of different sizes are produced and there is a waiting time between two consecutive avalanches. The statistics of waiting times follows a Γ distribution and the avalanche distribution shows power-law scaling, similar to what has been observed in the case of earthquake events and bursts in fracture experiments. We propose a prediction scheme that can tell when the system is expected to reach the continuous fracturing point from the intermittent phase.

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Rupture and breakdown [1,2] are complex processes that occur both in micro- and macroscales. Natural rupture phenomena like earthquake, landslide, mine collapse, and snow avalanches often appear catastrophic to human society. It is therefore a fundamental challenge to understand the underlying rupture process so that the losses in terms of properties and lives can be minimized by providing early alarms. The same crisis persists in construction engineering and the material industry, where detailed knowledge of the strength of the materials and their failure properties are essential. But the physical processes, which initiate rupture, help its growth, and finally result in breakdown, are not completely understood yet.

Fiber bundle model (FBM) has become a useful tool for studying rupture and failure [3] of composite materials under different loading conditions. The simple geometry of the model and clear-cut load-sharing rules allow one to achieve analytic solutions [4–6] to an extent that is not possible in any of the fracture models studied so far by the fracture community. FBM was introduced first in connection with textile engineering [7] and recently physicists took interest in it, mainly to explore the critical failure dynamics and avalanche phenomena in this model [8–10]. Not only the classical fracture failure (stress induced) in composites, FBM has been used successfully for studying noise-induced (fatigue) failure [11–15], creep [16–18], and thermally induced failures [19,20]. The statistics of avalanches in these types of failure models show similarities with results for acoustic emissions [21] (during material failure) and earthquakes [22–24].

In this work, through waiting time and avalanche statistics, we analyze a noise-induced intermittent fracturing process in composite materials under fixed external loading. The waiting time is defined as the time (Monte Carlo steps) between two consecutive avalanches in the avalanche time series for the entire failure process. Through a mean-field calculation we show that in the stress-noise space, there exists a robust phase boundary between continuous (no waiting time) and intermittent fracturing regimes and that can be verified by numerical simulations. In the intermittent fracturing regime we study the distributions of avalanches and waiting times for different types of fiber strength distributions. Finally, we mention and discuss studies on waiting-time statistics in other fracture models, earthquake events, and fracture experiments.

We consider first a bundle of N parallel fibers—and a load ($W = \sigma N$) is applied on the bundle. The fibers have different individual strengths (x) which are drawn from a probability distribution and the bundle has a critical strength σ_c [3], so that, without any noise, the bundle does not fail completely for stress $\sigma \leqslant \sigma_c$, but it fails immediately for $\sigma > \sigma_c$. We now assume that each fiber having strength x_i has a finite probability $P(\sigma,T)$ of failure at any stress σ induced by a nonzero noise T:

$$P(\sigma,T) = \begin{cases} \exp\left[-\frac{1}{T}\left(\frac{x_i}{\sigma} - 1\right)\right], & 0 \leqslant \sigma \leqslant x_i, \\ 1, & \sigma > x_i. \end{cases} \quad (1)$$

Here $P(\sigma,T)$ increases as T increases and for a fixed value of T and σ_c, as we increase σ, the bundle breaks more rapidly. We simulate this failure phenomenon following Eq. (1) in discrete time t. After each failure (at the fixed stress σ), the total load $N\sigma$ is redistributed among the remaining fibers equally and we check, at time $t+1$, if the present stress $\sigma(t+1) = W/N(t+1)$ can induce any further failure following Eq. (1). When the value of σ is considerably large, it so happens that at every time step at least a single fiber breaks until the complete collapse of the bundle. This is a single avalanche and there is no waiting time [15]. But as we decrease the initial value of σ, at a limiting value, in a particular time step t not a single fiber breaks. We consider this as a single waiting time ($t_W = 1$) and the limiting value of σ, at which the waiting time appears for the first time is denoted by σ_0. This is the onset of the intermittent fracturing process. After one waiting time, again

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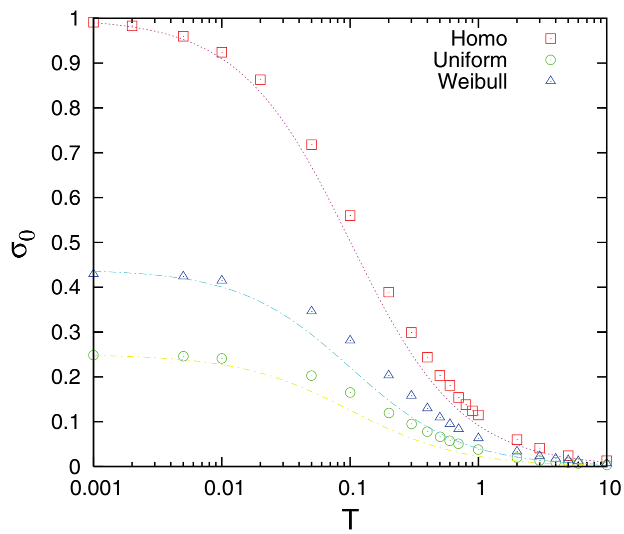

FIG. 1. (Color online) Phase boundary (σ_0 vs T plot) for three different type of fiber strength distributions with $N = 20\,000$. Data points are simulation results (averages are taken over 100 samples) and solid lines are analytic estimates [Eqs. (3) and (4)] based on mean-field arguments.

another avalanche starts and eventually all the fibers break after such a finite number of avalanches. The number of fibers broken during a single avalanche is counted as the avalanche size (m). It is obvious that as we increase the value of T, the value of σ_0 decreases. When the noise is large, the initial applied load has to be smaller for the emergence of a waiting time. Thus stress (σ) and noise (T) values determine whether the system is in continuous rupture phase or in the intermittent rupture phase. It may be mentioned that T can be interpreted as a measure of thermal noise in the system and similar thermally activated breakdown in the fiber bundle model had been studied experimentally [13] and theoretically [14].

To determine the phase boundary we can give a mean-field argument that, at $\sigma = \sigma_0$, at least one fiber must break to trigger the continuous fracturing process. After this single failure the load has to be redistributed on the intact fibers and the effective stress must be more than σ_0—which in turn enhances failure probability for all the intact fibers. Therefore, in the case of a homogeneous bundle where all the fibers have identical strength $x_i = 1$ (therefore, $\sigma_c = 1$), at the phase boundary $N P(\sigma_0, T) \geqslant 1$ giving

$$N \exp\left[-\frac{1}{T}\left(\frac{1}{\sigma_0} - 1\right)\right] \geqslant 1, \tag{2}$$

which gives

$$\sigma_0 \geqslant \frac{1}{1 - T \ln(1/N)}. \tag{3}$$

In the absence of noise T, $\sigma_0 = 1 = \sigma_c$, which is consistent with the static FBM results [3]. This analytic estimate coincides with the data obtained from simulation (Fig. 1). It shows a nice phase boundary between the continuous and intermittent fracturing regimes.

For heterogeneous cases where fibers have different strength and the whole bundle has a critical strength σ_c, we make the conjecture that

$$\sigma_0 \geqslant \frac{\sigma_c}{1 - T \ln(1/N)}, \tag{4}$$

keeping in mind that in the absence of noise T, $\sigma_0 = \sigma_c$. To verify our conjecture we choose heterogeneous bundles of N fibers where strength of the fibers are drawn from a statistical distribution. We have considered two different kinds of fiber strength distributions: (1) uniform distribution of fiber strength having cumulative form $Q(x) = x$ for $0 < x \leqslant 1$ and (2) Weibull distribution $Q(x) = 1 - \exp(-x^k)$, where k is the Weibull index (we have taken $k = 2.0$ and 5.0). Each fiber has a finite probability $P(\sigma, T)$ of failure at any stress σ induced by a nonzero T as mentioned before. Similar to the homogeneous case, for a particular value of T, below a certain value of σ, the waiting time appears here. One can see that the theoretical estimate of the phase boundary agrees with the numerical data for the heterogeneous cases (Fig. 1). However, this agreement was much better for the homogeneous case. This difference can be explained through the amount of randomness involved in the respective systems. In the case of homogeneous bundle there is no randomness in the fiber strength—the only randomness is coming from the noise term, whereas in the case of heterogeneous bundles—there are two sources of randomness—in the fiber strengths and in the noise term.

The mean-field calculation (3) suggests that σ_0 value depends on the number of intact fibers in the bundle (N). It increases with decrease in the number of intact fibers at time t (N_t). Therefore, when we start with a much lower stress value ($\sigma < \sigma_0$) the σ_0 value increases slightly with time (as N_t decreases with small individual failures). But the effective stress value follows a strict relation with applied stress and number of intact fibers as

$$\sigma_t = \sigma N/N_t. \tag{5}$$

These two equations [(3) and (5)] allow us to make a theoretical prediction of σ_0 value for a particular bundle of homogeneous fibers. If we plot together $-\sigma_0$ vs N_t and σ_t vs N_t for a particular σ—then the point of intersection will give the σ_0 value for that particular σ value (Fig. 2). Therefore, during a fracturing process if we can measure the effective stress or the number of intact elements in the system, we can always predict the onset of continuous fracturing.

Existence of such a phase boundary has important consequences on fracturing study in material failure and other fracture-breakdown phenomena. In real situations of material or rock fracturing, acoustic emission measurements can show clearly whether an ongoing fracturing process belongs to a continuous or intermittent fracturing phase. Acoustic emissions [21] are basically sound waves produced during a microcrack opening within the material body due to external stress and noise factors. Once a system enters into a continuous fracturing phase the breakdown must be imminent. Thus the identification of rupture phase can predict the fate of a system correctly.

In the intermittent fracturing phase avalanches of different sizes are produced separated by waiting times (t_W) of different magnitudes. This happens for a stress value σ below σ_0 at a certain noise (T) level. We have studied the waiting time

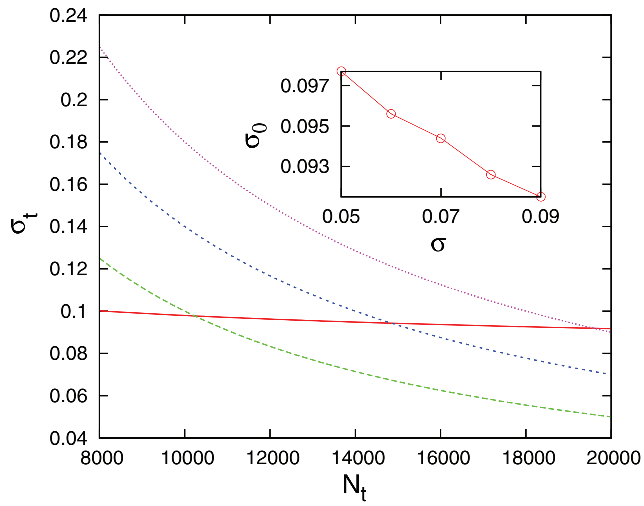

FIG. 2. (Color online) Prediction scheme for homogeneous bundle: σ_t vs t are plotted for three different initial stress values with $N = 20\,000$. The intersection points with red line [Eq. (3)] indicates the starting of continuous fracturing. The inset shows the variation of σ_0 value with initial stress (σ) in the same bundle.

distribution for both homogeneous and heterogeneous bundles with $N = 20\,000$. Each curve can be fitted with a Γ distribution [22–24]

$$D(t_W) \propto \exp(-t_W/a)/t_W^{1-\gamma}, \tag{6}$$

where $\gamma = 0.15$ for the homogeneous case and $\gamma = 0.26$ for heterogeneous cases (Fig. 3). As shown in the inset of

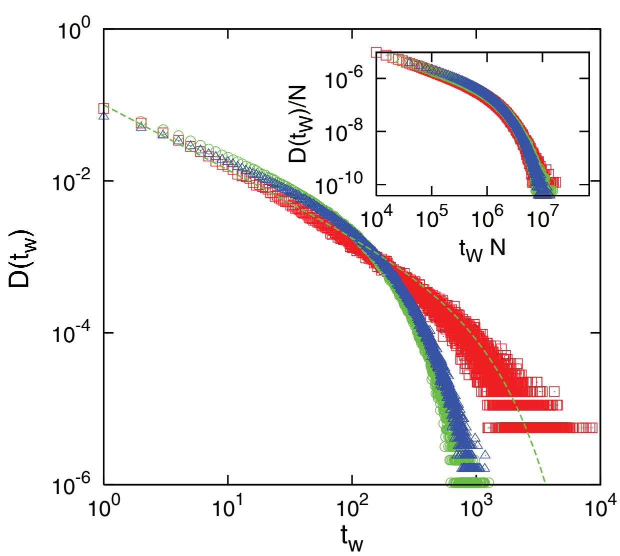

FIG. 3. (Color online) Simulation results for the waiting time distributions for three different types of fiber strength distributions (square, circle, and triangle symbols are used for homogeneous, uniform, and Weibull distributions, respectively) with $N = 20\,000$. All the curves can be fitted with the Γ form $\exp(-t_W/a)/t_W^{1-\gamma}$ (dashed line), where $\gamma = 0.15$ for the homogeneous case (averages are taken over 25 samples) and $\gamma = 0.26$ for uniform and Weibull distributions (averages are taken over 100 samples). In the inset we show the data collapse of the waiting time distributions with system sizes for uniform distribution.

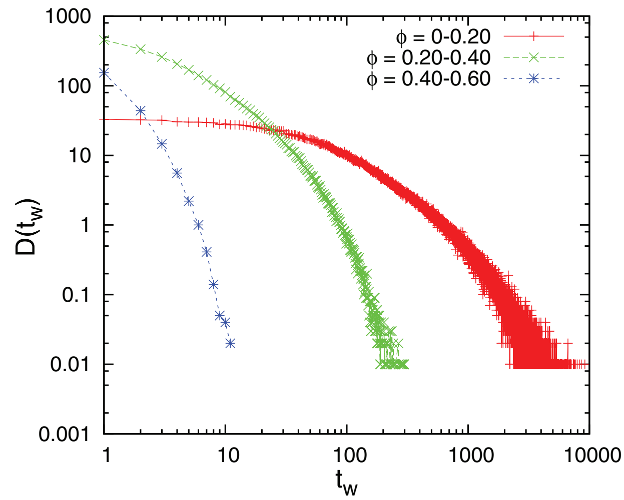

FIG. 4. (Color online) Evolution of waiting time for a homogeneous fiber bundle ($N = 20\,000$; average number = 25) at $T = 0.9$ and $\sigma = 0.062$.

Fig. 3, the plot of $D(t_W)/N$ against $t_W N$ gives good data collapse for different N values $[D(t_W) = A(1-P)^{t_W N} \sim A\exp(-Pt_W N)$, where P denotes individual failure probability and A is a constant; hence the normalization of $D(t_W)$ requires $A \sim N]$. Such a data collapse indicates the robustness of the Γ function form. The value of a is the measure of the extent of the power-law regime and it has different values for different types of strength distribution. As we increase N, the value of a gradually decreases. We have also studied the waiting time distribution for a fixed value of N, but different sets of values of T and σ, all of which show Γ distribution of the form of Eq. (6). For a fixed value of N and T as σ decreases, the power-law region extends longer and thus the value of a increases, but the exponent of power-law decay remains the same. Again for a certain value of N and σ as T decreases, the value of a increases without any change in the power-law exponent. These results imply that the power-law exponent remains unchanged with variation of σ, T, and N.

The noise-induced rupture process, modeled here, has two basic ingredients: external stress σ and noise T. The noise term triggers initial rupture which induces one or more load-redistribution cycles that finally enhance the effective stress level on the system. Therefore, the initial phase of the rupture process is dominated by the noise term and as the rupture process goes on the stress factor becomes more dominating. At the final stage the stress redistribution mechanism drives the system toward complete collapse through a big avalanche.

For finite values of N, we have studied the waiting time distribution at an interval of 0.20 of the fraction of the broken fibers (ϕ). It has been observed that within the intermittent regime for a homogeneous fiber bundle ($N = 20\,000$) the waiting time distribution is purely a Γ distribution during the first 0.20 fraction of fibers broken (Fig. 4). During the next 0.20 fraction of broken fibers (i.e., 0.20–0.40), the power-law portion diminishes and for the next interval (0.40–0.60) there is no power-law regime at all. For the next two intervals (0.60–0.80 and 0.80–1.00) no waiting time appears which implies that for a homogeneous fiber bundle the waiting time monotonously disappears with the breaking of fibers (Fig. 4).

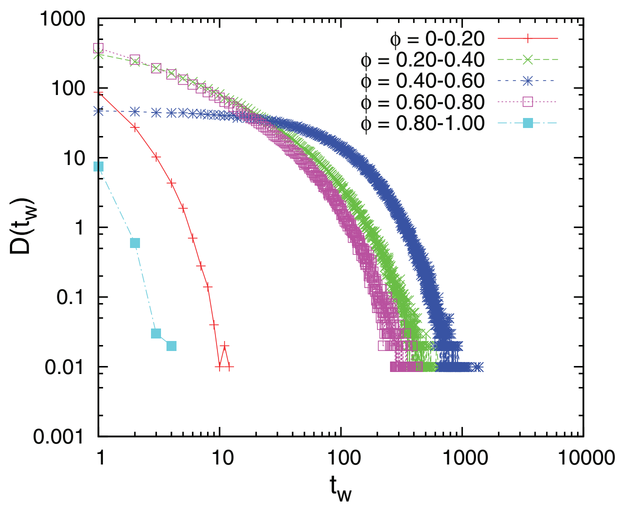

FIG. 5. (Color online) Evolution of waiting time for uniform fiber strength distribution ($N = 20\,000$; average number = 100) at $T = 0.7$ and $\sigma = 0.027$.

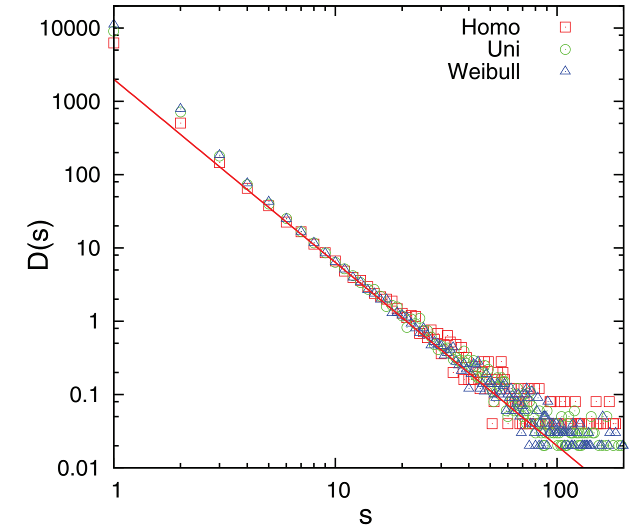

FIG. 6. (Color online) Numerical data for avalanche size distributions for three different types of fiber threshold distributions (averages are taken over 25 samples for homogeneous case and 100 samples for uniform and Weibull cases) with $N = 20\,000$. The straight line has a slope -2.5.

The nature of evolution of waiting time distribution for the uniform distribution is different from that of the homogeneous one. In the case of a uniform fiber bundle up to 0.60 fraction of fibers (at an interval of 0.20) the value of a increases and large waiting times appear as more fibers break. This is due to the fact that initially the fibers of very low strength break down instantaneously as soon as a finite stress is applied. But gradually those fibers of low strength become scarce and due to the presence of fibers of intermediate strengths and the moderately increased stress (due to gradual breakdown of fibers), waiting times of broad range appear. But the breaking of the consequent fibers are faster due to the increased stress and gradually the a value decreases (Fig. 5).

In general, avalanches or bursts bear important information of the dynamics of intermittent processes. In our model the noise T triggers a rupture process which continues through the load (or stress) redistribution mechanism. The avalanche size distributions follow a universal power-law [$D(s) \sim s^{-\xi}$] scaling with exponent $\xi = 2.5$. This result (Fig. 6) demands that such intermittent rupture process belongs to the quasistatic fracturing class, where the universality of the exponent value has already been established [8].

Instead of considering all the avalanches up to the complete failure of the system, if we gather avalanches within some window during the breaking process, the shape of the avalanche distributions changes as the system approaches complete failure. In the case of homogeneous strength distribution, there is a monotonic variation (Fig. 7), i.e., more and more large avalanches appear as the failure point is approached but bundle with uniform strength distribution shows a nonmonotonic variation (Fig. 8).

Our model for noise-induced rupture process is not limited to any particular system, rather it is a general approach and can model more complex situations. There are evidences of stress redistribution and stress localization around fracture-fault lines and several factors that can help rupture evolution are friction, plasticity, fluid migration, spatial heterogeneities, chemical reactions, etc. In our model such stress redistribution or stress localization can be taken into account through a proper load sharing scheme and noise term (T) can represent the combined effect of other factors.

We would like to mention here that waiting times and their statistics in different types of fracture models have also been discussed recently. Creep rupture in a nonlinear viscoelastic FBM was proposed and studied extensively by Hidalgo *et al.* in 2002 [16]. By construction it is a different class of fiber bundle model—there is no noise term and nonlinearity in material response has been introduced through an exponent in the constitutive equation. This model is different from our simple noise-induced FBM. It has been observed that the strain rate shows power-law relaxation in the creep regime followed

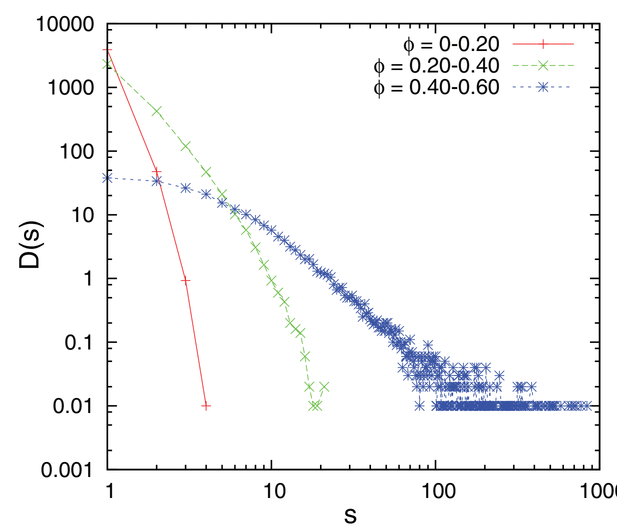

FIG. 7. (Color online) Evolution of the avalanche distributions with the fraction of broken fibers (ϕ) for a homogeneous fiber bundle ($N = 20\,000$; averages are taken over 25 samples) at $T = 0.9$ and $\sigma = 0.062$.

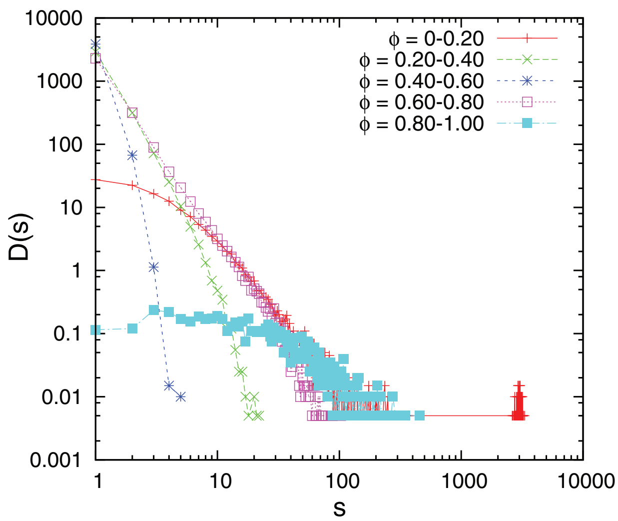

FIG. 8. (Color online) Evolution of avalanche distributions with fraction of broken fibers (ϕ) for uniform fiber strength distribution ($N = 20\,000$; averages are taken over 100 samples) at $T = 0.7$ and $\sigma = 0.027$.

by a power-law acceleration up to complete rupture [17] and the waiting time distributions in such creep models obey power laws [18].

Yoshioka *et al.* [19,20] discussed thermally activated failure in FBM introducing a Gaussian fluctuation in local force (stress) on individual fibers. Potentially this model goes back to classical FBM if the fluctuation term is zero. But if the fluctuation is nonzero, then the bundle can fail even when the external stress is zero which is confusing and not real. In that sense our noise-induced failure scheme in FBM (introduced in 2003 in Ref. [15]) is more robust and some exact analytic results (failure time and avalanche distribution) have already been calculated through this scheme.

Identification of phase boundary is crucial for any dynamical system because a system usually changes its behavior as it moves from one phase to another. As we can see in our model, there is no waiting time above the phase boundary (continuous rupture phase) and waiting time appears below the phase boundary (intermittent phase). One can also estimate the failure time of the system exactly [15] in the continuous rupture phase. In the case of fracturing in loaded rocks or materials, such study can help to identify reliable precursors which can warn of an imminent breakdown. We notice, in our model system, the magnitude of waiting time reduces gradually towards the breakdown point which is reflected in the variation of a in the functional form of the distribution. What is the exact form of this variation? Does it depend on the applied stress and noise level? Which one is the more sensitive parameter? These questions must be answered to develop a prediction scheme based on available precursors prior to failure or breakdown.

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