PHYSICAL REVIEW E **87**, 042403 (2013)

Measurements of liquid surface fluctuations at sub-shot-noise levels with Michelson interferometry

Takahisa Mitsui and Kenichiro Aoki

Research and Education Center for Natural Sciences and Dept. of Physics, Hiyoshi, Keio University, Yokohama 223–8521, Japan

(Received 12 June 2012; published 10 April 2013)

Surface fluctuation spectra of liquids are measured to unprecedented precision, down to 3 orders of magnitude below the shot-noise level using averaged correlations of interferometry measurements. This allows us to investigate the limits in our current theoretical understanding of these phenomena. The spectrum derived from hydrodynamical considerations agrees well with the observed results for water. However, for oil, deviations are seen at high frequencies ($\gtrsim$1 MHz), perhaps indicating its more complex underlying physics. The measurements are made possible by dualizing the Michelson interferometry measurements and employing the averaged correlation of signals, in which the shot noise is statistically reduced. The method works in the presence of the quantum noise of a coherent state of light. The optical part of the experimental setup is essentially the same as that of Michelson interferometry so that the method can be applied when Michelson interferometry can be used. Furthermore, the measurement method requires a relatively low light power and a short time so that it has a wide range of applicability.

DOI: 10.1103/PhysRevE.87.042403 PACS number(s): 68.03.Kn, 05.40.−a, 66.20.−d, 07.60.Ly

Interferometry is a most precise tool for detecting small displacements and hence is used in a very broad range of areas in physics, from microscopic spectroscopy such as ours, structural measurements of optical elements, to astrophysics [1], and arguably the most sensitive measurement, attempts to detect gravitational waves [2,3]. Furthermore, light interferometry can be noninvasive, making it suitable for delicate applications, including biology and medicine [4]. An essential obstacle to achieving precision in interferometry is extraneous noise that inevitably occurs in any measurement. Various types of noise occur but in particular, shot noise, whose origin is quantum, arises in any measurement involving photoelectric conversion. The spectrum of this noise in the power spectrum of the photodetector (PD) current, $I_{\rm PD}$, is $2eI_{\rm PD}$ and is frequency independent, where e is the electron charge [5]. There is a widespread belief that this shot noise limits the signal-to-noise ratio in light interferometry measurements of signals with no known periodicity, without the use of sub-Poissonian photon statistics [2,3,5]. Using squeezed light sources, a reduction of the shot noise by a factor of 2 has been achieved recently and a light source squeezing factor close to 20 has been attained [6] for a fixed measurement time. We show both theoretically and experimentally that the averaged correlations of the standard Michelson interferometer measurements can be used to improve the signal-to-noise ratio, even in the presence of shot noise associated with the quantum nature of light.

In this work, we measure surface thermal fluctuation spectra of simple liquids over a wide frequency range (1 kHz ∼ 40 MHz) and down to a few orders of magnitude below the shot-noise level by averaging the correlation of interferometry measurements to reduce the noise statistically. The properties of these interesting fundamental physics phenomena are thought to be understood analytically [7–9]. Our motivation is to investigate the limits of our current theoretical understanding by studying the surface fluctuation spectra with previously unachieved precision. We find that the traditional hydrodynamical description of these fluctuations works well for water over a wide frequency range and dynamic range ($\sim 10^8$ relatively), but small deviations from the theory are clearly observed for oil. Another important motivation in this work is to put the principle and the realization of our experimental scheme to a stringent test. In particular, the principle makes use of the quantum aspects of photons crucially and we would like to concretely test these notions. Surface thermal fluctuations have been measured with interferometry previously on mirrors using high-power lasers [10] and on liquid drop surfaces attached to fiber tips [11]. Spectral properties of surface waves on simple liquids have also been measured by using them as gratings [8,9]. However, this approach is difficult to apply to dissipative liquids such as oil, since they do not create well-defined waves that act effectively as gratings. Spectra of surface inclination fluctuations have been measured [12–15] using the surface as an optical lever. Also, exceptionally large surface fluctuations due to low surface tension have been observed using other methods [16].

Let us outline our experimental setup: Light is shone on a sample surface and the standard Michelson interferometry setup is used to obtain the light signal. This signal is then split in two and then fed into two detectors. We obtain the measurement as an averaged correlation of these two detector currents. In addition to the traditional Michelson interferometry, our method relies on the following two crucial elements to achieve sub-shot-noise level measurements—the noise reduction scheme and the independence of the shot noise in the two measurements, which are both explained below.

We briefly summarize the principle behind the noise reduction we perform [14,15]. This is not limited to optical measurements nor to shot noise. Given a single noisy measurement D_1 of a signal S with no known periodicity, $D_1 = S + N_1$, there is no way, even in principle, to distinguish the signal S from the noise N_1. Shot noise is a typical example of such noise. To overcome this, we perform an additional measurement $D_2 = S + N_2$ of the same signal, whose noise N_2 is not correlated to N_1. Then, by taking the correlation of $D_{1,2}$ and averaging over measurements, we obtain

$$\langle \overline{\tilde{D}_1} \tilde{D}_2 \rangle \longrightarrow \langle |\tilde{S}|^2 \rangle \qquad (\mathcal{N} \to \infty), \tag{1}$$

where $\mathcal{N}$ is the number of averagings and tildes denote Fourier transforms [14,15]. While the principle is simple, it

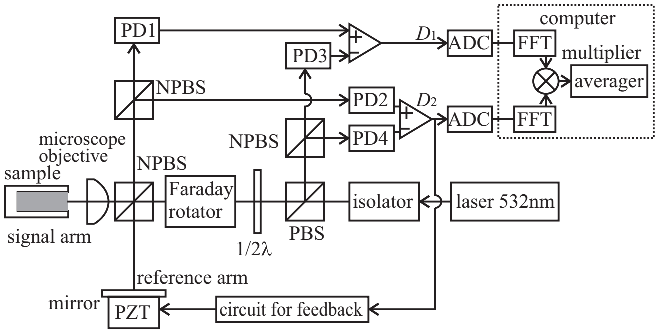

FIG. 1. Experimental setup: Two signals $D_{1,2}$ are obtained to be used in the averaged correlation equation (1). A Faraday rotator is used for separating out the light reflected back from the sample at the polarizing beam splitter (PBS). NPBS: nonpolarizing beam splitter. FFT: fast Fourier transformation.

is necessary to arrange multiple measurements of the same signal in such a way as to ensure that the noises in them are uncorrelated for this to work.

Our full experimental setup is shown in Fig. 1. Laser light from a solid-state diode-pump laser (Sapphire SF-532-50, Coherent, wavelength $\lambda = 532$ nm) is shone on the sample and on the reference mirror, each with a power 500 μW. The light from the sample and the reference are split and sent to PDs (S5973 Hamamatsu Photonics, Japan), whose signal currents are digitized by two analog-to-digital converters (ADCs, 8 bit, 125 Ms/s, PicoScope 5203, Pico Technology). The correlation equations (1) of these two digitized signals are then obtained and averaged, as explained in Fig. 2(a). A pair of PDs each is used for signals $D_{1,2}$ to implement differential detection [Fig. 2(b)]. This removes the light source amplitude modulation (AM) noise, which is common to the two signals and cannot be eliminated by taking their averaged correlation. To obtain maximal sensitivity in detecting sample surface displacements, the reference arm length is adjusted dynamically using a piezoelectric transducer (PZT) with feedback.

The shot noise in the spectrum can be derived from the phase noise $\langle \Delta\varphi^2 \rangle = 2e/I_{\rm PD}$ [2]. When the sensitivity in the interferometer is maximal, it is

$$S_{\rm SN}(f) = e\frac{\lambda^2}{32\pi^2}\frac{1+r^2}{r I_{\rm PD}}. \tag{2}$$

Here, r is the ratio of the reflectivities of the sample surface and the reference mirror. The quantum property of light plays a critical role in our experiment; the beam splitter randomly partitions the photons from the single signal beam to one of the two detection systems [Fig. 2(a)]. The photons giving rise to the shot noise are random when a coherent light source is used. Therefore the noise in the two detectors $N_{1,2}$ is uncorrelated and is averaged out when we compute their averaged correlation [Eq. (1)]. It is important to note that splitting the beam by itself does not guarantee the independence of the noise in the two measurements. Had we used a squeezed light source, for instance, the photons in the two measurements would have been correlated so that the noise could not have been eliminated by using the correlation of detector measurements. Therefore our approach is in contrast to those that use squeezed light sources to obtain sub-shot-noise measurements [3,17,18].

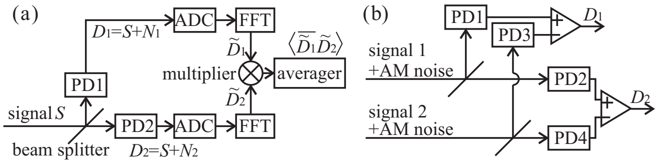

FIG. 2. Essential parts of the implementation. (a) The correlation of the two measurements $D_{1,2}$ are taken and the shot noise $N_{1,2}$ averages out in the signal. (b) To eliminate AM noise that inevitably occurs, we additionally use the differential detection of random signals 1 and 2 by adding PD 3 and 4. Signals 1 and 2 can be phase inverted or uncorrelated.

In another approach, two light sources with different wavelengths and two sets of detectors can also be used to obtain two independent measurements whose averaged correlation can be used to extract signals at sub-shot-noise levels. This was applied to inclination fluctuation spectra of surfaces previously [14,15]. In the current work, we use a single light source in the optical setup of the standard Michelson interferometer and just dualize the photodetection part of the setup to attain sub-shot-noise measurements. This leads to a simple effective setup. It also has more precision, which is important to our current work. Using a single light source is not only simpler but has an important practical advantage: to achieve maximal sensitivity in the interferometer, the path lengths need to be adjusted according to the wavelength of the light source, which is difficult to attain simultaneously for multiple wavelengths.

The spectral function of thermal surface fluctuations for a simple liquid is determined from hydrodynamical considerations by its density ρ, surface tension σ, and viscosity η and is

$$P(k,\omega) = \frac{k_{\rm B}T}{\pi}\frac{ku^2}{\rho\omega^3}\,{\rm Im}[(1-iu)^2 + y - \sqrt{1-2iu}]^{-1}, \tag{3}$$

where $u \equiv \rho\omega/(2\eta k^2), y \equiv \rho\sigma/(4\eta^2 k)$ [19]. Here, k,ω are the wave number and the angular frequency of the surface wave. Through Michelson interferometry, we measure fluctuations perpendicular to the liquid surface and its spectrum can be computed analogously to the inclination fluctuation spectra [14,15] as

$$S_{\rm h}(f) = 2\int_0^\infty dk\, k\, e^{-b^2k^2/8} P(k,2\pi f). \tag{4}$$

Here, $f = \omega/(2\pi)$ and b is the beam diameter. Surface waves with wavelengths larger than the sample size are cut off so that the effects of gravity can be ignored. Also, surface waves with wavelengths smaller than the beam spot size are suppressed due to averaging. In the measurements below, the size of the surface sample is 2.2 mm in diameter and $b = 0.96$ μm.

We compare the measured spectra against their theoretical predictions [Eq. (4)] for water and ethanol in Fig. 3. The physical properties of water and ethanol are known for a given temperature and are $(\rho\,[{\rm kg/m^3}],\sigma\,[{\rm kg/s^2}],\eta\,[{\rm kg/(m\,s)}],T[{\rm K}]) = (1.0\times10^3, 7.5\times10^{-2}, 1.7\times10^{-3}, 275)$, $(0.80\times10^3, 2.3\times10^{-2}, 1.7\times10^{-3}, 278)$, respectively. Given the temperature, these properties and the spectral function $P(k,\omega)$ are completely determined. The integrated spectrum $S_{\rm h}(f)$ [Eq. (4)] is uniquely determined by these properties and the beam size.

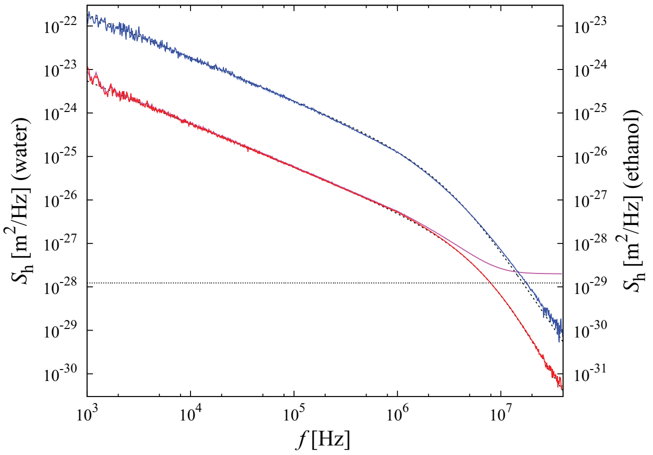

FIG. 3. (Color online) Experimentally observed surface height fluctuation spectra for water (red, lower) and ethanol (blue, upper). Respective theoretical spectra are also shown (black, dashed), which agree with the experimental results and are almost invisible. For comparison, observed data for a single differential detection $\langle|\tilde{D}_1|^2\rangle$ without using the correlation equation (1) are shown for water (magenta). This deviates from the correlation measurement in the region $f \gtrsim 10^6$ and is clearly dominated at higher frequencies by the shot noise, whose theoretical value [Eq. (2)] is also indicated (black, dotted).

For water surface fluctuations, the theoretical formula derived from hydrodynamic considerations agree almost perfectly with the observed spectrum. These measurements confirm the theoretical spectrum over a wide frequency range. For ethanol, good agreement is seen, except for the slightly slower falloff at frequencies above 10 MHz. We come back to this issue for the case of oil surface fluctuations.

Figure 3 contains a single differential measurement $\langle|\tilde{D}_1|^2\rangle$ of water surface fluctuations in which the shot noise dominates at higher frequencies. The observed shot noise has the expected frequency-independent behavior but its magnitude is roughly twice its theoretical value [Eq. (2)]. This indicates that the interferometer sensitivity is not at its theoretical maximum, whose most likely cause is the aberration of the objective lens. As can be seen in Figs. 3 and 4, this shot noise is essentially eliminated in the averaged correlation of the two differential measurements. There is a slight jittery behavior in the spectrum for $f \gtrsim 20$ MHz. This is due to the residual effect of the shot noise in the correlation Eq. (1), which has been reduced by a factor of 10^{-3}, as explained below. Since the noise is random, the contribution fluctuates around zero and adds to the measured spectrum.

Oil (Olympus immersion oil AX9602) surface thermal fluctuation spectra were obtained at various temperatures and compared to the theoretical spectra equation (4) in Fig. 4. There is a qualitative difference from the spectra of water and ethanol surface fluctuation spectra in Fig. 3 due to the high viscosity of oil. In particular, the spectral density decays much more slowly than that of water and oil at higher frequencies. For the oil, the temperature dependencies of σ,η are not known so that they were deduced from the spectra to be $(\rho\,[\mathrm{kg/m^3}],\sigma\,[\mathrm{kg/s^2}],\eta\,[\mathrm{kg/(m\,s)}],T\,[\mathrm{K}]) = (0.92\times10^3, 3.2\times10^{-2}, 0.15, 294)$, $(0.88\times10^3, 2.7\times10^{-2}, 0.025, 347)$. From the spectra, σ, η are determined to within 10%. For oil, we see that the fluctuation spectrum changes with the temperature, as it should. Most of this change is caused by the decrease in the viscosity with higher temperatures, making the fluctuations larger.

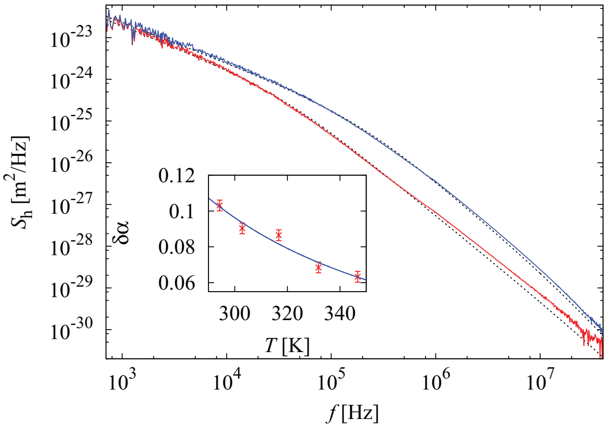

FIG. 4. (Color online) Experimentally observed surface fluctuation spectra for oil at $T = 294$ K (red) and 347 K (blue). Fluctuations are larger at higher temperatures. Their respective theoretical spectra [Eq. (4)] are also shown (black, dashed) and are slightly smaller than their corresponding experimental results at higher frequencies. (Inset) Temperature dependence of the deviation from theory of the falloff at high frequencies ($\gtrsim$1 MHz).

While the fluctuation spectra of liquids we study are usually regarded as well understood, the observed higher frequency ($\gtrsim$1 MHz) falloffs in the spectra for oil and ethanol are slightly slower than their theoretical predictions, especially at lower temperatures. It should be noted that these differences between the measurements and the theory do *not* occur for the water surface fluctuation spectrum, which has the smallest magnitude and was measured using the same setup. Furthermore, the differences appear in the region $f \lesssim 20$ MHz, where the residual effects of shot noise are insignificant. This slower falloff is perhaps due to the more complex nature of the liquids that cannot be explained just from the hydrodynamical considerations of simple liquids. The slower falloff of the spectrum has also been observed for solid materials and a gradual transition to such a dependence was seen for complex fluids [14]. We obtain the dependence $S_h(f) \sim f^\alpha$ for higher frequencies ($10^6 - 10^7$ Hz) and its deviation $\delta\alpha$ from the theoretical value in Eq. (4). In Fig. 4 (inset), the temperature dependence of $\delta\alpha$ is shown. The dependence can be reasonably well described by $\delta\alpha = C\exp(U/k_BT)$, with $U = 7\,\mathrm{kJ/mol}$. This energy scale U is comparable to the latent heat for oil, which is consistent with more complex behavior, such as molecular interactions causing visible effects in the spectrum.

Let us briefly describe the physical properties of the spectra equation (4). When $16\sqrt{2}\pi\eta^3 f/(\rho\sigma^2) \gtrsim 1$, the liquid can be regarded as being highly viscous so that any liquid is dissipative at high enough frequencies. In the frequency region we study, water and ethanol have low viscosity and oil has high viscosity. The spectra have qualitatively different f dependence for low and high viscosities, as explained below [14,15]. In both cases, $S_h(f) \sim k_BT/(\sigma f)$ at low frequencies. For a liquid with low viscosity, the spectrum crosses over at $f \sim \sqrt{\sigma/(\rho b^3)}$ to $S_h(f) \sim k_B T\eta/(\rho^2 b^5 f^4)$. For a liquid with high viscosity, the spectrum crosses over at $f \sim \sigma/(\eta b)$ to $S_h(f) \sim k_B T/(\eta b f^2)$. While outside the region of our measurements, at even higher frequencies, $f \gtrsim \eta/(\rho b^2)$, the spectrum changes to $S_h(f) \sim k_B T\eta/(\rho^2 b^5 f^4)$. For both low and high viscosities, the behavior of $S_h(f)$ is dominated by σ at low frequencies and is independent of η since the dynamical time scale is relatively large. At high frequencies, the behavior is governed by η and is independent of σ, to leading order.

In this work, we have proposed and implemented statistical noise reduction using the averaged correlation to achieve sub-shot-noise level measurements using Michelson interferometry. We applied the method to height fluctuations of the water liquid surface and found that the experimental results agreed well with the theory, down to few orders below the shot-noise level. For slightly more complex fluids, we find that the theory agrees well but with small deviations. We make multiple measurements of a single signal light, whose shot noise is independent due to its quantum nature. The noise can then be reduced statistically in the averaged correlation. While both random, the distinct difference between the thermal fluctuations and the shot noise, which are classical and quantum in their origins, allow for their separation. The measurement requires only a small light power, a relatively short time, and a small sample surface (500 μW, a few seconds, diameter 1 μm, in this work).

Let us discuss the efficacy of our noise reduction, including its limitations. How much the noise can be reduced depends on the properties of the signal to be measured. Our approach is effective when the number of averagings $\mathcal{N} = \Delta f T$ is large, where Δf is the frequency resolution in the spectrum and T is the total measurement time. The noise can then be reduced by the factor $1/\sqrt{\mathcal{N}}$. As a concrete example, let us take the water surface fluctuation measurement in Fig. 3. In this case, the total measurement time is 3.5 s and $\Delta f = 100$ kHz at $f = 10$ MHz, so that we achieve a statistical noise reduction factor of $1/\sqrt{\mathcal{N}} = 1.7 \times 10^{-3}$. Δf need not be the same across the spectrum, and if a same relative resolution is used, this statistical error is smaller at higher frequencies. Longer measurement times can lead to higher resolution, more averaging, and less statistical error. While the integration is effective for stationary signals, the situation is more subtle for transient signals, such as a single event. In such situations, the restriction that the measurement time needs to be within the time when the signal is present can be demanding. The duration of the signal needs to be long enough compared to the inverse of the desired frequency resolution to reduce the noise using the correlation of measurements. Roughly speaking, many periods of wave are necessary for averagings to reduce uncorrelated noise. Interferometry is perhaps the most commonly used method for precision measurements, and shot noise can often be the major limiting factor in its accuracy [2,3,5]. Our experimental setup is essentially the same as the standard Michelson interferometry, except that the photodetection is dualized so that the correlation of the signals can be averaged to reduce uncorrelated noise, such as shot noise. Therefore it can be applied to any situation to which Michelson interferometry can be applied. Our noise reduction can also be added to higher power measurements such as those in [10] to reduce the noise further.

K.A. was supported in part by a Grant-in-Aid for Scientific Research (No. 20540279) from the Ministry of Education, Culture, Sports, Science, and Technology of Japan.

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