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Inverse bremsstrahlung in relativistic quantum plasmas

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We study the absorption of an intense electromagnetic wave in a plasma by inverse bremsstrahlung, in the relativistic quantum regime, by using the Klein-Gordon (KG) equation. We examine the following points: (1) the solutions of the KG equation in the absence of collisions; (2) the transition probabilities between electron momentum states, and (3) the effective collision frequency in the weak and strong field limits.

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I. INTRODUCTION

Relativistic quantum plasmas are relevant to a large range of applications, from astrophysics to intense laser plasma experiments to be performed in the near future [1–3]. Quantum plasmas have also recently been reviewed by Haas [4]. In such a wide field, some progress has recently been made, by extending the well-known Volkov solutions, determining the electron (and positron) states in a plane electromagnetic wave in vacuum [5,6], to the case of particle-wave interaction in a plasma, and by deriving approximate Dirac solutions for both electrostatic and electromagnetic waves [7]. Dispersion relations for isotropic quantum relativistic plasmas have also been derived, using both Klein-Gordon [8] and Dirac formulations [9]. Another important area of application is that of free electron lasers in the quantum regime [10].

Here we propose to study the problem of wave absorption for intense waves in a plasma, due to the inverse bremsstrahlung process. The electrons are forced to oscillate in the field of the incident wave. This oscillatory motion is reversible and cannot lead to a net energy absorption. However, dissipation of the incident wave energy can occur when the oscillating electrons collide with the plasma ions. Inverse bremsstrahlung is considered one of the main processes of laser energy absorption for plasma heating and compression in laser fusion schemes [11], and has been approached by several authors in the past. Relevance should be given to the classical work by Silin [12], and Zel'dovich and Raiser [13], with further extensions by Seely and Harris and others [14–17]. Here we treat the problem for a generic relativistic quantum plasma, where electron recoil effects are retained and the usual dipole approximation is not possible. Quantum relativistic models of inverse bremsstrahlung in vacuum are known for many years [18–20], and the validity of the dipole approximation was discussed by [21]. In our present formulation, plasma dispersion effects in the electron quantum states will also be included.

For simplicity, we use the Klein-Gordon (KG) equation to determine the quantum electron states in the field of the intense wave in a plasma. With such a description we retain plasma dispersion but neglect spin effects, which can be assumed negligible for unmagnetized plasmas. We are also unable to describe the coupling between the electron and positron fields, but this is irrelevant in most laser plasma interactions. The same procedure can easily be extended to the Dirac solutions, which are formally similar and available.

A complete understanding of the relativistic quantum effects associated with intense laser plasma interaction should necessarily include spin and electron-positron coupling. However, it is known that both effects only become significant if the incident laser photons have an energy close to twice the electron rest energy, or $\hbar\omega \sim 2mc^2$. It is also known that creation of electron-positron pairs from vacuum will occur for ultrahigh laser intensities, when the laser electric field becomes close to the Schwinger limit, $E_{cr} = 1.3 \times 10^{18}$ V/m. This means that the present approach based on the KG equation will stay approximately valid for laser frequencies and field amplitudes well below these two limits, which is compatible with the existing intense laser sources, and to those foreseen in a near future. A detailed analysis of the various aspects associated with relativistic quantum plasmas can be found in a recent review [3].

The structure of the paper is the following. In Sec. II, we derive the KG solutions for the electron wave functions in a plasma, in the field of an intense wave, and in the absence of collisions. These solutions are then used, in Sec. III, to derive an expression for the transition probabilities between electron momentum states, induced by the presence of electron-ion collisions. The interaction potential is treated as a perturbation. The Yukawa potential is used, where the purely Coulomb interactions are modified by the Debye screening. Such a screening effect could be important for intense laser fields, leading to a significant amplitude of the driven electron oscillations. The transition probabilities are then used, in Sec. IV, to derive a general expression for the effective collision frequency, which characterizes the bremsstrahlung process. Our results are discussed in the nonrelativistic or low intensity, and the ultrarelativistic or high intensity wave limits, and comparison is made with previous results known in the literature. Finally, in Sec. V, we state some conclusions.

II. ELECTRON STATES IN A WAVE

In contrast with the usual approach, we consider plasma dispersive effects, by following the method developed in Ref. [7]. For simplicity, the electron quantum states are described by using the Klein-Gordon (KG) equation. This implies the neglect of spin effects and electron-positron coupling, which is valid for isotropic plasmas and laser intensities well below the Schwinger limit. We start by writing the KG equation in natural units ($\hbar = 1, c = 1$), describing the space time evolution of the electron wave function ψ, as

$$(i\partial_t + eV)^2\psi = [(i\nabla - e\mathbf{A})^2 + m^2]\psi, \tag{1}$$

where $-e$ and m are the electron charge and mass, V and $\mathbf{A}$ are the scalar and vector potentials. We consider the electron in an electromagnetic wave pulse, with frequency ω and wave number $\mathbf{k}$, such that

$$V = 0, \quad \mathbf{A} \equiv \mathbf{A}(\tau) = \mathbf{A}_0 f(\tau), \tag{2}$$

where $\mathbf{A}_0$ is the wave amplitude and $f(\tau)$ is a given function to be defined later, which only depends on the variable $\tau = t - (\mathbf{k}\cdot\mathbf{r})/\omega$. This is an useful and generic description of the wave pulse, which is valid if the pulse shape is not strongly deformed along propagation, and if the phase slippage inside the pulse is small [22]. We can solve the KG equation for such a field by assuming a solution of the form

$$\psi(\mathbf{r},t) = \varphi(\tau)\exp(i\mathbf{q}\cdot\mathbf{r} - i\omega_q t), \tag{3}$$

where $\mathbf{q}$ and ω_q are constants, to be specified later. Replacing Eqs. (2) and (3) in the wave equation (1), we obtain

$$-\Omega_p^2\varphi'' + 2i\Omega_q\varphi' - [2e(\mathbf{A}\cdot\mathbf{q}) + e^2A^2]\varphi = 0, \tag{4}$$

where we have introduced the auxiliary quantities,

$$\Omega_p^2 = \left(\frac{k^2}{\omega^2} - 1\right), \quad \Omega_q = \left(\omega_q - \frac{\mathbf{q}\cdot\mathbf{k}}{\omega}\right). \tag{5}$$

We notice that, in vacuum, the first term in (4) is absent, because we have $\omega = k$ and $\Omega_p^2 = 0$. But, in a plasma, we have the dispersion relation [8],

$$\omega^2 = k^2 + \frac{\omega_p^2}{\gamma_a}, \quad \gamma_a = \sqrt{1 + a^2}, \tag{6}$$

where ω_p is the electron plasma frequency, γ_a is the relativistic gamma factor in the wave field, and $a = e|A_0|/m$ is the normalized wave amplitude. In this dispersion relation we have retained the effect of self-induced transparency, which is relevant for intense laser pulses, such that $a \gg 1$. It means that we have $\Omega_p^2 = (\omega_p^2/\gamma_a\omega^2)$. In deriving Eq. (4) we have also taken the advantage of the fact that $\mathbf{q}$ and ω_q are arbitrary constants, and have taken the particular choice,

$$\omega_q^2 = q^2 + m^2. \tag{7}$$

This allowed us to drop the mass term. In order to find a simple analytical solution for Eq. (4), we can use an envelope approximation, by assuming that

$$\Omega_p^2|\varphi''| \ll |\Omega_q\varphi'|. \tag{8}$$

This is less stringent than the usual envelope approximation of nonlinear optics, where the additional factor $\Omega_p^2 < 1$ is missing, and can easily be justified for intense electromagnetic wave pulses, due to the additional relativistic factor $\gamma_a \gg 1$. Using an iterative approach where we first neglect the second derivative term in (4), we can obtain $\varphi(\tau)$ by integration. The resulting wave-function solution can then be written in the form,

$$\psi(\mathbf{r},t) = \psi_0 \exp[iS(\tau) + i(\mathbf{q}\cdot\mathbf{r} - \omega_q t)], \tag{9}$$

where the phase function $S(\tau)$ resulting from such an integration is defined by

$$S(\tau) = -\int^{\tau}\left\{F(\tau') - \frac{\Omega_p^2}{2\Omega_0}[F^2(\tau') + iF'(\tau')]\right\}d\tau'. \tag{10}$$

where we have used the auxiliary function,

$$F(\tau) = \frac{1}{\Omega_q}\left[e(\mathbf{A}\cdot\mathbf{q}) + \frac{1}{2}e^2A^2\right]. \tag{11}$$

This KG solution should be compared with the generalized Volkov solutions of the Dirac equation, valid for electrons in a plasma, and recently discussed by [7], in the limit $\Omega_p \to 0$. In this case, the above wave function ψ will be replaced by a bi-spinor, which contains information on both the spin effect and the electron-positron coupling. Spin contributions to the phase function $S(\tau)$ will also appear. But, apart from these important qualitative changes, the present KG solution is formally similar to the Volkov solutions, but generalizes it to include plasma dispersion effects. A detailed comparison between the two types of solutions is outside the scope of the present work.

As an example, let us consider the simple case of a linearly polarized sinusoidal wave, such that $\mathbf{A} = A_0\mathbf{e}$, where $\mathbf{e}$ is the unit polarization vector, and $f(\tau) = \cos(\omega\tau)$. In this case, and retaining the dominant plasma dispersion corrections, the phase function $S(\tau)$ can be written as

$$S(\tau) \simeq \eta\left[\omega\tau + \tfrac{1}{2}\sin(\omega\tau)\right] + \zeta\,\sin[\omega(\tau - \tau_0)], \tag{12}$$

where we have used $\tau_0 = \Omega_p^2/2\Omega_0$, and defined

$$\zeta = \frac{e}{\Omega_q}\frac{A_0}{\omega}(\mathbf{q}\cdot\mathbf{e}), \quad \eta = \frac{e^2}{4\Omega_q}\frac{A_0^2}{\omega} - \frac{\Omega_p^2}{4\Omega_0}\zeta^2\omega. \tag{13}$$

Performing the integration, and using Bessel expansions, we obtain

$$\exp[iS(\tau)] = e^{i\eta\theta}\sum_{n,m}J_n(\zeta)J_m(\eta/2)e^{i(n+2m)\theta}, \tag{14}$$

with $\theta \equiv \theta(\mathbf{r},t) = (\mathbf{k}\cdot\mathbf{r} - \omega t)$. For simplicity, we neglect the small correction associated with τ_0. Similarly, we can consider a circularly polarized wave. In this case, the result is

$$\exp[iS(\tau)] = e^{2i\eta\theta}\sum_n J_n(\zeta)e^{in\theta}. \tag{15}$$

These solutions for the electron wave function in the field of an intense wave can now be used to calculate the transition probabilities between different electron momentum states, induced by collisions with the ions, as shown in the next section.

III. TRANSITION PROBABILITIES

We have seen that, in the absence of collisions, the electron quantum states in the field of an intense laser pulse with frequency ω can be given by

$$\psi_q(\mathbf{r},t) = \tilde{\psi}_q e^{i\theta_q} e^{iS_q(\tau)}, \tag{16}$$

where $\tilde{\psi}_q$ is a normalization constant, and $\theta_q = (\mathbf{q}\cdot\mathbf{r} - \omega_q t)$. For circularly polarized light, this can be written in a more explicit form as

$$\psi_q(\mathbf{r},t) = \tilde{\psi}_q e^{i(\theta_q + 2\eta_q\theta)} \sum_n J_n(\zeta_q) e^{in\theta}. \tag{17}$$

These wave-function solutions satisfy the orthogonality relation,

$$\begin{aligned}\langle\psi_q|\psi_{q'}\rangle &\equiv \int \psi_q^*(\mathbf{r},t)\psi_{q'}(\mathbf{r},t)d\mathbf{r}\\ &= \tilde{\psi}_q^*\tilde{\psi}_{q'} \sum_{n,n'} J_n(\zeta_q)J_{n'}(\zeta_{q'}^*)I_{n,n'}(\mathbf{q},\mathbf{q}'),\end{aligned} \tag{18}$$

where we have introduced the integral,

$$I_{n,n'}(\mathbf{q},\mathbf{q}') = \int e^{-i(\theta_q-\theta_{q'})} e^{-2i(\eta_q-\eta_{q'})\theta} e^{-i(n-n')\theta} d\mathbf{r}. \tag{19}$$

At this point, it should be noticed that, in the usual dipole approximation corresponding to $\mathbf{k}\cdot\mathbf{r} \to 0$, this would reduce to

$$I_{n,n'}(\mathbf{q},\mathbf{q}') = 2\pi e^{i(n-n')\omega t}\delta(\mathbf{q}-\mathbf{q}'). \tag{20}$$

If we now impose a time averaging over a time interval $T \gg 1/\omega$ much larger than the wave period, use the Bessel functions addition theorem, and choose an appropriate value for the normalization constant $|\tilde{\psi}_q|^2$, we obtain

$$|\langle\psi_q|\psi_{q'}\rangle| = \delta(\mathbf{q}-\mathbf{q}'). \tag{21}$$

We can easily verify that this normalization condition stays valid for the general case, where $\mathbf{k}\cdot\mathbf{r} \neq 0$. The general solution for the electron wave equation is then a linear superposition of these orthonormal quantum states. Let us now assume the presence of a scalar potential, due to an ion located at $\mathbf{r} = 0$, as described by the Yukawa potential,

$$V(\mathbf{r}) = \frac{Ze^2}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right), \tag{22}$$

where Ze is the charge of the ion, $\lambda_D = v_{\text{the}}/\omega_p$ is the electron Debye length, and v_{the} the electron thermal velocity. The electron quantum states become coupled by this collision potential, and the electron wave function can be represented by a superposition of quantum states, with time-dependent amplitude coefficients $C_q(t)$, as given by

$$\psi(\mathbf{r},t) = \int C_q(t)\tilde{\psi}_q e^{i\theta_q} e^{iS_q(\tau)} \frac{d\mathbf{q}}{(2\pi)^3}. \tag{23}$$

In order to determine the coefficients $C_q(t)$, we replace this general solution in the KG equation (1), where the electrostatic potential V is determined by Eq. (22), and describes the Debye screened electron-ion collision process. For very intense laser fields, we can consider the potential V as a small perturbation, and use a perturbative approach. The result is

$$i\frac{dC_q}{dt} = -e\int d\mathbf{r}\int \frac{d\mathbf{q}'}{(2\pi)^3} V(\mathbf{r})C_{q'}(t)e^{-i(\theta_q-\theta_{q'})-i(S_q-S_{q'})}. \tag{24}$$

Let us now consider the explicit dependence of the exponents in this equation on $\mathbf{r}$ and t. Assuming a circularly polarized laser pulse and integrating in time, we obtain a result, valid for long time scales as compared with the wave period, such that

$$C_q(t) = i\pi \sum_n \delta(\omega_{q'} - \omega_n'') \int C_{q'}(t)H_n(\mathbf{q},\mathbf{q}')\frac{d\mathbf{q}'}{(2\pi)^3}, \tag{25}$$

where we have introduced the interaction matrix elements associated with the electron-ion collisions, as

$$H_n(\mathbf{q},\mathbf{q}') = eV(\mathbf{q}_n'')J_n(\zeta), \tag{26}$$

with $\zeta = \zeta_q + \zeta_{q'}^*$. Here, $V(\mathbf{q}_n'')$ is the Fourier transform of the Yukawa potential,

$$V(\mathbf{q}) = \int V(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}, \tag{27}$$

calculated for the particular value $\mathbf{q} = \mathbf{q}_n''$. The expression for $V(\mathbf{q})$ is well known and will not be explicitly written here. We have also introduced the definitions:

$$\begin{aligned}\omega_n'' &= \omega_q + n\omega + 2(\eta_q - \eta_{q'})\omega,\\ \mathbf{q}_n'' &= (\mathbf{q}-\mathbf{q}') + n\mathbf{k} + 2(\eta_q - \eta_{q'})\mathbf{k}.\end{aligned} \tag{28}$$

For initial conditions such that, at $t = 0$, the electron momentum state is well defined and $C_{q'} = \delta(\mathbf{q}' - \mathbf{q}_i)$, we can define a transition probability between the initial state $\mathbf{q}_i$ and a final state $\mathbf{q}$, as equal to $T(\mathbf{q}' \to \mathbf{q}) \equiv |C_q(t)|^2$, or in a more explicit form,

$$\begin{aligned}T(\mathbf{q}' \to \mathbf{q}) &= \sum_n T(n,\mathbf{q}' \to \mathbf{q}),\\ T(n,\mathbf{q}' \to \mathbf{q}) &= \pi^2|H_n(\mathbf{q},\mathbf{q}')|^2\delta(\omega_{q'} - \omega_n''),\end{aligned} \tag{29}$$

where $T(n,\mathbf{q}' \to \mathbf{q})$ are the transition probabilities between the two momentum states by absorption or emission of n laser photons. This expression is one of the main results of the present work, and it generalizes previously obtained transition probabilities in many respects. In particular, they take into account the recoil effects due to photon absorption, as well as the Debye screening and rest mass effects.

Notice that, in the dipole approximation, the dependence of H_n on the photon momentum would vanish, and the quantity $\mathbf{q}_n''$ would imply reduce to $\mathbf{q}'' = (\mathbf{q} - \mathbf{q}')$. As a result, the relevant Fourier component of the Yukawa potential would reduce to $V(\mathbf{q}'')$, and would be independent of the number of absorbed or emitted photons. The recoil effects associated with the inverse bremsstrahlung process would vanish in this limit, and the transition probability would be reduced to

$$T(n,\mathbf{q}' \to \mathbf{q}) = \pi^2|V(\mathbf{q}'')|^2 J_n^2(\zeta)\delta(\omega_{q'} - \omega_n''). \tag{30}$$

Even taken in this dipole approximation, the present results still generalize those of [17], by retaining the Debye screening and electron rest mass effects in the interaction.

IV. EFFECTIVE COLLISION FREQUENCY

An important feature of the inverse bremsstrahlung process is that it can be characterized by an effective collision frequency. This quantity determines the rate at which the energy of the incoming laser pulse is dissipated by electron-ion collisions. In order to derive such a quantity, we first introduce the electron kinetic equation with the collision terms associated with the laser-induced electron-ion collisions. Using a detailed balance equation for the electron momentum states, we can easily get a kinetic equation of the form [17],

$$\frac{\partial}{\partial t} f_e(\mathbf{q}) = \sum_{n\neq 0} \int \frac{d\mathbf{q}'}{(2\pi)^3} [T(n,\mathbf{q}' \to \mathbf{q}) f_e(\mathbf{q}') - T(n,\mathbf{q} \to \mathbf{q}') f_e(\mathbf{q})]. \tag{31}$$

Here we have a Boltzmann type of collision integral, due to absorption or emission of $n \neq 0$ laser photons, with transition probabilities determined by Eq. (30). Notice that, in general, we have $T(n,\mathbf{q}' \to \mathbf{q}) \neq T(n,\mathbf{q} \to \mathbf{q}')$, because the Fourier transform of the Yukawa potential, $V(\mathbf{q}'')$, is not symmetric with respect to the exchange of $\mathbf{q}$ and $\mathbf{q}'$. However, symmetry is recovered in the dipole approximation.

Let us assume that the plasma is nearly in equilibrium at a given temperature T, and that we can use the relativistic Maxwell distribution,

$$f_e(\mathbf{q}) = C_e \exp[-(\omega_q - m)/T]. \tag{32}$$

For simplicity we use $C_e = 1$ in our discussion, and drop the Boltzmann constant k_B by writing the temperature in energy units. Noting the existence of delta functions $\delta(\omega_{q'} - \omega_n'')$ in the expression of the transition probabilities, we can write

$$\begin{aligned} f_e(\mathbf{q}') &= f_e(\mathbf{q})\delta_n(\mathbf{q},\mathbf{q}'), \\ \delta_n(\mathbf{q},\mathbf{q}') &= \exp\left\{-\frac{\omega}{T}[n + 2(\eta_q - \eta_{q'})]\right\}. \end{aligned} \tag{33}$$

Replacing this in the above kinetic equation, we can transform it in

$$\frac{\partial}{\partial t} f_e(\mathbf{q}) = f_e(\mathbf{q}) \int \frac{d\mathbf{q}'}{(2\pi)^3} \nu_{\rm coll}(\mathbf{q},\mathbf{q}'), \tag{34}$$

where we have introduced an effective collision frequency determined by

$$\nu_{\rm coll}(\mathbf{q},\mathbf{q}') = \sum_n [\delta_n(\mathbf{q},\mathbf{q}')\, T(n,\mathbf{q}' \to \mathbf{q}) - T(n,\mathbf{q} \to \mathbf{q}')]. \tag{35}$$

An average collision frequency could then be defined from here, as

$$\langle \nu_{\rm coll} \rangle = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\mathbf{q}'}{(2\pi)^3} \nu_{\rm coll}(\mathbf{q},\mathbf{q}') f_e(\mathbf{q}). \tag{36}$$

However, a slightly different definition has already been introduced in the literature, related to the electron energy gain in the laser field, which is given by

$$\frac{d}{dt}\langle \epsilon \rangle = \langle \epsilon \rangle \nu_{\rm eff}, \tag{37}$$

where $\langle \epsilon \rangle \simeq m\gamma_a$ is the electron energy in the laser field. The evolution of this quantity can be determined by using

$$\frac{d}{dt}\langle \epsilon \rangle = \int \frac{d\mathbf{q}}{(2\pi)^3} \omega_q \frac{\partial}{\partial t} f_e(\mathbf{q}). \tag{38}$$

Using Eq. (37), we can then define the effective collision frequency as

$$\nu_{\rm eff} = \frac{1}{\langle \epsilon \rangle} \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\mathbf{q}'}{(2\pi)^3} \omega \nu_{\rm coll}(\mathbf{q},\mathbf{q}') f_e(\mathbf{q}). \tag{39}$$

Comparing with (36), we can see that $\nu_{\rm eff} \simeq \langle \nu_{\rm coll} \rangle$. In order to get a more explicit expression for the effective collision frequency $\nu_{\rm eff}$, let us go back to the expression for the transition probabilities $T(n,\mathbf{q}' \to \mathbf{q})$. Noting that they are nonsymmetrical with respect to the exchange between $\mathbf{q}$ and $\mathbf{q}'$, we are led to the following result:

$$\nu_{\rm coll}(\mathbf{q},\mathbf{q}') = \pi^2 e^2 \sum_n \{\delta_n(\mathbf{q},\mathbf{q}') - R_n\} |V(q_n'')|^2 \times J_n^2(\eta)\delta(\omega_{q'} - \omega_n''), \tag{40}$$

where we have used the Fourier transform of the Yukawa potential $V(q_n'')$, and the asymmetry factor R_n is determined by

$$\begin{aligned} |V(q_n'')|^2 &= \left(\frac{Ze}{\epsilon_0}\right)^2 \frac{\lambda_D^4}{\left(\lambda_D^2 q_n''^2 + 1\right)^2}, \\ R_n &= \frac{(\mathbf{q} - \mathbf{q}') + n\mathbf{k} + 2(\eta_q - \eta_{q'})\mathbf{k}}{(\mathbf{q}' - \mathbf{q}) + n\mathbf{k} - 2(\eta_{q'} - \eta_q)\mathbf{k}}. \end{aligned} \tag{41}$$

This completely determines the inverse bremsstrahlung process. In order to illustrate the importance of this result, let us now discuss the nonrelativistic and ultrarelativistic limits. First we consider the nonrelativistic case, which corresponds to weak incident laser fields, such that $a < 1$. In this case, we can use $\omega_q \simeq m + q^2/2m$, and

$$(\eta_q - \eta_{q'}) \simeq \frac{a^2}{4}\left[\frac{m}{\omega^2}\mathbf{k}\cdot(\mathbf{q} - \mathbf{q}') - \frac{(q^2 - q'^2)}{2m\omega}\right]. \tag{42}$$

We can see that these terms can only be neglected in the limit of very weak laser amplitudes, $a^2 \ll 1$. On the other hand, an estimate of the parameter $\zeta \equiv \zeta_q + \zeta_{q'}^*$ leads to

$$\zeta = \frac{\lambda}{\omega}, \quad \lambda = a\Delta p_\perp, \tag{43}$$

where $\Delta p_\perp$ is the variation in the perpendicular atomic momentum due to the inverse bremsstrahlung process. It can also be easily realized that, for weak laser fields, only single-photon transitions are important. Assuming that $\zeta \ll 1$, we can then use the asymptotic expression for the Bessel functions, $J_1^2(\lambda/\omega) \simeq (\lambda/2\omega)^2$, and neglect the terms $n \geqslant 1$. Finally, the quantities (28) become

$$\omega_n'' \simeq \omega_q + n\omega, \quad \mathbf{q}_n'' \simeq (\mathbf{q} - \mathbf{q}') + n\mathbf{k}. \tag{44}$$

These simplifications allow us to write the effective collision frequency (40) in the low laser intensity limit, as

$$\nu_{\rm coll}(\mathbf{q},\mathbf{q}') \simeq \pi^2 e^2 \left(\frac{\lambda}{2\omega}\right)^2 |V(q_1'')|^2 \left\{\left(e^{-\omega/T} - R_1\right)\delta(\Delta + \omega) - \left(e^{\omega/T} - \frac{1}{R_1}\right)\delta(\Delta - \omega)\right\}, \tag{45}$$

where $\Delta \equiv (\omega_q - \omega_{q'})$. This result strongly contrasts with previous results obtained for a low intensity laser field. In our case, even for a weak laser field, Debye shielding and purely quantum effects associated with the electron recoil are retained. Only in the extreme situation where we can ignore the electron recoil and screening, $\omega \ll \Delta$ and $\lambda_D^2 q_1''^2 \ll 1$, we recover the well-known results of Silin [12,17]. The corrections included in Eq. (45) for low intensities could be important, for instance, in the creation of plasmas by low or moderate intensity x-ray beams [24].

To complete our discussion, let us also discuss the ultrarelativistic limit of a very strong laser field, such that $a \gg 1$. In this case, (42) is replaced by

$$(\eta_q - \eta_{q'}) \simeq \frac{a^2}{4}\frac{m^2}{\omega^2 q^2}\mathbf{k}\cdot(\mathbf{q}-\mathbf{q}'). \tag{46}$$

It is clear that, in this limit, such a quantity cannot be neglected. In what concerns the parameter ζ, we now have

$$\zeta = \frac{\lambda'}{\omega}, \quad \lambda' = a\frac{m}{q}. \tag{47}$$

It is now useful to take the limit $\lambda' \gg \omega$. In the case of intense fields, the multiphoton processes with $n \gg 1$ will dominate. In this limit, we can take $R_n = 1$ and use the development [17],

$$\sum_n J_n^2\left(\frac{\lambda'}{\omega}\right)\delta(\Delta - n\omega) \simeq J_x^2\left(\frac{\lambda'}{\omega}\right)\sum_n \delta(\Delta - n\omega)$$
$$\simeq \frac{1}{2}[\delta(\Delta - \lambda') - \delta(\Delta + \lambda')], \tag{48}$$

where $x = \Delta/\omega$, and J_x^2 is maximum for $\Delta = \pm\lambda'$. This is formally identical to the result of [17] for the high intensity laser pulse, if we replace in their results Ω by Δ, and λ by λ', respectively.

V. CONCLUSIONS

In this work, we have examined the loss of energy of a laser pulse in a plasma by inverse bremsstrahlung, under the relativistic quantum regime. This effect could be the dominant absorption mechanism for intense laser plasma interaction. Our approach is valid for a relativistic quantum plasma, if spin effects and electron-positron pair production can be neglected. This could be valid for most experimental situations using the present state of the art laser systems. Our results are based on solutions of the KG equation for the electron wave functions, and can be easily generalized to take spin and pair effects by using the generalized Volkov solutions recently derived for electrons in a plasma [7].

General expressions for the transition probabilities between different electron momentum states due to the existence of a Yukawa potential describing electron-ion collisions in the presence of Debye screening, and for the corresponding effective collision frequency were obtained. In contrast with the classical results by [12,17], our results retain Debye screening and quantum recoil effects even in the low intensity limit. For these reasons they could be appropriate for the description of plasma creation by x-rays [23], where high-energy photons are absorbed. Other possible applications of the approach proposed here are high-harmonic generation [24] and the generation of gamma rays by laser backscattering of relativistic electron bunches [25]. The same methods can also be extended to the case of inverse bremsstrahlung by two laser fields, and to turbulence-induced laser absorption, as discussed in a future work.

Finally, it should be noticed that intrinsic relativistic quantum effects, such as those associated with spin and with electron-positron coupling, were ignored in the present work. They can only be understood by replacing the Klein-Gordon description used here by a more complete Dirac description. However, given the formal similarities between the present KG solutions and generalized Volkov solutions of the Dirac equation, as those defining for electron quantum states in a plasma [7], it is not very difficult to envisage an upgrade of the present model, rewriting it in terms of the appropriate Volkov solutions.

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