# Diversity-induced resonance in the response to social norms

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In this paper we focus on diversity-induced resonance, which was recently found in bistable, excitable, and other physical systems. We study the appearance of this phenomenon in a purely economic model of cooperating and defecting agents. An agent's contribution to a public good is seen as a social norm, so defecting agents face a social pressure, which decreases if free riding becomes widespread. In this model, diversity among agents naturally appears because of the different sensitivities towards the social norm. We study the evolution of cooperation as a response to the social norm (i) for the replicator dynamics and (ii) for the logit dynamics by means of numerical simulations. Diversity-induced resonance is observed as a maximum in the response of agents to changes in the social norm as a function of the degree of heterogeneity in the population. We provide an analytical, mean-field approach for the logit dynamics and find very good agreement with the simulations. From a socioeconomic perspective, our results show that, counterintuitively, diversity in the individual sensitivity to social norms may result in a society that better follows such norms as a whole, even if part of the population is less prone to follow them.

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## I. INTRODUCTION

The ever-increasing interest by physicists to contribute to understanding collective phenomena in social systems [1] has mostly concentrated around highly stylized models, often directly borrowed from physics, using vague plausibility arguments to justify their social context [2]. In this paper, we follow a less common route, namely, to work with a model which is established in, and directly taken from, the social sciences. It studies the effect of social norms on the emergence of cooperation. We study its dynamics from the physical perspective of diversity-induced resonance to shed new light on sustainable cooperation in a society where some fractions do not adhere to support it.

In a system consisting of distinct and nonidentical elements, diversity-induced resonance can be defined as the appearance of a maximum response to an external signal, dependent on the degree of diversity. This phenomenon was first reported in Ref. [3], in the context of coupled bistable or excitable systems that are subject to a subthreshold signal. It was shown that there is an optimum level of the diversity (quenched noise) of the coupled units that maximizes the response to the signal. Subsequent works [4–13] showed that similar behavior can be observed in other physical systems, thus reinforcing the notion that this type of resonance can be a quite general phenomenon. In fact, diversity-induced resonance was also shown to appear in models related to sociophysics: It was found in discrete models of opinion formation [14], such as the Galam model [15] (related to the random-field Ising model at zero temperature [16]), and in continuous ones [17], of which the Deffuant model [18] is a paradigmatic example. In all cases, the average opinion synchronizes to external signals or influences when the diversity in the preferred opinions attains an optimum value. In a broader context, diversity-induced resonance can be generalized to other sources of *disorder* in the internal dynamics of the system constituents. Interestingly, even repulsive and evolving patterns of interactions can trigger a common collective behavior, be it synchronization [19,20], an amplification of an external signal [17,21], or a nonlinear increase in the volatility of the global dynamics [22]. In a social context, these repulsive interactions would represent contrarians, i.e., individuals that oppose any type of consensus [23,24] or that intend to destabilize the system itself, such as the joker-like players studied in the context of social dilemmas [25].

The research reported here generalizes diversity-induced resonance by demonstrating its appearance in a purely economic model of social norms and their effect on cooperation [26]. Instead of relying on a model rooted in physics, we study an established model from the economics literature in which diversity and external driving are introduced based on economic considerations. In this model, diversity appears naturally as an idiosyncratic propensity to follow a social norm. We demonstrate for this model that there is an optimal range of diversity, which leads the society to follow such norms as a whole. Different from the setup of diversity-induced resonance models usually studied in the physics literature, in this case diversity appears in a multiplicative manner, and its dynamics are given by approaches typical of evolutionary game theory.

This paper is organized as follows: Sec. II presents our model and its economic context. Section III summarizes our simulation results, obtained for two different types of evolutionary dynamics, to demonstrate the robustness of the observations. To better understand the origin of the collective dynamics, we present our findings for three levels of increasing modeling complexity, without and with diversity and with external forcing. Subsequently, Sec. IV A improves our understanding by means of an analytical approach for the stationary level of cooperation, and Sec. IV B investigates the response to the external signal. Finally, Sec. V summarizes our conclusions and discusses the implications of this work.

 

## II. MODEL

### A. Economic context

In this paper, we model *conditional cooperation*, a phenomenon observed in many human interactions. This term was introduced by Keser and van Winden [27] and Fischbacher and E. Fehr [28] to refer to the fact that people often condition their cooperation on the cooperativeness of others or on their beliefs about others' behavior. In the specific context of prisoner's dilemma [29,30] or public good games [31], this means that people are ready to contribute more to the common welfare if others contribute as well. Furthermore, this willingness increases with the number of contributors in the game. There is a large body of experimental evidence supporting the existence of this type of behavior [32], even in structured populations [33–35]. It is only consistent to ask (i) for a deeper theoretical understanding of these observations and (ii) what their consequences are for economic reasoning. The first question is partly answered by the theory of social preferences [36], which posits that nonmonetary contributions to the utility function arise from social considerations, such as, e.g., inequity aversion or reciprocity. It has been argued that social preferences arise through social norms, i.e., rules of conduct that are enforced by internal or external sanctions [37]. Explanations for the emergence and robustness of such norms in evolutionary terms have been advanced [26,38,39], thus closing the rationale to explain conditional cooperation in terms of social preferences.

In this paper, we focus on the issue of norms and on the consequences of having a diverse population of conditional cooperators interacting in a public good setup. Thus, we investigate how diversity influences the response to exogenous efforts to promote cooperation through social norms. Following Spichtig and Traxler [26], we consider that a norm against free riding is enforced (internally or externally). This is achieved by adding a contribution to the utility function such that free riding (i.e., not contributing to the public good while benefiting from it) is heavily punished when rare, but the punishment weakens as free riding becomes more abundant in the population. This norm leads to conditional cooperation because of more willingness to cooperate when the population is mostly cooperative, and the propensity to cooperate decreases if less participants cooperate.

In the above context, we address the following question: How does the behavior of the population change if the social norm responsible for establishing a conditionally cooperative strategy varies in time? This question is important for two reasons. First, social norms are known to change in time, endogenously or exogenously, in periodic or random manners [40,41]. Therefore, it is most important to understand how those changes affect the observed behavior in order to assess the stability of cooperative environments. Second, understanding the response of the population to changes in the current social norm can help policy makers to design incentives or new norms that lead to more cooperative outcomes. However, it should also be realized that the effort of steering the norms towards a preferred direction is costly, and at some point, the benefit of improving the behavior of the populations may be lower than that of continuing changing the norm. Therefore, assessing the optimum amount of effort invested in modifying a given norm is a very relevant issue. Finally, we will come to the issue of diversity-induced resonance by considering that the sensitivity to the social norm depends on the individual through a specific coefficient to be introduced in the utility function. In the following, we will show that these issues can be addressed and are related to the phenomenon of diversity-induced resonance in this system.

### B. Model definition

Let us now implement the ideas above in a well-defined model built on the original proposal by Spichtig and Traxler [26]. We consider a population of $N$ agents which can take one of two possible (opposite) actions $\sigma_i \in \{0,1\}$ for $i = 1 \cdots N$. We assume that "cooperative" agents take action $\sigma = 1$, in this way contributing to a public good, while "free riders" take action $\sigma = 0$ and do not contribute to the public good. Defining the density of cooperators as $n_c \equiv N_c/N$ and the density of free riders as $n_f \equiv 1 - n_c = N_f/N$, the utility (or payoff) function per agent is defined as

$$u_i(\sigma_i,\theta_i;n_c) \equiv -c\,\sigma_i + \frac{r}{N}\sum_{j=1}^{N}\sigma_j + (\sigma_i - 1)\,\theta_i s(n_c). \quad (1)$$

The first term in Eq. (1) represents the cost $c$ per agent for providing the public good, which applies only if agent $i$ is cooperative, $\sigma_i = 1$. The second term represents the benefit $r/N$ per agent resulting from the public good. It applies regardless of the agent's action $\sigma_i$. Both terms describe the utility function of a classical public good game. The third term, new to the model, describes an additional effect resulting from the existence of a social norm, or social pressure, to cooperate. Free riders with $\sigma_i = 0$ face an internal or external sanction [37], which does not apply for cooperators with $\sigma_i = 1$. We assume that the strength of the social pressure $s(n_c)$ depends on the density of cooperators. If $n_c$ is small, i.e., if free riding is widespread, then agents deviating from cooperation may face weaker sanctions. Hence, $s(n_c)$ is assumed to increase monotonously with $n_c$, with $s(1) > 0$ and $\lim_{n_c \to 0} s(n_c) = 0$. In the following, we simply choose a linear function $s(n_c) = \alpha\, n_c$, with $\alpha > 0$.

Eventually, we consider that not all agents may be prone to social pressure in the same manner. To cope with this individual sensitivity to the social norm, we introduce a new variable $\theta$ with realizations $\theta_i$ drawn from a probability distribution function $g(\theta)$ with mean $\Theta$ and standard deviation $\Delta\theta$. Note that negative values of $\theta_i$ imply a positive contribution to the *perceived* agent's utility by violating the social norm. This reflects the presence of contrarians or jokers [25] in the population that are willing to go against the system in order to benefit. Such agents would more likely not contribute to the public good in the presence of social pressure, but as we will see below, their presence turns out not to be an obstacle for the general population to conform to the social norm.

With this utility function, the (bounded) rational choice of an agent on what action to take depends on the density of cooperators $n_c$ and on her individual sensitivity $\theta_i$. Introducing $\tilde{c} \equiv c - r/N$, it is easy to see that agents' decisions can be classified as three types: (i) agents will always cooperate, $\sigma_i = 1$, if $\theta_i > \tilde{c}/s(N^{-1})$, (ii) agents will always free ride, $\sigma_i = 0$,

if $\theta_i < \tilde{c}/s(1)$, and (iii) agents are conditional cooperators dependent on the density of free riders in the population, i.e., they cooperate if $\tilde{c}/s(N^{-1}) > \theta_i > \tilde{c}/s(1)$. Note that because $\lim_{n_c \to 0} s(n_c) = 0$, the criterion for the existence of cooperators is quite tight, and often they will be absent from the population. Hence, the diversity in the individual sensitivity $\theta$, precisely, the standard deviation $\Delta\theta$, will play an important role in deciding the size of the three groups defined above. The final level of cooperation (as well as the influence of the social norm) will, to a large extent, be governed by the conditional cooperators.

Finally, we will consider that the cooperation-fostering norm changes in time, which is modeled by assuming a time dependence of $\alpha \to \alpha(t)$. This corresponds to a change of the slope of the social pressure function, representing periods in history where free riding is less tolerated than in others but it is always tolerated if widespread. If we further assume that agents can change their action depending on their expected utility, i.e., the density of cooperators $n_c$ has a dynamics defined like in the following section, the third term in Eq. (1) representing the social pressure becomes $(\sigma_i - 1)\theta_i \alpha(t) n_c(t)$. Hence, we have a signal $\alpha(t)$ that changes over time because of external influences. In the present paper, for the sake of simplicity and without altering the main results [22], we will consider a periodic change in the amplitude of the social norms. In the absence of cooperators, the effect of this signal vanishes as well. The diversity in responding to the signal is given by the individual variables $(\sigma_i - 1)\theta_i$; i.e., only free riders will face the social pressure, but they are prone to it in a heterogeneous manner.

Studying the model in the setting of diversity-induced resonance allows us to use standard techniques for quantifying the response of the population to, for example, a change in the social pressure induced by a policy change. If the period of the signal $\alpha(t)$ is long enough, the results of a periodic forcing become equivalent to a one-time modification. Moreover, in contrast to previous studies of this phenomenon, the signal enters multiplicatively on the heterogeneous term.

### C. Evolutionary dynamics

As mentioned above, we implement a dynamics that allow agents to change their actions dependent on the utility expected. For this dynamics, we use a standard evolutionary game-theoretical setup with one-shot games; i.e., agents have no memory of their previous action. We consider a well-mixed population; i.e., all agents interact together. This is dynamically equivalent to considering a mean-field version of the public good game, already reflected in the sum term in Eq. (1). After each round of the game, agents collect their payoff and subsequently update their strategies according to two different dynamical rules, which we explain in detail below. From the various propositions for update rules in the literature [42,43], we have chosen (i) the replicator dynamics [44,45], which is widely used and has a well defined limit for $N \to \infty$, namely, the celebrated replicator equation [46,47], and (ii) the logit dynamics [48], which allows for the possibility of errors or mistakes in choosing actions and whose deterministic limit coincides with the best-response rule, widely used in economics [49].

From a socioeconomic context, both dynamic rules have a different interpretation. On the one hand, the replicator dynamics involves some degree of social interaction (the process is driven by imitation of successful strategies). On the other hand, the logit dynamics is simply based on strategic behavior. By choosing these quite different kinds of dynamics, we demonstrate the generality and the robustness of the results presented in this paper.

Regarding the formal description, it is important to notice that the diversity for $\theta$ introduced in our model no longer allows us to write down the macroscopic dynamics in terms of a single master equation. Instead, the system dynamics has to be split into the dynamics of groups of agents with the same value of $\theta$. Let $n(\theta,\sigma)\delta\theta$ be the number of agents with an individual sensitivity in the interval $[\theta - \delta\theta/2, \theta + \delta\theta/2]$ choosing action $\sigma$ at time $t$ (for simplicity, we also say agents are in state $\sigma$ at time $t$, i.e., "state" refers to "action"). Then, the rate equation for the density of cooperators with a sensitivity $\theta$ is given by

$$\dot{n}(\theta,1) = n(\theta,0)\,\omega_+(\theta) - n(\theta,1)\,\omega_-(\theta). \tag{2}$$

The transition rate $\omega_+(\theta)$ $[\omega_-(\theta)]$ specifies the overall transition into the state $\sigma = 1$ $(\sigma = 0)$ for the two subpopulations with a given sensitivity $\theta$ but different states. These transition rates depend on the dynamic rules chosen and are specified below.

#### 1. *Replicator dynamics*

With this update rule, after every time step all agents revise their action simultaneously by selecting one neighbor at random, e.g., agent $j$, and comparing their own payoff $u_i$ with their neighbor's payoff $u_j$. If $u_i > u_j$, agent $i$ keeps her action, whereas in the opposite case she adopts the action of the more successful agent $j$ with a probability proportional to $(u_j - u_i)$. Replicator dynamics is purely imitative, meaning that actions not present currently in the system cannot appear spontaneously. This in turn implies that states in which *all* agents defect or *all* contribute are absorbing states. In order to let the system leave those absorbing states, we have introduced noise to the dynamics: with a small probability $\epsilon$ an agent can switch her action spontaneously at every time step. Subsequently, all payoffs are reset to zero, and a new round of the game proceeds.

For agents with an individual sensitivity $\theta$, the overall transition rate towards the opposite state depends on the possible pairings with agents in the opposite state and equipped with individual sensitivity $\theta'$. This yields

$$\omega_-(\theta) = \epsilon + \int \omega_-(\theta|\theta')\, n(\theta',0)\, g(\theta')\, d\theta', \tag{3}$$

$$\omega_+(\theta) = \epsilon + \int \omega_+(\theta|\theta')\, n(\theta',+1)\, g(\theta')\, d\theta', \tag{4}$$

where $g(\theta)$ is the distribution function of $\theta$. The conditional transition rates $\omega_+(\theta|\theta')$ and $\omega_-(\theta|\theta')$ are equal to the differences in payoff if the payoff of the agent with $\theta'$ is larger, i.e.,

$$\omega_-(\theta|\theta') = \begin{cases} u(\theta',0) - u(\theta,+1) & \text{if } u(\theta',0) > u(\theta,+1), \\ 0, & \text{otherwise} \end{cases}$$

and

$$\omega_+(\theta|\theta') = \begin{cases} u(\theta',+1) - u(\theta,0) & \text{if } u(\theta',+1) > u(\theta,0), \\ 0, & \text{otherwise.} \end{cases}$$

In these expressions we used, without loss of generality, a dimensionality constant of 1 to match the transition rates with the payoff functions. Using the utility function of our model, Eq. (1), these expressions become

$$\omega_-(\theta|\theta') = \begin{cases} -\theta'\, s(n_c) + \tilde{c} & \text{if } \theta' < c/s(n_c), \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

and

$$\omega_+(\theta|\theta') = \begin{cases} -\tilde{c} + \theta\, s(n_c) & \text{if } \theta > c/s(n_c), \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Now, inserting Eqs. (5) and (6) into Eqs. (3) and (4) and choosing $g(\theta)$ to be a uniform distribution, we get

$$\omega_-(\theta) = \epsilon + \int_{\theta-\Delta\theta}^{\tilde{c}/s(n_c)} [-\theta'\, s(n_c) + \tilde{c}]\, \frac{n(\theta',0)}{2\Delta\theta}\, d\theta', \tag{7}$$

$$\omega_+(\theta) = \epsilon + \begin{cases} [-\tilde{c} + \theta\, s(n_c)]\frac{n_c}{2\Delta\theta} & \text{if } \theta > \tilde{c}/s(n_c), \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

We emphasize that, in the presence of other distributions for the idiosyncratic term, the transition rates become more sophisticated and closed form equations cannot be written in general.

#### 2. Logit dynamics

When considering bounded rational agents, economics literature often assumes that they do not imitate their neighbors but follow a strategy or action that would yield the best payoff for them. In line with this assumption, one possible rule would be to change the action into a cooperative ($\omega_+$) or defective ($\omega_-$) state with a transition rate

$$\omega_\pm(\theta) = \frac{1}{1 + \exp\{\mp\beta[u(\theta,1) - u(\theta,0)]\}}. \tag{9}$$

It is important to note that in this case the agent does not compare her payoff with that of another agent but with the payoff she would obtain by using the opposite action. As there is no other agent involved, there is also no interaction term in the above equation, which makes the transition rates much simpler than in the previous case. This will be advantageous for an analytical approach, as we will see below.

The parameter $\beta$ in Eq. (9) quantifies the randomness in the process: When $\beta$ is small, the agent is more likely to select another action at random, even if that action is not more successful. On the other hand, when $\beta \to \infty$, the rule becomes deterministic, and the action that yields the maximum payoff is always chosen, as posited by Ellison [49] when introducing his (myopic) best-response rule.

## III. RESULTS

### A. Setup for computer simulations

In order to present our results in a clear manner, we will deal first with the original model as introduced in [26], without considering diversity or external forcing. This will be the baseline scenario against which we will subsequently illustrate the effects of diversity to proceed to our main result, namely, the influence of an external driver and the concomitant appearance of diversity-induced resonance.

As described in the preceding section, the model has several parameters to specify. We start by measuring utilities as a function of the cost of contributing to the public good, i.e., by taking $c = 1$. For the multiplication factor we fixed $r = 5$, which, in a population of many agents, is too small to induce agents to contribute to the public good. Therefore, without the third term in Eq. (1) referring to the social norm, the only evolutionarily stable strategy is defection. For the population size, we have chosen $N = 10^3$ agents (some runs were repeated with $N = 10^4$ for the sake of comparison, yielding the same results).

Subsequently, we have chosen the following parameter values related to the social norm. The strength of the norm is given by the slope $\alpha$, which, in the absence of an external influence, is set as a constant $\alpha = 1$, although changes in this parameter do not qualitatively modify our conclusions. Finally, for the sensitivity to the norm, we need to specify the parameters of the distribution $g(\theta)$. In the following, we consider two cases: (a) There is a sensitivity to the social norm equal for all agents, which is given by the mean value $\Theta$ of the distribution (homogeneous model). We will choose different values of $\Theta$. (b) The sensitivity to the social norm is different for all agents and is randomly chosen from a uniform distribution in $[\Theta - \sqrt{3}\Delta\theta, \Theta + \sqrt{3}\Delta\theta]$, where $\Delta\theta$ is the standard deviation (heterogeneous model). Note that our choice allows for negative sensitivities with effects as described in Sec. II.

To monitor the evolution of the system, we have measured the time-dependent density of cooperators $n_c(t) = (1/N)\sum_i \sigma_i(t)$. To determine the stationary level of cooperation, we compute the time-average number of cooperation, $n_c = \langle n_c(t)\rangle_t$. Subsequently, we also compute the second moment of $n_c(t)$, i.e., $\xi^2 = \langle (n_c(t) - n_c)^2\rangle_t$, which is the susceptibility of the system.

### B. Dynamics in the unforced model

#### 1. Model without diversity

In the homogeneous model, the sensitivity to the social norm is equal for all agents, $\theta \equiv \Theta$. Starting from an initial condition where half of the population acts as cooperators and half as free riders, Fig. 1 shows the asymptotic results of computer simulations for the two update dynamics introduced in Sec. II C. As can be clearly seen in the top panels, an increase in the parameter $\Theta$, which controls the influence of the social norm, results in an increase in the density of cooperators. For the replicator dynamics and for large values of randomness $\epsilon$, this effect becomes less visible as the width of the transition increases. The results for the logit dynamics point in the same direction, with $\beta^{-1}$ being the parameter that controls the randomness or the frequency of mistakes. Note that Fig. 1 is obtained for equal initial densities of contributors and free riders, but extensive simulations show that the value of $\Theta$ at which the transition occurs does not depend on the initial condition.

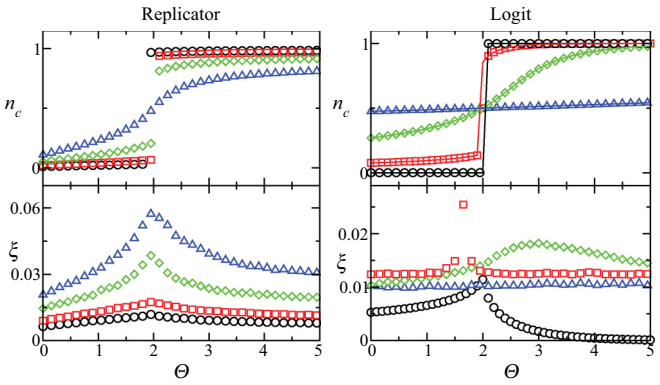


FIG. 1. (Color online) (top) Asymptotic fraction of cooperators $n_c$ dependent on the sensitivity to the social norm, $\theta \equiv \Theta$, which is equal for all agents in the model without diversity. (bottom) Fluctuations $\xi$ around the expected action, which is free riding ($\sigma = 0$) for $\Theta < 2$ and cooperation for $\Theta > 2$. (left) Replicator dynamics. Different curves correspond to different values of $\epsilon$: circles, 0.01; squares, 0.02; diamonds, 0.05; triangles, 0.10. (right) Logit dynamics. Different curves correspond to different values of $\beta$: circles, 0.1; squares, 1.0; diamonds, 2.5; triangles, 10. The other parameters are described in the main text. In the top right panel, the lines correspond to the analytical treatment, developed in Sec. IV.

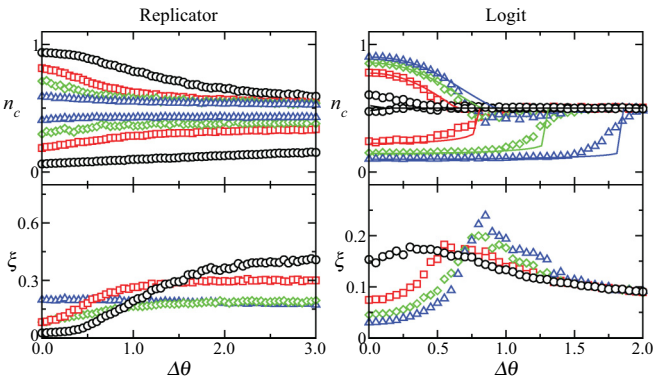


FIG. 2. (Color online) (top) Asymptotic fraction of cooperators $n_c$ and (bottom) susceptibility $\xi^2$ dependent on the sensitivity to the social norm $\theta$, which is different for all agents in the model with diversity. $\Delta\theta$ is the variance of the distribution $g(\theta)$ with mean value $\Theta = 2$. (left) Replicator dynamics. Different curves correspond to different values of $\epsilon$: circles, 0.02; squares, 0.05; diamonds, 0.07; triangles, 0.10. (right) Logit dynamics. Different curves correspond to different values of $\beta$: circles, 2.0; squares, 2.25; diamonds, 2.50; triangles, 2.75. In the top panels, we have selected two different initial conditions [$n_c(0) = 0.1$ and 0.9] for both kinds of dynamics. The other parameters are described in the main text. In the top right panel, the lines correspond to the analytical treatment, developed in Sec. IV.

It is interesting to note the peak of the susceptibility (bottom panels) close to the transition towards cooperation, both for replicator and logit dynamics. This is reminiscent of bistable systems, which change their stability at the transition. It is the archetypal situation where diversity-induced resonance has already been demonstrated, and as we will see below, it will give rise to the same behavior in this socioeconomic context. It is worth noting that the results for the susceptibility become very similar for both dynamics only when the noise intensity is very small.

#### 2. *Model with diversity*

Using the results from the model without diversity as a reference case, we now focus on the role of diversity in the sensitivity to the social norm. That means that instead of a fixed value $\theta$ we consider an individual value for each agent which is drawn from the uniform distribution $g(\theta)$ specified in Sec. III A. The standard deviation $\Delta\theta$ varies the degree of diversity. The results of computer simulations are shown in Fig. 2. From the previous discussion (cf. Fig. 1) we know that, for the chosen set of parameters, the transition from free riding to cooperation occurs at a value $\Theta = 2$. Therefore, in all the curves of Fig. 2, we have fixed the average sensitivity to this value in order to investigate the role of diversity. When plotting the stationary number of cooperators, the two curves for the same parameter set correspond to different initial conditions with a majority of cooperators or defectors. From the simulation results, we can clearly conclude that diversity alone does not favor the transition towards cooperation. Also, increasing noise does not enhance this situation. From Fig. 2 we see that, for the replicator dynamics, the lower the noise is, the lower the cooperation is in the asymptotic state, reaching the random level of $n_c = 0.5$ for very high values (agents make mistakes every other time step on average). For logit dynamics the results are similar, but for low noise we observe an asymmetric bifurcation in which the stationary state of low cooperation merges onto the $n_c = 0.5$ state only for very large values of the diversity; higher noise values change the bifurcation towards a more symmetric form.

### C. Dynamics under driving

So far we have only discussed the role of the idiosyncratic sensitivity $\theta$ to the social norm and have found that it does not induce a transition to cooperation. Now, as an important new ingredient, we consider that the influence of the norm changes in time, expressed by the time-dependent parameter $\alpha(t)$. Basically, any time dependence can be considered. For simplicity we have chosen a periodic function in the form of a square wave defined as

$$\alpha(t) = \begin{cases} \alpha + \Delta\alpha & \text{if } 2nT < t < (2n+1)T, \\ \alpha - \Delta\alpha & \text{if } (2n+1)T < t < 2(n+1)T, \end{cases} \tag{10}$$

with $n = 0,1,2,\ldots$ In an adiabatic limit, where the period $T$ is large such that the system reaches the stationary equilibrium in a period, this situation is equivalent to the application of a single change in the social pressure as perceived by agents (by external means, like a change in policy, for example). We have verified that using a sinusoidal function basically leads to the same results, qualitatively, as those shown in this paper. So we will focus on the expression given in Eq. (10).

We already defined the global density of cooperators $n_c(t)$ to be used as the order parameter. In particular, in the following, we will instead plot both the minimum and maximum values reached by the density of cooperators over time. To further quantify the collective response of the system to the externally changing influence of the social norm, we introduce the

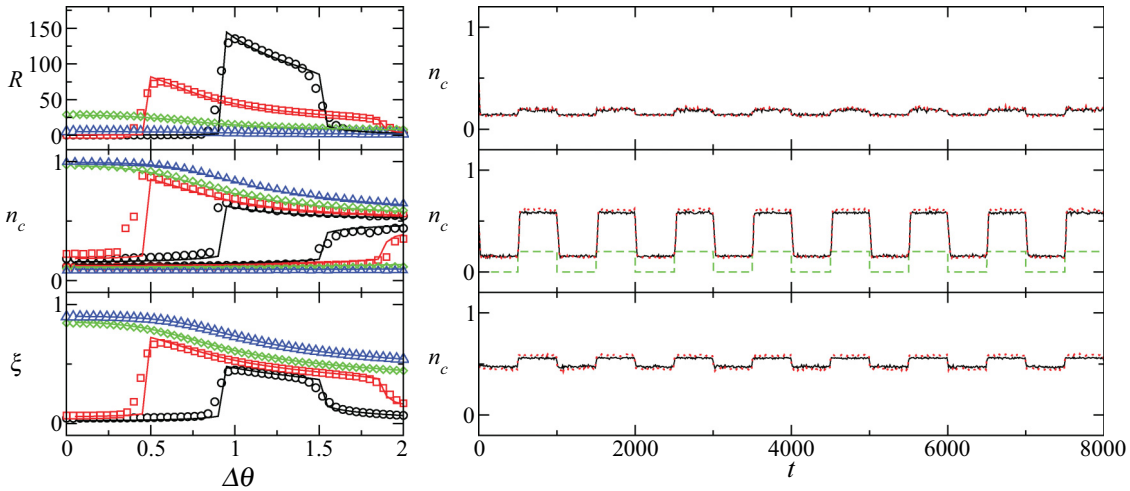


FIG. 3. (Color online) Response of the system in the presence of a periodic square-wave forcing with logit dynamics. In all the plots, $\beta = 2.5$. The left column shows (top) spectral amplification factor $R$, Eq. (11), (middle) maximum and minimum levels of cooperation attained during the evolution of $n_c$ for the system, and (bottom) the susceptibility. Each symbol corresponds to a different signal amplitude: $\Delta\alpha = 0.05, 0.1, 0.2, 0.5$ are shown as circles, squares, diamonds, and triangle, respectively. Analytical results (see main text) are represented with solid lines. In the right column, we depict the time dependency of the macroscopic state $n_c$ (solid black lines) for three different values of the parameter $\Delta\theta$. The values are $\Delta\theta = 0.7, 1.2, 1.7$ in the top, middle, and bottom plots, respectively. The dotted line represents the social pressure ($\Delta\alpha = 0.1$), while the thin green dashed line (only in the middle plot) shows the signal applied (not on the same scale, for clarity). Other parameters are $T = 10^3$, $N = 10^4$, $r = 5$, $\Theta = 2$, $\alpha = 1$.

spectral amplification factor (SAF), $R$, defined as [50]

$$R = 4\frac{|\langle n_c(t)e^{i2\pi t/T}\rangle_t|^2}{\Delta\alpha^2}. \tag{11}$$

Now, in addition to the variance $\Delta\theta$ of the sensitivity to the social norm, which describes an *individual* feature, we further have the change $\Delta\alpha$ in the social pressure caused by *external* influences.

A summary of our numerical results is presented in Fig. 3. The left column shows the spectral amplification factor $R$, the maximum and minimum values of cooperation $n_c$, and $\xi$ as a function of the standard deviation of the diversity $\Delta\theta$ for different values of the amplitude of the external driving $\Delta\alpha$. These results correspond solely to the logit dynamics; the results for the replicator dynamics are qualitatively similar to those presented in the plot and are not shown. We further noticed that, for all choices of parameters, the results are independent of the initial conditions. As can be seen from the plots, for low $\Delta\theta$, the response $R$ is largely independent of $\Delta\theta$. In this limit, it is possible to see the existence of superthreshold signal intensities $\Delta\alpha$, which are those values exhibiting large oscillations in the limit $\Delta\theta \to 0$. For the parameters in the plot, this corresponds to $\Delta\alpha \geqslant 0.2$. On the other hand, for smaller values of signal amplitude, we find that (in the limit of small heterogeneity) the system responds simply linearly to changes in the social norm. From a dynamical point of view, responses of the system to the external influence for low $\Delta\theta$ are depicted in the right column in the top panel.

However, intermediate values of $\Delta\theta$ do provide evidence for resonant behavior if the driving intensity is small: $R$ shows a peak for values of $\Delta\theta \simeq 1$, which becomes more noticeable for smaller signals. The oscillations of $n_c(t)$ are centered around $1/2$, a value much larger than the one obtained for lower values of diversity. Moreover, the application of a one-time increase in the strength of the external signal may yield a nonlinear response in terms of the growth of cooperating agents. From a policy making point of view, this translates to low incentive costs being able to enforce the cooperative state throughout the population. When the driving amplitude $\Delta\alpha$ is much larger, the system may be able to follow the signal simply because the signal is superthreshold, and the same thing happens even in the absence of diversity. Therefore, there is a true resonance phenomenon, which can be observed for low external signals, that elicits a strong response. In the middle panel of the right column, we show the dynamic response for a small applied signal, showing the large excursions in the number of cooperators when successively activating and deactivating the external signal.

Finally, for very large values of diversity, no response to the external influence is observed. The number of agents with very heterogeneous responses to the external signal does not allow a significant portion of the population to react to the external signal, and the system's response becomes linear again. The latter result can be observed in a vanishing response $R$ and small oscillation amplitudes in $n_c$ (the latter is shown in the bottom panel of the right column). In all the previous analyses, it is worth noticing that a peak in the susceptibility signals also the diversity-induced resonance in this system.

Analyzing the role of noise for a fixed driving strength $\Delta\alpha$, we observe another interesting feature of the dynamics under driving. Figures 4 and 5 show for both the replicator and the logit dynamics the appearance of stochastic resonance [50]. That means for an intermediate noise intensity (temperature or randomness in the proposed dynamics) the diversity-induced resonance peak is more clearly observed, whereas smaller or larger values of the randomness mostly suppress it. As with most stochastic phenomena, the resonant behavior is clearly marked also in the fluctuations of the system. The observation

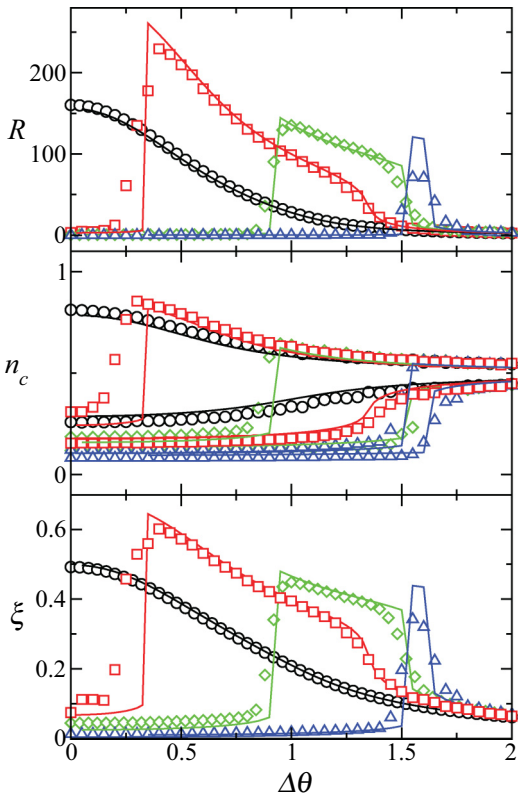


FIG. 4. (Color online) Response of the system in the presence of a periodic forcing with logit dynamics. In all the plots, $\Delta\alpha = 0.05$. (top) Spectral amplification factor $R$, Eq. (11). (middle) Maximum and minimum levels of cooperation attained during the evolution of $n_c$ for the system. (bottom) The system susceptibility. Each symbol (and color) corresponds to a different value of the inverse randomness: $\beta = 2, 2.32, 2.5, 2.75$ are shown by circles, squares, diamonds, and triangles, respectively. With symbols we represent the results obtained by means of computer simulations, while the analytical results are presented as solid lines. Other parameters are $T = 10^3$, $N = 10^4$, $r = 5$, $\Theta = 2$, $\alpha = 1$.

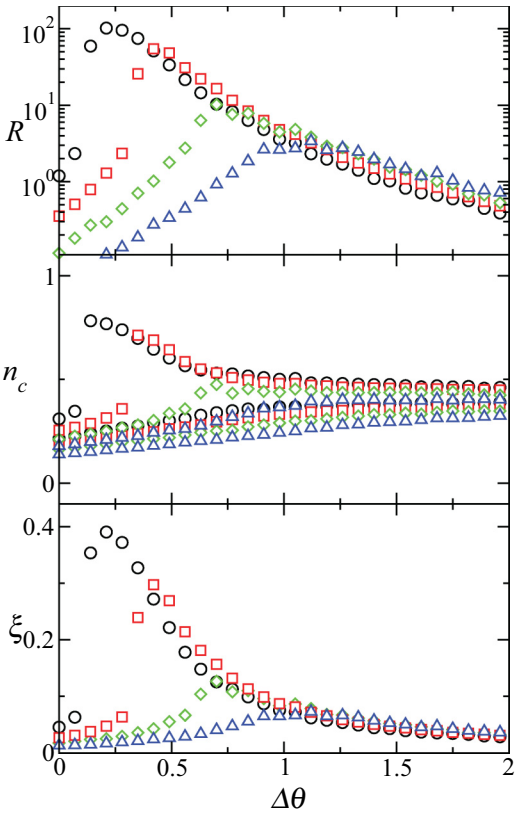


FIG. 5. (Color online) Effect of a periodic signal applied to the social norm as a function of the population diversity $\Delta\theta$ for the replicator dynamics. In all plots, $\Delta\alpha = 0.05$. (top) Spectral amplification factor $R$, Eq. (11). (middle) Maximum and minimum levels of cooperation attained during the evolution of $n_c$ for the system. (bottom) The system's susceptibility. Each curve corresponds to a different value randomness: $\epsilon = 0.04, 0.045, 0.05, 0.055$ are shown as circles, squares, diamonds, and triangles, respectively. Other parameters are $T = 10^3$, $N = 10^4$, $r = 5$, $\theta = 2$, $\alpha = 1$.

of stochastic resonance is remarkable because it shows up not in a physical but in a socioeconomic context. It indicates that some level of imperfections in the adoption of the better performing strategies may lead to larger responses to external stimuli.

It is also worth mentioning that the presence of contrarians, i.e., agents which defect even in the presence of social pressure, can be beneficial for finding the diversity-induced resonance phenomenon. In some extreme cases, as shown in Fig. 4, the resonance peak may appear only in the presence of contrarians for $\beta = 2.75$ [51]. This finding is against our intuition that contrarians would hamper the adoption of a cooperative state in the system. It reminds us of the positive influence of destructive agents on the emergence of cooperation in social dilemma situations as discussed in [25], where this phenomenon was termed "the joker effect." Turning to replicator dynamics, further increasing the noise intensity would lead to a (small) maximum of response. This corresponds to values of diversity where contrarians are pervasive inside the population. However, in such a situation agents' actions are very often randomly taken. Therefore, in this regime it is of little importance whether they behave as contrarians or not. We want to emphasize that there is another side to heterogeneity: for large values of $\Delta\Theta$, some agents become very sensitive to the social norm $\alpha(t)$. Thus, even if the signal is small, these agents start to cooperate, this way increasing the effective value of the social norm. This in turn feeds back by recruiting larger numbers of agents for cooperation.

## IV. ANALYTICAL APPROACH

### A. Dynamics without driving

To further understand the phenomenon of diversity-induced resonance in our model, we now develop an analytical approach that should be compared to the numerical simulations presented in the previous section. While the transition rates for the replicator dynamics, Eqs. (7) and (8), are too complicated for a tractable analytical approach, the situation is different for the logit dynamics. In this case, the density of agents with a given sensitivity $\theta$ depends only on the total number of cooperators in the population, which is a macroscopic variable.

Consequently, with the transition rates of Eq. (9) and the equilibrium condition for the payoff function, Eq. (1), we find for the transition rate towards the cooperative (and defective) states the following expression:

$$\omega_{\pm}(\theta) = \frac{1}{1+\exp\{\mp\beta\,[\tilde{c} - \theta\, s(n_c)]\}}. \tag{12}$$

From the above equation, we can trivially compute the density of cooperators by integrating over the complete population of agents,

$$n_c = \int d\theta'\, g(\theta') \frac{1}{1+\exp\{\mp\beta\,[\tilde{c} - \theta'\, s(n_c)]\}}, \tag{13}$$

which, by using the uniform distribution of the sensitivity $\theta$, reduces to

$$n_c = \frac{1}{2\Delta\theta}\int_{\Theta-\Delta\theta}^{\Theta+\Delta\theta} d\theta' \frac{1}{1+\exp\{\mp\beta\,[\tilde{c} - \theta\, s(n_c)]\}}. \tag{14}$$

Expanding this equation, one readily obtains for the density of cooperators

$$n_c = s(n_c) + \frac{\ln(1+\exp\{-\beta[\tilde{c} - s(n_c)(\theta-\Delta\theta)]\})}{2s(n_c)\,\beta\,\Delta\theta} - \frac{\ln(1+\exp\{-\beta[\tilde{c} + s(n_c)(\theta-\Delta\theta)]\})}{2s(n_c)\,\beta\,\Delta\theta}. \tag{15}$$

This equation can be solved self-consistently to obtain the stationary value of $n_c$. The corresponding results are shown as solid lines in Figs. 1 and 2 (right columns). We find a very good agreement between the numerical simulations and the prediction of our analytical approach, thus further supporting the validity of our results.

In agreement with our discussion in Sec. III B, the system exhibits a pitchfork bifurcation. When increasing the control parameter $\Delta\theta$, the solution $n_c = 1/2$ changes its stability from unstable to stable, when the two branches (one with a majority of cooperators and the other with a majority of free riders) collapse in the center point. As observed in the simulations, and now confirmed by the analytical treatment, the solutions are asymmetrical with respect to the stable point, with the lowest branch being less dependent on the value of the control parameter.

This is key to understanding the mechanism behind the diversity-induced resonance phenomenon in this socioeconomic system: For intermediate values of the diversity $\Delta\theta$, small perturbations are sufficient to overcome the separatrix, i.e., the unstable solution $n_c = 1/2$ that divides the attractor basins of the two stable solutions. Thus, a signal which is usually too small to cause transitions between those states can be sufficient to trigger such a transition near the bifurcation point. Farther from this critical point, a small signal only causes a linear response of the system, around a stable fixed point. This fully confirms the discussion of the numerical results for the system with driving in the previous section.

### B. Relaxational dynamics with driving

After considering the dynamics without driving in the previous section, we now turn to the dynamics with driving to better understand the response of the system to the external change of the norm. We note that the change in the density of cooperators after one state has been selected for update is given by

$$n_c(t+\delta t) = n_c(t) + \frac{1}{N}\left\langle \sigma_i(t+\delta t) - \sigma_i(t) \middle| \{\sigma(t)\}\right\rangle, \tag{16}$$

where $\langle\cdot\rangle$ represents the ensemble average, which is conditional on $\{\sigma(t)\}$, i.e., all those states that did not change. Going over to small $\delta t \equiv 1/N$, we arrive at the continuous dynamics:

$$\frac{dn_c(t)}{dt} \cong \left\langle \sigma_i(t+\delta t) \middle| \{\sigma(t)\}\right\rangle - n_c(t).$$

The expected value for the selected state $\sigma_i$ after update can be expressed as

$$\left\langle \sigma_i(t+\delta t) \middle| \{\sigma(t)\}\right\rangle = \mathrm{Prob}[\sigma_i(t+\delta t) = 1]. \tag{17}$$

Without loss of generality, the probability that $\sigma_i(t+\delta t)$ is $+1$ is given by $(1 - \mathrm{Prob}[+1 \to 0]) + \mathrm{Prob}[0 \to +1]$, which for this system is given by

$$\left\langle \sigma_i(t+\delta t) \middle| \{\sigma(t)\}\right\rangle = \int d\theta'\, g(\theta')[1 - \omega_-(\theta) + \omega_+(\theta)]. \tag{18}$$

Restricting ourselves again to the particular case of the uniform distribution for $\theta$ and logit dynamics, we have

$$\frac{dn_c(t)}{dt} = f(n_c) = \frac{1}{2} - n_c(t) - \frac{\ln\{\cosh[\beta\tilde{c} + \beta s(n_c)(\Delta\theta - \theta)]\}}{4\beta\, s(n_c)\Delta\theta} + \frac{\ln\{\cosh[\beta\tilde{c} - \beta s(n_c)(\Delta\theta + \theta)]\}}{4\beta\, s(n_c)\Delta\theta}. \tag{19}$$

If the external signal given by $\alpha(t)$ is slow enough, we can determine $R$ by assuming that $n_c(t)$ reaches its stationary state quickly compared to changes in $\alpha$. Then, $n_c(t) = n_c^*(\alpha(t))$. For a squared signal, the spectral amplification factor is simply given by

$$R(n_c^*) = \frac{\pi[n_c^*(\alpha+\Delta\alpha) - n_c^*(\alpha-\Delta\alpha)]^2}{\Delta\alpha^2}. \tag{20}$$

For this forcing, the average number of cooperators reduces to $n_c^* = [n_c^*(\alpha+\Delta\alpha) + n_c^*(\alpha-\Delta\alpha)]/2$. Then, the susceptibility can be computed as

$$\xi^2 = \int_0^{T/2} dt\,[n_c^*(\alpha+\Delta\alpha) - n_c]^2 + \int_{T/2}^{T} dt\,[n_c^*(\alpha-\Delta\alpha) - n_c]^2,$$

from which we get for the susceptibility

$$\xi^2 = [n_c^*(\alpha+\Delta\alpha) - n_c^*(\alpha-\Delta\alpha)]^2. \tag{21}$$

Figures 3 and 4 present a comparison between the analytical and numerical results. As with the previous comparisons, the match is very satisfactory. While our socioeconomic model is quite different from a physics model, the dynamic observations have similar underlying mechanisms as known in physical systems with diversity-induced resonance, which makes it possible to apply a standard analytical approach. For the

replicator dynamics, we cannot apply the same techniques to calculate the observables. But the fact that we find in the simulations similarities between the logit dynamics, for which we have analytical confirmation, and the replicator dynamics allows us to conjecture similarities in the underlying mechanisms.

## V. DISCUSSION AND CONCLUSION

In this paper, we have studied a socioeconomic model of cooperation to understand the effect of social pressure on the contribution to a public good [26]. We tried to point out analogies with the phenomenon of diversity-induced resonance in bistable physical systems reported in Ref. [3]. This was to show that methodological input from physics can be beneficial for social sciences, in particular with respect to the vast knowledge about complex nonlinear dynamical systems. By adopting an already existing model, we avoided imposing a physics-inspired toy model that may not have fit the modeling paradigms of social sciences.

Our analytical and numerical results demonstrate that our approach has been largely successful. Indeed, we found strong evidence of diversity-induced resonance, i.e., of the fact that the response of the system to a weak external signal is stronger in a certain range of the parameters governing the disorder in the system. Importantly, such strong signals are subcritical, meaning that these alone would not be able to drive a homogeneous system, whereas diversity on its own would lead to an undesired behavior (in our case, to a decrease in cooperation). Furthermore, we have pursued another analogy to a physical phenomenon, namely, stochastic resonance [50]. We found evidence that there is an optimal range of noise or randomness to obtain the response of the system to the external signal.

It is most interesting to interpret the above results in terms of the original socioeconomic model. In that context, diversity means different sensitivities to the influence of the social pressure towards behaving in a cooperative manner. If an external signal is emitted (e.g., changing laws or incentives by the government) that leads to changes of the social pressure, the population will follow these directions only if its corresponding sensitivity to such pressure is diverse, but not too little or too much. Homogeneous populations will simply ignore the new norms, whereas very heterogeneous populations will end up behaving in some kind of "average" manner that does not follow the change. This is in agreement with the fact that strongly homogeneous groups, like gangs or sects can be considered to be, are very insensitive to external influences trying to bring them to contribute to the general welfare (although the fact that such groups may have low global sensitivity to the norm is also an issue). In an optimally diverse population, on the contrary, we would see that the most sensitive people would abide by the social pressure and start contributing to the common good, thus leading to an increment of the social pressure that pushes other agents and so forth. These results are in line with the seminal work on the threshold model by Granovetter [52], where agents only act in a certain way if the proportion of the population behaving this way exceeds a given threshold. Granovetter's model shows that heterogeneity at the population level is a possible mechanism to extend a given behavior across the population.

In this context, it is important to stress that the phenomenon is robust against the kind of dynamics considered for the transition towards cooperation. This is particularly meaningful as the two cases studied in our paper, i.e., replicator and logit dynamics, correspond to two completely different approaches to decision making from the agent's viewpoint. While the former is based on a social, imitative component, the second describes a purely strategic behavior, even a myopic one. Finally, we have observed that in some cases the required degree of heterogeneity for the appearance of the resonance leads to the existence of contrary individuals in the population, who would benefit from going against the norm. This resembles the case of diversity-induced resonance arising from repulsive interactions and related results in social dilemmas, as mentioned in Sec. II.

It is also worth noticing that the phenomenon of diversity-induced resonance only uses a weak signal to obtain the desired results. Strong signals would drive the population irrespective of its degree of diversity, but the external effort of the "driver" has to be much larger. This may be important for policy-making decisions where costly interventions in the society are not desirable because their benefit may, in the end, be smaller than the incurred cost. Of course, the requirement of diversity implies that these easily implemented policies may not be possible for all groups or societies, which in itself is another hint to policy makers about the need to estimate costs prior to specific interventions. It goes without saying that applications of these ideas in real life may need more complete models. For instance, one could think of endogenously generated norm changes, involving a feedback between actions and utility functions or including the affective dimension of agents by considering their emotional response [53]. On the other hand, applying these ideas to organizations may require a careful consideration of hierarchical effects [54]. Such improved models would lead to results that would be much more amenable to comparison with actual social group dynamics or even with specifically designed experiments and thus would contribute to our knowledge of the mechanics of social improvement. Work along these lines is in progress.

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[1] C. Castellano, S. Fortunato, and V. Loreto, Rev. Mod. Phys. **81**, 591 (2009).
[2] D. Stauffer, Comput. Sci. Eng. **5**, 71 (2003).
[3] C. J. Tessone, C. R. Mirasso, R. Toral, and J. D. Gunton, Phys. Rev. Lett. **97**, 194101 (2006).
[4] C. J. Tessone, A. Scirè, R. Toral, and P. Colet, Phys. Rev. E **75**, 016203 (2007).
[5] R. Toral, C. J. Tessone, and J. V. Lopes, Eur. Phys. J. Spec. Top. **143**, 59 (2007).
[6] H. Chen, Y. Shen, Z. Hou, and H. Xin, Chaos **19**, 033122 (2009).
[7] H. Chen, Z. Hou, and H. Xin, Physica A **388**, 2299 (2009).
[8] N. Komin, L. Lacasa, and R. Toral, J. Stat. Mech. Theory Exp. (2010) P12008.
[9] H. Calisto and M. G. Clerc, New J. Phys. **12**, 113027 (2010).
[10] T. Pérez, C. R. Mirasso, R. Toral, and J. D. Gunton, Philos. Trans. R. Soc. A **368**, 5619 (2010).
[11] Q. Y. Wang, M. Perc, Z. S. Duan, and G. R. Chen, Int. J. Mod. Phys. B **24**, 1201 (2010).
[12] D. Wu, S. Zhu, and X. Luo, Physica A **390**, 1835 (2011).
[13] M. D. McDonnell and L. M. Ward, Nat. Rev. Neurosci. **12**, 415 (2011).
[14] C. J. Tessone and R. Toral, Eur. Phys. J. B **71**, 549 (2009).
[15] S. Galam, Physica A **238**, 66 (1997).
[16] J. P. Sethna, K. Dahmen, S. Kartha, J. A. Krumhansl, B. W. Roberts, and J. D. Shore, Phys. Rev. Lett. **70**, 3347 (1993).
[17] T. Vaz Martins, M. Pineda, and R. Toral, Europhys. Lett. **91**, 48003 (2010).
[18] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, Adv. Complex Syst. **3**, 87 (2000).
[19] C. J. Tessone, D. H. Zanette, and R. Toral, Eur. Phys. J. B **62**, 319 (2008).
[20] C. J. Tessone and D. H. Zanette, Europhys. Lett. **99**, 68006 (2012).
[21] T. Vaz Martins, V. N. Livina, A. P. Majtey, and R. Toral, Phys. Rev. E **81**, 041103 (2010).
[22] G. Harras, C. J. Tessone, and D. Sornette, Phys. Rev. E **85**, 011150 (2012).
[23] S. Galam, Physica A **333**, 453 (2004).
[24] D. Stauffer and J. S. Martins, Physica A **334**, 558 (2004).
[25] A. Arenas, J. Camacho, J. A. Cuesta, and R. J. Requejo, J. Theor. Biol. **279**, 113 (2011).
[26] M. Spichtig and C. Traxler, J. Econ. **102**, 237 (2011).
[27] C. C. Keser and F. van Winden, Scand. J. Econ. **102**, 23 (2000).
[28] S. G. U. Fischbacher and E. Fehr, Econ. Lett. **71**, 397 (2001).
[29] A. Rapoport and M. Guyer, Gen. Syst. **11**, 203 (1966).
[30] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).
[31] J. H. Kagel and A. E. Roth, *The Handbook of Experimental Economics* (Princeton University Press, Princeton, NJ, 1995).
[32] S. Gächter, in *Economics and Psychology: A Promising New Cross-Disciplinary Field*, edited by B. S. Frey and A. Stutzer (MIT Press, Cambridge, MA, 2007).
[33] J. Grujić, C. Fosco, L. Araujo, J. A. Cuesta, and A. Sánchez, PLoS ONE **5**, e13749 (2010).
[34] J. Grujić, B. Eke, A. Cabrales, J. A. Cuesta, and A. Sánchez, Sci. Rep. **2**, 638 (2012).
[35] C. Gracia-Lázaro, A. Ferrer, G. Ruiz, A. Tarancón, J. A. Cuesta, A. Sánchez, and Y. Moreno, Proc. Natl. Acad. Sci. USA **109**, 12922 (2012).
[36] E. Fehr and K. Schmidt, in *Handbook on the Economics of Giving, Reciprocity and Altruism*, edited by S. C. Kolm and J. M. Ythier, Vol. 1 (North-Holland, Amsterdam, 2006), pp. 616–691.
[37] J. Coleman, *Foundations of Social Theory* (Harvard University Press, Cambridge, MA, 1990).
[38] J. M. Galán and L. R. Izquierdo, J. Artif. Soc. Soc. Simul. **8**, 2 (2005).
[39] F. Mengel, J. Econ. Behav. Organ. **67**, 608 (2008).
[40] E. Ostrom, J. Econ. Perspect. **14**, 137 (2000).
[41] E. A. Posner, *Law and Social Norms* (Harvard University Press, Cambridge, MA, 2000).
[42] G. Szabó and G. Fáth, Phys. Rep. **446**, 97 (2007).
[43] C. P. Roca, J. Cuesta, and A. Sánchez, Phys. Life Rev. **6**, 208 (2009).
[44] D. Helbing, Physica A **181**, 29 (1992).
[45] K. H. Schlag, J. Econ. Theory **78**, 130 (1998).
[46] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge, 1998).
[47] H. Gintis, *Game Theory Evolving*, 2nd ed. (Princeton University Press, Princeton, NJ, 2009).
[48] C. Alós-Ferrer and N. Netzer, Games Econ. Behav. **68**, 413 (2010).
[49] G. Ellison, Econometrica **61**, 1047 (1993).
[50] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[51] Assuming a uniform distribution of the sensitivity $\theta$, contrarians can be found in the population if $\sqrt{3}\Delta\theta > \Theta$.
[52] M. Granovetter, Am. J. Sociol. **83**, 1420 (1978).
[53] F. Schweitzer and D. García, Eur. Phys. J. B **77**, 533 (2010).
[54] K. Cronin and A. Sánchez, Adv. Complex Syst. **15**, 1250066 (2012).