Plasma core at the center of a sonoluminescing bubble

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Considering high temperature and pressure during single bubble sonoluminescence collapse, a hot plasma core is generated at the center of the bubble. In this paper a statistical mechanics approach is used to calculate the core pressure and temperature. A hydrochemical model alongside a plasma core is used to study the bubble dynamics in two host liquids of water and sulfuric acid 85 wt % containing Ar atoms. Calculation shows that the extreme pressure and temperature in the plasma core are mainly due to the interaction of the ionized Ar atoms and electrons, which is one step forward to sonofusion. The thermal bremsstrahlung mechanism of radiation is used to analyze the emitted optical energy per flash of the bubble core.

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I. INTRODUCTION

Cavitation is the formation, growth, and collapse of microbubbles occurring in a small period of time, which subsequently results in large magnitudes of energy focusing [1]. Cavitation can be produced by acoustic waves, lasers [2,3], or particles such as a beam of neutrons [4]. Sonoluminescence (SL) is a physical phenomenon in which acoustic phonons are converted to optical photons. Single bubble sonoluminescence (SBSL) has been studied more by researchers [4] since its demonstration by Gaitan [5]. Multibubble SL (MBSL) [6,7] and moving SBSL were also studied extensively [8–11]. SBSL in cavitation physics plays the role of the hydrogen atom in atomic physics. The isolation of SBSL from perturbation and the fixed position of the bubble, without any translational movement, makes it simpler to study.

The gas atoms and intra-bubble molecules form plasma with broadband photon spectra, because of the extreme pressure and temperature at the collapse time of the SBSL. It is claimed that neutrons are detected during acoustic cavitation in deuterated acetone [12]; however, the results have been challenged by other researchers [13].

Recently, observation of the Ar atomic lines shows that optically opaque plasma is generated at the center of the bubble during cavitation [14–16]. It is claimed that high-energy electron impact from the hot opaque plasma core is responsible for the detected atomic and ionic lines. The generated plasma, in the core of a sonoluminescing bubble, has amazing features but its thermodynamic conditions cannot be deduced by emission spectroscopy. Direct measurement of the pressure and temperature of the plasma core is impossible; they can only be modeled.

In this paper, a statistical mechanics approach is used to model and calculate the temperature and the pressure of the generated plasma at the core of the SBSL. The calculations are substantially based on the hydrochemical model for dynamics of the bubble alongside a plasma core at the center of the bubble. It is found that the temperature and pressure of the plasma core are much higher than the other regions of the bubble, which is mainly due to the interaction of the ionized Ar atoms and electrons. The emitted spectral power radiance is also calculated using the bremsstrahlung mechanism of radiation for the plasma core. Our calculations of the emitted optical energy per flash of the bubble core are in good agreement with the available experimental data.

II. MODEL

Before proceeding further, it is profitable to review the available models of the SBSL. The isothermal-adiabatic model (adiabaticity behavior only near the minimum radius and isothermal for the rest of the cycle) is very simple; many parameters of the bubble cannot be found and some parameters of the host liquid and the bubble parameters, such as chemical reactions, are ignored [17,18].

Another way to investigate the SBSL is molecular dynamics (MD) simulation [19,20]. However, due to the essence of MD, which is time consuming and is only applicable to the systems that are smaller than their corresponding ones in reality, it is profitable to study SBSL using other models.

The hydrochemical model is another model which is used to study SBSL [21–23]. The time evolution of the bubble radius is described by

$$\left(1-\frac{\dot{R}}{C_l}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3C_l}\right)\dot{R}^2 = \left(1+\frac{\dot{R}}{C_l}\right)\frac{1}{\rho}(P_g-P_a-P_0)+\frac{\dot{R}}{\rho C_l}\dot{P}_g-4\mu\frac{\dot{R}}{R}-\frac{2\sigma}{\rho R}, \tag{1}$$

$$P_g=\frac{N_{\rm tot}k_BT}{V-N_{\rm tot}B}. \tag{2}$$

In these equations R is the bubble radius; $\dot{R}$ and $\ddot{R}$ are its time derivatives. ρ, C_l, μ, and σ are the density of liquid, the velocity of the sound wave, liquid viscosity, and liquid surface tension, respectively. P_0, P_a, and P_g are ambient pressure, driving pressure, and gas pressures, respectively. $N_{\rm tot}$, T, B, and k_B are the total number of particles inside the bubble, the gas temperature inside the bubble, the hard core parameter, and the Boltzmann constant, respectively.

Chemical reactions in the bubble, particle diffusion from the bubble wall, evaporation and condensation of molecules, heat conduction, and the bubble stabilities are considered in this model. By using this complete model, a lot of properties

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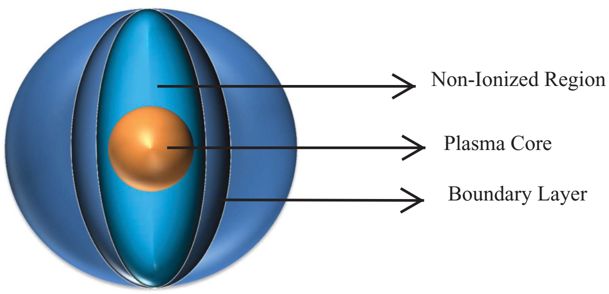

FIG. 1. (Color online) Three regions named as the boundary layer, nonionized, and the plasma core of the sonolominescencing bubble.

of the bubble can be investigated. This model of SBSL has been discussed in detail [21–23].

In this paper a hydrochemical-plasma model is proposed to study SBSL. As depicted in Fig. 1, the bubble is divided into three regions: (i) the outermost region (boundary layer)—the procedure of consideration of this region has been discussed previously [19–21]; (ii) the innermost area where the plasma core is created, which will be discussed later; and (iii) the area between these two regions where nonionized Ar atoms exist. There are N argon atoms in the bubble; only N^+ of them are ionized at the collapse instant. It should be noted that each region at the collapse time is assumed to be uniform and the bubble density is assumed to be homogeneous; it means $N/N^+ = V/V_p$. It is assumed that the Ar atom is partially ionized and a plasma core of volume V_p is produced at the center of the bubble. The Coulomb interaction between charged particles in a sphere of volume V_p at the center of the bubble is considered. Starting from the partition function of such system, one can derive the properties of the generated plasma. The derivation of the equation is discussed in the Appendix.

The total pressure of the plasma core is the sum of the calculated pressure from the plasma part and the van der Waals counterpart:

$$P_p^{\mathrm{tot}} = \frac{N_{\mathrm{tot}} k_B T}{V - N_{\mathrm{tot}} B} + \frac{2N^+ k_B T}{V_P} - \frac{e^2}{4\pi\varepsilon_0}\frac{N^+}{3V_P^{4/3}}. \quad (3)$$

In this equation ε_0 is vacuum permittivity and e is the electron charge.

The assumed equation of state of the plasma core is given by

$$T_p = \frac{P_p^{\mathrm{tot}}(V_p - N^+ B)}{N^+ k_B}. \quad (4)$$

The electron-ion bremsstrahlung is the radiation from an accelerated electron in the Coulomb field of an Ar ion. The spectral power radiance per unit volume of such radiation is given as follows [1]:

$$dP = 1.57 \times 10^{-40} q^2 n^2 A T^{-1/2} \lambda^{-2} \mathrm{e}^{-A/\lambda T} d\lambda, \quad (5)$$

where $A = 1.44 \times 10^{-2}$ mK; P, q, and λ are spectral power radiance per unit volume, first degree of ionization, and wavelength, respectively. In this equation n is the number density of the argon atoms.

The physical parameters of water and sulfuric acid (SA) 85 wt % are the same as in our previous paper [23]. The ultrasound frequency used in the calculations was 38 kHz and host liquids are saturated with 3 Torr partial pressure of Ar gas. Now, the entire model has been addressed and the calculation will be discussed in detail.

III. RESULTS AND DISCUSSIONS

In order to have stable bubbles, one should consider three dominant instabilities: shape, diffusion, and position instability. The procedure of considering these instabilities in the simulation has been discussed before [23]; it should be remembered that the boundary condition values are extracted from the phase diagrams in each case. A bubble saturated with 3 Torr argon is studied in this paper, due to the higher sonoluminescence radiation. In Fig. 2, the phase diagrams of water and SA 85 wt % saturated with 3 Torr Ar are shown. It should be realized that the SL region is the part of the diffusion curve with a positive slope and is below the shape and Bjerknes threshold instabilities. The maximum stable bubble phase parameter is determined from the intersection of the diffusion curve with a dominant instability curve in each case. The initial radii and acoustic pressures for the intersection points for water and SA 85 wt % are derived by this procedure and the selected points can give the brightest SBSL emission on the SBSL phase space. The expansion ratio, which is a measure of the intensity of bubble collapse, is determined by the Ar concentration relative to saturation concentration [24]. In these conditions the expansion ratios are around 8.

The time evolution of the bubble radius (Rayleigh-Plesset equation) and the generated plasma at the bubble center at the collapse instant, using the homogeneity assumption, is shown in Figs. 3(a) and 3(b). As one can see, the volume of the generated plasma core is only about 1% of the bubb-le at the time of collapse (the core radius is about one-fifth of the bubble itself). This suggests that the size of the formed core is much smaller than the related bubble size. Furthermore, it is noticed that the plasma radius in SA is about 2.5 times greater than that of water which leads to a larger plasma surface. Therefore, high-energy electron impact from the hot, opaque plasma core in SA is more probable than that of water.

The time evolution of the temperature of the bubble and the plasma core at the collapse time are depicted in Figs. 4(a) and 4(b) for water and SA 85%, respectively. These figures denote that the core temperature is approximately three times higher than the outside temperature. The difference between chemical

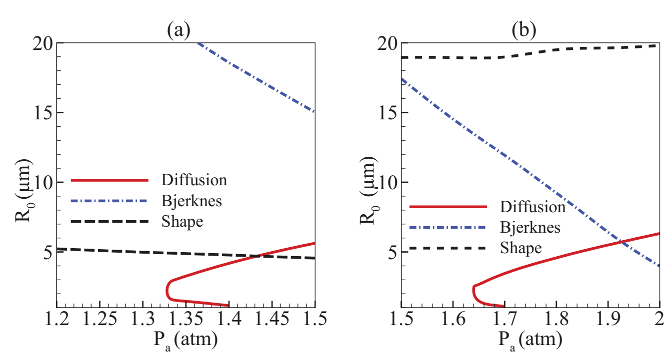

FIG. 2. (Color online) Phase diagrams in (R_0–P_a) space of the Ar bubble for (a) water and (b) SA 85 wt % at room condition.

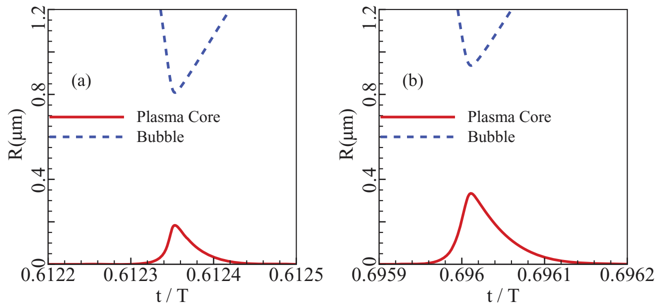

FIG. 3. (Color online) Time evolution of the bubble radius and the generated plasma at the bubble center at the collapse instant for (a) water and (b) SA 85 wt %.

and physical properties of water and SA causes the Ar bubble of the same expansion ratio to have different behavior in these host liquids. The increase in the SBSL temperature of SA is due to absorption of a larger amount of the collapse energy in the SA than that of water. The bubble in SA is greater and derived by higher acoustic pressure. As a consequence, the Ar bubble in SA has a larger volume and a hotter plasma core. In Figs. 4(c) and 4(d) the maximum pressure of the bubble and the plasma core at the collapse time are shown. One can notice that the pressure at the center of the bubble is much (approximately three times) larger than the pressure at the nonionized region.

Using Eq. (A10), the number of the ionized Ar atoms at the center of the bubble and the degree of ionization of Ar atoms is depicted in Figs. 5(a) and 5(b). It can be realized that the number of ionized argon atoms is about 1% of the total Ar atoms in the bubble. In Eq. (1) the plasma pressure is proportional to the number of ions; therefore, more ionized Ar leads to higher plasma pressure and consequently to greater

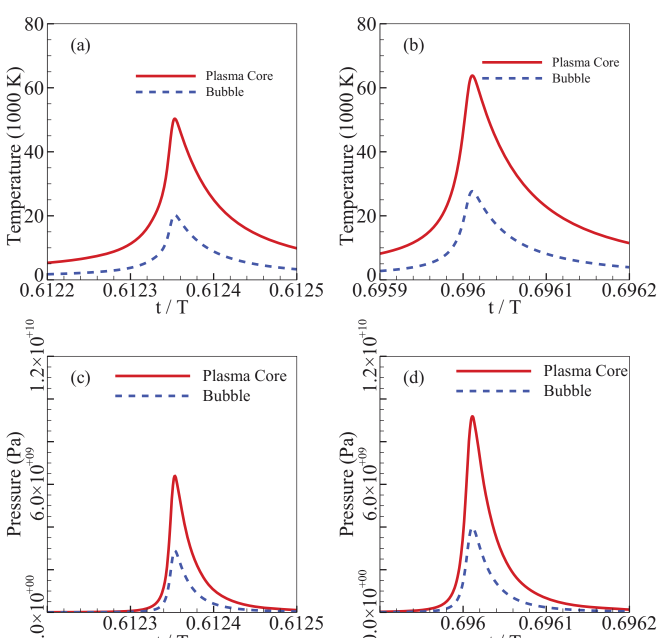

FIG. 4. (Color online) Thermodynamic properties of SBSL and the generated plasma core at its center. Time evolution of the temperature for (a) water and (b) SA 85 wt % and the time evolution of the pressure for (c) water and (d) SA 85 wt %.

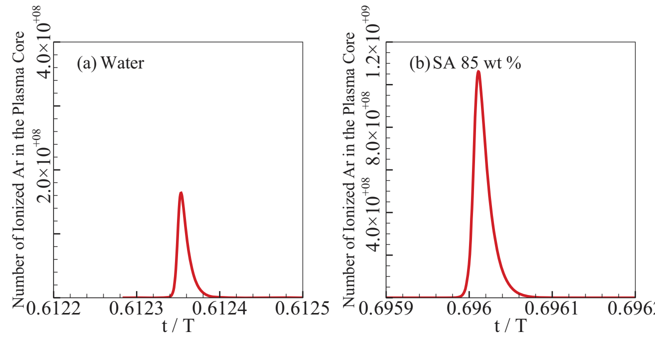

FIG. 5. (Color online) Number of ionized Ar atoms inside the plasma core of the SBSL bubble at the maximum achievable intensity for (a) water, (b) SA 85 wt %.

plasma pressure. Hence, the ionized Ar plays a central role in this approach.

The radiation from an accelerated electron in the Coulomb field of ionized Ar atoms is calculated using Eq. (5). The spectral power radiance can be integrated to obtain the optical energy per flash. The maximum emitted optical energy per flash for water and SA 85% are 35 pJ/flash and 624 pJ/flash, respectively. The results are interesting and in good accordance with the available experimental data which state that the emitted optical energy per flash for a sonoluminescing bubble is in the order of a picojoule [1].

IV. CONCLUSIONS

A hydrochemical-plasma model is proposed to study the generated plasma at the core of a sonoluminescing bubble and its thermodynamic conditions. By considering the effect of an inhomogeneous bubble interior one can reach the higher temperatures and pressures which results in higher bremsstrahlung SL radiation and spectral power radiance from the hot plasma core. The calculated temperatures result in reasonable amounts of radiation which are in better agreement with the experimental values. Based on the statistical approach, the extreme pressure and temperature are mainly due to the interaction of charged particles at the bubble center. Extending this approach to other host liquids such as deuterated acetones can produce much higher temperatures which may justify the claimed measured amount of neutron radiation from these systems.

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APPENDIX: CALCULATION OF THE THERMODYNAMIC PROPERTIES OF PLASMA CORE AT THE CENTER OF SBSL

As discussed before, the bubble is divided into three regions: the boundary layer, the plasma core, and the area between these two regions. A mixture of N^+ ionized Argon atoms and N^+ electrons in a sphere of volume V_p is assumed in the plasma core at the center of the sonoluminescing bubble (see Fig. 1). The energy is stored in a Coulomb interaction potential between these charged particles. According to this model, the charges are distributed in a continuous homogeneous medium. This distribution of charges within this medium gives rise to an electric potential in the form of $\Phi = \sum_{i<j}^{2N^+} \frac{c_i c_j}{|q_i - q_j|}$, in which q_i and q_j are the spatial coordinates of each charge and $c_i = +c_0 = \sqrt{\frac{e^2}{4\pi\varepsilon_0}}$ for $i = 1, \ldots, N^+$ and $c_i = -c_0 = -\sqrt{\frac{e^2}{4\pi\varepsilon_0}}$ for $i = N^+ + 1, \ldots, 2N^+$. The high density of the plasma core makes the Debye length smaller than $|q_i - q_j|$, which suggests that the cluster expansions are no longer valid; therefore, it is more convenient to solve the problem in a more general method.

The Hamiltonian of such system is

$$H = \sum_{i=1}^{2N^+} \frac{p_i^2}{2m_i} - \sum_{i<j}^{2N^+} \frac{c_i c_j}{|q_i - q_j|}, \tag{A1}$$

where p_i and m_i are the momentum and mass of particle i.

Recalling the definition of the partition from [25] the partition function of the system is

$$Z = \frac{1}{(N^+!)^2} \int \prod_{i=1}^{2N^+} \frac{d^3 q_i d^3 p_i}{h^3} e^{-\beta\left(\sum_{i=1}^{2N^+} \frac{p_i^2}{2m_i} - \sum_{i<j}^{2N^+} \frac{c_i c_j}{|q_i - q_j|}\right)}. \tag{A2}$$

In Eq. (A2) $\beta = 1/k_b T$. After taking the integral over momenta Eq. (A2) is written as below:

$$Z = \frac{1}{h^{6N^+}(N^+!)^2(\lambda_e \lambda_i)^{3N^+}} \int \prod_{i=1}^{2N^+} d^3 q_i \, e^{-\sum_{i<j}^{2N^+} \frac{\beta c_i c_j}{|q_i - q_j|}}, \tag{A3}$$

where $\lambda_e = \frac{h}{\sqrt{2\pi m_e k_b T}}$ and $\lambda_i = \frac{h}{\sqrt{2\pi m_{\text{ion}} k_b T}}$. In these equations m_e and m_{ion} are the mass of the electron and mass of the ion, respectively, and h is Planck's constant.

It is impossible to perform the integral over q_i exactly, but the dependence of Z on V_p can be obtained by a simple rescaling of coordinates $q_i' = q_i/l$.

$$Z = \frac{1}{h^{6N^+}(N^+!)^2(\lambda_e \lambda_i)^{3N^+}} \int \prod_{i=1}^{2N^+} l^3 d^3 q_i' \prod_{i<j}^{2N^+} e^{-\frac{\beta c_i c_j}{l|q_i' - q_j'|}}. \tag{A4}$$

There are N^{+2} pairs of opposite charges and the numbers of like pairs are $2\binom{N^+}{2} = N^+(N^+ - 1)$, so there are N^{+2} terms for which the interaction is attractive, $\beta c_i c_j = -\beta c_0^2$, and $N^+(N^+ - 1)$ terms for which the interaction is repulsive, $\beta c_i c_j = \beta c_0^2$. Thus

$$Z = \frac{(l^3)^{2N^+} e^{\frac{\beta N^+ c_0^2}{l}}}{h^{6N^+}(N^+!)^2(\lambda_e \lambda_i)^{3N}} \int \prod_{i=1}^{2N^+} d^3 q_i' \prod_{i<j}^{2N^+} e^{-\frac{1}{|q_i' - q_j'|}}, \tag{A5}$$

$$Z = \frac{V_P^{2N^+} e^{\frac{\beta N^+ c_0^2}{V_P^{1/3}}}}{h^{6N^+}(N^+!)^2(\lambda_e \lambda_i)^{3N^+}} Z_0, \tag{A6}$$

$$Z_0 = \int \prod_{i=1}^{2N^+} d^3 q_i' \prod_{i<j}^{2N^+} e^{-\frac{1}{|q_i' - q_j'|}}. \tag{A7}$$

Now the pressure of a sphere of interacting charged particles due to their interaction can be calculated as follows:

$$\begin{aligned} P_p &= \frac{1}{\beta}\frac{\partial \ln Z}{\partial V_P} = \frac{1}{\beta}\left(\frac{2N^+}{V_P} - \frac{\beta N^+ c_0^2}{3V_P^{4/3}}\right) \\ &= \left(\frac{2N^+ k_B T}{V} - \frac{N^+ c_0^2}{3V_P^{4/3}}\right) = \left(\frac{2N^+ k_B T}{V_P} - \frac{e^2}{4\pi\varepsilon_0}\frac{N^+}{3V_P^{4/3}}\right). \end{aligned} \tag{A8}$$

Therefore, the total pressure on the plasma core is due to the van der Waals and the interaction of charges which can be written as

$$P_p^{\text{tot}} = \frac{N_{\text{tot}} k_B T}{V - N_{\text{tot}} B} + \frac{2N^+ k_B T}{V_P} - \frac{e^2}{4\pi\varepsilon_0}\frac{N^+}{3V_P^{4/3}}. \tag{A9}$$

To calculate the number of the ionized Ar atoms at any time the Saha equation is used [26]; the first degree of ionization ($q = N^+/N$) and the number of free electron or ions inside the bubble core are

$$\frac{q^2}{1-q} = \frac{2.4 \times 10^{21} T^3 e^{-E_{\text{Ar}}/k_B T}}{N}, \tag{A10}$$

$$n_e = qN/V_p. \tag{A11}$$

In these equations, E_{Ar} is the ionization energy of Ar and n_e is the electron density in the plasma core.

[1] F. R. Young, *Cavitation* (McGraw-Hill, London, 1989).
[2] O. Baghdassarian, B. Tabbert, and G. A. Williams, Phys. Rev. Lett. **83**, 2437 (1999).
[3] R. Sadighi-Bonabi, F. Razeghi, H. Ebrahimi, S. Fallahi, and E. Lotfi, Phys. Rev. E **85**, 016302 (2012).
[4] M. P. Brenner, S. Hilgenfeldt, and D. Lohse, Rev. Mod. Phys. **74**, 425 (2002).
[5] D. F. Gaitan, L. A. Crum, C. C. Church, and R. A. Roy, J. Acoust. Soc. Am. **91**, 3166 (1992).
[6] F. R. Young, *Cavitation* (CRC Press, Boca Raton, FL, 2005).
[7] R. Sadighi-Bonabi, N. Rezaee, H. Ebrahimi, and M. Mirheydari, Phys. Rev. E **82**, 016316 (2010).
[8] Y. T. Didenko, W. B. McNamara, and K. S. Suslick, Nature (London) **407**, 877 (2000).
[9] S. D. Hopkins, S. J. Putterman, B. A. Kappus, K. S. Suslick, and C. G. Camara, Phys. Rev. Lett. **95**, 254301 (2005).
[10] R. Toegel, S. Luther, and D. Lohse, Phys. Rev. Lett. **96**, 114301 (2006).
[11] R. Sadighi-Bonabi, M. Mirheydari, N. Rezaee, and H. Ebrahimi, Phys. Rev. E **84**, 026301 (2011).
[12] R. P. Taleyarkhan, C. D. West, J. S. Cho, R. T. Lahey, Jr., R. I. Nigmatulin, and R. C. Block, Science **295**, 1868 (2002).
[13] B. Naranjo, Phys. Rev. Lett. **97**, 149403 (2006).
[14] D. J. Flannigan and K. S. Suslick, Nat. Phys. **6**, 598 (2010).

[15] D. J. Flannigan and K. S. Suslick, Phys. Rev. Lett. **95**, 044301 (2005).

[16] D. J. Flannigan and K. S. Suslick, Nature **434**, 52 (2005).

[17] S. Hilgenfeldt, S. Grossmann, and D. Lohse, Nature **398**, 402 (1999).

[18] S. Hilgenfeldt, S. Grossman, and D. Lohse, Phys. Fluids **11**, 1318 (1999).

[19] S. J. Ruuth, S. Putterman, and B. Merriman, Phys. Rev. E **66**, 036310 (2002).

[20] M. Matsumoto, K. Miyamoto, K. Ohguchi, and T. Kinjo, Prog. Theor. Phys. Suppl. **138**, 728 (2000).

[21] R. Toegel and D. Lohse, J. Chem. Phys. **118**, 1863 (2003).

[22] R. Toegel, S. Hilgenfeldt, and D. Lohse, Phys. Rev. Lett. **88**, 034301 (2002).

[23] Kh. Imani, F. Bemani, M. Silatani, and R. Sadighi-Bonabi, Phys. Rev. E **85**, 016329 (2012).

[24] G. E. Vazquez and S. J. Putterman, Phys. Rev. Lett. **85**, 3037 (2000).

[25] M. Kardar, *Statistical Physics of Particles* (Cambridge University Press, New York, 2007).

[26] Y. B. Zeldovich and Y. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic* (Academic Press, New York, 1966).