PHYSICAL REVIEW E **87**, 062136 (2013)

Difference of reciprocity effect in two coevolutionary models of presumed two-player and multiplayer games

Jun Tanimoto*

Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan

(Received 9 February 2013; revised manuscript received 3 May 2013; published 25 June 2013)

Unlike other natural network systems, assortativity can be observed in most human social networks; however, it has been reported that a social dilemma situation represented by a 2×2 prisoner's dilemma game favors dissortativity to enhance cooperation. Our simulations successfully reveal that a public goods game with coevolution for both agents' strategy and network topology encourages assortativity, although it only slightly enhances cooperation as compared to a 2×2 donor and recipient game with a strong dilemma to be solved. This outcome occurs because the network dynamics in a multiplayer game discourages emerging cooperation unlike its beneficial result in a 2×2 prisoner's dilemma game.

DOI: 10.1103/PhysRevE.87.062136 PACS number(s): 02.50.Le, 89.65.−s, 87.23.Kg

I. INTRODUCTION

The mechanism of cooperation evolution is a mysterious puzzle that remains unsolved. Nowak and May [1] identified five mechanisms that result in the evolution of cooperation (C) instead of defection (D): kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Note that these mechanisms share a common feature, a means of decreasing the anonymity of game-participating agents, called "social viscosity" [2].

Among Nowak's five mechanisms, network reciprocity has been considered the most important from the biology and physics viewpoints. Because network reciprocity relies on only a simple mechanism that allows agents to interact with neighbors and copy their strategy, defined as C or D, it may imply that many animal species with unsophisticated information processing have nevertheless developed cooperative social systems. In fact, since 1992, when Nowak and May [3] conducted the first study on the spatial prisoner's dilemma, hundreds of studies have reported the mechanism through which network reciprocity functions as a key protocol for developing cooperation. Researchers have understood that networks with heterogeneous degree distribution, such as scale-free networks, can better enhance cooperation than homogeneous networks because heterogeneous networks allow the existence of hub C-agents, perhaps having high payoffs, which compel cooperation among their neighbors, thus causing strong, stable cooperation (e.g., [4–7]); however, network reciprocity is more strongly influenced by the assumed strategy updating and dynamics than by the assumed underlying network topology [8,9]. Note also that "heterogeneous topology" is represented with not only degree distribution but also average path length, average cluster coefficient, and other network parameters. Among those parameters, assortativity has been frequently studied in emerging cooperation. Rong *et al.* [10] noted that the cooperative phase in prisoner's dilemma (PD) games would be more robust in a heterogeneous network with a negative assortative coefficient [11].[1] This finding might be plausible because hub C-agents who connect with low-degree neighbors in a dissortative network convert less frequently from C to D by copying from neighbors than do assortative network hub C-agents who have high-degree (high payoff) neighbors likely including defectors. This finding was validated (e.g., [12,13]) by assuming coevolutionary models (e.g., [14–16]).

Newman, who defined the assortative coefficient [11], observed an interesting difference between human social networks and natural network systems. Networks such as those in physics, biology, mathematics coauthorships, and film-actor collaborations; networks of company directors; and those of teenagers' sexual relationships have positive assortative coefficients. In contrast, networks such as the Internet, protein interactions, neural networks, and marine and freshwater food-chain systems have negative assortative coefficients [11].

These observations raise the challenging question of why human society has been able to develop cooperation even on assortative heterogeneous networks. One possible answer is that interactions among human networks might be different from those among natural systems. Along with the previous studies, one might infer whether human interactions can be represented by 2×2 PD games. If that is possible, we must then investigate the concrete model based on a 2×2 PD, where both assortative heterogeneity and cooperation can emerge as in human social systems.

In our previous study [17], we presented one possibility, where agents play 2×2 PD games, and additionally assumed two different agents' features: learner and teacher agents [18]. Through coexisting learners—who copy strategy from neighbors as do models assumed in former studies—and teachers—who impose their strategy upon neighbors—the coevolutionary process allows emerging assortative networks with an enhanced cooperation phase among those heterogeneous agents.

---

*tanimoto@cm.kyushu-u.ac.jp

[1] The *assortativity coefficient* (AC) is the Pearson correlation coefficient of degree between pairs of linked nodes, defined by $\mathrm{AC} = \frac{\sum_{jk} jk(e_{jk} - p_j q_k)}{\sigma_q^2}$, where p_k is the degree distribution; q_k is the distribution of the remaining degree, given by $q_k = \frac{(k+1)p_{k+1}}{\sum_j j p_j}$; e_{jk} refers to the joint probability distribution of the remaining degrees of the two vertices. This quantity is symmetric on an undirected graph, and follows the sum rules $\sum_{jk} e_{jk} = 1$ and $\sum_j e_{jk} = q_k$.

In the current study, we present another possibility by using a different coevolutionary model. One important difference from our previous study is that the assumed game played by agents is not 2×2 but PGG (e.g., [19–22]). We observe that this little difference causes interesting and substantially different results from what we know about the coevolutionary model of 2×2 games. Recently, Moreira *et al.* [23] reported a novel result by an analytical approach assuming a coevolutionary situation based on a public goods game (PGG), but noted nothing on network characteristics, including assortativity, observed in emerging networks.

This paper contains four sections. Section II presents the model description and assumptions for numerical simulations. Section III reports results and discussion of the numerical simulations. Section IV summarizes our findings.

II. MODEL SETUP

A. Public goods game

At each time step, an agent on a network plays a public goods game (PGG), which is a multiplayer prisoner's dilemma. Agent i's payoff, depending on her strategy $s_i = 1$ for a cooperator (C) or 0 for a defector (D), is defined as follows:

$$\pi_{s_i}^{i} = r\frac{n_C^i}{G_i} - s_i = r\frac{n_C^i}{k_i+1} - s_i, \tag{1}$$

where k_i is the degree of Agent i, n_C^i is the number of her cooperative neighbors including Agent i, and r is the game parameter indicating dilemma weakness. G_i indicates the game size (number of game participants) that Agent i hosts. That is, $G_i = k_i + 1$. Agent i also obtains payoffs resulting from games that her neighbors host. Assuming one of her k_i neighbors, Agent j, we define Agent i's payoff through link i-j as follows:

$$\pi_{s_i}^{i\text{-}j} = r\frac{n_C^j}{k_j+1} - s_i. \tag{2}$$

Thus, the total payoff obtained by Agent i is

$$\Pi_i = \pi_*^i + \sum_{j\in\{N_i\}} \pi_*^{i\text{-}j}, \tag{3}$$

where $\{N_i\}$ denotes Agent i's neighbor set, and $*$ indicates a wildcard of either C or D, which is Agent i's strategy. The first term indicates the payoff of the game Agent i hosts, and the second term represents those of the games that Agent i's respective neighbors host. The total number of games Agent i plays is $k_i + 1$. Because $r\frac{n_C}{k+1} - 1 < r\frac{n_C}{k+1}$ is always true, a PGG becomes a multiplayer PD game when $r\frac{n_C}{k+1} - 1 < r\frac{n_C-1}{k+1}$ (that is, $r/G < 1$) is satisfied.

For comparison, we can define another PGG by assuming $G_i = 2$ instead of $G_i = k_i + 1$, where each multiplayer game is always played by two agents connected by a link. This structure characterizes a donor and recipient (D&R) game, a subclass of 2×2 PD games. The relationship of dilemma strength between PGG and D&R games can be drawn as follows: $r = 2(1 - D_g) = 2(1 - D_r)$ when we define a 2×2 payoff matrix [24] as $\left(\begin{smallmatrix} R & S \\ T & P \end{smallmatrix}\right) = \left(\begin{smallmatrix} 1 & -D_r \\ 1+D_g & 0 \end{smallmatrix}\right)$. In D&R games, $D_g = D_r$ is satisfied. The two parameters $D_g = T - R$ and $D_r = P - S$ imply a chicken-type dilemma and stag-hunt dilemma, respectively [24]. The PD game class is achieved by assuming $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$. In D&R games, the total number of games Agent i plays is k_i.

B. Strategy and network adaptations

After games at each time step, either strategy updating with the probability p_{str} or network adaptation occurs. Throughout this study, we assume that $p_{\mathrm{str}} = 0.5$, which means that we assume the same time scales for both strategy and network adaptations.

As strategy updating, we assume imitation max (IM), where the focal player i imitates the strategy, either C or D, with the maximum payoff among all strategies taken by the focal player and her immediate neighbors [9]. We assume the synchronous procedure for strategy updating.

At the beginning of each simulation episode, agents are connected by a random network [based on the Erdős-Rényi graph [25] (E-R random graph)] having $\langle k\rangle N/2$ links. N denotes the number of nodes in the network (population count of agents) and $\langle k\rangle$ denotes the average degree. A double-connected link is prohibited. Each simulation episode generates a different E-R random graph.

Network adaptation consists of two procedures: severing a link with one neighbor and creating a new link with an unknown agent. An agent who severs a link must create a new link, thus preserving her degree during the adaptation process.

In 2×2 game research, several studies have addressed defining the protocol for severing links. Zimmermann *et al.* [14] assumed that D-D links should be disconnected. Tanimoto then investigated differences from Zimmermann's result if both C-D and D-D links are disconnected [15]. Tanimoto posed another criterion for disconnecting a link: if the link has the minimum payoff, and if its payoff is less than $(P + R + S + T)/4$ [12]. These previous studies share the basic concept for disconnecting links: The focal agent's link most significantly malfunctioning to gain her payoff should be disconnected because disconnecting "bad" links (links with defectors) assures reciprocity by using the "game exit" option (e.g., [26]). A PGG requires a different protocol for disconnecting links because the game situation of what occurs around the focal agent differs from that of a 2×2 game. Therefore, we define the following severing protocol:

Agent i disconnects the link with Agent j if it is the link with the minimum payoff among her neighbor hosting games, and Agent j has a lower fraction of cooperators among the game participants in her hosting game than p_C^{th}.

The parameter p_C^{th} is defined to be globally constant. In this study, we fix $p_C^{th} = 0.5$. One may consider the latter part of the protocol to be inappropriate if we believe that an agent playing a PGG knows only local information obtained through the link with that particular neighbor if one assumes that the data on how many cooperative neighbors that particular neighbor has seems secondary or is indirectly known by the focal agent. However, this assumption is incorrect. The focal Agent i can know whether her neighbor, Agent j, has a lower or greater fraction of cooperators than p_C^{th} by evaluating whether the following inequality is true:

$$\pi_{s_i}^{i\text{-}j} \leqslant r p_C^{th} - s_i. \tag{4}$$

Either $\pi_C^{i\text{-}j}$ or $\pi_D^{i\text{-}j}$ is observable by Agent i, and p_C^{th} is disclosed to every agent.

Following the disconnection, the agent creates a new link with another agent; however, the new link is never the same as any existing link. Thus, we assume a random connection.

C. Simulation procedure

Each simulation runs as follows. Initially, an equal percentage of strategies is randomly distributed to the players allocated on different vertices of the network, as previously described. Several simulation time-step generations are run until the frequency of cooperation arrives at quasiequilibrium. If the cooperation frequency continues to fluctuate, we obtain the average frequency of cooperation for the last 250 generations over a 10 000-generation run. We vary r to control dilemma strength. The result shown below is drawn from 100 runs; that is, each ensemble average is drawn from 100 simulation episodes. We assume $N = 4900$ and $\langle k \rangle = 8$ (meaning $\langle G \rangle = 9$ for PGG). $N = 4900$ is confirmed as sufficiently large to yield simulation results insensitive to system size.

III. RESULTS AND DISCUSSION

Figure 1 depicts correlation of dilemma weakness with (a) cooperation fraction, (b) maximum degree, (c) assortative coefficient, (d) maximum degree of cooperators, and (e) maximum degree of defectors. Each panel depicts the ensemble average with standard deviation among 100 runs. Figure 2 compares the cooperation fraction by the proposed coevolution model [i.e., Fig. 1(a)] with that by the conventional (monoevolution) model, considering only strategy adaptation among agents playing a PGG (hereafter, monoevolution). Figure 2 also depicts the comparison between coevolution and monoevolution results when agents play a 2×2 D&R game instead of PGG. To ensure the validity of the comparison between the PGG and D&R games, we normalize dilemma weakness by taking r/G instead of r in Fig. 2.

Figure 1 indicates that the present model encourages evolving assortative and heterogeneous topology in its coevolutionary process. Specifically, when we compare the coevolution model currently presented (red closed circle) with monoevolution (blue open square) in Fig. 2, we observe that coevolution exhibits cooperation slightly inferior to monoevolution for the stronger dilemma region (gray dotted line) despite much better cooperation for the weaker dilemma region. We discuss the point in greater detail later in this section.

Figure 3 indicates the evolutionary processes when cooperators successfully survive to the game's end. Figure 3 depicts ensemble-averaged time evolution only when agents can attain cooperative coexisting equilibrium with $r = 5.0$, which is on the border between the cooperative phase and the all-defectors phase, as Fig. 1(a) depicts. This gap in the cooperation fraction happens in a discontinuous manner, because all evolutionary trails are attracted by either the all-defectors equilibrium for $r \leqslant 4.5$ or the almost all-cooperators equilibrium for $r \geqslant 5.5$. For $r = 5.0$ (we took $\Delta r = 0.5$ in our simulations), though, roughly 30% of trails are attracted by the almost all-cooperators state while the remaining 70%

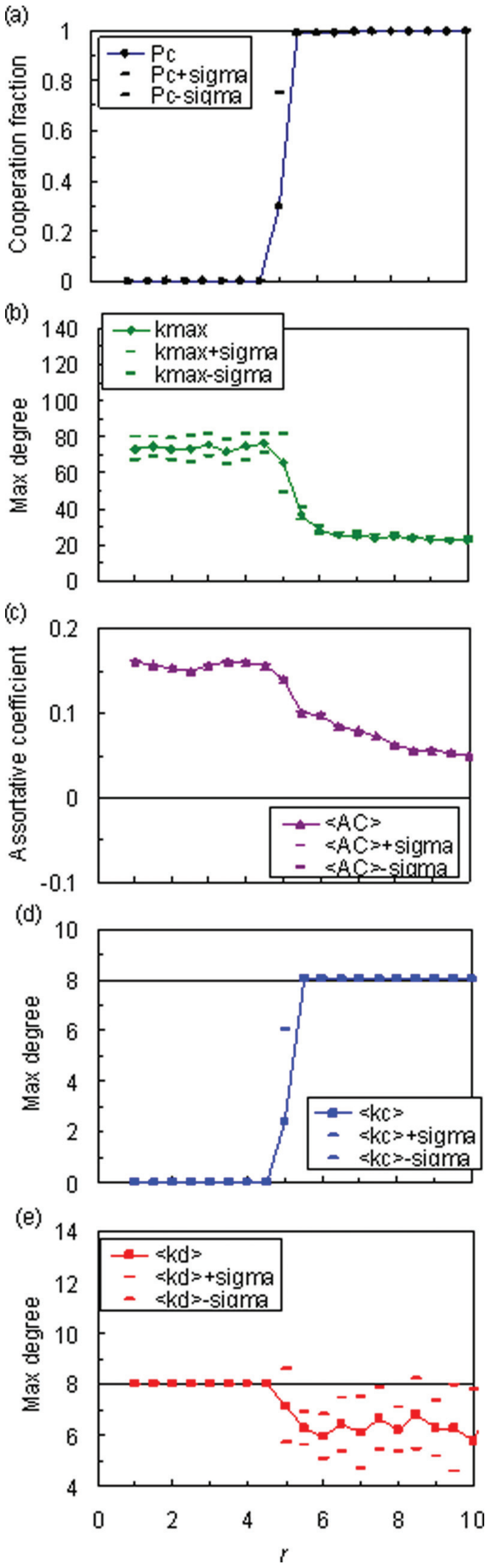

FIG. 1. (Color online) Relations of dilemma weakness r with (a) cooperation fraction, (b) maximum degree, (c) assortative coefficient, (d) maximum degree of cooperators, and (e) maximum degree of defectors. Sigma means standard deviation.

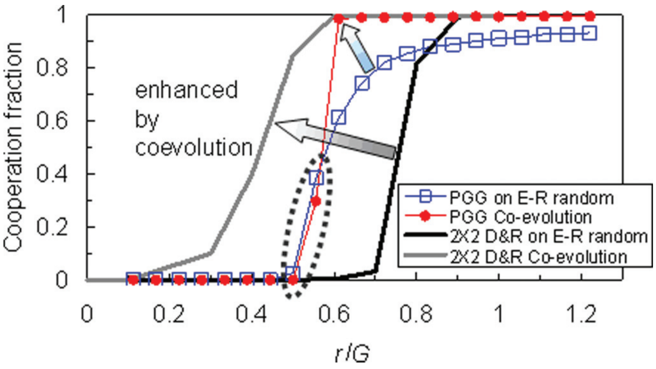

FIG. 2. (Color online) Comparison of cooperation fractions by the proposed coevolution model for PGG [red closed circle, same as Fig. 1(a)]; the monoevolution model for PGG (blue open square); and coevolution (gray bold line) and monoevolution (black bold line) models for a 2×2 D&R game.

are attracted by the all-defectors state. Figure 3 depicts the average time evolution of those 30% of trails' cooperation fraction, assortative coefficient, and cooperation fractions of neighbors to cooperators and defectors (a), and the maximum degree among all agents and average degrees of cooperators and defectors (b).

Initially, the cooperation fraction rapidly decreases from 0.5 because neighboring defectors rapidly invade cooperators originally randomly allocated on an E-R random network. If the trail's equilibrium would not be attracted by the all-defectors state, some cooperators must survive during this initial ordeal by preventing assault from the surrounding defectors. In fact, a certain number of cooperative clusters survive, in which the cooperators are heavily connected to each other. The rapid increase in the average degree of cooperators and slower decrease in the cooperation fraction of cooperators' neighbors than that of defectors' neighbors before the minimum point of global cooperation fraction demonstrates this behavior (dotted-line box in Fig. 3). This situation is followed by the rapid increase in assortativity, global cooperation fraction, and global maximum degree, as well as the slightly decreasing defectors' average degree (broken-line box in Fig. 3) after the relatively invariant period of 20 time steps. This fact suggests that the surviving cooperative clusters composed of heavily connected cooperators belonging to the same clusters are linked to each other during the relatively invariant period, and then those liaised cooperative clusters gradually change the surrounding defectors to cooperators. Defectors directly neighboring the cooperative clusters become cooperators. Furthermore, by the network dynamics, hub cooperators in cooperative clusters connect with relatively high-degree defectors, and convert those defectors into cooperators through strategy adaptation. This behavior triggers quick conversion of lower-degree defectors underlying those high-degree defectors to cooperators. To this end, a hub cooperator connects to another hub cooperator, which raises assortativity and the global maximum degree as well as the global cooperation fraction, and produces cooperative equilibrium. This phenomenon might crucially explain why our model achieves assortativity in emerging heterogeneous networks, in contrast to all coevolutionary models presented in previous studies where emerging networks exhibit disassortativity instead of heterogeneity. As the evolutionary trails depicted in Fig. 3 suggest, existence of the surviving cooperative clusters with dense mutual relations triggers assortativity. Those cooperative clusters bias the assortative coefficient positively because all cooperators belonging to a cluster have relatively high degree. The assortative coefficient also increases as a result of the process in which hub defectors convert to cooperators when they interact with high-degree cooperators belonging to the initial dense cooperative clusters.

Figure 4 depicts one of the representative degree distributions of cooperative trails at the 100th time steps, which is almost the same as the distribution at the equilibrium of this particular episode. Obviously, the coevolution process drives heterogeneous topology.

Note that the PGG coevolution model allows evolving cooperation despite causing assortative topology. This study marks this feature's first observation in ordinal coevolution models.

Let us return to Fig. 2. As noted, for a stronger dilemma region, the PGG coevolution model depicts inferior cooperation compared to the monoevolution model, although the coevolution model results are superior to those of the monoevolution model for a weaker dilemma region. Interestingly, Fig. 2 confirms that the coevolution model for a 2×2 D&R, which is a $G = 2$ PGG, results in network reciprocity superior to the monoevolution model for all dilemma regions. Thus, we can state that, unlike a

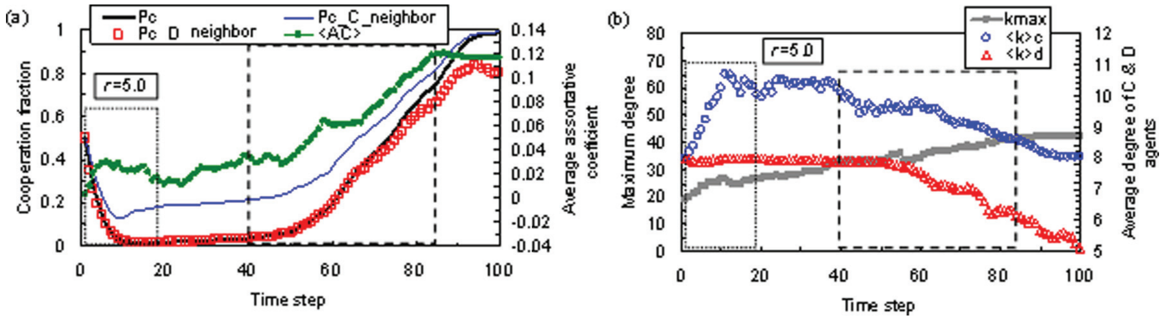

FIG. 3. (Color online) Ensemble time evolution episode only when agents can attain cooperative coexisting equilibrium when $r = 5.0$: (a) cooperation fraction, assortative coefficient, and cooperation fractions of neighbors to cooperators and defectors; and (b) maximum degree among all agents and average degrees of cooperators and defectors.

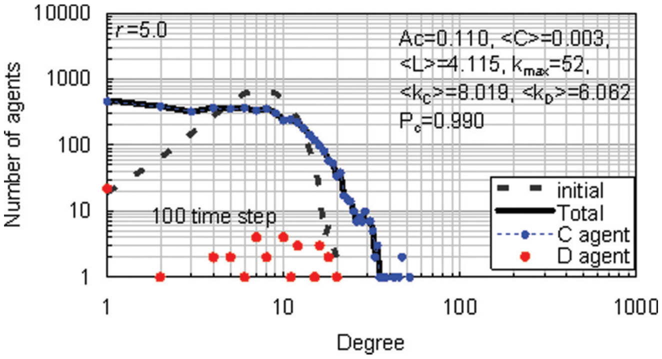

FIG. 4. (Color online) Degree distribution at time step = 100 of one of the episodes, which attain cooperative coexisting phase (showing ensemble average time evolution). Cooperation fraction P_c, assortative coefficient Ac, average cluster coefficient $\langle C\rangle$, average path length $\langle L\rangle$, maximum degree $k_{\max}$, and average degree of cooperators and defectors $\langle k_C\rangle$, $\langle k_D\rangle$ at time step = 100 are shown. The broken line represents the initial distribution (Poisson distribution), the bold line represents the distribution of all agents, and the red and blue plots represent those of defectors and cooperators, respectively.

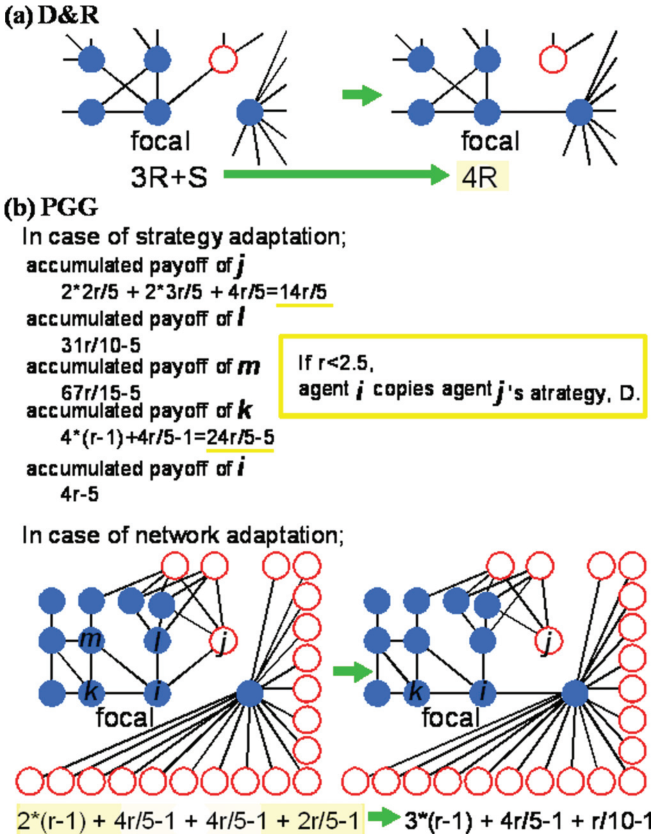

FIG. 5. (Color online) Schematic of what happens in respective coevolutionary processes for a 2 × 2 D&R and PGG to explain why coevolution destroys cooperation in a stronger dilemma situation. In a 2 × 2 D&R (a), network adaptation may improve the focal agent's payoff by rewiring a cooperator. In a PGG (b), the situation seems more complicated. The maximum cooperator among the Agent i's neighbor and Agent i herself is Agent k. Hence, $\frac{24}{5}r - 5 < \frac{14}{5}r \Leftrightarrow r < \frac{5}{2}$ is valid, Agent j becomes "maximum". Under a strong dilemma, the focal i copies D from the defective neighbor j who gains maximum payoff among i's neighbors and i when focal i updates her strategy. Thus, when focal i updates her links with neighbors, the link with j is severed even if the cooperative fraction among the participants of the game j hosts is not as low as the global cooperative fraction. In addition, focal i encounters difficulty in establishing a new link with an appropriate agent who has relatively denser cooperative neighbors than does agent j. Consequently, focal i builds a new link with the inappropriate agent who herself is a cooperator but has a high percentage of defective neighbors.

2 × 2 game, coevolution, where agent behavior changes the network as well as the strategy, does not always enhance cooperation as compared to monoevolution when assuming PGG.

Figure 5 schematically explains this phenomenon. This example depicts both network adaptations for 2 × 2 D&R and PGG around the cooperative focal agent with four neighbors, three cooperators, and one defector. Here, the focal cooperator occupies the boundary of the cooperative cluster encountering the defector. Let us assume that the global cooperative fraction P_c is low because of a strong dilemma. For a 2 × 2 D&R, this focal cooperator severs the link with the defective neighbor, and randomly reconnects with a cooperator. As long as P_c is not too small (say, not almost zero), the possibility of connecting with a cooperator is not too small (which is an order of P_c according to mean field approximation). Thus, in a 2 × 2 D&R, she successfully improves her total payoff by these network dynamics (network adaptation process). Alternatively, we can say that the cooperative cluster can expand by dint of network adaptation. However, in a PGG, it deteriorates, as Fig. 5(b) reveals, by the following chain of causation. The focal cooperator (Agent i) severs the link with the defector (Agent j) that has the lowest percentage of cooperative game participants among that of focal cooperator i's other neighbors. In fact, this percentage is not as low as the global cooperative fraction P_c, although it is less than p_C^{th}. Thus, the focal cooperator i finds it difficult to establish a new link with either a cooperator or defector who has a higher percentage of cooperative game participants than that of Agent j, from whom she has just defected to play. To this end, she may build a new link, as Fig. 5(b) depicts, with a new neighbor who has a high percentage of defective neighbors, even though this new agent herself acts as a cooperator. Unless the focal cooperator i adapts her network by the above-mentioned process, she must change her strategy by copying defection from defector j. In short, in a PGG, the cooperative cluster can be corroded by coevolution in a strong dilemma environment. In summary, when assuming a multiplayer game as an interaction model instead of a two-player game, the cooperation fraction of the focal player's secondary neighbors inevitably affects her total payoff, which implies greater entanglement or deeper interdependence among agents' influences on social efficiency in multiplayer games. This model may explain the phenomena in human social networks.

IV. CONCLUSIONS

To explain why a human network prefers an assortative topology for building a cooperative society, which differs from the explanations offered by 2 × 2 game models, we establish a coevolutionary model based on a multiplayer situation PGG.

The simulation results reveal that the coevolutionary model encourages cooperation through establishing assortative networks. However, unlike a 2×2 game, the coevolutionary PGG model does not always enhance cooperation as compared to monoevolution, where only strategy adaptation occurs. That is, assuming a PGG, under relatively strong dilemma situations, coevolution causes inferior cooperation compared to monoevolution. This occurs because the focal agent's secondary as well as direct neighbors influence coevolution in a PGG.

One implication drawn from the present study's findings is that the human social network interactions might be simulated in a multiplayer game, where, unlike in a 2×2 game, secondary as well as immediate neighbors influence the focal agent's payoff. We can thus state that human interactions are not as simple as pairwise relationships, but actually more entangled and interdependent.

ACKNOWLEDGMENTS

This study was supported partially by a Grant-in-Aid for Scientific Research by Japan Society for the Promotion of Science, awarded to Professor Tanimoto (Grant No. 25560165), the Hayao Nakayama Foundation for Science and Technology and Culture, and the Pfizer Health Research Foundation. We express our gratitude to these funding sources.

[1] M. A. Nowak, Science **314**, 1560 (2006).
[2] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature **441**, 502 (2006).
[3] M. A. Nowak and R. M. May, Nature **359**, 826 (1992).
[4] F. C. Santos and J. M. Pacheco, Phys. Rev. Lett. **95**, 098104 (2005).
[5] F. C. Santos, J. M. Pacheco, and T. Lenaerts, Proc. Natl. Acad. Sci. USA **103**, 3490 (2006).
[6] M. Perc, J. Gomez-Gardenes, A. Szolnoki, L. M. Floria, and Y. Moreno, J. R. Soc., Interface **10**, 20120997 (2013).
[7] Z. Wang, A. Szolnoki, and M. Perc, Sci. Rep. **3**, 1183 (2013).
[8] A. Yamauchi, J. Tanimoto, and A. Hagishima, BioSystems **102**, 82 (2010).
[9] A. Yamauchi, J. Tanimoto, and A. Hagishima, BioSystems **103**, 85 (2011).
[10] Z. Rong, X. Li, and X. Wang, Phys. Rev. E **76**, 027101 (2007).
[11] M. E. J. Newman, Phys. Rev. Lett. **89**, 208701 (2002).
[12] J. Tanimoto, Physica A **388**, 953 (2009).
[13] J. Tanimoto, Physica A **389**, 3325 (2010).
[14] M. G. Zimmermann and V. M. Eguiluz, Phys. Rev. E **72**, 056118 (2005).
[15] J. Tanimoto, Phys. Rev. E **76**, 021126 (2007).
[16] M. Perc and A. Szolnoki, BioSystems **99**, 109 (2010).
[17] J. Tanimoto, Physica A **392**, 2955 (2013).
[18] J. Tanimoto, M. Brede, and A. Yamauchi, Phys. Rev. E **85**, 032101 (2012).
[19] Z.-J. Xu, Z. Wang, and L.-Z. Zhang, Phys. Rev. E **80**, 061104 (2011).
[20] H.-F. Zhang, R.-R. Liu, Z. Wang, H.-X. Yang, and B.-H. Wang, Europhys. Lett. **94**, 18006 (2011).
[21] Z. Wang, Z.-J. Xu, J.-H. Huang, and L.-Z. Zhang, Chin. Phys. B **19**, 10020 (2010).
[22] A. Szolnoki and M. Perc, Phys. Rev. E **85**, 026104 (2012).
[23] J. A. Moreira, J. Pacheco, and F. Santos, Sci. Rep. **3**, 1521 (2013).
[24] J. Tanimoto and H. Sagara, BioSystems **90**, 105 (2007).
[25] B. Ballobas, *Random Graphs* (Academic Press, London, 1985).
[26] R. Schuessler, J. Conflict Resolut. **33**, 728 (1989).