Reduction of compaction force in a confined bidisperse granular media

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(Received 21 December 2012; published 31 May 2013)

Experiments and simulations of the compaction force in a confined binary granular mixture column were conducted. We measured the resistance force encountered by a piston pushing a vertical granular mixture in this confined arrangement. Granular mixtures with two different particle sizes were considered; the size ratio and the size fraction were both varied. An important decrease of the compaction force was found for volumetric fractions between 15% and 40% and size ratios larger than 3. By conducting some supplementary discrete element simulations, we found that the force chain network is fractured and redistributed when small particles are present. Hence we argue that the reduction of compaction force results from the redistribution of force within the granular column.

DOI: 10.1103/PhysRevE.87.052210 PACS number(s): 45.70.Ht, 45.70.Cc

I. INTRODUCTION

Granular flows are relevant because of their ubiquitous presence in nature and industry, such as pharmaceuticals, food, mining, and construction [1,2]. Although there are many applications where the flow of granular media is involved, most problems are still solved based on empirical knowledge. In order to have a better understanding of granular materials behavior, many theoretical and experimental studies have been conducted. The main obstacle to establishing a theoretical description of granular materials is its complex behavior, which depends on local flow conditions, packing, surface roughness, temperature, and even humidity [3]. A container filled with a granular medium is a common problem. Here the presence of boundaries is a natural initiator of a pressure screening effect discovered by Janssen [4]. Since then, many experimental works have studied this phenomenon [5–9].

An important aspect in granular media, directly related to the Janssen effect, is the formation of force chains [10]. Through these force networks, the stress within a granular material can propagate across distances much larger than the typical grain size. Many experimental, numerical, and theoretical studies, or a combination of them, have been conducted to characterize the behavior of granular media. Particulary, the force chain phenomenon is an important aspect in granular media compaction; this effect has been investigated by theoretical and analytical methods [11], and these results have been compared with the experimental images of a monodisperse birefringent granular medium obtained by Clément *et al*. [12]. Similar results have been found by Majmudar and Behringer [13]; they carried out measurements of contact force to visualize internal stresses in a bidisperse granular medium. In order to understand the formation and evolution of force chains, the discrete element method (DEM) has been used. Particularly, Peters *et al*. [10] used a method to identify force chains based on two assumptions: first, the force chain is a quasilinear arrangement of three or more particles, and second, stress concentration within each grain is characterized by the vector delineating the most compressive principal stress. Under these assumptions, realistic force chains were detected which corresponded to strong contact force pathways.

Charalampidou *et al*. [14] conducted comparisons between experiments and DEM simulations to study the stress-strain relation for granular materials. In our study we will use a similar approach to understand the compaction force in confined columns. The study of compaction forces in granular columns has been addressed by others [3,6,15]. All studies, with the exception of Charalampidou *et al*. [14], have considered monodisperse grain sizes. A few studies have considered the effect of not having a single grain size in a variety of granular flows, such as avalanches [16], chute flows [17], and simple shear [18]; in all cases, the behavior of the granular flow is modified significantly by the presence of a second size species. In particular, the study of Alam and Luding [18] shares some similarities with the present investigation despite the fact that they do not consider the same regime. They found that both pressure and viscosity increase or decrease with increasing size ratio, depending on whether the particles are of the same mass or density, respectively. They found, in other words, that the mixture become more or less resistant to the shearing motion with the addition of a small amount of another particle species, compared with its equivalent monodisperse counterpart. In our experiments and simulations we found less resistance to vertical motion against gravity for bidisperse mixtures of the same density, compared with columns of the same mass but with only one particle size. It is important to note that our study is for dense granular matter, while that of Alam and Luding [18] is for dilute-collisional shear flows.

There are some studies that have investigated the concept of granular lubrication [19–21], but in different contexts to that considered here. For instance, [19] used a thermoelasto-hydrodynamic lubrication theory to explain the influence of a of solid lubricant dispersed in liquid-based lubricant; [20,21] studied the lubrication of granular flows produced by the collisional nature of agitated dilute granular flows. To our knowledge, studies that consider bidisperse granular columns under compaction and the resulting reduction of compaction force have not been reported to date. Our interest, which is the main motivation of this study, is to find a situation for which

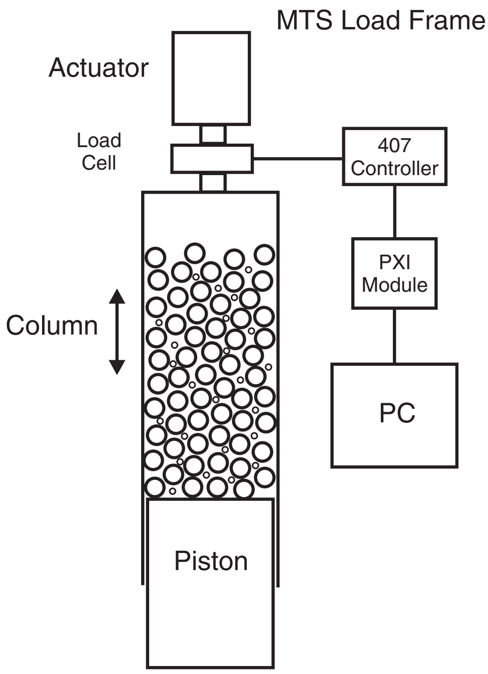

FIG. 1. Experimental setup.

the compaction force in a granular column can be reduced. Such a reduction could have important practical implications.

II. EXPERIMENTAL SETUP

The granular medium, which consisted of mixtures of glass beads of two distinct mean sizes, was placed inside a cylindrical glass container of diameter $D = 50.8$ mm. A piston with a diameter $D_p = 50.3$ mm was inserted through the bottom end of the column, which was held fixed at the base of the test instrument, as depicted in Fig. 1. The experiments were carried out in a MTS load frame Minibionix 858 and its 407 MTS controller. The typical load cell used in this equipment (25 kN) was replaced by a 2.2 kN interface load cell to improve accuracy. The glass cylinder was connected to the load cell and to the displacement sensor. The displacement signal was acquired from the LVDT sensor of the MTS machine. The force sensor is located between the actuator and the cylinder. Hence the compaction force measures the force on the side wall of the container as it moves downwards, compacting the granular column. Note that there is a clearance of 0.5 mm between the cylinder and the piston to avoid friction between them; hence only the contribution of compaction force is measured. In order to register the force and displacement signals, a National Instruments PXI chassis and a NI-PXI 6285 board were used. The acquisition rate was 10 000 samples/s, registered by a programmed LABVIEW virtual instrument and a PC.

The cylinder velocity v_c was set to be 1 mm/s for all experiments; some experiments were conducted at lower velocities, and the results did not appear to be affected by this variable. For the granular mixtures, we studied three cases with different size ratio s, defined by

$$s = \frac{d_l}{d_s}, \tag{1}$$

where d_l and d_s correspond to the diameters of the large and small species, respectively. The three cases are (A) $s = 3$ with 3 and 1 mm particles, (B) $s = 6$ with 6 and 1 mm particles, and (C) $s = 2$ with 6 and 3 mm particles. In all cases the volumetric fraction of small grains Vf_{ds} was varied from 0% to 100% with 5% increments. The volumetric fraction of small particles is defined by

$$Vf_{ds} = \frac{V_{ds}}{V_{dl} + V_{ds}}, \tag{2}$$

where V_{dl} and V_{ds} are the volumes of the large and small species and $V_{dl} + V_{ds}$ corresponds to the total volume of the granular mixture in the column. For all cases the height of the granular medium was kept constant ($H = 200$ mm). One experimental cycle consists of a compression period during which the cylinder and the granular column move downward at constant velocity; here all measurements are obtained. Then, the experiment stops, and the cylinder and the column of granular material return to their initial position. This return motion helps us avoid the progressive compaction of the column; the experiment goes back approximately to its initial conditions. All experiments were repeated three times. All data were postprocessed to obtain the force as a function of displacement.

III. NUMERICAL SIMULATION

Numerical simulations were conducted to try to understand experimental results. The simulations were performed using LIGGGTHS, an open source software [22] based on the DEM, commonly used to simulate flows of discrete systems such as granular materials [23]. The algorithm numerically integrates Newton's equations of motion for each grain of the system according to prescribed interactions described as elastic and viscous forces in the normal and tangential direction. Grains interact only at contact, allowing a small relative overlap $\xi_{ij}^n = R_i + R_j - r_{ij} > 0$, where R_k is the radius of the kth grain, from which the magnitude of the normal force is obtained as

$$f_{ij}^n = k_n \xi_{ij}^n - \gamma_n v_{ij}^n, \tag{3}$$

while the tangential force is

$$f_{ij}^t = k_t \xi_{ij}^t - \gamma_t v_{ij}^t, \tag{4}$$

where v_n (v_t) are the relative normal (tangential) velocities of the interacting grains. Figure 2 shows a schematic diagram of this spring-dashpot model for the interaction. Friction is implemented via a Coulomb type model characterized by a

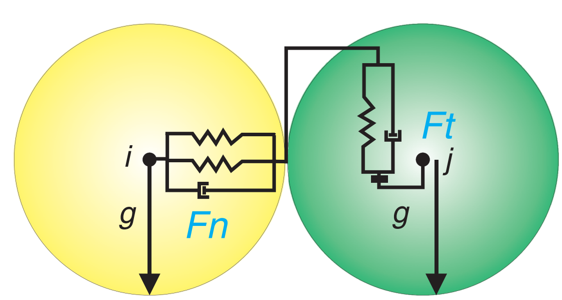

FIG. 2. (Color online) Schematic of the interaction model [22].

friction coefficient μ such that

$$f_{ij}^t \leqslant \mu f_{ij}^n;$$

that is, the tangential force grows according to model (4) until $f_{ij}^t/f_{ij}^n = \mu$, and then it is maintained at $f_{ij}^t = \mu f_{ij}^n$ until the grains lose contact. The interaction with the walls is the same spring-dashpot model, considering the walls as grains of an infinite radius.

The elastic and viscous coefficients k_n (k_t), γ_n (γ_t) of the interaction model given by Eqs. (3) and (4) are related to material properties [22] that characterize the grains. For this work values are chosen that correspond to laboratory glass beads with a density of $\rho = 2500$ kg/m^3 and a coefficient of restitution $e = 0.25$. The columns are prepared by randomly pouring two-dimensional (2D) grain disks with diameters of 3 and 0.75 mm into a rectangular region 50 mm wide and 200 mm high with the desired volume fraction of small and large grains. At $t = 0$ the bottom wall starts moving upwards with a velocity of 1 mm/s, and the average force on it is measured until $t = 50$ s.

IV. RESULTS

Figure 3 shows the experimental results for mixtures A, B, and C for all concentrations of small particles. In all cases, a rapid transient is observed: the compaction force rapidly increases. After this period, a mean force is attained for all displacements. These curves are similar in shape to those found for monodisperse granular media which have been previously reported in the literature [3,6,7,15]. The obtained curves in our work registered the fluctuations of the pushing force as a function of displacement in accordance with the behavior reported by Kolb *et al.* [15]. These fluctuations in force correspond to a succession of stick-slip events.

Figure 3(a) shows the case for which the size ratio is equal to 2 (mixture A). For most concentrations of small particles, the force reaches a mean value after a few millimeters of displacement. The value of the mean force is dependent on the number of small particles. For the case in which no small particles are present, the evolution of the compaction force is slower than the other cases. The increase of force with displacement occurs over larger distances, and a clear asymptotic state is not reached. This behavior is consistent with that observed by Kolb *et al.* [15]. For the case of mixture B, shown in Fig. 3(b) ($s = 3$), all tests show a clear asymptotic value of the force after a certain displacement. As in the previous case, the force fluctuates slightly around a mean value. Also in this case, the mean force changes with the number of small particles in the mixture. Finally, Fig. 3(c) shows the tests conducted for mixture C, for which the size ratio is 6. In this case, the value of the force continues to slightly increase for all displacements. More importantly, we find that the mean force reaches a minimum value for a certain number of small particles.

To quantify the reduction of compaction force phenomena effectively, all curves were processed to obtain an average compaction force discarding the initial transient period. The average force was calculated with the acquired force values that registered between 5 and 20 mm of displacement (about 3 times the grain size) for each experiment. The average

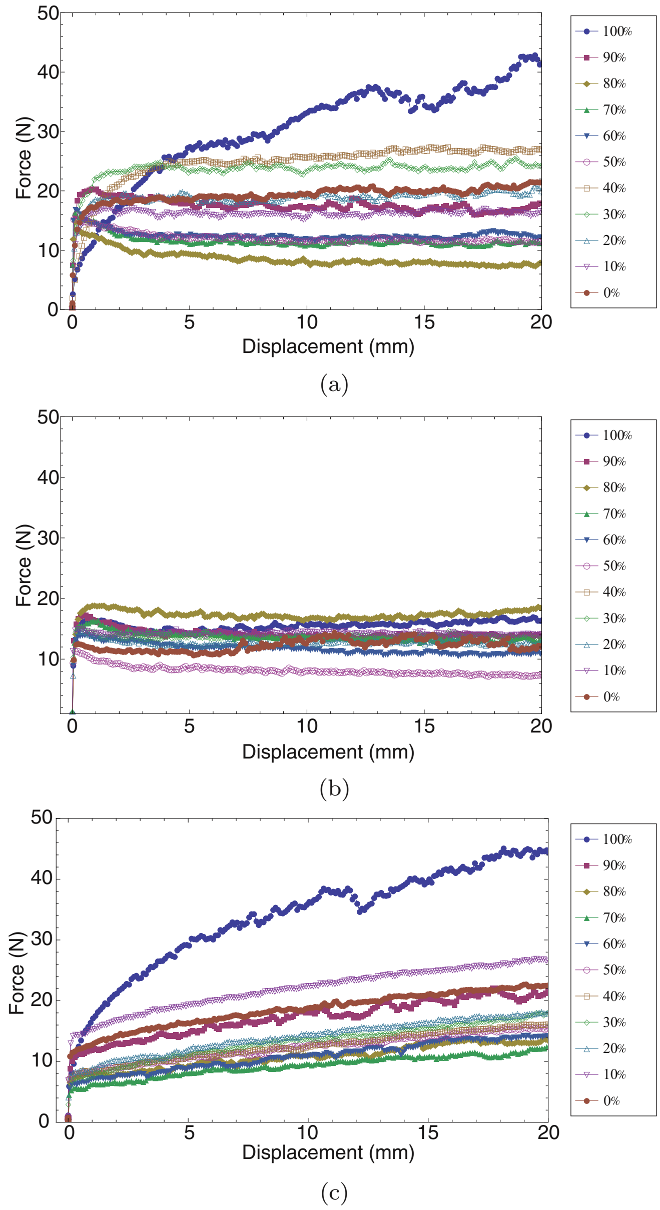

FIG. 3. (Color online) Force as a function of displacement curves for different mixtures of granular media. (a) Particle size ratio $s = 2$. The diameter of the lager particle $d_l = 6$ mm. (b) Particle size ratio $s = 3$. The diameter of the lager particle $d_l = 3$ mm. (c) Particle size ratio $s = 6$. The diameter of the lager particle $d_l = 6$ mm.

compaction force as a function of the volumetric fraction of small particles is presented in Fig. 4(a). The percentage of reduction of compaction force (RCF) is defined by

$$\mathrm{RCF} = \left(1 - \frac{f_{\text{mixture}}}{f_{\text{monodisperse}}}\right) \times 100, \qquad (5)$$

where f_{mixture} is the minimum required force to push the granular mixture ($V_{dl} + V_{ds}$) through the column against gravity force and $f_{\text{monodisperse}}$ is the force required to push just large diameter species under the same condition. The reduction of the compaction force effect as a function of particle size ratio can be observed in Fig. 4(b). Clearly, there is a strong reduction of the compaction force effect in our experimental results. In

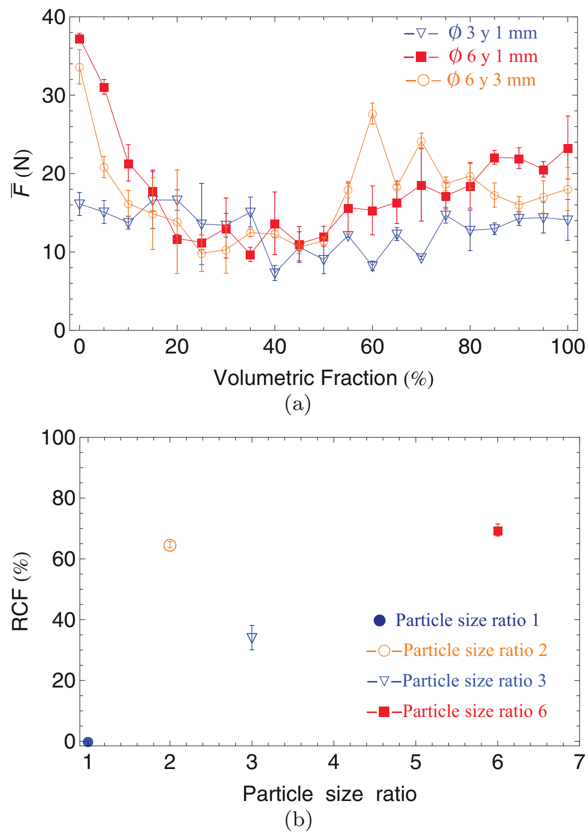

FIG. 4. (Color online) Evidence of reduction of compaction force as a function of particle size and volumetric fraction in a bidisperse granular medium under compaction force. (a) Average compaction force as a function of volumetric fraction of small particles and (b) reduction of compaction force percentage as a function of particle size ratio.

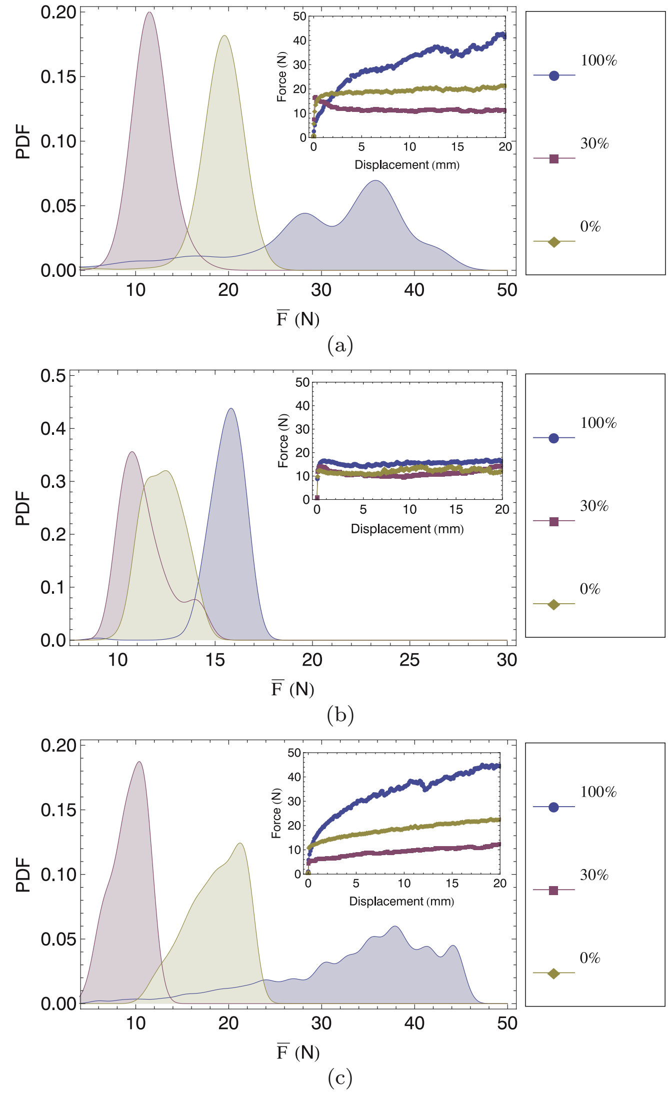

FIG. 5. (Color online) Histograms of compaction force behavior for different values of s and three values of Vf_{ds} with force vs displacement curves shown in the insets. (a) PDF vs compaction force for $s = 2$; the minimum required force to push the granular media was around 10 N for $Vf_{ds} = 30\%$, while 100% of large grains $d_l = 6$ mm needed a force ranging between 25 and 40 N. (b) PDF vs compaction force for $s = 3$; in this case the minimum required force was around 10 N for $Vf_{ds} = 30\%$, but the required force to push pure large grains $d_l = 3$ mm was narrowed to 15 N, showing less dispersion than the largest grains. (c) PDF vs compaction force for $s = 6$, the minimum required force to push the granular media was around 10 N for $Vf_{ds} = 30\%$, while 100% of large grains $d_l = 6$ mm needed a force ranging between 30 and 45 N.

all cases the maximum reduction of compaction force occurred between 15% and 45% of the volumetric fraction for mixtures A, B, and C. In all cases the average force was reduced up to 4 times with respect to the force required to compress the column made of only large grains. A reduction of 2 times was observed when comparing with the compaction force of only small grains.

To further analyze the compaction force behavior, three cases were selected for each mixture: a comparison of the force distribution among the cases was conducted for the columns made of grains of only one size (with large and small) and that corresponding to the mixture for which the minimum compaction force was attained. The probability density function (PDF) of the force signal was calculated for each case, shown in Fig. 5. The insets show the cases analyzed. These curves suggest that reduction of the compaction force effect is a result of a process of redistribution of forces. When the column consists only of large grains (in particular for the cases when $d_l = 6$ mm), the force is distributed over a wide range of values. In contrast, when the column is formed only by small grains, the force distribution is narrow and symmetrical. When the granular column is composed by a mixture of sizes, the mean value of the force is reduced, as shown in Fig. 4(a), and the distribution is narrow. A similar behavior can be observed for the three cases shown in Fig. 5. The dots always correspond to 100% of the largest grains, and the diamonds correspond to 100% of the smallest grains; furthermore the squares represent the maximum lubrication state. For all cases illustrated in Figs. 5(a)–5(c) the mixture of particles for 30% in volumetric fraction showed a decrease in the compaction force.

We have clearly observed a reduction of the compaction force phenomenon in a bidisperse granular medium; however, the experiments alone do not explain why this occurs. To answer this question, numerical simulations were conducted.

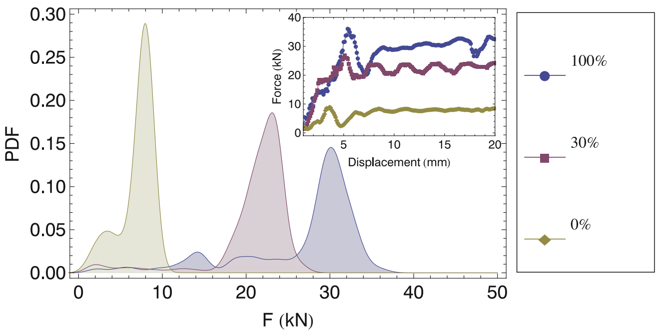

FIG. 6. (Color online) Histograms of compaction force behavior calculated from simulation data for different volumetric fractions with $s = 1$. The inset shows the force vs displacement curves from which we calculated the PDFs.

From these simulations it was possible to obtain curves equivalent to those obtained experimentally. We found the same qualitative behavior as in the experiments: there is an intermediate concentration of small particles for which the mean compaction force is significantly reduced with respect to that obtained for only one size of particles. However, the comparison is only qualitative because the numerical simulation is in a 2D arrangement (disks). Nevertheless, the simulation results can be used to visualize the force among particles inside the column. It was possible to visualize the force chain distribution in the column. In addition, it was possible to obtain the curves of force as a function of displacement from the numerical simulations; the results were qualitatively similar (shown in Fig. 6). However, the compaction force did not increase again for the case when the column is composed only by small particles. We attribute this change of behavior to the fact that the simulation is 2D. The case for only small grains required less force than the others. Despite this different behavior, the same lubrication effect was present when the volumetric fraction of small grains was increased.

From simulation data, it was also possible to calculate and to draw the force chains for several cases. The force chains are obtained between pairwise particle contacts and displayed according to the magnitude. The comparison between cases is shown in Fig. 7(a). It can be observed that, in the steady state, the force chains for the column filled only with large grains traverse the whole width of the column; i.e., a single force chain can extend from one wall to the other. As a result the force needed to push the grains upwards is increased. When small particles are added to the mixture, the number, length, and strength of the force chains are significantly modified, as shown in Figs. 7(b) to 7(d).

The force chains are disrupted; the small particles redistribute the force. This effect can be observed in Fig. 7(e), where the total number of contact points decays exponentially while the force magnitude increases. When small grains were added, the number of large particles in contact was reduced; nevertheless, the total number of contact points of small-small, small-large, and large-large particles increased significantly. As a result of this, the number of force chains and their magnitude were modified. Clearly, the number of force chains increased, but their magnitudes were reduced. Note that only

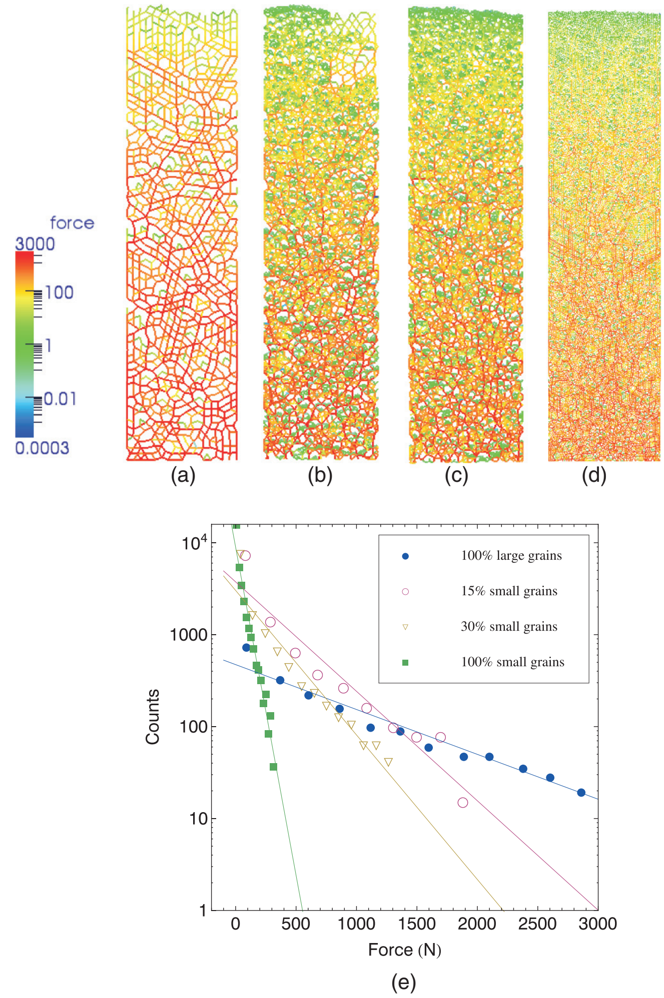

FIG. 7. (Color online) Force chain distribution in a bidisperse granular medium: Chain network in the asymptotic steady state for a column (a) filled entirely of large grains, (b) filled 15% with small grains, (c) filled 30% with small grains, and (d) filled with 100% small grains. (e) Distribution of force magnitude, showing an exponential decay when small particles are added. The lines are the best fits of data.

a few of the remaining force chains presented an increment in magnitude for Figs. 7(b)–7(d). Since the grains move slightly inside the cylinder, the distribution of force magnitudes changed gradually; with the increase of total number of contact points between particles and the redistribution of force magnitudes, some of the force chains were disrupted, producing a lubrication effect.

We can argue that the same force is distributed over several contact points, each one supporting forces of smaller magnitude. Therefore the net horizontal force over the wall is smaller, resulting in an effective lubrication. The probability of jamming and its relation to force chains has been studied by several authors [10,24–26]. We can argue that the probability a force chain will jam decreases when small particles are present, considering the same confinement, since there are many nonjammed configurations that the grains may choose. However, the structure of the chains and jamming has not been studied in any detail for the case of bidispersed granular mixtures. Clearly, this analysis deserves more in depth study. We intend to pursue this study in a future work.

V. CONCLUSIONS

Experiments were conducted to study the compaction properties of bidisperse granular media. We found that the presence of a second species of particles significantly reduced the compaction force needed to displace the containing walls of the column. For all cases, the compaction force reached a minimum for a certain volumetric fraction of small particles of about 35%. For the parameters explored in this study, the largest size ratio (6:1) resulted in the largest force reduction. With the aid of numerical simulations, we attributed this effect to the redistribution of force chains within the granular medium. To our knowledge such an effect has not been reported in the literature, at least for the case of column compaction. These results could have important practical applications.

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