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Contrarian behavior in a complex adaptive system

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Contrarian behavior is a kind of self-organization in complex adaptive systems (CASs). Here we report the existence of a transition point in a model resource-allocation CAS with contrarian behavior by using human experiments, computer simulations, and theoretical analysis. The resource ratio and system predictability serve as the tuning parameter and order parameter, respectively. The transition point helps to reveal the positive or negative role of contrarian behavior. This finding is in contrast to the common belief that contrarian behavior always has a positive role in resource allocation, say, stabilizing resource allocation by shrinking the redundancy or the lack of resources. It is further shown that resource allocation can be optimized at the transition point by adding an appropriate size of contrarians. This work is also expected to be of value to some other fields ranging from management and social science to ecology and evolution.

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I. INTRODUCTION

Complex adaptive systems (CASs) [1] are a dynamic network of many agents (which may represent species, individuals, companies, nations, etc.) constantly acting and reacting to what the other agents are doing. Resource allocation is one of the most fundamental issues these agents are facing. In reality, they must compete against others for sharing limited resources with an unbiased or a biased distribution, in order to survive and develop. Thus, there are plenty of competitions in CASs [1–9]. The purpose of such competitions is essentially to get enough resources such as living spaces, food, money, etc. Examples include the survival competition among different species, the competition of individuals in stock markets, the competition of companies in the markets of different sizes, the competition of nations in world trade, and so on. For engaging in competitions, agents in the CASs often utilize various kinds of strategic behaviors, one of which is contrarian behavior. Contrarian behavior means figuring out what the herd is doing, and doing the opposite [10]. Contrarian behavior can be regarded as a kind of self-organization, which is one of the characteristics which distinguish CASs from other types of complex systems. To determine the nature of contrarian behavior is also of practical importance when one faces the relevant problems of resource allocation, say, risk evaluation and crisis management. Thus, contrarian behavior has been an active subject of studies in various fields like finance and economics [11], complexity science [12], and social science [13–16]. In social fields, previous contrarian studies using a Galam model of two-state opinion dynamics [13–15] aimed at the effect of contrarian choices on the dynamics of opinion forming, which shed a significant light on hung elections. In this work, we designed a procedure to study the effect of contrarian behavior on social resource allocation. It is a common belief that contrarian behavior always stabilizes resource allocation by shrinking the redundancy or the lack of resources (positive role). However, is this common belief true? Here we specially raise this question because unbiased or biased distributions of resources are everywhere in nature where contrarians are often needed. In other words, to comply with the real world, we need to investigate the role of contrarian behavior as the environment (which here is defined by the ratio between two resources, namely, resource ratio) varies.

The above-mentioned CASs involving competitions of agents for various kinds of resources can be modeled as a typical class of artificial well-regulated market-directed resource-allocation systems (simply denoted as "resource-allocation systems" in the following) [7,8], as an extension of the original minority game [5]. Such resource-allocation systems can reflect some fundamental characteristics of the above CASs in the real world [4–8], say, a resource-allocation balance that emerged as a result of system efficiency [7,8]. Thus, without loss of generality, we shall investigate the role of microscopic agents' contrarian behavior in the macroscopic properties of the resource-allocation system. In the process, we identify a class of transition points which help to distinguish the positive role (stabilizing, etc.) and the negative role (unstabilizing, etc.) of contrarian behavior for an unbiased or weakly biased and a strongly biased distribution of resources, respectively. Comparing our work with the contrarian study by Galam [13], which also shows the transition point at a critical value of the contrarian proportion to identify opinion group forming, here the transition points in this work help us to reveal that the allocation of resources can be optimized at the transition point by adding an appropriate size of contrarians which is observed in human experiments. To proceed, based on the extensively adopted methods of both statistical analysis [17–20] and agent-based modeling [5–8,21,22], we shall resort to three complementary tools: human experiments (producing data for statistical analysis), heterogeneous-agent-based computer simulations (of agent-based modeling), and statistical-mechanics-based theoretical analysis (of agent-based modeling).

II. HUMAN EXPERIMENTS

We design and conduct a series of computer-aided human experiments on the basis of the resource-allocation system [4–8]. As revealed in Refs. [7,8], the system can reach a macroscopic dynamic balance that corresponds to the most stable state where the resources are allocated most efficiently

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and the total utilities of the system are maximal due to the absence of macroscopic arbitrage opportunities. Here we add a proportion of contrarians, in order to observe how contrarian behavior affects the macroscopic properties of the resource-allocation system. For the experiments, we recruited 171 subjects, all of whom are students and teachers from several departments of Fudan University. The experiments were conducted in a big computer laboratory, and each subject had a computer to work with. All of the subjects were given a leaflet interpreting how the experiment would be performed before the experiment started. In the computer-aided online experiment, there are two virtual rooms: Room 1 and Room 2. Each room owns a certain amount of resources marked as M_1 or M_2 accordingly. The subjects do not know the exact resource ratio, M_1/M_2, at every experimental round. In the experiment, any kind of communication is not allowed, and every subject chooses to enter Room 1 or Room 2 independently to share the resources in it. Meanwhile, the computer program secretly adds contrarians into the system whose behaviors are controlled by the following settings. In every round of the experiment, each contrarian randomly chooses five subjects as his or her group. Then the contrarian will choose to enter the less-entered room according to the group. For example, if most of the subjects in a contrarian's group choose to enter Room 1, the contrarian will choose to enter Room 2. The total number of the subjects and the contrarians entering Room 1 and Room 2 are denoted as N_1 and N_2, respectively. After every experimental round, if $M_1/N_1 > M_2/N_2$, we say Room 1 (or Room 2) is the winning (or losing) room, because the subjects and contrarians entering Room 1 obtain more resources per capita, and vice versa. The subjects in the winning room will be granted 10 scores, and those in the losing room will be given 0 score. The final rewards are based on the scores each subject obtains in all the experimental rounds according to the exchange rate: 10 scores = 1 Chinese RMB. In addition, we will pay every subject 30 Chinese RMB as the attendance fee, and reward the top 10 subjects (having the highest scores), each with extra 100 Chinese RMB. More details are explained in the Appendix.

In the experiment, we adjusted two parameters: one is the resource ratio, M_1/M_2, and the other is the ratio between the number of contrarians and subjects, β_c. Thirty experimental rounds are repeated under each parameter set: M_1/M_2 and β_c. Let us denote the number of subjects as N_n and the number of contrarians as N_c, thus yielding $\beta_c = N_c/N_n$. In addition, the total number of all the subjects and contrarians is $N = N_n + N_c = N_1 + N_2$.

The experiment was conducted on two successive days: 88 subjects on the first day and 83 on the second day. The different number or different subjects show no influence on the results of the experiment. The experimental results are shown in Fig. 1, where $\langle N_1\rangle/\langle N_2\rangle$ is plotted as a function of M_1/M_2. When the distribution of resources is weakly biased up to $M_1/M_2 = 3$, the experimental results of $\langle N_1\rangle/\langle N_2\rangle$ are approximately located on the line with slope = 1 for the three values of β_c. In such cases, the system reaches dynamic balance at which the total utilities of the system are maximal due to the elimination of the macroscopic arbitrage opportunities. Nevertheless, for the strongly biased resource ratio, say, $M_1/M_2 = 10$, the balance is broken as shown by the three experimental values that deviate far from the "slope = 1" line. In other words, as the resource ratio is unbiased or weakly biased, adding a small proportion of contrarians does not hurt the system balance. In contrast, as the resource ratio is biased enough, the contrarians of the same proportion break the balance instead.

Then we analyze the experimental results from both individual and overall aspects of preference. As we know,

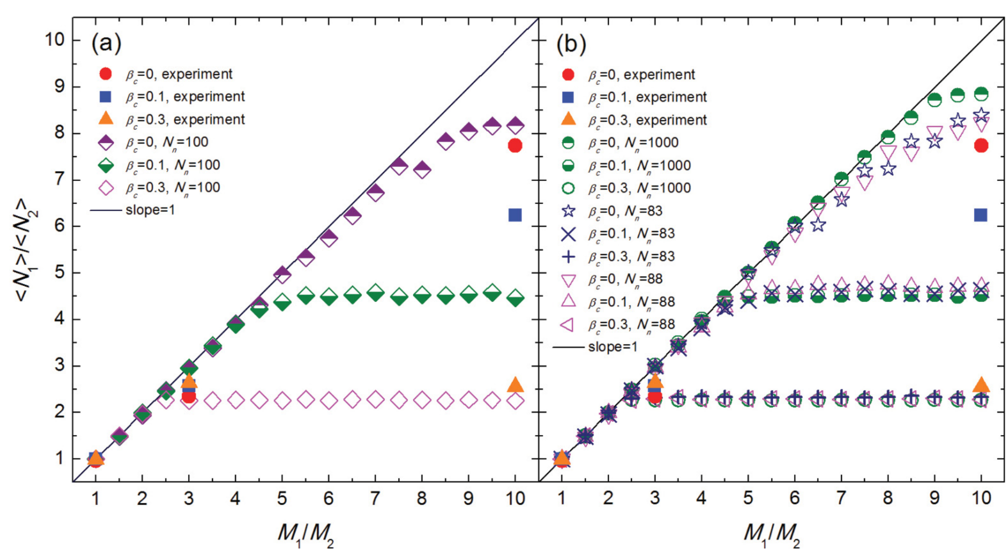

FIG. 1. (Color online) Population ratio, $\langle N_1\rangle/\langle N_2\rangle$, as a function of resource ratio, M_1/M_2. The line with "slope = 1" indicates the balance state where $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2$. Each experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics). Simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In (a), the number of normal agents in simulations is 100. In (b), the numbers of normal agents are respectively 1000, 83, and 88. $\langle\cdots\rangle$ denotes the average over the last 24 rounds for the experiment or the last 200 time steps for the simulations. The three experimental data at $M_1/M_2 = 1$ are overlapped. Parameters for the simulations: $S = 8$ and $P = 64$.

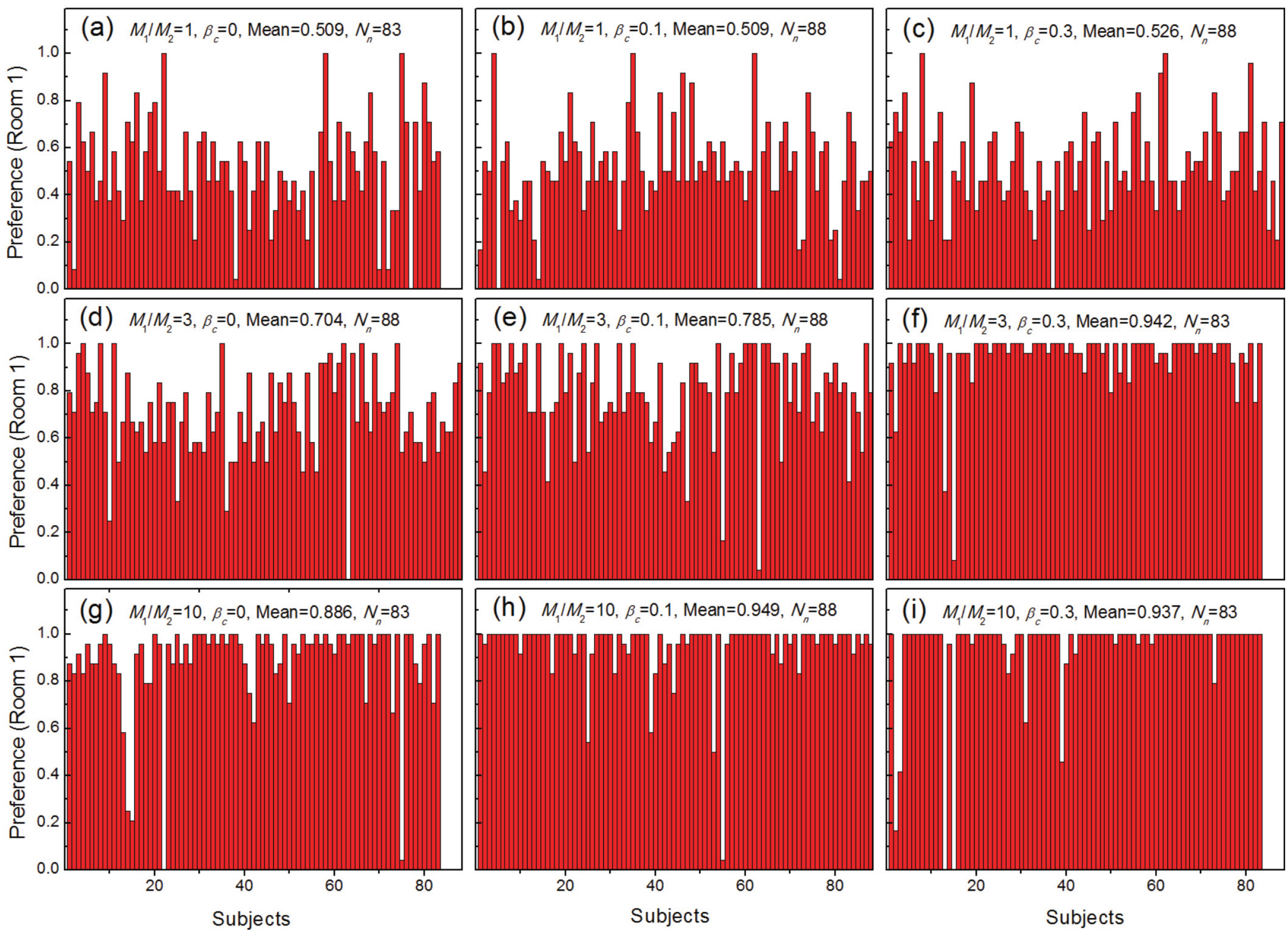

FIG. 2. (Color online) Experimental data of the preference of each subject to Room 1 for the nine parameter sets with $M_1/M_2 = 1$ (a–c), 3 (d–e), and 10 (g–i) and $\beta_c = 0$ (a, d, g), 0.1 (b, e, h), and 0.3 (c, f, i). For each parameter set, the experiment lasts for 30 rounds (the first six rounds for equilibration and the last 24 rounds for statistics). In the figure, "Mean" denotes the average preference of all the subjects.

different individuals have different preferences for a resource, which reflects heterogeneity of preferences. The heterogeneity has a remarkable influence on achieving the balance of the system. Here, the preference of each subject is defined as his or her average rate of entering Room 1 in the 30 rounds of experiments. The statistical results are shown in Fig. 2. Figure 2(a) shows the result for $M_1/M_2 = 1$ and $\beta_c = 0$. The preferences of the subjects are different except for the unbiased distribution of the two resources, $M_1/M_2 = 1$. We see that the third subject preferred Room 1 while the second player preferred Room 2. Such heterogeneity of preferences remains after introducing contrarians in Figs. 2(b)–2(c). As for the larger resource ratios in Figs. 2(d)–2(f) and Figs. 2(g)–2(i), the subjects still have different preferences. However, the average preference of all the subjects varies with M_1/M_2, which illustrates the environmental adaptability of the subjects.

Next, in order to clearly observe the influence of contrarians on the macroscopic system, we calculated the stability of the system, $f = \frac{1}{2N}\sum_{i=1}^{2}\langle (N_i - \tilde{N}_i)^2\rangle$ [7], where $\langle\cdots\rangle$ denotes the average of time series $\cdots$. This definition describes the fluctuation in the room population away from the balance state at which the optimal room population, $\tilde{N}_i = M_i N/(M_1 + M_2)$, can be realized. Clearly the smaller value of f is, the closer the system approaches to the dynamic stability. Figure 3(a) displays that, for small M_1/M_2, the fluctuations of the system decrease after introducing contrarians. Namely, the system becomes more stable. However, for large M_1/M_2, adding contrarians makes the system more unstable. Thus, we generally conclude that M_1/M_2 has a threshold, which distinguishes the different role of contrarians in the stability of the system. This experimental phenomenon will be further interpreted in the following part about the computer simulations and theoretical analysis about transition points.

To further evaluate the performance of the overall system, we have also calculated the efficiency and the predictability of the resource-allocation system. Here the efficiency is defined as $e = |\frac{\langle N_1\rangle}{\langle N_2\rangle} - M_1/M_2|/(M_1/M_2)$ [7]. Evidently, a larger value of e means a lower efficiency of resource allocation and vice versa. Figure 3(b) shows the change of e when adding contrarians into the experiment. When M_1/M_2 is 1 or 3, the adding of contrarians makes the resource-allocation system more efficient. However, for $M_1/M_2 = 10$, the presence of contrarians reduces the efficiency. Figure 3(c) shows the predictability of the system, which is represented by the

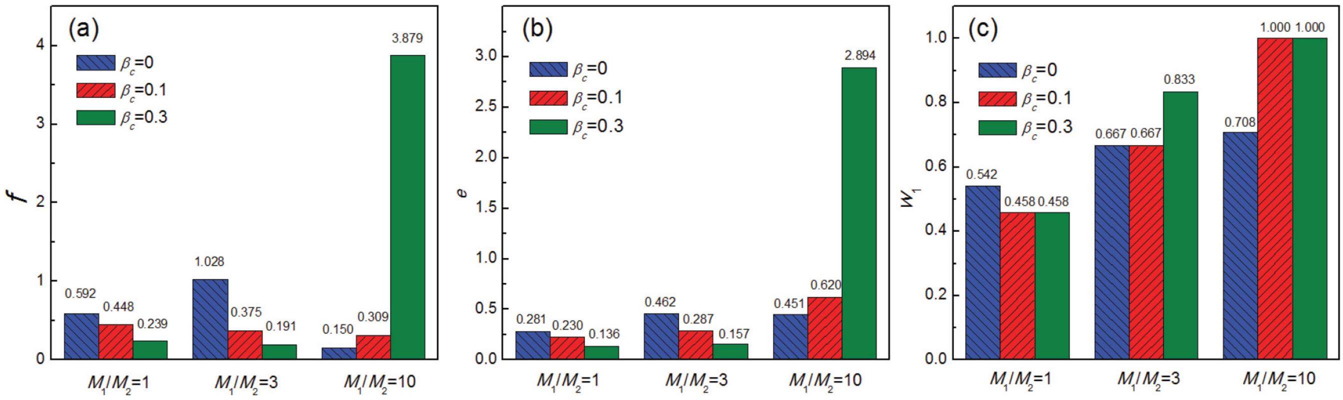

FIG. 3. (Color online) Experimental data for (a) stability f, (b) efficiency e, and (c) predictability w_1 at $M_1/M_2 = 1$, 3, and 10. Each experiment lasts for 30 rounds (the first six rounds for equilibration and the last 24 rounds for statistics).

winning rate of Room 1, w_1 [8]. Note $w_1 = 0.5$ means the winning rate is the same for both Room 1 and Room 2, which is hard for the subjects to predict. If w_1 deviates from 0.5, the winning rate of one room is higher than the other, so the subjects can predict the results easily. According to Fig. 3(c), when $M_1/M_2 = 1$, the winning rate w_1 fluctuates around 0.5, which means it is hard to make the prediction. But if M_1/M_2 becomes larger, the subjects are easy to predict for the winning room for the next round, especially when enough contrarians are added.

III. HETEROGENEOUS-AGENT-BASED COMPUTER SIMULATIONS

Clearly the above experiment has some unavoidable limitations: specific time, specific experiment avenue (a computer room in Fudan University), specific subjects (students and teachers of Fudan University), and the limited number of subjects. Now we are obliged to extend the experimental results (Figs. 1–3) beyond such limitations. For this purpose, we establish an agent-based model on the basis of the resource-allocation system. In this model, we denote N_n as normal agents and N_c as contrarians. Normal agents correspond to the subjects in the experiment, and each of them decides to enter one of the two rooms using their strategy table, which is the same as the one designed in the agent-based model of a market-directed resource-allocation game [7,8]. In particular, the table of a strategy is constructed by two columns. The left column represents P potential situations, and the right column is filled with 0 and 1 according to the integer, L, which characterizes the heterogeneity in the decision making of normal agents. For a certain value of L, ($L \in [0,P]$), there is a probability of L/P to be 1 in the right column of the table and a probability of $(P - L)/P$ to be 0. Here 0 and 1 represent entering Room 2 and Room 1, respectively. At each time step, normal agents choose to enter a room according to the right column of the strategy tables directed by the given situation P_i, ($P_i \in [1,P]$). Before the simulation starts, every normal agent will randomly choose S strategy tables, each determined by an L. At the end of every time step, each normal agent will score the S strategy tables by adding 1 (or 0) score if the strategy table predicts correctly (or incorrectly). Then, the strategy table with the highest score will be used for the next time step. In addition, because contrarians have no strategy tables, their behavior is set to be the same as that already adopted in the experiment.

For the computer simulations, we use 100 normal agents and set $S = 8$ and $P = 64$. The result of $\langle N_1\rangle/\langle N_2\rangle$ versus M_1/M_2 is shown in Fig. 1(a). Clearly, qualitative agreement between experiments and simulations is displayed. In order to confirm this result, we conduct more simulations with different numbers of normal agents to compare with experimental results, which are shown in Fig. 1(b). We choose to use 83 and 88 normal agents, which are consistent with experiments, and 1000 normal agents, which represent the case of a remarkable different size. Comparing the different simulations in Figs. 1(a) and 1(b), their results show no qualitative differences though the number of normal agents varies. Therefore, we can say that the number of agents has no influence on our simulation results. This means that the experimental results reported in Fig. 1 are general (at least to some extent), being beyond the above-mentioned experimental limitations. Thus, we are confident to do more simulations in the following. For convenience, we use 100 normal agents in the remainder of this work.

In order to compare with the experiment, the preferences of 100 normal agents are also calculated; see Fig. 4. The simulation results are very similar to the experimental results in Fig. 2. That is, normal agents also show the heterogeneity of preferences and the environmental adaptability.

Then we are in a position to scrutinize the role of contrarians. To compare with the experimental results in Fig. 3, we also calculate stability (f), efficiency (e), and predictability (w_1); see Fig. 5.

From Fig. 5, we find that the resource-allocation system clearly exhibits a transition point when taking M_1/M_2 and w_1 as the tuning parameter and order parameter, respectively. This agrees with what we have reported in Ref. [8]. In the meantime, at the transition point, $(M_1/M_2)_t$, f reaches the lowest value, which means the system becomes the most stable. In detail, for a small β_c, increasing M_1/M_2 will increase the system stability until f has the minimum value at $(M_1/M_2)_t$, which corresponds to the most stable state of the system. Once the minimum value is passed, the stability of the system will worsen for larger M_1/M_2. The former (or the latter) is positive

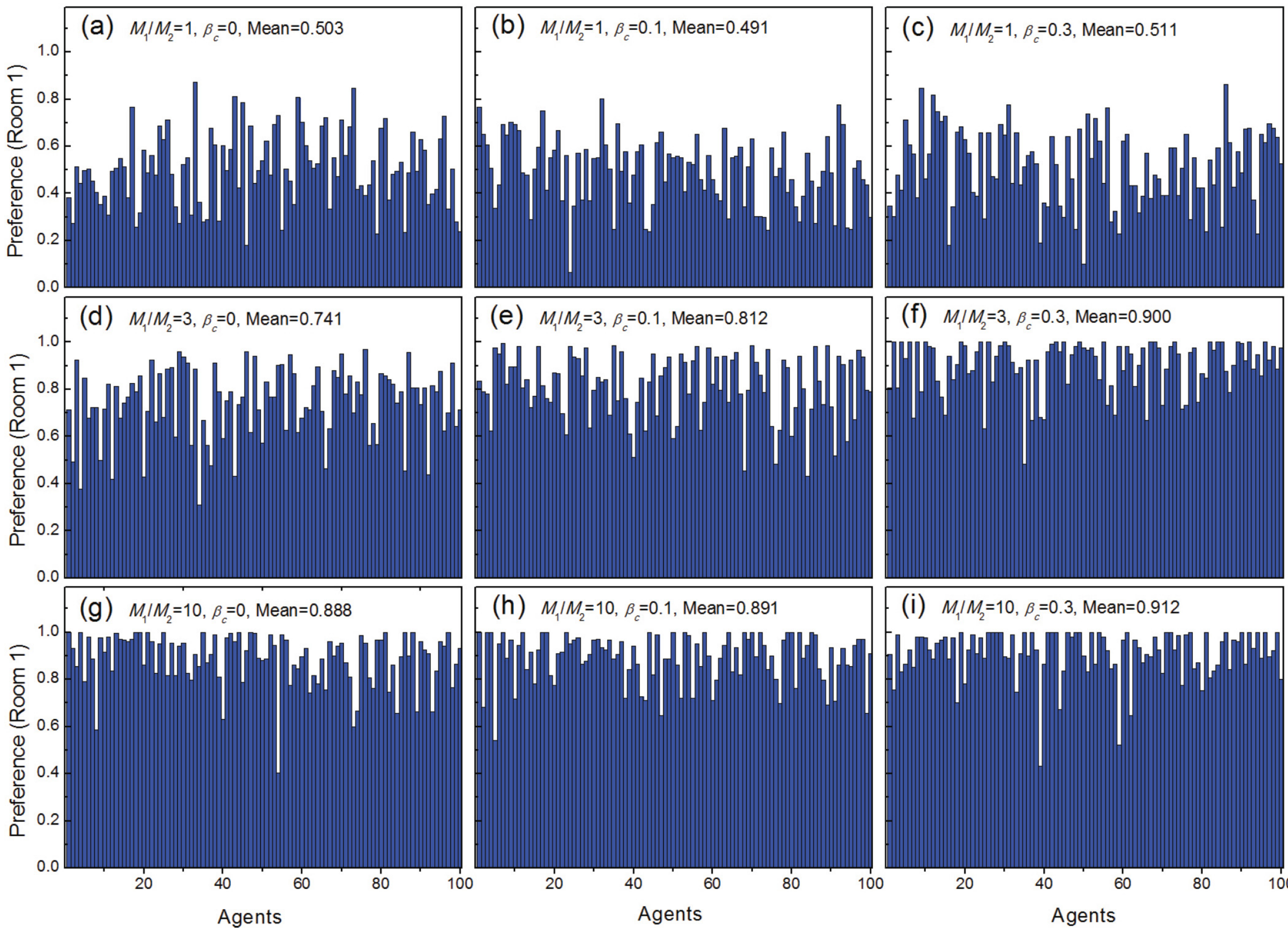

FIG. 4. (Color online) Simulation data of the preference of each normal agent to Room 1 for the nine parameter sets with $M_1/M_2 = 1$ (a–c), 3 (d–e), and 10 (f–i) and $\beta_c = 0$ (a, d, g), 0.1 (b, e, h), and 0.3 (c, f, i). For each parameter set, simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In the figure, "Mean" denotes the average preference of the 100 normal agents.

(or negative) role of contrarians. As for large β_c, increasing M_1/M_2 will always make the system more unstable (negative role). In addition, as β_c increases, $(M_1/M_2)_t$ moves toward the direction of decreasing M_1/M_2. We shall discuss the movement of $(M_1/M_2)_t$ in the following theoretical analysis.

Figure 5(b) shows the simulation results for the change of system efficiency, e. When M_1/M_2 is small, increasing contrarians can make the system more efficient at a certain range. In contrast, for large M_1/M_2, adding contrarians always reduces the efficiency. Such simulation results echo with those experimental results as shown in Fig. 3(b).

Figure 5(c) displays the predictability of Room 1. Similarly, we can see from Fig. 5(c) that when M_1/M_2 is very small (close to 1), the winning rate of two rooms remains almost unchanged at 0.5 or so, even though β_c varies. That is, in this case, the system is unpredictable. When M_1/M_2 is gradually increasing, adding more contrarians will cause w_1 to increase from the value for $\beta_c = 0$; namely, it becomes more easy for the agents to predict the winning room. Again, these simulation results agree with those experimental results in Fig. 3(c).

Now, we can understand the role of contrarians in the resource-allocation system. On one hand, contrarians have positive roles as M_1/M_2 is small. Namely, adding contrarians can help to not only improve the system stability, but also increase the system efficiency while keeping the system unpredictable. On the other hand, contrarians have negative roles as M_1/M_2 becomes large enough. That is, adding contrarians can hurt the system stability and efficiency while making the system more predictable. Both positive and negative roles have been well distinguished by identifying a transition point, $(M_1/M_2)_t$. Further, it is clear that the transition points identified herein also help to reveal that the allocation of resources can be optimal (i.e., stable, efficient, and unpredictable) at $(M_1/M_2)_t$ by adding an appropriate size of contrarians.

IV. STATISTICAL-MECHANICS-BASED THEORETICAL ANALYSIS

In order to get a better understanding of the underlying mechanics of the agent-based model, we conduct theoretical analysis. When S and P are fixed, the system of our interest could reach the most stable state only at the transition point, i.e., a particular ratio between the two resources, $(\frac{M_1}{M_2})_t$. If we

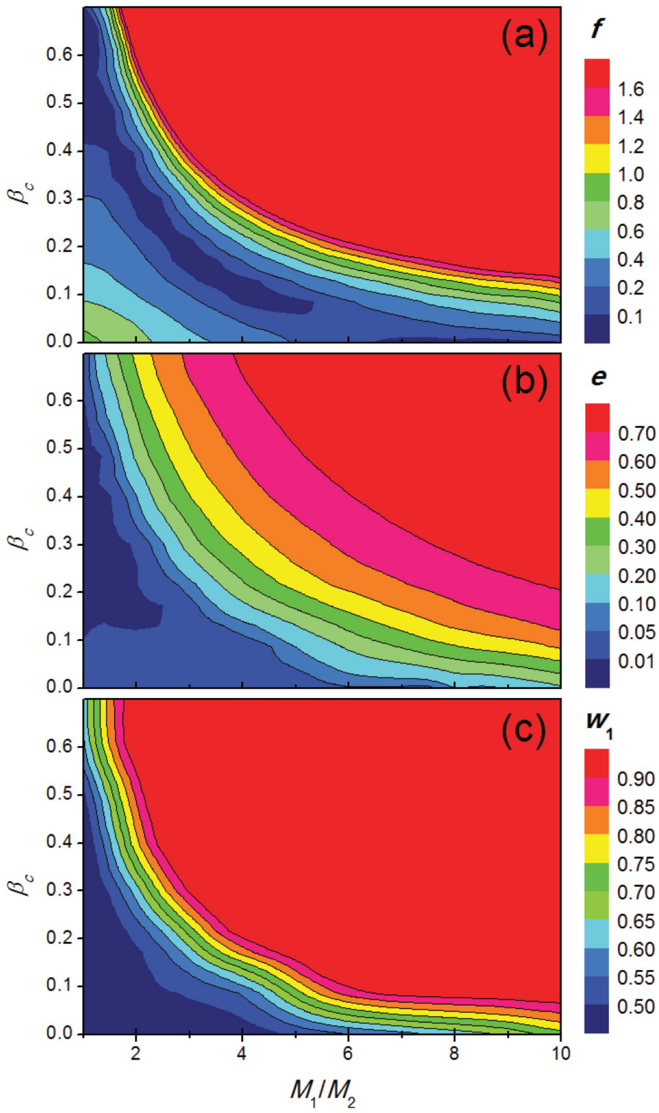

FIG. 5. (Color online) $\beta_c - M_1/M_2$ contour plots for (a) stability f, (b) efficiency e, and (c) predictability w_1. For each parameter set, simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration).

adjust the values of β_c, the transition point, $(\frac{M_1}{M_2})_t$, will change accordingly.

A. The properties of the transition point, $(\frac{M_1}{M_2})_t$

(a) Without contrarians: It can be proven that for the agent-based model, the transition point has two properties: 1) every normal agent uses the strategy with the largest preference, $(L_i)_{\max}$, in his or her hand; 2) the system is in the balance state, which means the ratio between the numbers of agents in the two rooms is equal to the ratio between the two resources [7]. We first define

$$N_1 = \sum x_i,$$

where the choice of agent i is denoted as $x_i = 1$ (Room 1) or 0 (Room 2). Then, at the transition point, the expected ratio of normal agents who chooses to enter Room 1 is

$$\frac{\langle N_1\rangle}{N_n} = \frac{\sum\langle x_i\rangle}{N_n} = \frac{\sum_i^{N_n}(L_i)_{\max}}{PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_t, \quad (1)$$

where $\langle\cdots\rangle$ denotes the averaged value of $\cdots$. Equation (1) shows that when $\frac{M_1}{M_1+M_2} > (\frac{M_1}{M_1+M_2})_t$, Room 1 will become unsaturated. This means the system does not stay at the balance state.

(b) With contrarians: From the properties of the transition point and the behavior of the contrarians, it can be shown that, all the normal agents still use the largest-preference strategy $(L_i)_{\max}$ at the transition point when contrarians are added. Every contrarian follows the minority in his or her group to make a choice denoted as x_c. Then the expected ratio of agents (both normal agents and contrarians) who choose to enter Room 1 at the transition point becomes

$$\frac{\langle N_1\rangle}{N} = \frac{\sum_i^{N_n}(L_i)_{\max} + P\sum_c^{N_c}\langle x_c\rangle}{(1+\beta_c)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}, \quad (2)$$

where $\beta_c = \frac{N_c}{N_n}$ and $(\frac{M_1}{M_1+M_2})_{t'}$ stands for the new transition point with contrarians added.

B. Finding the expressions of $\sum_i^{N_n}(L_i)_{\max}$ and $\sum_c^{N_c}\langle x_c\rangle$

(a) Without contrarians: The probability that L_i takes a certain integer from the range 0 to P is $\frac{1}{P+1}$. Then, the probability of $(L_i)_{\max}$ being a certain value of L is

$$p(L) = \left(\frac{L+1}{P+1}\right)^S - \left(\frac{L}{P+1}\right)^S.$$

If N_n is large enough, there is

$$\sum_i^{N_n}(L_i)_{\max} = \sum_{L=0}^{P} N_n p(L)L$$
$$= PN_n\left[1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S\right]. \quad (3)$$

In the absence of contrarians, the substitution of Eq. (3) into Eq. (1) leads to

$$\frac{\langle N_1\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n, \quad (4)$$

where m_n represents the transition point for the system with only normal agents.

(b) With contrarians: Since the normal agents still use their strategy with $(L_i)_{\max}$ at the transition point after adding contrarians into the resource-allocation system. Therefore, for normal agents, we have

$$\frac{\langle N_{n1}\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n.$$

When contrarian c chooses k normal agents as his or her group, the probability to get a normal agent who chooses Room 1 can be expressed approximately as $\frac{\langle N_{n1}\rangle}{N_n} = (\frac{M_1}{M_1+M_2})_t \equiv m_n$. Then, the probability for $x_c = 1$ (or 0) is

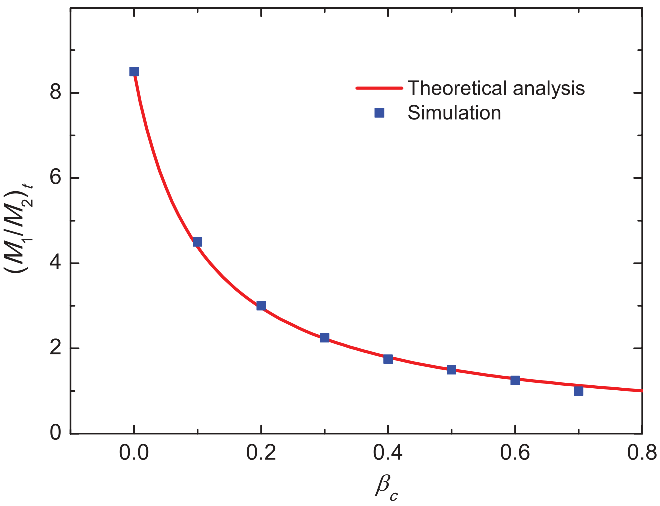

FIG. 6. (Color online) Transition point $(M_1/M_2)_t$ versus β_c, as a result of theoretical analysis [curve obtained according to Eq. (6)] and simulation [data extracted from Fig. 5(a)]. Parameters: $S = 8$ and $P = 64$.

$\sum_{q=0}^{y} C_k^q [(\frac{M_1}{M_1+M_2})_t]^q [(\frac{M_2}{M_1+M_2})_t]^{k-q} \equiv m_c$ (or $1 - m_c$), where $(\frac{M_2}{M_1+M_2})_t = 1 - m_n$, $y = \frac{k-1}{2}$, and k is odd. Thus, we have the average of x_c, $\langle x_c \rangle$, as

$$\langle x_c \rangle = m_c = \sum_{q=0}^{y} C_k^q (m_n)^q (1 - m_n)^{k-q}. \quad (5)$$

Plugging Eq. (5) into Eq. (2) yields

$$\frac{\langle N_1 \rangle}{N} = \frac{P N_n m_n + P N_c m_c}{(1 + \beta_c) P N_n} = \left(\frac{M_1}{M_1 + M_2} \right)_{t'},$$

and then we have

$$\frac{\langle N_1 \rangle}{N} = \frac{m_n + \beta_c m_c}{1 + \beta_c} = \left(\frac{M_1}{M_1 + M_2} \right)_{t'}. \quad (6)$$

Clearly, by adjusting β_c, we can change the transition point of the resource-allocation system. Figure 6 shows the monotonically decreasing trend of $(\frac{M_1}{M_2})_t$ for increasing β_c, which displays an excellent agreement between theoretical and simulation results.

In both experiments and computer simulations, we have found that when the system is in the balance state [$M_1/M_2 < (M_1/M_2)_t$], the fluctuations of the system decrease after introducing a small number of contrarians. Because in both experiments and simulations, the behavior of contrarians is set to follow the same rule, it is necessary to further analyze the influence of this behavior on the stability of the whole system. Equation (5) describes the probability of contrarians choosing to enter Room 1 when the system reaches balance. It is known that at this balance state, the number of subjects in the experiments (or normal agents in the simulations) choosing to enter each room still varies at every time step due to fluctuations. Hence we replace m_n in Eq. (5) with N_{n1}/N_n and get $\langle x_c \rangle = \sum_{q=0}^{y} C_k^q (N_{n1}/N_n)^q (1 - N_{n1}/N_n)^{k-q}$, where N_{n1} is the number of subjects or normal agents who choose to enter Room 1, and the average of x_c, $\langle x_c \rangle$, represents the expected probability of contrarians choosing Room 1. Note that $\langle x_c \rangle$ is a random variable due to the fluctuations of N_{n1}.

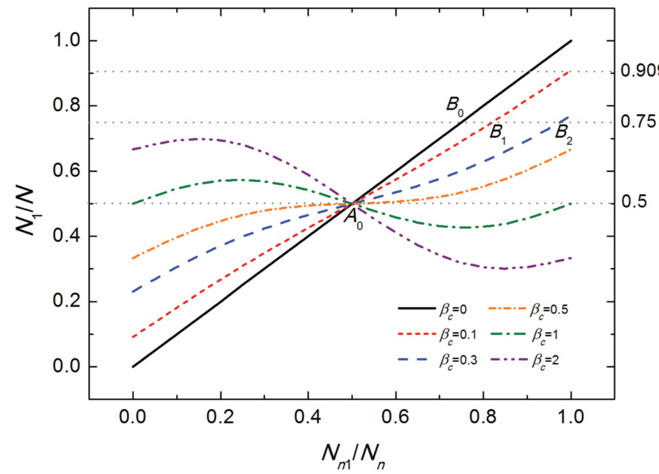

FIG. 7. (Color online) N_1/N versus N_{n1}/N_n according to Eq. (7). The three horizontal gray dot lines are given by $N_1/N = 0.5$, 0.75, and 0.909, which are respectively related to the balance state of three resource ratios, $M_1/M_2 = 1$, 3, and 10.

Then, according to Eq. (6), we obtain

$$\frac{N_1}{N} = \frac{N_{n1}/N_n + \beta_c \langle x_c \rangle}{1 + \beta_c}. \quad (7)$$

By drawing N_1/N versus N_{n1}/N_n, we achieve Fig. 7, which shows the influence of the deviations of N_{n1}/N_n on N_1/N under different values of β_c. For $M_1/M_2 = 1$, it is shown that the balance point of the system lies on A_0 (0.5,0.5) when $\beta_c = 0, 0.1, 0.3$, and 0.5. And the deviations of N_{n1}/N_n can cause the system to vibrate around A_0 along a certain line in Fig. 7, which is determined by β_c. Then, Fig. 7 shows that, under the same range of deviations of N_{n1}/N_n, by increasing β_c, we can bring down the vibration of N_1/N around $N_1/N = 0.5$. In addition, we can see from Fig. 7 that, when β_c becomes too large, such as $\beta_c = 1$ or 2, A_0 is no longer a stable point. The state of the system tends to move to the right end of the associated line because now more subjects or normal agents choosing to enter Room 1 will make Room 1 easier to win. That is, when β_c is too large, adding more contrarians will lead the system to a more unstable state. For a biased distribution of resources, say, $M_1/M_2 = 3$, Fig. 7 shows that the balance point of the system lies on different points for different values of β_c, i.e., B_0 (0.75,0.75), B_1 (0.82,0.75), and B_2 (0.97,0.75) for $\beta_c = 0$, 0.1, and 0.3. It can be shown that adding a small number of contrarians makes the system with a biased distribution of resources more stable due to the following two reasons: 1) under the same deviations of N_{n1}/N_n, the vibration of N_1/N (say, around B_0, B_1, or B_2 for $M_1/M_2 = 3$) decreases slightly when adding more contrarians; and 2) when adding more contrarians, the values of N_{n1}/N_n at the balance points (e.g., B_0, B_1, and B_2 for $M_1/M_2 = 3$) increase; in this case, subjects or normal agents will be more certain to choose Room 1, which reduces the deviation range of N_{n1}/N_n, thus decreasing the vibration of N_1/N.

V. CONCLUSION

In summary, using the three tools, we have investigated the role of contrarian behavior in a resource-allocation system. In contrast to the common belief that contrarian behavior always plays a positive role in resource allocation (say, it stabilizes resource allocation by shrinking the redundancy or the lack of resources), the transition points have helped us to reveal that the role of contrarian behavior in resource-allocation systems can be either positive (to stabilize the system, to improve the system efficiency, and to make the system unpredictable) or negative (to unstabilize the system, to reduce the system efficiency, and to make the system predictable) under different conditions. Further, the transition points identified herein have also helped us to show that resource allocation can be optimized by including an appropriate size of contrarians.

Our work is also expected to be of value to other fields. In management and social science, administrators should not only conduct contrarianism when finding the formation of a herd, but also need to consider system environment and timing to see whether contrarianism is globally positive or negative. In ecology and evolution, it is not only necessary to study the mechanism of contrarian formation, but also to pay more attention to the effect of contrarianism on the whole ecological system and evolution groups.

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APPENDIX: ABOUT THE EXPERIMENT

The existence of contrarians is not told to the subjects in the experiment. The contrarians generated by the computer program play the online game together with the subjects. The parameters, M_1/M_2 and β_c, are controlled by the experiment organizer via the control panel (Fig. 8), and every parameter set (i.e., each pair of M_1/M_2 and β_c) lasts for 30 rounds. The values of the parameter set are not told to the subjects either. The experiment organizer only lets every subject know whether he or she wins or loses after each experimental round (Fig. 9). Details can be found in the following leaflet, which

FIG. 8. (Color online) The control panel for the experiment organizer to adjust parameters.

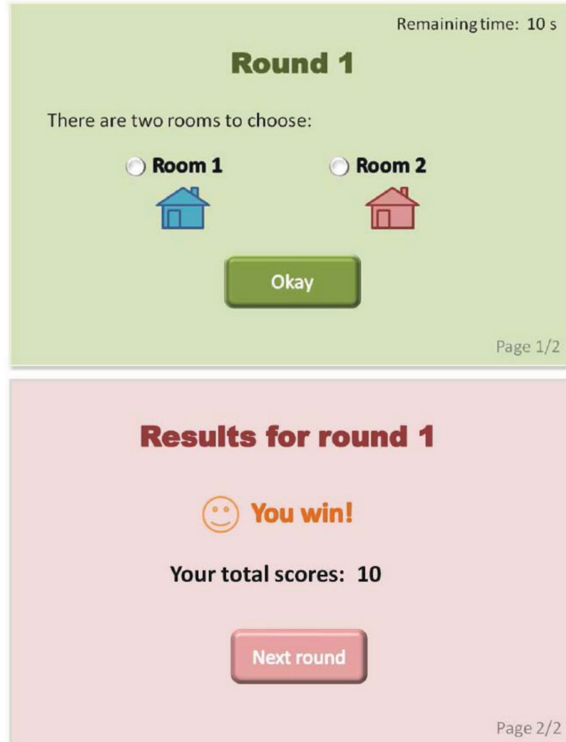

FIG. 9. (Color online) The two panels for subjects in the experiments.

was explained to the subjects who participated in the computer-aided online human experiment.

Leaflet to the Experiment

Thank you for participating in this experiment! Please read the instructions of the experiment carefully before starting to play. If you have any questions, please feel free to ask. No communication is allowed once the experiment starts.

Everyone will be allocated with an anonymous account in the experiment. You will use the account throughout the experiment. After logging in, you will see page 1/2 (in Fig. 9) with two options: Room 1 and Room 2. Each room will own an amount of resources, labeled as M_1 and M_2. You can choose to enter either Room 1 or Room 2, and then click "Okay" and wait. The page will automatically turn to page 2/2 after all the players have finished. The result of this round and the current score will be shown in page 2/2. You will have 15 seconds to check the results. After that, the page will automatically change to page 1/2 again and the next round starts.

The total number of players entering Room 1 is N_1, and N_2 for Room 2. After all players finish, the computer program will choose the winners according to the resource per capita determined by $\frac{M_1}{N_1} > \frac{M_2}{N_2}$ or $\frac{M_1}{N_1} < \frac{M_2}{N_2}$.

If $\frac{M_1}{N_1} > \frac{M_2}{N_2}$, those who choose Room 1 win.

If $\frac{M_1}{N_1} < \frac{M_2}{N_2}$, those who choose Room 2 win.

Example:

Suppose the resources in Room 1 and Room 2 are both 100 units. If 30 players choose to enter Room 1 and 70 players choose to enter Room 2, each player in Room 1 will have more resources per capital, and he or she wins. Suppose the resources in Room 1 and Room 2 are 100 units and 200 units, respectively. If 50 players choose to enter Room 1 and 50 players choose to enter Room 2, then each player in Room 2 will have more resources per capital, and he or she wins.

Notice:

The resources in Room 1 and Room 2 (M_1 and M_2) and the number of players entering Room 1 and Room 2 (N_1 and N_2) will not be announced. You cannot see the other players' options. Only your results will be shown on your computer screen after every round. You can use this information to decide which room to enter in the next round. Every account's original score is set to 0. Ten scores will be added in every round if you win and 0 added if you lose. We will pay you cash with the exchange rate, 10 scores = 1 Chinese Yuan, after the experiment finishes. In addition, we will pay every player 30 Chinese Yuan as the attendance fee, and reward the top 10 players (with the highest scores after all the experiment is completed) each with extra 100 Chinese Yuan.

[1] J. H. Holland, *Adaptation in Natural and Artificial Systems* (MIT Press, Cambridge, MA, 1992).
[2] M. Paczuski, K. E. Bassler, and Á. Corral, Phys. Rev. Lett. **84**, 3185 (2000).
[3] J. Halloy *et al.*, Science **318**, 1155 (2007).
[4] W. B. Arthur, Am. Econ. Assoc. Papers & Proc. **84**, 406 (1994).
[5] D. Challet and Y. C. Zhang, Physica A **246**, 407 (1997).
[6] N. F. Johnson, P. Jefferies, and P. M. Hui, *Financial Market Complexity* (Oxford University Press, Oxford, 2003).
[7] W. Wang, Y. Chen, and J. P. Huang, Proc. Natl. Acad. Sci. USA **106**, 8423 (2009).
[8] L. Zhao, G. Yang, W. Wang, Y. Chen, J. P. Huang, H. Ohashi, and H. E. Stanley, Proc. Natl. Acad. Sci. USA **108**, 15058 (2011).
[9] K. Y. Song, K. N. An, G. Yang, and J. P. Huang, PLoS ONE **7**, e33588 (2012).
[10] D. Dreman, *Contrarian Investment Strategies* (Random House, New York, 1979).
[11] A. Park and H. Sabourian, Econometrica **79**, 973 (2011).
[12] L. X. Zhong, D. F. Zheng, B. Zheng, and P. M. Hui, Phys. Rev. E **72**, 026134 (2005).
[13] S. Galam, Physica A **333**, 453 (2004).
[14] S. Galam, Qual. Quant. J. **41**, 579 (2007).
[15] C. Borghesi and S. Galam, Phys. Rev. E **73**, 066118 (2006).
[16] Q. Li, L. A. Braunstein, S. Havlin, and H. E. Stanley, Phys. Rev. E **84**, 066101 (2011).
[17] V. Plerou, P. Gopikrishnan, and H. E. Stanley, Nature (London) **421**, 130 (2003).
[18] R. Mantegna and H. E. Stanley, *Introduction to Econophysics* (Cambridge University Press, Cambridge, 2000).
[19] F. Lillo, J. D. Farmer, and R. N. Mantegna, Nature (London) **421**, 129 (2003).
[20] B. Podobnik and H. E. Stanley, Phys. Rev. Lett. **100**, 084102 (2008).
[21] T. Lux and F. Westerhoff, Nat. Phys. **5**, 2 (2009).
[22] J. D. Farmer and D. Foley, Nature (London) **460**, 685 (2009).