Twisted electrostatic ion-cyclotron waves in dusty plasmas

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(Received 23 August 2012; revised manuscript received 21 November 2012; published 4 January 2013)

We show the existence of a twisted electrostatic ion-cyclotron (ESIC) wave carrying orbital angular momentum (OAM) in a magnetized dusty plasma. For our purposes, we derive a 3D wave equation for the coupled ESIC and dust ion-acoustic (DIA) waves from the hydrodynamic equations that are composed of the continuity and momentum equations, together with Poisson's equation. The 3D wave equation reveals the formation of a braided or twisted ESIC wave structure carrying OAM. The braided or twisted ESIC wave structure can trap and transport plasma particles in magnetoplasmas, such as those in Saturn's F-ring and in the forthcoming magnetized dusty plasma experiments.

DOI: 10.1103/PhysRevE.87.015101 PACS number(s): 52.25.Xz

Charged dust grains in plasmas or dusty plasmas are ubiquitous [1–12], ranging from the cosmic and astrophysical environments (e.g., interstellar media, molecular dust clouds, star forming dust clouds, Eagle Nebula, and supernovae remnants) to planetary ring systems (e.g., E and F rings of Saturn) and interplanetary media with cometary dust particles, as well as to the Martian atmosphere in the form of dust devils and in the surrounding of the surface of Sun and moon, as well as in the Earth's ionosphere and mesosphere. Charged dust debris also appear in space due to the destruction of satellites and near space propulsion vehicles due to rocket exhausts. Charged dust grains of different sizes and shapes are common in industrial processing plasmas for nanotechnology and in magnetic fusion reactors. Clearly, studies of the dynamics of charged dust grains and collective dust-plasma interactions are of great importance in a variety of diverse physical systems, including astrophysical settings and Sun-Earth connection, laboratory experiments on ground, and onboard the International Space Station for fundamental and applied research in cutting-edge areas of modern sciences. Accordingly, over a quarter century, much effort has been directed to studying dust grain charging [13,14] and numerous collective phenomena at kinetic levels [4,8–10,15–22] that naturally occur in astrophysical and low-temperature laboratory dusty plasma systems.

The dust acoustic wave (DAW) [15] and the dust ion-acoustic wave (DIAW) [16] are the two fundamental eigenmodes of an unmagnetized dusty plasma composed of electrons, ions, and charged dust grains. In the low-phase speed (in comparison with the electron and ion thermal speeds) and low-frequency (in comparison with the dust plasma frequency) DAW, the restoring force comes from the pressures of the inertialess electrons and ions that follow the Boltzmann distribution, while the dust mass provides the inertia. The DAW phase speed is $C_D = [Z_d P k_B T_i/m_d(1 + n_{e0}T_i/n_{i0}T_e)]^{1/2}$, where Z_d is the number of electrons on a dust grain, $P = Z_d n_{d0}/n_{i0} = 1 - n_{e0}/n_{i0}$ is the dust parameter, n_{j0} is the number density of the particle species j (j equals i for the ions, e for electrons, and d for dust grains), k_B is the Boltzmann constant, and T_i (T_e) is the ion (electron) temperature. We note that there does not exist a counterpart of the DAW in an electron-ion plasma without charged dust grains. On the other, in the low-phase speed (in comparison with the electron thermal speed) and low-frequency (in comparison with the ion plasmas frequency) DIAW, the restoring force comes from the pressure of the inertialess Boltzmann-distributed electrons, while the ion mass provides the inertia to sustain the wave. The dust effect appears here through the modification of the equilibrium quasineutrality condition [15–17]

$$n_{i0} = n_{e0} + Z_d n_{d0}, \tag{1}$$

which exhibits that there is depletion of the electrons from the background plasma, because a dust grain absorbs electrons to be negatively charged [23]. Since the ion number density is now larger than the electron number density in the presence of negative dust grains (which are stationary on the timescale of the ion plasma period), the phase speed of the DIAW is somewhat enhanced [by a factor $(n_{i0}/ne0)^{1/2}$] in comparison with the ion acoustic speed $c_s = (k_B T_e/m_i)^{1/2}$ of the IAW in an electron-ion plasma without charged dust grains. Recently, Shukla [22] has illuminated the underlying of the DAW and has also discovered a twisted DAW in an unmagnetized dusty plasma. We recall that both the dust acoustic and dust ion acoustic waves have been observed in several low-temperature laboratory dusty plasma discharges [24–29], and also in space dusty plasmas [21].

However, dusty plasmas in the Saturn's F-ring system and in the forthcoming laboratory experiments have ambient magnetic fields. In the presence of the latter, both the electrons and ions can be magnetized, which, in turn, participate in the dynamics of such new modes as linearly coupled dust

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ion-acoustic and ESIC modes [30,31], modified lower-hybrid [17] and dust lower-hybrid waves [32], the dust magnetoacoustic [33] and dust Alfvén [17,34], and the dust whistler [17,35] in a uniform dusty magnetoplasma.

In this Brief Report, we show that a 3D dispersive ESIC wave can propagate in the form of a twisted vortex beam or in the form of a braided electrostatic potential structure. The latter, which is also associated with a tornado of an electrical potential distribution, can trap and transport the plasma particles from one region to another location in a uniform dusty magnetoplasma. In fact, the present result may help to understand the salient features of a braided electrical potential structure of the Saturn's F ring that has been conjectured by Hill and Mendis [36].

Let us consider a magnetized electron-ion plasma in the presence of the low-frequency (in comparison with the electron gyrofrequency $\omega_{ce} = eB_0/m_e c \gg \nu_{ei} \ll \nu_{en}$, where e is the magnitude of the electron charge, B_0 the strength of the ambient magnetic field $B_0\hat{\mathbf{z}}$, m_e the electron mass, c the speed of light in vacuum, $\hat{\mathbf{z}}$ the unit vector along the z axis in a Cartesian coordinate system, and ν_{en} is the electron-neutral frequency), long wavelength (in comparison with the ion gyroradius) dispersive ESIC waves with the electric field $\mathbf{E} = -\nabla\phi$, where ϕ is the scalar potential. We assume that the charged dust grains of uniform sizes are immobile, since we are concerned with the occurrence of a twisted ESIC wave (with the phase speed much larger than the ion and dust thermal speeds) on a timescale much shorter than the dust plasma and dust gyroperiods. Thus, the charged dust grains do not have time to respond to the ESIC oscillations, and subsequently there are insignificant dust number density perturbations. The effect of the dust component then appears through the equilibrium quasineutrality condition, given by Eq. (1). Hence, in the ESIC wave field, the electron fluid velocity is

$$\mathbf{u}_e \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla\varphi_e + \frac{c}{B_0\omega_{ce}}\left(\frac{\partial}{\partial t} + \nu_{en}\right)\varphi_e + \hat{\mathbf{z}}u_{ez}, \tag{2}$$

where $\varphi_e = \phi - k_B T_e n_{e1}/n_{e0}$, n_{e1} ($\ll n_{e0}$) is a small electron number density perturbation in the equilibrium electron number density n_{e0}, k_B is the Boltzmann constant, T_e is the electron temperature, and the magnetic field-aligned electron fluid velocity u_{ez} is determined from the z component of the electron momentum equation

$$\left(\frac{\partial}{\partial t} + \nu_{en}\right)u_{ez} = \frac{e}{m_e}\frac{\partial \varphi_e}{\partial z}. \tag{3}$$

The electron number density perturbation n_{e1} is determined from the electron continuity equation

$$\frac{\partial n_{e1}}{\partial t} + n_{e0}\nabla\cdot\mathbf{u}_e = 0, \tag{4}$$

which, together with Eqs. (2) and (3), yields

$$\left(\frac{\partial^2}{\partial t^2} + \nu_{en}\frac{\partial}{\partial t} - V_{Te}^2\frac{\partial^2}{\partial z^2}\right)n_{e1} + \frac{n_{e0}c}{B_0\omega_{ce}}\left(\frac{\partial}{\partial t} + \nu_{en}\right)^2\nabla_\perp^2\phi + \frac{n_{e0}e}{m_e}\frac{\partial^2\phi}{\partial z^2} = 0, \tag{5}$$

where $V_{te} = (k_B T_e/m_e)^{1/2}$ is the electron thermal speed. We have assumed that $(V_{Te}/\omega_{ce})^2\nabla_\perp^2 n_{e1} \ll n_{e1}$. The perpendicular (to $\hat{\mathbf{z}}$) component of the ion fluid velocity perturbation $\mathbf{u}_{i\perp}$ is determined from

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)\mathbf{u}_{i\perp} = \frac{c\omega_{ci}^2}{B_0}\hat{\mathbf{z}} \times \nabla\varphi_i - \frac{c\omega_{ci}}{B_0}\frac{\partial \nabla_\perp\varphi_i}{\partial t}, \tag{6}$$

which is obtained by manipulating the perpendicular component of the ion momentum equation, where $\omega_{ci} = eB_0/m_i c$ ($\gg \nu_{ie}, \nu_{in}$) is the ion gyrofrequency, m_i is the ion mass, and ν_{ie} (ν_{in}) is the ion-electron (ion-neutral) collision frequency. Furthermore, we have denoted $\varphi = \phi + \gamma_i k_B T_i n_{i1}/n_{i0}$, with γ_i being the adiabatic index for the ion fluid, and $n_{i1}(\ll n_{i0})$ is the ion number density perturbation.

The magnetic field-aligned ion fluid velocity perturbation u_{iz} is determined from

$$\frac{\partial u_{iz}}{\partial t} = -\frac{e}{m_i}\frac{\partial\varphi_i}{\partial z}, \tag{7}$$

where $\partial u_{iz}/\partial t \gg \nu_{in}u_{iz}$ has been assumed. We also assume that the number density of immobile neutral particles in our dusty plasmas is rather low, and therefore there are insignificant interactions between neutrals and the charged species of our magnetized dusty plasma.

Equations (6) and (7) can be combined with the linearized ion continuity equation to obtain an equation that relates n_{i1} and ϕ. We have

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)\frac{\partial^2 n_{i1}}{\partial t^2} - \frac{n_{i0}c\omega_{ci}}{B_0}\frac{\partial^2\nabla_\perp^2\phi}{\partial t^2} + \frac{n_{i0}e}{m_i}\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)\frac{\partial^2\phi}{\partial z^2} = 0. \tag{8}$$

Equations (5) and (8) are closed by Poisson's equation,

$$\nabla^2\phi = 4\pi e(n_{e1} - n_{i1}). \tag{9}$$

We now consider coupled 3D DIA and ESIC waves with $\partial n_{e1}/\partial t \ll \nu_{en}n_{e1}$, $\nu_{en}\partial n_{e1}/\partial t \ll V_{Te}^2\partial^2 n_{e1}/\partial z^2$, and $\nu_{en}\partial\nabla_\perp^2\phi/\partial t \ll \omega_{ce}^2\partial^2\phi/\partial z^2$. Here, Eq. (5) yields the Boltzmann law for the electron number density perturbation

$$n_{e1} \approx \frac{n_{e0}e\phi}{k_B T_e}. \tag{10}$$

Equation (10) dictates that in our magnetized dusty plasma with frequent electron-ion collisions, inertialess electrons rapidly thermalize along the external magnetic field direction $B_0\hat{\mathbf{z}}$ in order to establish the Boltzmann law for the electron number density perturbation. The magnetic field-aligned rapid motion of the electrons ensures their couplings with the ions in order to maintain overall quasineutrality on the time scale of the ion gyroperiod.

Invoking the quasineutrality condition $n_{e1} \approx n_{i1}$, which is valid for $\lambda_{\rm De}^2\nabla^2\phi \ll \phi$, where $\lambda_{\rm De} = (k_B T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye radius, we can eliminate n_{i1} from Eq. (8) by using Eq. (10), obtaining the coupled DIA-ESIC wave equation

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)\frac{\partial^2\phi}{\partial t^2} - C_s^2\frac{\partial^2\nabla_\perp^2\phi}{\partial t^2} - \left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)C_s^2\frac{\partial^2\phi}{\partial z^2} = 0, \tag{11}$$

where $C_s = (n_{i0}k_BT_e/n_{e0}m_i)^{1/2}$ is the modified ion-acoustic (m-IA) speed [16]. In deducing Eq. (10), we have assumed that $T_e \gg T_i$. We see that the the m-IA speed C_s is larger by a factor $\sqrt{n_{i0}/n_{e0}}$ in comparison with $\sqrt{k_BT_e/m_i}$.

Within the framework of a plane-wave approximation, assuming that ϕ is proportional to $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where ω and $\mathbf{k}(=\mathbf{k}_\perp + \hat{\mathbf{z}}k_z)$ are the angular frequency and the wave vector, respectively, we can Fourier analyze Eq. (11) to obtain

$$\omega^2 = \frac{\Omega_{ic}^2}{2} \pm \frac{1}{2}\left(\Omega_{ic}^4 + 4\Omega_s^2\omega_{ci}^2\right)^{1/2}, \tag{12}$$

which reveals a linear coupling between the ESIC and DIA waves. Here, $\Omega_{ic} = (\omega_{ci}^2 + k^2C_s^2)^{1/2}$ and $\Omega_s = k_zC_s$ are the ESIC and DIA wave frequencies, respectively, $k^2 = k_\perp^2 + k_z^2$, with $\mathbf{k}_\perp$ and k_z being the components of $\mathbf{k}$ across and along $\hat{\mathbf{z}}$.

The plane-wave approximation has to be abandoned for 3D coupled ESIC and DIA waves that are twisted. Accordingly, we consider a twisted ESIC (T-ESIC) wave with the frequency ω_k and the magnetic field-aligned propagation wave number k_z, which has the potential structure of the form

$$\phi = \Phi(r)\exp(ik_zz - i\omega_kt), \tag{13}$$

where $\Phi(r)$ is a slowly varying function of z, and $r = (x^2 + y^2)^{1/2}$. By using Eq. (13), which reveals that a propagating T-ESIC wave will have a radial mode structure, we can write Eq. (11) in a paraxial approximation (viz. $\partial^2\Phi/\partial z^2 \ll k_z^2\Phi$) as

$$2i\frac{\partial\Phi}{\partial\xi} + \nabla_\perp^2\Phi = 0, \tag{14}$$

where $\xi = z\lambda_z^3/8\pi^3\rho_s^2(1 + 4\pi^2\rho_s^2/\lambda_z^2)$, $\lambda_z = 2\pi/k_z$, and $\rho_s = C_s/\omega_{ci}$ is the modified ion-acoustic gyroradius. In obtaining Eq. (14), we used $\omega_k = (\omega_{ci}^2 + 4\pi^2C_s^2/\lambda_z^2)^{1/2}$. Moreover, we have denoted the operator $\nabla_\perp^2\Phi = (1/r)(\partial/\partial r)(r\partial\Phi_l/\partial r) + (1/r^2)\partial^2\Phi_l/\partial\theta^2$ and introduced the cylindrical coordinates with $\mathbf{r} = (r,\theta,z)$. The choice of $\nabla_\perp^2\Phi$, as above, ensures that in 3D space dimension, a T-EIC wave will possess orbital angular momentum (OAM) because of the dependence of Φ on θ.

The solution of Eq. (14) can be written as a superposition of Laguerre-Gaussian (LG) modes [37–39], each of them representing a state of OAM, characterized by the quantum number l, such that

$$\Phi = \sum_{pl}\Phi_{pl}F_{pl}(r,z)\exp(il\theta), \tag{15}$$

where the mode structure function is

$$F_{pl}(r,z) = H_{pl}f^{|l|}L_p^{|l|}(f)\exp(-f/2), \tag{16}$$

with $f = r^2/w^2(\xi)$, and $w(\xi)$ is the ESIC beam width. The normalization factor H_{pl} and the associated Laguerre polynomial $L_p^{|l|}(f)$ are, respectively,

$$H_{pl} = \frac{1}{2\sqrt{\pi}}\left[\frac{(l+p)!}{p!}\right]^{1/2}, \tag{17}$$

and

$$L_p^{|l|}(f) = \frac{\exp(f)}{f^lp!}\frac{d^p}{df^p}[f^{l+p}\exp(-f)], \tag{18}$$

where p and l are the radial and angular mode numbers of the ESIC orbital angular momentum state. In a special case with $l = 0$ and $p = 0$, we have a Gaussian ESIC beam.

The LG solutions, given by Eq. (15), describe the salient feature of a twisted ESIC vortex (ESICV) beam carrying OAM ($l = \pm1, l = \pm2, \ldots$). In a twisted ESICV beam, the phase fronts rotate, clockwise for positive l values and anticlockwise for negative l values, around the beam's propagation direction in a spiral that looks like fusilli pasta (or a bit like a DNA double helix), creating an ESICV. The ESICV beam will have zero intensity at its center and be strongest at its edges, which cause orbital angular momentum. A twisted ESICV beam in a magnetized dusty plasma can occur naturally or it can be spontaneously created artificially by two oppositely propagating 3D ESIC waves that are colliding. Twisting of the ESIC waves occurs because different sections of the wavefront bounce off different steps, introducing a delay between the reflection of neighboring sections and, therefore, causing the wavefront to be twisted due to entanglement of the wavefronts.

In summary, we have shown that a 3D ESIC wave in a uniform dusty magnetoplasma can propagate as a twisted ESICV beam or as a braided electric potential structure. A twisted ESICV beam can trap the plasma particles and transport them from one region to another in planetary systems (e.g., through a braided electric potential structure in the F ring of Saturn [36] and in the forthcoming laboratory dusty magnetoplasma experiments [40]). A twisted ESICV beam can be identified as observational signatures of a rapidly rotating helical-shaped electric potential structure with twisted wavefronts, which can provide an alternative mechanism for the transport of the plasma particles in the planetary system, in the Earth's ionosphere, and in low-temperature laboratory experiments with strong external magnetic fields [40]. Furthermore, the present investigation of a twisted ESICV beam can also be exploited for diagnostic purposes, since the frequency and the magnetic field-aligned phase speed of a twisted ESIC wave are $(\omega_{ci}^2 + 4\pi^2C_s^2/\lambda_z^2)^{1/2}$ and $2\pi C_s/\lambda_z(\omega_{ci}^2 + 4\pi^2C_s^2/\lambda_z^2)^{1/2}$, respectively. In closing, we mention that long- and short-wavelength (in comparison with the ion gyroradius) twisted electrostatic dust-cyclotrons (T-ESDC) may also arise in a magnetized dusty plasma. Here, the electrons, ions, and charged dust particles are magnetized in strong magnetic fields (say of the order of several Tesla), and one has to account for the dust number density perturbation [41] on the timescale of the dust gyroperiod. The governing 3D equation for a T-ESDC wave will be different than Eq. (10), but essential solutions may resemble Eq. (14). A detailed investigation of a T-ESDC wave is outside the scope of the present Brief Report.

This research was partially supported by the Deutsche Forschungsgemeinschaft (DFG), Bonn, through Project No. SH21/3-2 of Research Unit 1048.

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