Nature of laminar-turbulence intermittency in shear flows

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(Received 5 August 2012; revised manuscript received 20 April 2013; published 18 June 2013)

In pipe, channel, and boundary layer flows turbulence first occurs intermittently in space and time: at moderate Reynolds numbers domains of disordered turbulent motion are separated by quiescent laminar regions. Based on direct numerical simulations of pipe flow we argue here that the spatial intermittency has its origin in a nearest neighbor interaction between turbulent regions. We further show that in this regime turbulent flows are intrinsically intermittent with a well-defined equilibrium turbulent fraction but without ever assuming a steady pattern. This transition scenario is analogous to that found in simple models such as coupled map lattices. The scaling observed implies that laminar intermissions of the turbulent flow will persist to arbitrarily large Reynolds numbers.

DOI: 10.1103/PhysRevE.87.063012 PACS number(s): 47.20.Ft, 47.27.Cn, 47.27.N−, 47.52.+j

I. INTRODUCTION

In fluid flow inertia has in general a destabilizing effect, whereas viscous forces tend to quickly restore smooth motion. The balance between the two is expressed in the dimensionless Reynolds number $\mathrm{Re} = LU/\nu$, where L and U are a characteristic length and velocity, respectively, and ν the kinematic viscosity of the fluid. As Re is increased laminar flows tend to be unstable, and in some special cases it is even possible to determine a critical Reynolds number by linearizing the governing equations. However, in shear flows such as pipe, channel, and Couette flows turbulence occurs in experiments for sufficiently strong perturbations [1–4] in parameter regimes where the steady laminar flow is stable to infinitesimal disturbances.

One of the striking features of linearly stable flows, first recognized by Osborne Reynolds, is that disordered and smooth motion can coexist [1]. Here turbulence is at onset restricted to localized patches (called puffs in pipe flow) embedded in a laminar background. Recently streamwise-localized solutions of the Navier–Stokes equations that share key spatial properties with puffs but are time-periodic have been discovered [5]. Although turbulent puffs are transient [6–9], spatial proliferation in the form of splitting can balance the decay of individual puffs and lead to an overall sustainment of turbulence [10,11]. The persistent turbulent state emerging through this nonequilibrium phase transition, occurring at $\mathrm{Re}_c \simeq 2040$ [11], consists of a spatially temporally intermittent flow where individual turbulent clusters are transient but proliferate faster than they decay. At the same time turbulence can only be sustained locally, driven by the upstream laminar motion [12]. At the downstream end the pluglike velocity profile needs to recover towards the laminar parabolic profile before a new turbulent region can arise. This results in a minimum spacing of turbulent clusters [13] and an effective recovery length [14]. At higher Re puffs are superseded by continuously growing structures called slugs [15], and it is usually assumed that given sufficient time turbulence will fill the entire pipe. This state depleted of laminar intermissions will in the following be referred to as fully turbulent flow.

Althoguh laminar-turbulent intermittency was first reported in the early experiments of Reynolds [1], probably the first quantitative characterization of intermittency in pipes was provided by Rotta [16]. In his experiments he continuously disturbed the flow at the pipe inlet to generate turbulence and monitored its evolution at different downstream distances. He quantified the turbulent fraction and concluded that a state of fully turbulent flow would always be reached above the critical Reynolds number Re_c, which he speculated to be about 2000 [16]. Wygnanski and coworkers later recognized the intermittent nature of turbulence in the transitional regime and concluded that a fully turbulent flow would be realized at about $\mathrm{Re} \gtrsim 2700$ [15]. More recently the increase in computing power has made it possible to numerically simulate intermittent flows in spatially extended domains [10,11,17]. One advantage of simulations is that due to periodic boundary conditions in the axial direction the dynamics of turbulent patches can be studied for very long times, whereas in experiments they are convected out. Using simulations in pipes of up to 125 diameters in length, Moxey and Barkley [10] argue that no laminar islands are found in turbulent flow beyond $\mathrm{Re} \simeq 2600$. However, results from a model of pipe flow, allowing for very long domains and observation times, suggest that neither the transition to fully turbulent flow nor the transition from puffs to slugs is sharp [14]. In this paper we show that laminar-turbulence intermittency is an intrinsic feature of shear flow. As the Reynolds number increases, laminar regions become scarcer, yet they do not disappear entirely. The underlying physical process has its roots in a nearest neighbor interaction between turbulent regions.

II. NUMERICAL SIMULATIONS OF LONG PERIODIC PIPES

A clear constraint of all previous investigations is the limited observation times and pipe length accessible to numerical simulation and laboratory experiment. In order to estimate the dynamical behavior of pipe flow in the limit of infinite length and time, we performed numerical simulations of periodic pipes of up to 500 diameters in length and considered observation times of up to $25\,000 D/U$ (D is the

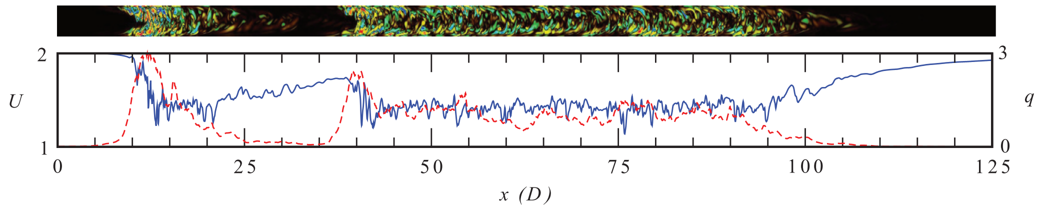

FIG. 1. (Color online) Coexistence of a puff and a slug at Re = 2600 in direct numerical simulations of pipe flow. The flow is from left to right, and 125 diameters (D) of a $176D$-long pipe are shown. (Top) Snapshot of streamwise vorticity in a (x,y) plane. (Bottom) Cross-sectionally averaged streamwise vorticity q (dashed) and streamwise velocity U (solid) along the pipe axis.

pipe diameter and U the mean speed; the Reynolds number is $\mathrm{Re} = DU/\nu$). The Navier-Stokes equations were solved in cylindrical coordinates (r,θ,x) using the hybrid spectral finite-difference method of Willis [18]. The numerical discretization consists of a nonequispaced nine-point finite-difference stencil in r and of Fourier modes in θ and x. The results for $\mathrm{Re} \leqslant 2800$ have been obtained with up to $N = 64$ radial points and $M = \pm 36$ azimuthal Fourier modes, whereas $K = \pm 1024$ axial Fourier modes have been used for a pipe length of $32\pi D \simeq 100D$. For $\mathrm{Re} > 2800$ we used $N = 64$, $M = \pm 48$, and $K = \pm 1280$.

The spreading of turbulence along the pipe was systematically studied by using the following procedure. First, a puff was generated at low Reynolds number, e.g., Re = 2100, from a localized perturbation. Subsequently, the Reynolds number was impulsively changed to a prescribed value, and the dynamics was monitored in time. Figure 1(a) shows a snapshot of streamwise vorticity in a (x,y) plane at $t \simeq 75D/U$ after the Reynolds number was increased to Re = 2600. Although initially the localized puff appears to spread continuously, as a slug, it later splits into two smaller turbulent structures, the first one resembling a slug and the second one a puff. Between them the cross-sectionally averaged streamwise vorticity $q = \sqrt{\langle \omega_x^2 \rangle_{r,\theta}}$ [dashed line in Fig. 1(b)] rapidly approaches zero, whereas the streamwise velocity at the centerline (solid line) slowly increases towards the laminar value $u = 2U$ but does not quite reach it. Our simulations confirm previous experimental observations, which identified mixed occurrences of puffs and slugs originating from single localized perturbations [2,19] and suggest that the coexistence of localized and expanding structures (i.e., puffs and slugs) is intrinsic to pipe flow.

III. STATISTICAL ANALYSIS OF LAMINAR-TURBULENCE INTERMITTENCY

In order to study the asymptotic behavior of the system, simulations started from a localized disturbance were continued until the entire domain was filled with spatiotemporal intermittency. The flow dynamics thereafter is shown in Fig. 2(a), which is a space-time diagram of $q(x,t)$ at Re = 2500. At a given instant in time t, an x-constant cross section was considered laminar if $q(x,t) < q^* = 0.3U/D$. We note that our threshold choice yields a puff size of $10D$ at Re = 2100, which is consistent with results from previous experimental studies considering the active part of the puff [12]. Note that when streamwise velocity or pressure is considered, the length of a puff is typically much longer due to the slow recovery of the parabolic profile in the downstream direction [Fig. 1(b)]. Using this criterion the lengths of turbulent segments Δx were determined from each of the snapshots in Fig. 2(a), and a collection of turbulent lengths $l^i = \Delta x^i$ was generated. The cumulative distribution of l^i is shown as left triangles in a logarithmic vertical axis in Fig. 2(b) and is found to follow an exponential law for lengths $l \gtrsim L_t^0 = 15D$ (patches longer than a puff). At low Reynolds number ($\mathrm{Re} \lesssim 2200$ at the time

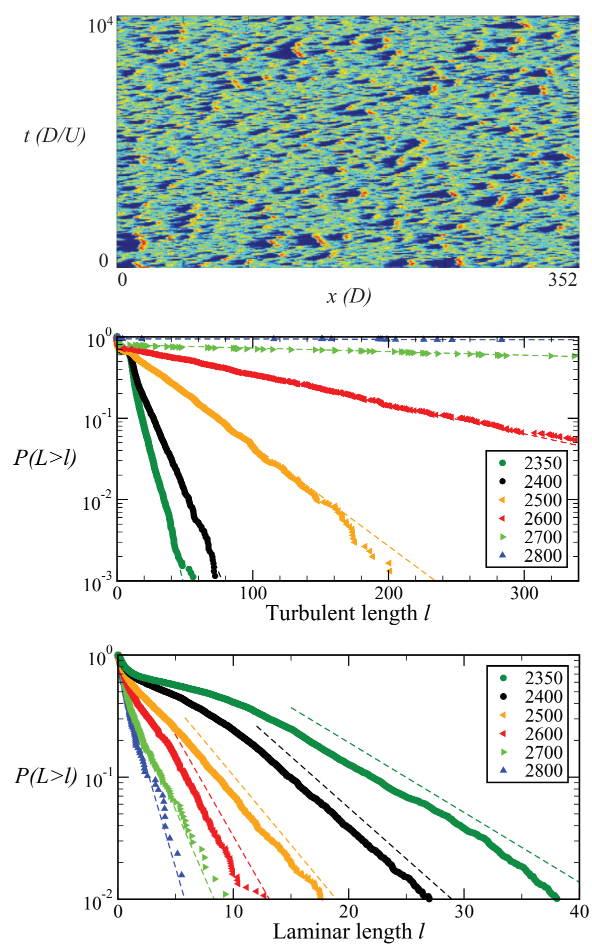

FIG. 2. (Color online) Top: Space-time diagram of cross-sectionally averaged streamwise vorticity q in a frame comoving at $0.93U$ at Re = 2500. Dark blue corresponds to laminar flow and red to intense turbulence. Middle: Cumulative distribution functions of turbulent lengths at $\mathrm{Re} \in [2350,2800]$ in a vertical log scale. Bottom: Cumulative distribution functions of laminar lengths at $\mathrm{Re} \in [2350,2800]$ in a vertical log scale.

scales considered here) spreading events (puff splitting) are very rare. In the absence of these events the length distributions are Gaussian-like about the average puff length. At higher Reynolds number this behavior is still observed for short patches $l \lesssim L_t^0$.

The curves in Fig. 2(b) show turbulent length distributions in the range Re ∈ [2350,2800]. As the Reynolds number increases, turbulent patches become increasingly longer because relaminarization of pipe sections becomes more infrequent. The same analysis applied to the laminar gaps between turbulent sections renders exponential distributions as well [Fig. 2(c)]. Here also distributions are exponential only for laminar gaps longer than the typical interaction length at which puffs strongly influence each other [12,13]. At Re = 2350 this interaction length is about $l \gtrsim L_l^0 = 15D$, whereas it rapidly decreases as Re increases (at Re $\gtrsim$ 2700, $L_l^0 \lesssim 1D$). We note that exponential distributions of the size of chaotic and laminar clusters are a typical signature of spatiotemporal intermittency and were previously reported in simulations of coupled map lattices [20], convection in an annulus [21], in the Taylor-Dean system [22], and in torsional Couette flow [23]. Although the distribution of laminar lengths is expected to be scale invariant (algebraic) at the onset of sustained spatiotemporal intermittency ($\mathrm{Re}_c \simeq 2040$ in pipe flow), the relevant physical decay and spreading processes occur at a time scale $> 10^7 D/U$ [11], clearly out of reach in the simulations. Using very long simulations of a reduced model of plane Couette flow, Manneville [24] has demonstrated that laminar size distributions are algebraic at onset. More recently, scale invariance at the onset of turbulence has been shown from simulations of the Navier–Stokes equations for Couette flow in a tilted domain [25]. In pipe flow, reduced models that recover the main features of the transitional dynamics have been also proposed [18] and might prove useful in addressing this point. Here we checked that the laminar size distributions are indeed exponential for Re $\gtrsim$ 2350, whereas we found that for Re $\lesssim$ 2300 our simulations cannot be used to quantitatively estimate the distributions. Exploring this low Re regime remains a challenge and would require time integrations and domains substantially beyond those used in this work.

The distributions of the form $\exp[(l - L_t^0)/L_t]$ in Fig. 2(b) naturally define a characteristic turbulent length L_t at each Reynolds number. The variation of the scale parameter L_t as a function of Re is shown in a logarithmic vertical axis in Fig. 3(a). As Re increases L_t grows strikingly fast [Fig. 3(a)]. The data are best approximated by a super-exponential fit of the shape $L_t = \exp[\exp(a + b\,\mathrm{Re})]$ and indicate, if extrapolated, that no divergence of turbulent lengths occurs at a finite Re. This result suggests that no continuously growing slugs exist, but rather these split into smaller slugs after perhaps a very long, but finite, time. Moreover, the fit approximates the data well into the puff regime and hence provides a quantitative link between localized and pipe-filling turbulent flow. In the case of laminar gaps it was found that the analogous scale parameter L_l decreases algebraically with Reynolds number [see the inset in Fig. 3(a)], also supporting that fully turbulent flow is reached only in the asymptotic Re limit. Note that L_l is not the average laminar gap length because it does not contain information about the part of the distribution which is not exponential. For example, $L_l \sim 7.5D$ at Re = 2350, but the average laminar gap length is $L_l + L_l^0 = 22.5D$.

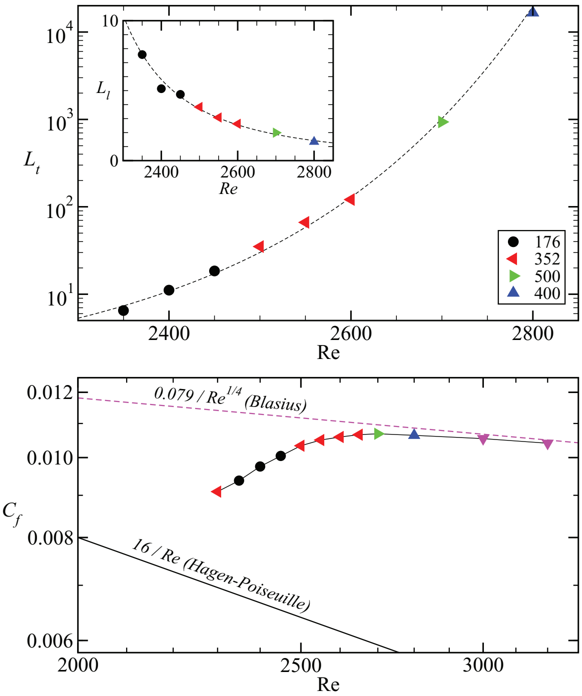

FIG. 3. (Color online) Top: Characteristic turbulent size, corresponding to the slope of the dashed lines in Fig. 2(c), as a function of Re. The computational domain length is indicated in the legend. Inset: Characteristic laminar gap length as function of Re. Bottom: Skin friction coefficient $C_f = -\langle \partial_z p \rangle_{r\theta z t} D/(2\rho U^2)$, where ρ is the fluid density.

We verified that the length distributions are independent of initial conditions by starting simulations with turbulent flow at large Reynolds number and quenching to the desired Re. These lead to the same size distributions as the simulations started from a single puff. Secondly, we repeated the simulations for different pipe lengths. Again convergence to the same distributions was achieved. Overall, the results show that the asymptotic size distributions are independent of the pipe length and initial conditions, and so they are an intrinsic property of the system in the "thermodynamic limit." Although lengths (durations) of laminar and turbulent regions in pipe flow had been previously investigated in experiments [15,16,26], the observation times of $< 500D$ were too short to reach size distributions which are in statistical equilibrium (see especially Fig. 26 of Ref. [16]). Hence, the lengths of the observed laminar episodes was likely controlled by the time scale at which turbulence was spontaneously triggered by the disturbance (obstacle) at the inlet.

While a variation of the cutoff q^* discriminating laminar from turbulent flow necessarily leads to a shift of absolute values, the qualitative scaling remains unchanged. Regardless of the choice, the characteristic turbulent (laminar) size scale super-exponentially (algebraically) with Re. It was found that turbulent distributions are more sensitive to the threshold value than laminar ones, which remain almost unchanged.

The gradual transition from spatiotemporal intermittency to fully turbulent flow is further illustrated in Fig. 3(b),

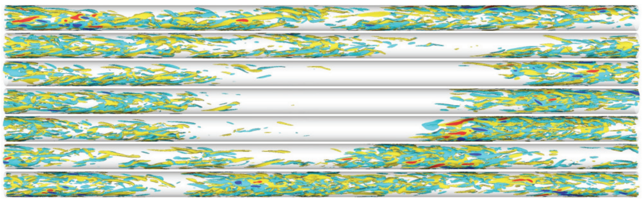

FIG. 4. (Color online) Streamwise vorticity isosurfaces showing the development of a laminar island in turbulent flow at Re = 2800. The time snapshots are separated by $28D/U$ in a frame comoving at the mean speed U. Here $23D$ out of a domain of $400D$ are shown.

showing the skin friction dependence on Re. In contrast to previous simulations [17] we do not observe an overshoot of the Blasius curve as this is approached. This discrepancy is likely due to numerical resolution. It is noted that to accurately resolve friction values high numerical resolutions are required. Here convergence was checked by independently increasing the resolution in all directions until the difference between computed frictions was below 0.5%. Beyond Re > 3000 the flow rapidly converges to the Blasius friction law, whereas for Re $\lesssim$ 2300 it was not possible to reliably compute friction values due to the extremely slow time scales at which puffs merge, split, and annihilate each other to open laminar gaps [11].

IV. TRANSIENT LAMINAR ISLANDS IN TURBULENT FLOW

The development of a laminar island at Re = 2800 is shown in Fig. 4 and illustrates the collapse of streamwise vortices in an extended part of the flow domain. A similar phenomenon was observed in minimal channels, where occasionally streamwise vortices weaken across the entire domain and turbulent activity temporarily ceases [27]. Our observation that laminar islands keep emerging in the flow as the Reynolds number is increased supports that quiescent regions are intrinsic to turbulent shear flow in realistically long domains. Unlike in Couette flows, where laminar-turbulent patterns have been suggested to emerge as a wavelength instability of the fully turbulent flow [28–30], in pipe flow laminar domains are found to appear at random locations and times in the turbulent flow [see Fig. 2(a)]. Our results are in agreement with recent experiments in long pipes [13] which found random spacing between puffs following Reynolds number reductions from fully turbulent flow. The essence of this process is captured by excitable and bistable media models [14]: the emergence of a laminar gap depends here only on the state of turbulence in the nearby region. The exponential size distributions of laminar and turbulent lengths further suggest that laminar turbulent patterns in pipe flow are the manifestation of a contact process only asymptotically giving rise to fully turbulent flow. The results presented here can be used to test and calibrate models of pipe flow that have been recently developed following different approaches [14,31,32].

V. DISCUSSION

The traditional view of equilibrium puffs giving way to a regime of puff splitting and eventually to one of expanding slugs [15,33] has originated from observations over time scales typically accessible in laboratory experiments. While over these time scales this picture appears to reflect the sequence of events, it nevertheless obscures some of the most relevant physics. Only the observation of a large number of events in extended domains and over long times reveals that a uniform state of turbulence (or fully turbulent flow) does not exist. Instead strong spatiotemporal fluctuations are intrinsic to the flow, including the extreme case where turbulence collapses in some part of the domain. As we have pointed out elsewhere [11], localized turbulent patches are also never in equilibrium; i.e., they either grow or decay, and an equilibrium puff regime does not exist either.

In conclusion, only one state of turbulence exists which is that of a spatiotemporally intermittent flow exhibiting large fluctuations. At low Re > $\mathrm{Re}_c \approx 2040$ fluctuations manifest themselves in sequences of laminar and turbulent regions, whereas at Re > 3000 laminar events become so scarce that turbulence flow will here appear as space filling but with large fluctuations of intensity in space and time providing the familiar turbulence intermittency still observed at Reynolds numbers many orders of magnitude larger.

ACKNOWLEDGMENTS

We thank A. P. Willis for sharing his hybrid spectral finite-difference code. We thank K. Avila for correcting our interpretation of Rotta's paper in an earlier draft. We are grateful to D. Barkley, N. Goldenfeld, M. Holzner, A. de Lozar, and A. P. Willis for discussions. The research leading to these results has received funding from the Max Planck Society and the European Research Council under the European Union's Seventh Framework Programme (FP/2007-2013)/ERC Grant Agreement 306589. We acknowledge computing resources from GWDG and the Jülich Supercomputing Centre (grant HGU16).

[1] O. Reynolds, Proc. R. Soc. London **35**, 84 (1883).
[2] A. Darbyshire and T. Mullin, J. Fluid Mech. **289**, 83 (1995).
[3] F. Daviaud, J. Hegseth, and P. Bergé, Phys. Rev. Lett. **69**, 2511 (1992).
[4] B. Hof, A. Juel, and T. Mullin, Phys. Rev. Lett. **91**, 244502 (2003).
[5] M. Avila, F. Mellibovsky, N. Roland, and B. Hof, Phys. Rev. Lett. **110**, 224502 (2013).
[6] B. Hof, J. Westerweel, T. M. Schneider, and B. Eckhardt, Nature (London) **443**, 59 (2006).
[7] B. Hof, A. de Lozar, D. J. Kuik, and J. Westerweel, Phys. Rev. Lett. **101**, 214501 (2008).
[8] M. Avila, A. P. Willis, and B. Hof, J. Fluid Mech. **646**, 127 (2010).
[9] D. J. Kuik, C. Poelma, and J. Westerweel, J. Fluid Mech. **645**, 529 (2010).

[10] D. Moxey and D. Barkley, Proc. Natl. Acad. Sci. USA **107**, 8091 (2010).
[11] K. Avila, D. Moxey, A. De Lozar, M. Avila, D. Barkley, and B. Hof, Science **333**, 192 (2011).
[12] B. Hof, A. De Lozar, M. Avila, X. Tu, and T. M. Schneider, Science **327**, 1491 (2010).
[13] D. Samanta, A. de Lozar, and B. Hof, J. Fluid Mech. **681**, 193 (2011).
[14] D. Barkley, Phys. Rev. E **84**, 016309 (2011).
[15] I. J. Wygnanski and F. H. Champagne, J. Fluid Mech. **59**, 281 (1973).
[16] J. Rotta, Arch. Appl. Mech. **24**, 258 (1956).
[17] A. P. Willis and R. R. Kerswell, Phys. Rev. Lett. **100**, 124501 (2008).
[18] A. P. Willis and R. R. Kerswell, J. Fluid Mech. **619**, 213 (2009).
[19] M. Nishi, B. Ünsal, F. Durst, and G. Biswas, J. Fluid Mech. **614**, 425 (2008).
[20] H. Chaté and P. Manneville, Physica D **32**, 409 (1988).
[21] S. Ciliberto and P. Bigazzi, Phys. Rev. Lett. **60**, 286 (1988).
[22] M. M. Degen, I. Mutabazi, and C. D. Andereck, Phys. Rev. E **53**, 3495 (1996).
[23] A. Cros and P. Le Gal, Phys. Fluids **14**, 3755 (2002).
[24] P. Manneville, Phys. Rev. E **79**, 025301 (2009).
[25] L. Shi, M. Avila, and B. Hof, Phys. Rev. Lett. **110**, 204502 (2013).
[26] K. R. Sreenivasan and R. Ramshankar, Physica D **23**, 246 (1986).
[27] L. Xi and M. D. Graham, Phys. Rev. Lett. **104**, 218301 (2010).
[28] A. Prigent, G. Grégoire, H. Chaté, O. Dauchot, and W. van Saarloos, Phys. Rev. Lett. **89**, 014501 (2002).
[29] D. Barkley and L. S. Tuckerman, Phys. Rev. Lett. **94**, 014502 (2005).
[30] P. Manneville, Europhys. Lett. **98**, 64001 (2012).
[31] M. Sipos and N. Goldenfeld, Phys. Rev. E **84**, 035304 (2011).
[32] K. T. Allhoff and B. Eckhardt, Fluid Dyn. Res. **44**, 031201 (2012).
[33] I. Wygnanski, M. Sokolov, and D. Friedman, J. Fluid Mech. **69**, 283 (1975).