Langmuir oscillations in a nonextensive electron-positron plasma

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The Langmuir oscillations, Landau damping, and growing unstable modes in an electron-positron (EP) plasma are studied by using the Vlasov and Poisson's equations in the context of the Tsallis's nonextensive statistics. Logically, the properties of the Langmuir oscillations in a nonextensive EP plasma are remarkably modified in comparison with that of discussed in the Boltzmann-Gibbs statistics, i.e., the Maxwellian plasmas, because of the system under consideration is essentially a plasma system in a nonequilibrium stationary state with inhomogeneous temperature. It is found that by decreasing the nonextensivity index q which corresponds to a plasma with excess superthermal particles, the phase velocity of the Langmuir waves increases. In particular, depend on the degree of nonextensivity, both of damped and growing oscillations are predicted in a collisionless EP plasma, arise from a resonance phenomena between the wave and the nonthermal particles of the system. Here, the mechanism leads to the unstable modes is established in the context of the nonextensive formalism yet the damping mechanism is the same developed by Landau. Furthermore, our results have the flexibility to reduce to the solutions of an equilibrium Maxwellian EP plasma (extensive limit $q \to 1$), in which the Langmuir waves are only the Landau damped modes.

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I. INTRODUCTION

Electron-positron (EP) plasma play an important role in the physics of a number of astrophysical situations such as active galactic nuclei [1], pulsar and neutron star magnetosphere [2], solar atmosphere [3], accretion disk [4], black holes [5], the early universe [6], and many others. For example, the detection of circularly polarized radio emission from the jets of the archtypal quasar 3C297 indicates that EP pairs are an important component of the jet plasma [7]. Similar detections in other radio sources suggest that, in general, extragalactic radio jets are composed mainly of an EP plasma [7]. It has been suggested that the creation of pair plasma in pulsars is essentially by energetic collisions between particles, which are accelerated as a result of electric and magnetic fields in such systems [8]. Also, two terrestrial sources of EP plasmas are the interaction of high-power lasers with plasmas [9] and the laboratory experiments in plasma confinement devices [10]. It is observed that the annihilation time of EP pairs in typical experiments is often long compared with typical confinement times [11], showing that the lifetime of EP pairs in the plasma is much longer than the characteristic time scales of typical oscillations. The long lifetime of EP pairs against pair annihilation indicates that many collective oscillations can occur and propagate in a pure EP plasma.

Waves in EP plasmas have been extensively studied over the past two decades. These studies mainly have concentrated on the relativistic EP plasmas (see, e.g., the references given in Ref. [12]). However, there are many experiments that confirm the possibility of nonrelativistic EP plasmas in the laboratory [13]. Until now, some authors have studied the characteristics of possible modes in the EP plasmas. An excellent and comprehensive study on the collective modes in nonrelativistic EP plasmas was presented by Iwamato [14] and used a kinetic theory description. Zank and Greaves [15], by use of a two-fluid model, discussed the linear and nonlinear modes in nonrelativistic EP plasmas. Furthermore, some authors have studied some aspects of the nonlinear electrostatic and electromagnetic wave propagation in the EP plasmas (see, e.g., the references given in Ref. [16]). From a theoretical point of view, analysis of an EP plasma, as compared with the ordinary electron-ion (EI) plasmas, leads to considerable modifications in the mathematical description. As we know, an EP plasma is composed of species with the same absolute charge-to-mass ratio. Therefore, the same dynamics of the EP pairs and, hence, the involved symmetry imply that the physical properties of such a system would differ from those of an ordinary EI plasma.

Although pair plasmas consisting of electrons and positrons have been experimentally produced, because of fast annihilation and the formation of positronium atoms and also low densities in typical EP experiments, the identification of collective modes in such experiments is very difficult in practice. To resolve this problem, one may experimentally deal with a pure pair-ion plasma instead of a pure EP plasma for identification of the collective modes. An appropriate experimental method has been developed by Oohara and Hatakeyama [17] for the generation of pure pair-ion plasmas consisting of only positive and negative ions with equal masses by using fullerenes C_{60}^- and C_{60}^+. The fullerene pair plasmas are physically akin to the EP plasmas, without having to worry about fast annihilation. By drastically improving the pair-ion plasma source in order to excite effectively the collective modes, Oohara *et al.* [18] have experimentally examined the electrostatic modes propagating along the magnetic-field lines in a fullerene pair plasma.

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In the experiment of Oohara *et al.* [18], three kinds of electrostatic modes have been observed from the obtained dispersion curves: the acoustic waves in a relatively low-frequency band, an intermediate-frequency backwardlike mode, and the Langmuir-type waves in a relatively high-frequency band. There, they have briefly discussed some aspects of their experimental results by using a theoretical two-fluid model. To our knowledge, a satisfactory and accepted theoretical justification for explanation of the acoustic and the backward intermediate modes in a fully symmetric pure pair plasma (as the Oohara *et al.* [18] claim their experiment to be) does not exist. Particularly, it is obvious that the acoustic modes are not possible in a fully symmetric pair plasma where the particles have the same dynamics. Here, we want to discuss, in detail, the high-frequency Langmuir-type oscillations in a pure pair plasma by using a kinetic theory model and argue some properties of these modes in a subtler manner.

It is often observed that the physical distribution of particles in space plasmas as well as in laboratory plasmas are not exactly Maxwellian and particles show deviations from the thermal distribution [19,20]. Presence of nonthermal particles in space plasmas has been widely confirmed by many spacecraft measurements, e.g., see the references in Ref. [21]. In many cases, the velocity distributions show non-Maxwellian tails decreasing as a power-law distribution in particle speed. Several models for phase-space plasma distributions with superthermal wings or other deviations from purely Maxwellian behavior have become rather popular in recent years, like the so-called kappa (κ) distribution, which was introduced initially by Vasyliunas in 1968 [22] to describe plasmas far from the thermal equilibrium such as the magnetosphere environment and the solar winds (e.g., see Ref. [23]); the nonthermal model advanced by Cairns *et al.* in 1995 [24], which was introduced at first to explain the solitary electrostatic structures involving density depletions that have been observed in the upper ionosphere in the auroral zone by the Freja satellite [25]; and the nonextensive Tsallis model. In the following we want to briefly review the formalism of the Tsallis model and to argue why it is preferred, rather than that of the Carins and kappa models.

Generally, the standard Boltzmann-Gibbs (BG) extensive thermostatistics constitutes a powerful tool when microscopic interactions and memories are short ranged and the environment is a Euclidean space-time, a continuous and differentiable manifold. However, there are many studies which show the breakdown of the BG statistics to describe systems with long-range interactions, long-time memory, and fractal space-time structures (see, e.g., the references given in Ref. [26]). Basically, systems subject to long-range interactions and correlations and long-time memories are related to the non-Maxwellian distributions where the standard BG statistics do not apply. The plasma environments in the astrophysical systems, obviously, are subject to spatial and temporal long-range interactions evolving in a non-Euclidean space-time that make their behavior nonextensive. A suitable generalization of the Boltzmann-Gibbs-Shannon (BGS) entropy for statistical equilibrium was first proposed by Reyni [27] and subsequently by Tsallis [28], preserving the usual properties of positivity, equiprobability, and irreversibility but suitably extending the standard extensivity or additivity of the entropy to nonextensivity.

The nonextensive generalization of the BGS entropy which was proposed by Tsallis in 1988 [28] is given by the following expression:

$$S_q = k_B \frac{1 - \sum_i p_i^q}{q - 1}, \quad (1)$$

where k_B is the standard Boltzmann constant, $\{p_i\}$ denotes the probabilities of the microstate configurations, and q is a real parameter quantifying the degree of the nonextensivity. The most distinctive feature of S_q is its pseudoadditivity. Given a composite system $A + B$, constituted by two subsystems A and B, which are independent in the sense of factorizability of the joint microstate probabilities, the Tsallis entropy of the composite system $A + B$ satisfies $S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B)$. In the limit of $q \to 1$, S_q reduces to the celebrated logarithmic BGS entropy $S = -k_B \sum_i p_i \ln p_i$, and the usual additivity of the entropy is recovered. Hence, $|1 - q|$ is a measure of the lack of extensivity of the system. There are a number of evidences exhibiting that the nonextensive statistics, arising from S_q, is a better framework for describing many physical systems, such as galaxy clusters [29], plasmas [30,31], turbulent systems [32], and so on, in which the system shows a nonextensive behavior as a result of long-range interactions and correlations. The functional form of the velocity distribution in the Tsallis formalism may be derived through a nonextensive generalization of the Maxwell ansatz [33] or through maximizing the Tsallis entropy under the constraints imposed by normalization and the energy mean value [34]. Furthermore, from a nonextensive generalization of the "molecular chaos hypothesis," it is shown that the equilibrium q-nonextensive distribution is a natural consequence of the H theorem [35].

It is to be noted that the empirically derived κ distribution function in space plasmas is equivalent to the q-distribution function in the Tsallis nonextensive formalism, in the sense that the spectrum of the velocity distribution function in both models shows similar behavior and, in fact, that both the κ distribution and the Tsallis q-nonextensive distribution describe deviations from the thermal distribution. In particular, Leubner in 2002 [36] showed that the distributions very close to the κ distributions are a consequence of the generalized entropy favored by the nonextensive statistics and proposed a link between the Tsallis nonextensive formalism and the κ distribution functions. In fact, relating the parameter q to κ by use of a formal transformation $\kappa = 1/(1 - q)$ [36] provides the missing link between the q-nonextensive distribution and the κ-distribution function favored in space-plasma physics, leading to a required theoretical justification for the use of κ distributions from fundamental physics. Furthermore, Livadiotis and McComas in 2009 [37] examined how *kappa* distributions arise naturally from the Tsallis statistical mechanics. On the other hand, the nonthermal distribution function introduced by Cairns *et al.* [24] is a proposal function to model an electron distribution with a population of energetic particles. It is especially appropriate to describe the nonlinear propagation of large amplitude electrostatic excitations such as solitary waves and double layers, which are very common in the magnetosphere. However, the lack of a statistical foundation behind this proposal function is clearly seen, leading to it receiving less attention than the κ function and Tsallis distribution. The q-nonextensive formalism, with a powerful thermostatistics foundation and considerable experimental evidence, may cover many features of the other nonthermal models and provide a good justification for its preference over the other models. It has considerably extended both the statistical mechanics formalism and its range of applicability. The interested reader may refer to the references given in Refs. [38–41], where the significance, historical background, physical motivations, foundations, and applications of the nonextensive thermostatistics are discussed in detail.

The waves, Landau damping, and instabilities in the plasma may be investigated in the framework of nonextensive statistics. For example, Lima *et al.* [42] have studied the Langmuir oscillations and Landau damped waves in a collisionless EI plasma in the context of nonextensive statistics. In particular, they have stressed that, due to the long-range nature of Coulombic interactions in the plasma, the standard Maxwell-Boltzmann distribution may provide only a very crude description of such systems, even in the collisionless limit. Furthermore, by constraining the nonextensive statistics with plasma oscillation data [43], it is revealed that a good agreement between the theory and the experimental results for the standard Bohm-Gross dispersion relation is possible. Also, in the context of the nonextensive statistic, the nonlinear Landau damping of the electrostatic waves in an unmagnetized collisionless EI plasma has been investigated numerically by using a semi-Lagrangian Vlasov-Poisson code [44]. Furthermore, Liyan and Jiulin [45] have discussed the dispersion relation and Landau damping of ion-acoustic waves in a collisionless magnetic-field-free plasma in the nonextensive statistics. Particularly, they have emphasized that the physical state described by the q distribution in Tsallis's statistics is not the thermodynamic equilibrium. In fact, the deviation of q from unity quantifies the degree of the inhomogeneity of the temperature T via the formula $k_B\vec{\nabla}T + (1-q)Q_\alpha\vec{\nabla}\phi = 0$ [46], where Q_α denotes the electric charge of specie α. In other words, the nonextensive statistics describes systems in the nonequilibrium stationary state with inhomogeneous temperature that contains a number of nonthermal particles.

In the present work, our goal is to investigate the Langmuir oscillations in a field-free collisionless EP plasma in the context of the q-nonextensive statistics, emphasizing the possible damping and instability. A kinetic theory model based on the linearized Vlasov and Poisson's equations is used by which the general form of the dielectric function ($D(k,\omega)$) for weakly damped (or growing) longitudinal waves in an EP plasma is presented, as shown in Sec. II. The eigenvalues of $D(k,\omega) = 0$ for coherent oscillations in a nonextensive EP plasma are derived in Sec. III and then the real and imaginary parts of the frequency of Langmuir oscillations are obtained there. In Sec. IV, the possibility of normal modes, Landau damped, and growing unstable modes are discussed. Finally, the paper is summarized in Sec. V.

II. THE MODEL EQUATIONS

Here, we present a brief review of kinetic equations for describing the electrostatic collective modes specialized to an EP plasma with the constraint of weak damping or growth.

We consider a spatially uniform field-free EP plasma at the equilibrium state. If at a given time $t = 0$ a small amount of charge is displaced in the plasma, the initial perturbation may be described by

$$f_\alpha(t=0) = f_{\alpha 0}(\vec{v}) + f_{\alpha 1}(\vec{x},\vec{v},t=0), \quad f_{\alpha 1} \ll f_{\alpha 0},$$

where $f_{\alpha 0}$ corresponds to the unperturbed and time-independent stationary distribution and $f_{\alpha 1}$ is the corresponding perturbation about the equilibrium state, where α stands for electrons and positrons ($\alpha = e,p$). We assume that the perturbation is electrostatic and the displacement of charge gives rise to a perturbed electric but no magnetic field. With this assumption, the time development of $f_{\alpha 1}(\vec{x},\vec{v},t)$ is given by solution of the linearized Vlasov and Poisson's equations as follows [47,48]:

$$\frac{\partial f_{e1}}{\partial t} + \vec{v}\cdot\frac{\partial f_{e1}}{\partial \vec{x}} + \frac{e}{m}\vec{\nabla}\phi_1\cdot\frac{\partial f_{e0}}{\partial \vec{v}} = 0, \tag{2}$$

$$\frac{\partial f_{p1}}{\partial t} + \vec{v}\cdot\frac{\partial f_{p1}}{\partial \vec{x}} - \frac{e}{m}\vec{\nabla}\phi_1\cdot\frac{\partial f_{p0}}{\partial \vec{v}} = 0, \tag{3}$$

$$\nabla^2\phi_1 = 4\pi ne\int (f_{e1} - f_{p1})\,d\vec{v}, \tag{4}$$

where e, m, and n denote, respectively, the absolute charge, mass, and number density of the electron and positron and ϕ_1 is the electrostatic potential produced by the perturbation. This set of linearized equations for perturbed quantities may be solved simultaneously to investigate the plasma properties for the time intervals shorter than the binary collision times. Specially, we can study the properties of the plasma waves whose oscillations period are much less than a binary collision time. The standard technique for solving simultaneously the differential equations (2)–(4) is the method of integral transforms, as developed for the first time by Landau in the case of an ordinary EI plasma [47–49]. Another simplified method of solving the Vlasov-Poisson's equations for the longitudinal waves, with the frequency ω and the wave vector $\vec{k}$, is to assume that the solution has the form

$$f_{\alpha 1}(\vec{x},\vec{v},t) = f_{\alpha 1}(\vec{v})e^{i(\vec{k}\cdot\vec{x}-\omega t)}, \quad \alpha = e,p$$
$$\phi_1(\vec{x},t) = \phi_1 e^{i(\vec{k}\cdot\vec{x}-\omega t)}. \tag{5}$$

Without loss of the generality, we consider the x axis to be along the direction of the wave vector $\vec{k}$ and let $v_x = u$. Then, by applying Eq. (5) and solving Eqs. (2)–(4), we find the dispersion relation for longitudinal waves in an EP plasma as follows:

$$D(k,\omega) = 1 - \frac{4\pi ne^2}{mk^2}\int \frac{\frac{\partial}{\partial u}(f_{e0}(u) + f_{p0}(u))}{u - \frac{\omega}{k}}\,du = 0, \tag{6}$$

where $D(k,\omega)$ is the dielectric function of a field-free pair EP plasma for the longitudinal oscillations. We then can investigate the response of the pair plasma to an arbitrary perturbation via the response dielectric function $D(k,\omega)$. In general, the frequency ω which satisfies the dispersion relation $D(k,\omega) = 0$ is complex, i.e., $\omega = \omega_r + i\omega_i$. However, in many cases $\mathrm{Re}[\omega(k)] \gg \mathrm{Im}[\omega(k)]$, and the plasma responds to the perturbation a long time after the initial disturbance with oscillations at a range of the well-defined frequencies. The nontransient responses of the plasma to an initial perturbation are the normal modes of the plasma. We can determine the normal modes of the plasma from the dispersion relation $D[k,\omega(k)] = 0$, which gives the frequency of the plasma waves as a function of the wave number k or vice versa. It should be further mentioned that when we solve the Vlasov and Poisson's equations as an initial valve problem, here via $f_{e0} + f_{p0}$, it is possible to obtain the solutions with negative or positive values of ω_i, corresponding to the damped or growing waves, respectively. This can be explicitly seen from the electrostatic potential associated with the wave number k of the excitation as follows:

$$\phi_1(x,t) = \phi_1 e^{i(kx-\omega_r t)} e^{\omega_i t}, \tag{7}$$

where a solution with negative ω_i displays a damped wave, while the solution with positive one corresponds to an unstable mode.

With the constraint of the weak damping or growth, i.e., $\omega_i \ll \omega_r$, the dielectric function $D(k,\omega)$ given in Eq. (6) can be Taylor expanded in the small quantity ω_i, and then we may explicitly find the real and imaginary parts of the dielectric function as follows:

$$D_r(k,\omega_r) = 1 - \frac{4\pi n e^2}{mk^2}\mathrm{P.V.}\int \frac{\frac{\partial}{\partial u}(f_{e0}(u) + f_{p0}(u))}{u - \frac{\omega_r}{k}}\, du, \tag{8}$$

$$D_i(k,\omega_r) = -\pi\left(\frac{4\pi n e^2}{mk^2}\right)\left[\frac{\partial}{\partial u}(f_{e0}(u) + f_{p0}(u))\right]_{u=\frac{\omega_r}{k}}. \tag{9}$$

Here we have made the analytic continuation of the velocity integral in Eq. (6) over u, along the real axis, which passes under the pole at $u = \frac{\omega}{k}$ with the constraint of weakly damped waves, where P.V. $\int$ denotes the Cauchy principal value. By these relations and neglecting the terms of order $(\frac{\omega_i}{\omega_r})^2$, ω_r and ω_i can be computed, respectively, from the relations [48,49]

$$D_r(k,\omega_r) = 0, \tag{10a}$$

$$\omega_i = -\frac{D_i(k,\omega_r)}{\partial D_r(k,\omega_r)/\partial\omega_r}. \tag{10b}$$

III. LANGMUIR OSCILLATIONS WITH NONEXTENSIVE STATIONARY STATE

Now we want to obtain the formalism and some features of the Langmuir waves in an EP plasma in the context of the nonextensive statistics. For this purpose we assume that the stationary state of the plasma obeys the q-nonextensive distribution function, instead of a Maxwellian one, which merely describes a fully equilibrium stationary sate. The q-distribution function in one dimension is given by [33–35]

$$f_{\alpha 0}(u) = A_{\alpha,q}\left[1 - (q-1)\frac{m_\alpha u^2}{2k_B T_\alpha}\right]^{\frac{1}{q-1}}, \tag{11}$$

where m_α and T_α are, respectively, the mass and temperature of species α ($\alpha = e,p$) and k_B is the standard Boltzmann constant. The normalization constant $A_{\alpha,q}$ can be written as

$$A_{\alpha,q} = L_q\sqrt{\frac{m_\alpha}{2\pi k_B T_\alpha}}, \tag{12}$$

where the dimensionless q-dependent coefficient L_q reeds

$$L_q = \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)}\sqrt{1-q}, \quad \text{for} \quad -1 < q \leqslant 1, \tag{13a}$$

$$L_q = \left(\frac{1+q}{2}\right)\frac{\Gamma\left(\frac{1}{2} + \frac{1}{q-1}\right)}{\Gamma\left(\frac{1}{q-1}\right)}\sqrt{q-1}, \quad \text{for} \quad q \geqslant 1. \tag{13b}$$

One may examine that for $q > 1$, the q-distribution function (11) exhibits a thermal cutoff, which limits the velocity of particles to the values $u < u_{\max}$, where $u_{\max} = \sqrt{\frac{2k_B T_\alpha}{m_\alpha(q-1)}}$. For these values of the parameter q we have $S_{q>1}(A+B) < S(A) + S(B)$ referred to the *subextensivity*. This thermal cutoff is absent when $q < 1$, that is, the velocity of particles is unbounded for these values of the parameter q. In this case, we have $S_{q<1}(A+B) > S(A) + S(B)$ referred to the *superextensivity*. Moreover, the q distribution (11) is unnormalizable for the values of the $q < -1$. Moreover, the parameter q may be further restricted by the other physical requirements, such as finite total number of particles and consideration of the energy equipartition for contribution of the total mean energy of the system. Interestingly, in the extensive limit $q \to 1$, where $S(A+B) = S(A) + S(B)$, and by using the formula $\lim_{|z|\to\infty} z^{-a}[\frac{\Gamma(a+z)}{\Gamma(z)}] = 1$ [50], the distribution function (11) reduces to the standard Maxwell-Boltzmann distribution $f_\alpha(u) = \sqrt{\frac{m_\alpha}{2\pi k_B T_\alpha}} e^{-\frac{m_\alpha u^2}{2k_B T_\alpha}}$. In Fig. 1, we have depicted schematically the nonthermal behavior of the distribution function (11) for some values of the spectral index q in which the velocity u and the distribution function $f(u)$, respectively, have normalized by the standard thermal speed $v_{th} = \sqrt{\frac{2k_B T}{m}}$ and $\sqrt{\frac{m}{2\pi k_B T}}$. We can see that in the case of a superextensive distribution with $q < 1$ [Fig. 1(a)], compared with the Maxwellian limit (solid curve), there are more particles with the velocities faster than the thermal speed v_{th}. These are the so-called superthermal particles and we can see the q distribution with $q < 1$ behave like the κ distribution, the same as that introduced to describe the space plasmas far from the thermal equilibrium [22]. In fact, in a superthermal plasma modeled by a κ-like distribution (here, the cases in which $q < 1$), the particles have distributed in a wider spectrum of the velocities, in comparison with a Maxwellian plasma. In other words, the low values of the spectral index q correspond to a large fraction of superthermal particle populations in the plasma. On the other hand, in the case of a subextensive distribution with $q > 1$ [Fig. 1(b)], compared with the Maxwellian limit (solid curve), there is a large fraction of particles with velocities that are slower than the thermal speed v_{th}. Also, for these values of the q parameter, we can explicitly see the thermal cutoff which limits the velocity of particles, as mentioned before. In fact, the q distributions with $q > 1$ are suitable for describing the systems containing a large number of low-speed particles.

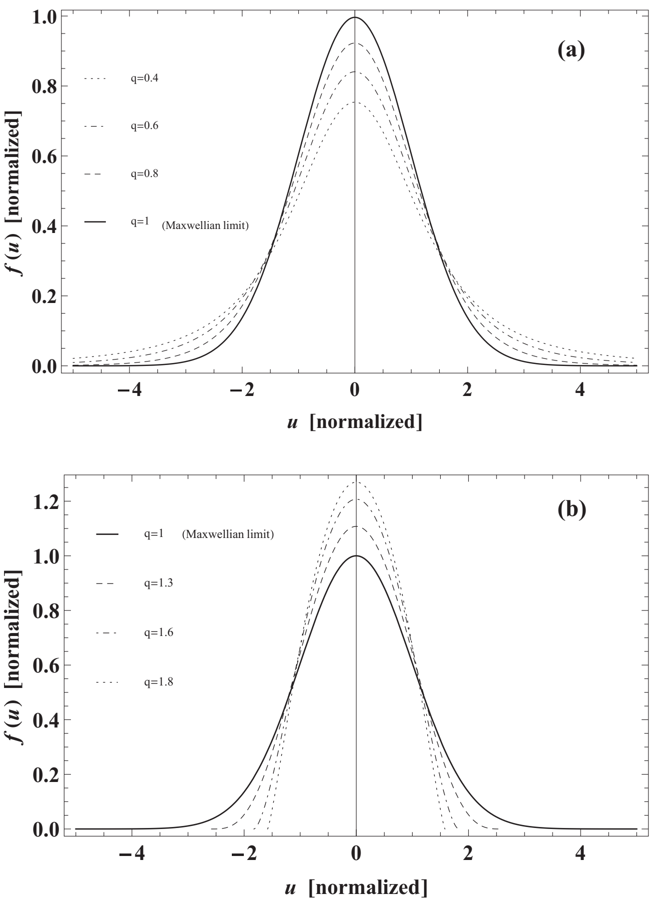

FIG. 1. The nonthermal behavior of the q-nonextensive distribution function and its comparison with the Maxwellian one (solid carve): (a) Superxtensive distribution with $q < 1$ that behaves like the κ distributions for superthermal plasmas. In this case, the particles have distributed in a wider spectrum of the velocities, in comparison with a Maxwellian distribution. (b) Subextensive distribution with $q > 1$, which is suitable for describing the systems containing a large number of low-speed particles. In this case, there is a thermal cutoff which limits the velocity of particles.

Before deriving the normal modes of the nonextensive EP plasma, it is necessary to discuss the condition for coherent oscillations in the plasma. In an EP plasma both of the electrons and positrons participate in the wave motion and they must oscillate coherently, in response to the local wave field. For coherent oscillations, a given group of the electrons and positrons must be influenced only by the local electric field of the wave. This requires that more distant fields are screened out, so the electrons and positrons only respond to the field that is set up within a distance which is much less than the wavelength, which indicates that $\lambda_D \ll \lambda$. This is the condition for coherent response to a perturbation by the electrons and positrons, where λ_D is the Debye screening length and is given in a charge-neutral EP plasma by

$$\lambda_D^{-2} = \frac{4\pi n e^2}{k_B}\left(\frac{1}{T_e} + \frac{1}{T_p}\right). \tag{14}$$

Furthermore, this condition becomes increasingly stringent as the plasma grows hotter. The reason in that the electric field remains unscreened over a greater distance, since λ_D increases with temperature by Eq. (14).

We consider the high-frequency oscillations with phase velocity much greater than the thermal speed of the electrons and positrons ($\frac{\omega}{k} \gg u$). Then, the Cauchy principal value of Eq. (8) may be evaluated by an expanding in u as follows:

$$\begin{aligned} &-\!\!\!\!\!\!\int_{-u_{\max}}^{+u_{\max}} \frac{\frac{\partial}{\partial u}(f_{e0}(u) + f_{p0}(u))}{u - \frac{\omega_r}{k}}\, du \\ &\quad = \frac{k}{\omega_r}\int_{-u_{\max}}^{+u_{\max}} \left(\frac{\partial f_{e0}(u)}{\partial u} + \frac{\partial f_{p0}(u)}{\partial u}\right) \\ &\quad\quad \times \left(1 + \frac{k}{\omega_r}u + \frac{k^2}{\omega_r^2}u^2 + \frac{k^3}{\omega_r^3}u^3 + \cdots\right) du. \end{aligned} \tag{15}$$

Here, in order to include both cases $q < 1$ (superextensivity) and $q > 1$ (subextensivity), we have denoted the integration limits in Eq. (15) by $\pm u_{\max}$. In fact, as discussed earlier, the integration limits are unbounded, i.e., $\pm u_{\max} = \pm\infty$, when $q < 1$, and they are given by the q dependent thermal cutoff $\pm u_{\max} = \pm\sqrt{\frac{2k_B T_\alpha}{m_\alpha(q-1)}}$ when $q > 1$.

With the Tsallis q distribution given in Eq. (11), noting that $f_{\alpha 0}(u)$ is an even function with argument u and $\frac{\partial f_{\alpha 0}}{\partial u}$ is an odd function, one may verify the following relations for all values of $q > \frac{1}{3}$:

$$\int_{-u_{\max}}^{+u_{\max}} \frac{\partial f_{\alpha 0}(u)}{\partial u}\, du = 0, \tag{16a}$$

$$\int_{-u_{\max}}^{+u_{\max}} u\frac{\partial f_{\alpha 0}(u)}{\partial u}\, du = -1, \tag{16b}$$

$$\int_{-u_{\max}}^{+u_{\max}} u^2\frac{\partial f_{\alpha 0}(u)}{\partial u}\, du = 0, \tag{16c}$$

$$\int_{-u_{\max}}^{+u_{\max}} u^3\frac{\partial f_{\alpha 0}(u)}{\partial u}\, du = -3\left(\frac{2}{3q-1}\right)\frac{k_B T_\alpha}{m_\alpha}, \tag{16d}$$

where α stands for electron and positron. The above integrals are computed by parts and we have calculated the average value of u^2 as follows:

$$\langle u^2\rangle = \int_{-u_{\max}}^{+u_{\max}} u^2 f_{\alpha 0}(u)\, du = \frac{2}{3q-1}\frac{k_B T_\alpha}{m_\alpha}, \tag{17}$$

which requires that the parameter q must restrict to the values of $q > \frac{1}{3}$. Note that for q values equal or lower than the critical value $q_c = \frac{1}{3}$, the mean value of u^2 diverges. Therefore, we see that the q parameter for the case $q < 1$ is further restricted to the values $\frac{1}{3} < q < 1$, in order that the physical requirement of energy equipartition is preserved. We emphasize that our results here are valid both for the case $\frac{1}{3} < q < 1$ where the value of $u_{\max}$ is unbounded and also in the case $q > 1$ in which $u_{\max}$ is given by the thermal cutoff $u_{\max} = \sqrt{\frac{2k_B T_\alpha}{m_\alpha(q-1)}}$. Note that in both cases the above integrals are evaluated by limits that are symmetric across the origin. The interested reader may easily check the validity of Eqs. (16) and (17) for all allowed values of q. Furthermore, in the extensive limit $q \to 1$, Eq. (17) reduces to the familiar energy equipartition theorem for each degree of freedom in the Boltzmann-Gibbs statistics as $\langle\frac{1}{2}m_\alpha u^2\rangle = \frac{1}{2}k_B T_\alpha$.

In the present work we shall consider $T_e = T_p = T$ which is in agreement with the experimental works in a pure pair plasma comprised of particles with the same dynamics [17,18]. In a different manner, comparing the electron-electron, positron-positron, and electron-positron relaxation time scales reveals that the creation of a pure EP plasma with a considerable difference in the temperature of the pairs is not possible in practice [14]. In other words, in the creation of a pure EP plasma the whole system reaches a common thermal state with $T_e = T_p = T$ signifying a temperature-symmetric EP plasma.

Furthermore, it is to be noted that the q distribution given in Eq. (11) describes the stationary state of the species α in the framework of the Tsallis nonextensive formalism. The value of the spectral index q is a measure that determines the slope of the energy spectrum of the nonthermal particles and measures the deviation from the standard thermal distribution (which is recovered at the limit $q \to 1$). The value of the spectral index q is determined as a result of long-range interactions and correlations of the whole system. Therefore, a distinction between electrons and positrons in q may depend on the physics of the system under consideration. Here, following El-Tantawy *et al.* [51], we make no distinction between electrons and positrons in q. From a physical standpoint, we have considered a charge-neutral and fully symmetric pair plasma in which the thermodynamic characteristics are equal both in T and in q.

Now, with Eqs. (15) and (16), the real part of the dielectric function in Eq. (8) reads as

$$D_r(k,\omega_r) = 1 - \frac{8\pi n e^2}{m\omega_r^2} - 3\left(\frac{8\pi n e^2}{m}\right)\frac{k^2}{\omega_r^4}\left(\frac{2}{3q-1}\right)\left(\frac{k_B T}{m}\right), \tag{18}$$

Without considering the thermal effects in Eq. (18), with $D_r(k,\omega_r) = 0$ we find the natural oscillation frequency in a charge-neutral EP plasma as $\omega_p = (\frac{8\pi n e^2}{m})^{\frac{1}{2}}$. However, by including the thermal correction, the solution of the equation $D_r(k,\omega_r) = 0$ yields the dispersion relation for Langmuir waves in a nonextensive EP plasma as follows:

$$\omega_r^2 = \omega_p^2\left[1 + 3(k\lambda_D)^2\frac{2}{3q-1}\right], \tag{19}$$

where λ_D is the Debye screening length given in Eq. (14). The dispersion relation (19) is quite compatible with the results presented by Iwamato [14] and Zank and Greave [15], by considering the extensive limit $q \to 1$.

On the other hand, by using Eq. (9) and for the q-distribution function (11), it is straightforward to obtain the imaginary part of the dielectric function as follows:

$$D_i(k,\omega_r) = \sqrt{\frac{\pi}{2}}L_q\frac{1}{k^3\lambda_D^3}\frac{\omega_r}{\omega_p}\left[1 - (q-1)\frac{\omega_r^2}{2k^2\lambda_D^2\omega_p^2}\right]^{\frac{2-q}{q-1}}. \tag{20}$$

By $D_r(k,\omega_r)$ and $D_i(k,\omega_r)$ given in Eqs. (18) and (20), we may obtain the explicit solution of the imaginary part of the frequency from Eq. (10b), noting that both $k\lambda_D$ and $\frac{\omega_i}{\omega_r}$ are assumed small. The result is as follows:

$$\omega_i = -\sqrt{\frac{\pi}{8}}L_q\frac{\omega_p}{(k\lambda_D)^3}\left[1 - (q-1)\times\left(\frac{1}{2(k\lambda_D)^2} + \frac{3}{3q-1}\right)\right]^{\frac{2-q}{q-1}}, \tag{21}$$

where L_q is that given in Eq. (13).

Here the solutions (19) and (21) have been derived for the high-frequency oscillations with $\frac{\omega}{k} \gg (\frac{2k_B T_\alpha}{m})^{\frac{1}{2}}$, $\alpha = e,p$, when the condition of weak damping or growth is satisfied with $k\lambda_D \ll 1$, indicating the coherent long-wave oscillations. In addition, in the Maxwellian limit $q \to 1$, the imaginary part of the frequency in the Eq. (21) reduces to that presented by Iwamato [14] as follows:

$$\omega_i = -\sqrt{\frac{\pi}{8}}\frac{\omega_p}{(k\lambda_D)^3}e^{-(\frac{1}{2(k\lambda_D)^2}+\frac{3}{2})}. \tag{22}$$

Note that in this extensive limit, the longitudinal oscillations have only the (Landau) damping and no growth, because of the negative value of the imaginary part of the frequency provided by Eq. (22). One basic feature of our analysis is the inclusion of the nonextensivity of the system, which is essentially a result of the long-range Coulombian interactions of charge particles in the plasma. The nonextensivity of the system is determined by the spectral index q and may lead to positive or negative values for ω_i in Eq. (21). Therefore, depending on the nonextensivity of the plasma, both the damping and growth may be happening for the electrostatic oscillations in an EP plasma.

IV. DISCUSSION

Equations (19) and (21) describe the Langmuir oscillations in a nonextensive EP plasma which satisfy the condition of weak damping or growth by $k\lambda_D \ll 1$. Now we can investigate the dispersion relation and the damping or growth of the Langmuir oscillations via these solutions.

A. Dispersion relation

In Fig. 2 we have plotted the ratio $\frac{\omega_r}{\omega_p}$ as a function of the dimensionless parameter $k\lambda_D$ for some values of the nonextensivity index q. The solid curve corresponds to the extensive limit $q = 1$ and the other ones show the deviations from the Maxwellian limit. It is seen that, for

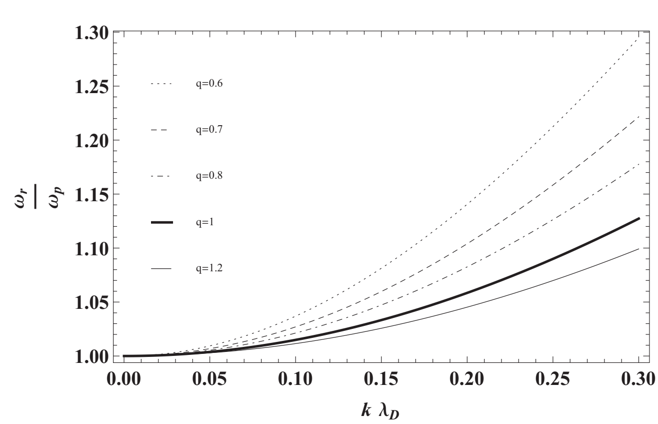

FIG. 2. The effect of the nonextensivity on the dispersion relation of Langmuir waves, where the solid curve corresponds to the extensive limit $q \to 1$ and the other ones show the deviations from the Maxwellian plasma.

a given wavelength, the phase velocity of the Langmuir waves increases with decreasing the value of q. We can discuss the physical meaning of this result in the context of the nonextensive statistics as follows. As mentioned earlier, the q-distribution function with $q < 1$, compared with the Maxwellian one ($q = 1$), indicates the systems with more superthermal particles (superextensivity). On the other hand, the q distribution with $q > 1$ is suitable to describe the systems containing a large number of low-speed particles (subextensivity). However, because of the long-range nature of Coulombian interactions in plasma environments and the presence of many superthermal particles in such systems, confirmed by many astrophysical measurements (see, e.g., the references given in Ref. [21]), a q distribution with $q < 1$ is strongly suggested for the real plasma systems or superthermal plasmas. It is obvious that in a plasma with more superthermal particles ($q < 1$), the phase velocity of the Langmuir waves should be larger than the case with lack of superthemal particles ($q > 1$), in agreement with our results here. Therefore, it is expected that our diagrams with $q < 1$ in Fig. 2 are more probable for space plasma systems than the results with $q \geqslant 1$. On the other hand, the experimental analysis of electrostatic modes in a pure pair-ion plasma [18] (that is physically akin to a pure EP plasma) confirms that a good fit with our dispersion relation with values $q < 1$ is provided.

B. Landau damping and growing oscillations

1. *Superextensive or superthermal plasmas ($q < 1$)*

The Landau damped and growing Langmuir-type modes in a superthermal EP plasma can be discussed via the imaginary part of the frequency given in Eq. (21) for the values of $q < 1$. In Fig. 3 we have plotted the ratio $\frac{\omega_i}{\omega_p}$ with respect to the nonextensivity index q for all allowed values of $q < 1$, i.e., $\frac{1}{3} < q < 1$, at the limit of long wavelengths (supported by, e.g., $k\lambda_D = 0.1$). It is explicitly seen that both of the damped ($\omega_i < 0$) and growing unstable oscillations ($\omega_i > 0$) are predicted in a superthermal EP plasma. Our numerical analysis shows that in two q regions, i.e., $0.34 \lesssim q \lesssim 0.6$ and $0.71 \lesssim q \lesssim 0.78$, the longitudinal oscillations are unstable,

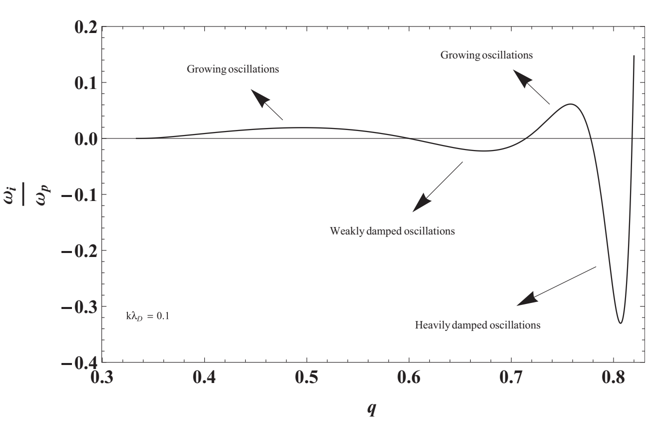

FIG. 3. The imaginary part of the frequency with respect to the nonextensivity index for $q < 1$ (superextensivity), which shows the q regions for damped and growing oscillations.

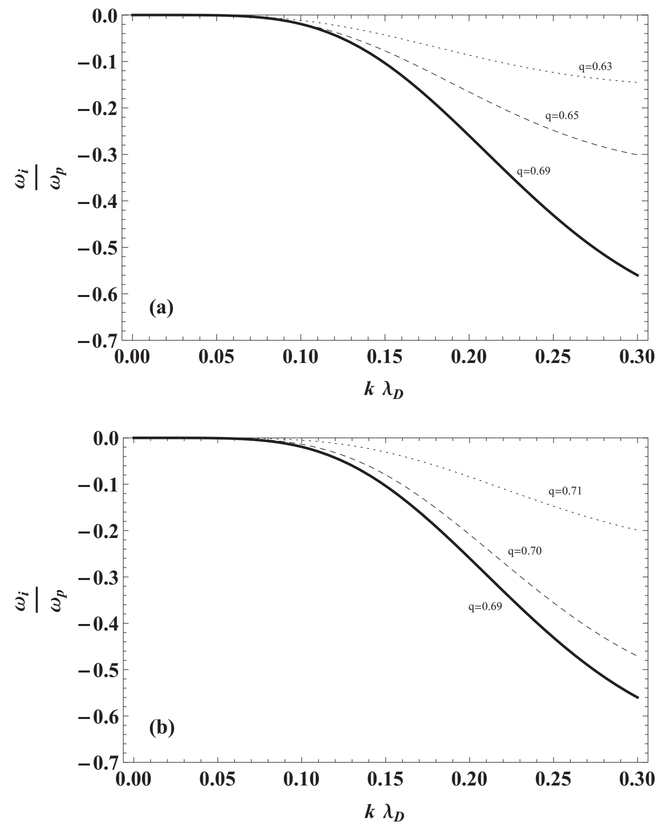

FIG. 4. The damping rate with respect to the wave number for weakly damped Langmuir waves in the case $q < 1$, where with increasing the nonextensivity index q (a) the damping rate increases up to a critical value at the vicinity of $q = 0.69$, after which (b) the damping rate decreases with q.

due to the fact that ω's have the positive imaginary parts and then the associated electrostatic modes will grow in time [Eq. (7) is noted]. The physical mechanism which leads to this instability may be explained as follows. As we expressed earlier, the q-nonextensive distribution with $q < 1$ describes a system with a large number of superthermal particles [see Fig. 1(a)]. Therefore, our solution for the Vlasov and Poisson's equations with $q < 1$ indicates an evolution which has started from a stationary state with a large portion of superthermal particles. The Langmuir modes may gain energy from these superthermal particles and result in growing oscillations in time. In other words, this instability arises from a stationary state which describes a superthermal plasma and, in fact, we have obtained the solution for a nonequilibrium stationary state.

On the other hand, the Langmuir waves have the Landau damping in two q regions $0.6 \lesssim q \lesssim 0.71$ and $0.78 \lesssim q \lesssim 0.82$, because ω's have the negative imaginary parts for these degrees of nonextensivity in the plasma. The Landau damping is a resonant phenomenon between the plasma particles (electrons and positrons) and the wave for the particles moving with nearly the phase velocity of the wave [47,48]. Noting that the q distribution is a decreasing function with u, there are, on average, more particles moving slightly slower than the wave than particles moving slightly faster than the wave; if the slower particles are accelerated by the wave, this must

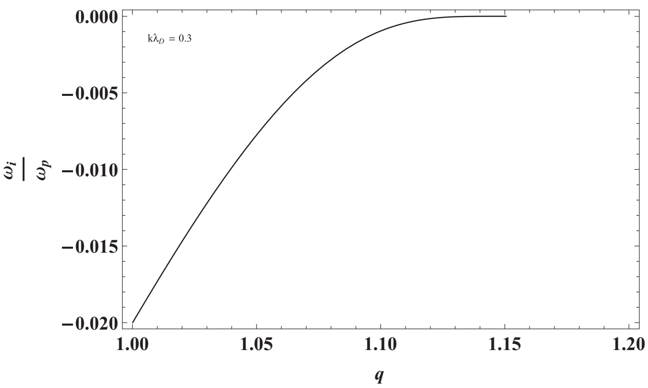

FIG. 5. The imaginary part of the frequency with respect to the nonextensivity index for $q > 1$ (subextensivity). For these values of the nonextensivity index q, the Lanqmuir waves have only the damping and no growth.

reduce the energy of the wave, and the wave damps. Our analysis reveals that the damping rate in the first q region, i.e., $0.6 \lesssim q \lesssim 0.71$, is small and the associated Langmuir oscillations are weakly damped. These are the normal modes of the plasma which would persist in several oscillation periods. But, in the q region $0.78 \lesssim q \lesssim 0.82$ the Langmuir oscillations are heavily damped and they would disappear after a few periods. Here we consider the weakly damped solutions which can be considered the normal modes of the plasma and abandon the heavily damped and the growing unstable oscillations. The damping rate with respect to the wave number is plotted in Fig. 4 for some values of the nonextensivity index q in the weakly damped q region. It is seen that the Landau damping is weak at the limit of long wavelengths ($k\lambda_D \ll 1$), as is expected. Also, Fig. 4 shows that by increasing the value of q, the damping rate increases up to a critical value at the vicinity of $q = 0.69$, after which the damping rate decreases with q.

2. *Subextensive plasmas* ($q > 1$)

Considering the values of $q > 1$, we can investigate the Langmuir oscillations in a subextensive EP plasma in which there are a number of low-speed particles. In Fig. 5, the imaginary part of the frequency with respect to nonextensivity index q is plotted for values of $q \geqslant 1$. From this graph, we find that the Langmuir oscillations have only the damping and no growth when $q \geqslant 1$. Additionally, the damping rate in this case is small, in comparison with the case of a superthermal plasma ($q < 1$), and also it decreases for higher values of q. The physical reason is that the number of particles participating in the resonance with the wave is small for a stationary state with $q > 1$. In fact, the slope of the velocity q-distribution function increases with q and there is even a thermal cutoff for the particles in the case of $q > 1$ [see Fig. 1(b)]. This corresponds to the presence of numerous low-speed particles in the plasma and weak resonance with the (high-frequency) Langmuir waves. In Fig. 6, we have plotted the damping rate versus the wave number for some values of $q > 1$, including the Maxwellian limit ($q = 1$). We see again that the Landau damping is weak for long-wavelength oscillations ($k\lambda_D \ll 1$) and it decreases with q.

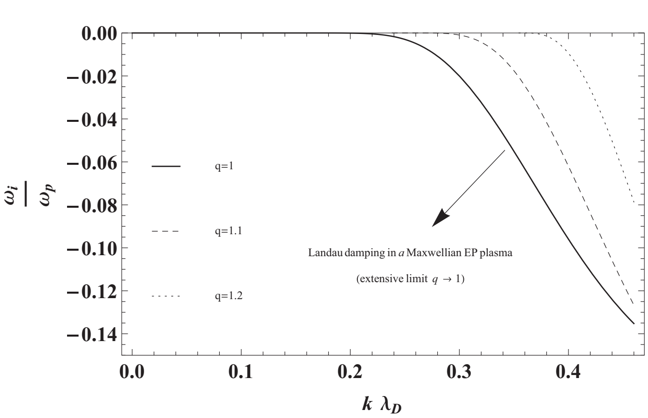

FIG. 6. The damping rate with respect to the wave number in the case $q > 1$, including the Maxwellian limit $q \to 1$.

V. SUMMARY AND CONCLUSIONS

In this work, we have investigated the Langmuir waves in a collisionless and magnetic-field-free quasineutral plasma composed of electrons and positrons on the basis of the nonextensive statistics. We have thereby used a kinetic theory model by employing the Vlasov and Poisson's equations to obtain the response dielectric function of the EP plasma to an arbitary perturbation. The dispersion relation and properties of the Langmuir waves are discussed here and it is shown that by decreasing the nonextensivity index q the phase velocity of the Langmuir waves increases, indicating a plasma with more superthermal particles. Furthermore, it is found that depending on the degree of nonextensivity of the plasma, both the damping and growth may occur for the longitudinal oscillations in a collisionless EP plasma, arising from a resonance phenomenon between the wave and nonthermal particles of the plasma. In the case of $q < 1$ (superextensivity), both the damped and growing unstable oscillations have been predicted in the plasma, while in the case of $q > 1$ (subextensivity) the Langmuir oscillations have only damping and no growth. The mechanism which leads to the damping is the same as that developed by Landau [47], arising from a decreasing distribution function with velocity, but the growing unstable oscillations are somewhat unexpected and its mechanism lies in the heart of the nonextensivity. We have postulated that the concerned instability can be associated with the presence of numerous superthermal particles (in the case $q < 1$), which may give energy to the wave in the resonance process and results in growing oscillations in time. This instability disappears in the case of $q > 1$, describing a plasma with a great number of low-speed particles. Additionally, the damping rate in the case $q > 1$ is smaller than in the case of $q < 1$, because of the difference in the number of the particles participating in the resonance with the wave.

We emphasize that in the present work, we have considered an inhomogeneous plasma in a nonequilibrium thermal state by considering the q-nonextensive distribution for the stationary state of the plasma. Therefore, it is reasonable that our results should differ from those of a homogeneous and equilibrium EP plasma, where the growing oscillations are not predicted [14].

Nevertheless, we have noted that our resultant solutions at the extensive limit $q \to 1$ are consistent with the ones for a homogeneous and equilibrium EP plasma in the BG statistics. In fact, the properties of the Langmuir oscillations derived here are suitable for plasmas in a nonequilibrium stationary state with inhomogeneous temperature which contain many superthermal or low-speed particles. We hope that this study would be useful for the explanation of the typical modes in a pure EP plasma (or pure pair-ion plasma) which are out of the scope of the BG statistics.

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