Patient and impatient pedestrians in a spatial game for egress congestion

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Large crowds evacuating through narrow bottlenecks may create clogging and jams that slow down the egress flow. Especially if people try to push towards the exit, the so-called faster-is-slower effect may occur. We propose a spatial game to model the interaction of agents in such situations. Each agent has two possible modes of play that lead to either patient or impatient behavior. The payoffs of the game are derived from simple assumptions and correspond to a hawk-dove game, where the game parameters depend on the agent's location in the crowd and on external conditions. Equilibrium configurations are computed with a myopic best-response rule and studied in both a continuous space and a discrete lattice. We apply the game model to a continuous-time egress simulation, where the patient and impatient agents are given different individual parameter values, which are updated according to the local conditions in the crowd. The model shows how threatening conditions can increase the proportion of impatient agents, which leads to clogging and reduced flows through bottlenecks, even when smooth flows would be possible.

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I. INTRODUCTION

The speed of pedestrian flow through bottlenecks is one of the key factors affecting the outcome of evacuations. The factors affecting these flows have been studied with evacuation experiments [1–4] and by computer simulations [5,6]. One of the key findings is the faster-is-slower effect [5], which indicates that if individuals push harder towards an exit, the flow through it may be reduced. This is due to the increased pressure, which increases the interpersonal friction forces and creates jams in front of bottlenecks.

The widely used social-force model describes the crowd with a self-driven many particle system. It produces realistic flows through bottlenecks and is able to create the faster-is-slower effect [5]. However, there is no model that explains or describes how, why, and when the crowd members adopt impatient pushing behavior. The reasons for a change in the behavior can be in the external conditions or in the behavior of other crowd members.

In the literature of social psychology, the pushing behavior is often related to panic [7]. Panic occurs in situations of scarce and dwindling resources and panicking people tend to behave irrationally and adopt a selfish attitude [8]. However, there has been a consensus for decades that actual panic occurs rarely in real crowds and evacuating people tend to behave rationally [9–11].

There are some systematic experimental studies on the factors leading to clogging [12,13]. The classic experiment of Mintz [12] had aluminium cones put inside a bottle, each with a string attached leading out of the bottle. Each participant held a string to one cone. Only one cone could pass the bottleneck at a time and more cones at the bottleneck would create a jam. The bottle started to slowly fill with water and the goal of the participants was to get their cones out before they got wet. Each participant was rewarded or fined according to the amount of water on his cone. Mintz found out that, with this setting, passing the bottleneck became uncoordinated, resulting in jams and the wetting of many.

According to what is meant by panic in current literature, it cannot be the reason for the outcome of Mintz's experiment. The participants' lives were not threatened nor did they start to behave completely irrationally. Rather, they observed the situation and tried to act as good as possible to maximize their reward.

The natural choice for computational modeling of strategic interactions is game theory, which has also been applied to evacuation modeling [14–18]. Brown [14] presented a game theoretic explanation for the emergence of pushing behavior in evacuation situations. He considered a prisoner's dilemma game with two players, the options of the players being either to rush to the exit or to take turns. The payoffs of the prisoner's dilemma game are such that the rational action for everybody is to rush, which results in jamming. Coleman [15] points out that in evacuation situations people are able to observe the others' actions, and thus, Brown's one-shot game model would not be adequate. Therefore, Coleman presents a thorough analysis of different extensions to Brown's model including, for example, iterated prisoner's dilemmas and contingent strategies.

The models of Brown and Coleman do not consider the spatial nature of escape situations. People are unable to observe the actions and interact with the whole crowd. Nevertheless, in reality interactions occur with the nearest neighbors. Also, not all crowd members are in the same situation. If no rushing

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occurs, the people in front of the exit escape fast, while the ones in the back have to wait a long time. If time is a limited resource, the agents at different locations are involved in completely different games.

Spatial game theory is a method for modeling agent interactions with their neighbors and the development of strategies over time. Spatial games are usually studied in two-dimensional lattices, where each individual occupies a site. Especially two game models, the prisoner's dilemma [19–22] and the hawk-dove game [23–25], also called the chicken game or the snowdrift game, have attracted lot of attention.

The emergence of strategies in evolutionary games has been studied with different dynamical models. In unstructured populations, the replicator dynamics [26,27] models the process of biological evolution. As generations proceed, high-payoff strategies have a higher probability to survive and replicate than low-payoff strategies. In spatial settings, the principles of the replicator dynamics can be transferred to learning rules, where successful strategies propagate to neighboring sites in the lattice. There are plenty of different ways in which such strategies can be implemented, but most of them are similar to the replicator dynamics. A vast majority of studies on spatial evolutionary games are based on some version of such learning models (see, e.g., Refs. [19,24]).

While the replicator dynamics describes the evolution of strategies over generations, some other learning models describe the dynamics of a group of individuals playing a game repeatedly. In reinforcement learning [28–31], agents tend to repeat the strategies that in the past led to satisfactory outcomes, while unsatisfactory experiences are avoided. Reinforcement learners do not need to know the strategic nature and the payoffs of the game they are playing. In the myopic best-response learning models [16,27,32,33], when updating their strategies, agents observe the actions of the others and adopt the strategy that would give them the highest payoff assuming the others stick to their current strategies. Best-response learning models have been previously used to model pedestrian interaction with others when selecting the exit in evacuations [16], but also, for example, to describe traffic flow in telecommunication networks [34,35]. Fictitious play [27,36] is quite similar to the best-response dynamics, but the players assume their opponents play a static mixed strategy and estimate it from the whole history of their play.

In this study, we present a spatial game theoretic model for pedestrian behavior in situations of exit congestion. The options of the agents are either to behave patiently or impatiently. The payoffs of our game are derived from natural assumptions on crowd dynamics, which turn out to result in a hawk-dove game matrix. Nevertheless, the parameters of the game depend on the agents' location in the crowd, and thus, the agents in front of the exit play a different game than the ones further back in the crowd. We apply the best-response learning scheme and study the equilibria of the game. The behavioral game model is coupled with the popular social-force egress simulation model. The individual parameters of the social-force model are set to depend on the agents' strategies. Simulation results show that the model gives an explanation for the clogging occurring at bottlenecks of egress routes under threatening conditions.

II. MODEL DESCRIPTION

For an agent queuing in front of an exit, the time it takes for him to pass the exit can be estimated. This time depends on the capacity of the exit, that is, how many agents are able to pass it within a given time interval, and on the number of the other agents that are closer to the exit [37]. We call this time the *estimated evacuation time* of agent i, denote it by T_i, and define it as

$$T_i = \frac{\lambda_i}{\beta}, \tag{1}$$

where λ_i is the number of other agents closer to the exit and β is the capacity of the exit.

We assume that agents have a *cost function* that describes the risk of not being able to evacuate before the conditions become lethal. This is a function of T_i and is denoted by $u(T_i;T_{\text{ASET}})$. The shape of the cost function depends on a parameter called the *available safe egress time* (ASET), which we denote by T_{ASET}. The term ASET is widely used in the literature and describes the time when fire-induced conditions within an occupied space or building become untenable [38]. The value of T_{ASET} depends on the fire and smoke conditions. Agents are assumed to observe their situation and these conditions and estimate T_i and T_{ASET}. For the rest of this paper, we denote the cost function only by $u(T_i)$ for simplicity. We also make the assumption that the cost function is increasing and convex:

$$u'(T_i) \geqslant 0, u''(T_i) \geqslant 0. \tag{2}$$

The convexity assumption means that the further the agents are from the exit, that is, the larger the T_i, the more they will gain from reducing the estimated evacuation time T_i.

The agents interact with their nearest neighbors. The set of agent i's nearest neighbors is denoted by N_i. We assume that each agent has two possible strategies of behavior: *patient* and *impatient*. The agents playing the strategy impatient try to push forward and overtake others while those playing patient move with the crowd and try to avoid physical contacts. In the interaction of two neighboring agents i and j, we define $T_{ij} = (T_i + T_j)/2$ as their average estimated evacuation time. When the two agents interact with each other, we assume that the agents expect the following outcomes.

(i) An impatient agent i can overtake its patient neighbor j. This reduces T_i and increases T_j by ΔT. As a result, the cost of agent i *decreases* by $\Delta u(T_{ij})$ and the cost of agent j *increases* by the same amount:

$$\Delta u(T_{ij}) = u(T_{ij}) - u(T_{ij} - \Delta T) \simeq u'(T_{ij})\Delta T. \tag{3}$$

Since $u'(T_{ij}) \geqslant 0$, we have $\Delta u(T_{ij}) \geqslant 0$.

(ii) Two patient agents do not compete with each other. They keep their respective positions and their costs do not change.

(iii) When two impatient agents interact, neither can overtake the other, but they will face a conflict where both have an equal chance of getting injured. We describe this risk of injury by a cost $C > 0$, which affects both players. We call the constant C the *cost of conflict*.

From these assumptions, we can construct a 2×2 game matrix (4), which is the basis of loss minimization of agent i when playing against its neighboring agent j. Player i's strategies are identified with the rows and player j's with the columns. In the cells of the matrix, the first (second) value describes the loss of agent i (j), when they play the strategies of the corresponding row and column. The total loss of the agent is calculated as the *sum of losses* against all of its neighbors, and based on this the agent selects the best strategy and plays it against all of them. We will discuss this in detail in Sec. III.

	Impatient	Patient
Impatient	C, C	$-\Delta u(T_{ij}), \Delta u(T_{ij})$
Patient	$\Delta u(T_{ij}), -\Delta u(T_{ij})$	$0, 0$

(4)

It is important to notice that the losses (gains) of game (4) are functions of the average estimated evacuation time of the two agents, T_{ij}. Hence, the agents that are closer to the exit play a different game than the ones in the back of the crowd. This is natural because the agents near the exit expect to evacuate faster, and thus, are in a less threatening situation than the ones further back in the crowd.

When each payoff of game (4) is divided by $\Delta u(T_{ij})$, the game can be written as

	Impatient	Patient
Impatient	$\frac{C}{\Delta u(T_{ij})}, \frac{C}{\Delta u(T_{ij})}$	$-1, 1$
Patient	$1, -1$	$0, 0$

(5)

The game is symmetric and it depends only on the ratio $C/\Delta u(T_{ij})$, that is, the ratio between the cost of conflict and the loss of getting overtaken.

Note that $C/\Delta u(T_{ij}) > 0$, since $\Delta u(T_{ij})$ and C are positive. We omit the case $\Delta u(T_{ij}) = 0$. The game to be played is

(i) *prisoner's dilemma* (PD) if $0 < \frac{C}{\Delta u(T_{ij})} \leqslant 1$,

(ii) *hawk-dove* (HD) if $\frac{C}{\Delta u(T_{ij})} > 1$.

The strategies impatient and patient correspond to hawk and dove in the HD, and defect and cooperate in the PD, respectively (see Appendix). Hence, the game is a PD if the loss of letting another agent pass is greater than the cost of conflict. In this case, the strategy impatient strictly dominates the strategy patient and the game has only one *Nash equilibrium* (NE) where both agents play impatient. Hence, in an area of a crowd where all agents play a prisoner's dilemma against all of their neighbors, the equilibrium outcome is that all agents end up playing impatient.

However, if the loss of letting another agent pass is smaller than the cost of conflict, neither of the strategies dominates the other and the game is a HD. This game has three Nash equilibria. Two of the equilibria are in pure strategies: (agent i plays impatient, agent j plays patient) and (agent i plays patient, agent j plays impatient). The third equilibrium is in mixed strategies, where the probability to play impatient is $P(\text{impatient}) = \Delta u(T_{ij})/C$ and the probability to play patient is $P(\text{patient}) = 1 - \Delta u(T_{ij})/C$ for both agents [39].

Due to the convexity assumption (2), the loss $\Delta u(T_{ij})$ increases with increasing T_{ij}. Hence, as the cost of conflict C is constant, the proportion of impatient agents in the mixed strategy NE increases when going further away from the exit. At some distance, $\Delta u(T_{ij})$ becomes greater than C and the game turns into a PD. To adopt the terminology used by Maynard Smith [26], we note that the mixed strategy equilibrium is an *evolutionarily stable strategy* (ESS), while the two pure strategy equilibria are not.

It should be emphasized that the outcomes of the above game are what the agents think will happen in different encounters. The agents' decision making is based on these expectations. Nevertheless, the realizations of the encounters in a real moving crowd may differ from the expectations as the crowd is a large complex system and all of the interactions among the crowd are not considered in the simple assumptions of this section. In Sec. VI, we couple the simple decision making model with a continuous-time egress simulation model. The agents' individual movement parameters are set to depend on their strategies; that is, the impatient agents are set to push forward more aggressively. Hence, when coupling the game strategies with egress simulation, the actual outcome of an interaction is not necessarily exactly what the agents' assumed when selecting their strategies. Before that, we study the properties of the derived game. In Sec. III, the form of the cost function $u(T_{ij})$ is derived from simple assumptions. Section IV describes the spatial setting and learning dynamics that are used in the computational part. In Sec. V, we compute equilibria of the game in a static spatial setting and study their properties.

III. THE COST FUNCTION

In Sec. II, we presented the general assumptions on the cost function $u(T_{ij})$. The cost depends on the average estimated evacuation time T_{ij} and the available safe egress time T_{ASET} and it describes the threat of not being able to evacuate before the conditions get lethal.

In the general description of the model, we only made the assumption of Eq. (2) that the cost function is increasing and convex. However, to be able to analyze the model computationally, the function needs to be specified. In the rest of this paper, we assume for simplicity that the time gained by overtaking, ΔT, has a constant value of 1 s.

To describe the shape of the cost function, we define another parameter denoted by T_0. It describes how short the time difference between T_{ij} and T_{ASET} has to be before the agents start playing the game. If $T_{ij} < T_{\text{ASET}} - T_0$, the game is not played and all agents behave patiently.

We derive the explicit form of the cost function satisfying the following assumptions (see Fig. 1).

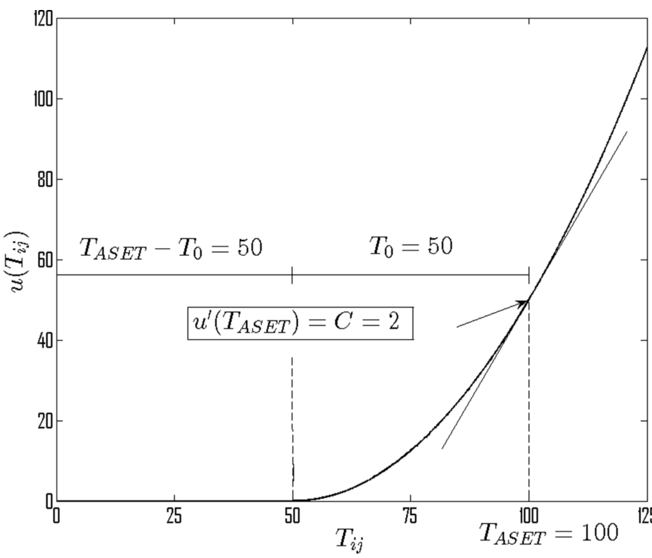


FIG. 1. Illustration of the parameters of the cost function. The function in the figure has the parameter values $T_{\mathrm{ASET}} = 100$, $T_0 = 50$, and $C = 2$.

(1) When $T_{ij} < T_{\mathrm{ASET}} - T_0$, the game is not played and $u(T_{ij}) \equiv 0$.

(2) When $T_{ij} > T_{\mathrm{ASET}} - T_0$, the cost function starts to increase quadratically.

(3) $u'(T_{\mathrm{ASET}}) = C$. This assumption means that when $T_{ij} \geqslant T_{\mathrm{ASET}}$, the game turns into a PD.

The general purpose of these assumptions is to make the model produce coherent results for different crowd sizes and different values of T_{ASET}. A cost function that meets these three assumptions is a quadratic function of T_{ij} defined by the given parameters. Hence, in all situations, the game to be played depends only on the difference $T_{ij} - T_{\mathrm{ASET}}$; that is, regardless of the size of the crowd or the value of T_{ASET}, the agents with a given value of $T_{ij} - T_{\mathrm{ASET}}$ will play the same game. With this property the model produces coherent results in all possible situations.

A cost function that meets the three assumptions is

$$u(T_{ij}) = \begin{cases} 0, & \text{if } T_{ij} < T_{\mathrm{ASET}} - T_0, \\ \frac{C}{2T_0}(T_{ij} - T_{\mathrm{ASET}} + T_0)^2, & \text{if } T_{ij} > T_{\mathrm{ASET}} - T_0, \end{cases} \tag{6}$$

and the derivative of the cost function is

$$u'(T_{ij}) = \begin{cases} 0, & \text{if } T_{ij} < T_{\mathrm{ASET}} - T_0, \\ \frac{C}{T_0}(T_{ij} - T_{\mathrm{ASET}} + T_0), & \text{if } T_{ij} > T_{\mathrm{ASET}} - T_0. \end{cases} \tag{7}$$

Now, using Eq. (3), we get the loss (gain) of overtaking:

$$\Delta u(T_{ij}) \simeq u'(T_{ij})\Delta T = \frac{C}{T_0}(T_i - T_{\mathrm{ASET}} + T_0)\Delta T. \tag{8}$$

Using the assumption that $\Delta T = 1$ s, we get the value of the parameter $C/\Delta u(T_{ij})$ of the game matrix (5):

$$\frac{C}{\Delta u(T_{ij})} \simeq \frac{T_0}{T_{ij} - T_{\mathrm{ASET}} + T_0}. \tag{9}$$

Hence, the losses (gains) of the game only depend on the parameters T_{ASET} and T_0 and not on C. Also note that (9) implies assumption 3 above. Figure 1 illustrates the shape and the parameters of the cost function.

IV. SPATIAL SETTING AND BEST-RESPONSE DYNAMICS

Each agent plays the presented game against his nearest neighbors. In an evacuation situation, it is natural to assume that agents are only able to interact with the agents next to them and not with the ones that are behind other agents.

In a cellular automata environment, the natural choice is to use the *Moore neighborhood*, where an agent's neighbors are the agents occupying the surrounding eight cells. Most studies on spatial games in square lattices use either the Moore neighborhood or the *von Neumann* neighborhood, in which each agent has four neighbors. In real life dense crowds, the number of immediate neighbors is usually closer to eight than to four, and thus, the selection of the Moore neighborhood is natural for this study.

The presented game model can be implemented as well in continuous space, where the agents can be located anywhere in a two-dimensional space. In this case, an agent's neighbors are the agents within a given radius r of the agent. To ensure interactions with only the nearest neighbors, a suitable condition for two agents to be counted as neighbors would be a skin-to-skin distance of less than 40 cm.

The evacuating agents are considered to observe the strategies of their neighbors and select their own strategies accordingly. The agents update their strategies frequently based on their best-response functions. The best responses are considered to be *myopic* in nature; that is, the agents do not consider the past or possible future updates but only react to the current strategies of their neighbors.

The best-response strategy $s_i^{(t)}$ of agent i on iteration period t is given by his best-response function BR_i, defined by

$$s_i^{(t)} = BR_i(s_{-i}^{(t-1)}; T_i, T_{-i}) = \arg\min_{s_i' \in S} \sum_{j \in N_i} v_i\left(s_i', s_j^{(t-1)}; T_{ij}\right). \tag{10}$$

The function $v_i(s_i', s_j^{(t-1)}; T_{ij})$ gives the loss defined by game (5) to agent i when he plays strategy s_i' and agent j has played strategy $s_j^{(t-1)}$ on period $(t-1)$; that is, $v_i(s_i', s_j^{(t-1)}; T_{ij})$ is equal to the corresponding matrix element. Here $s_{-i}^{(t-1)}$ is used to denote the strategies of all other agents than agent i on period $t-1$ and T_{-i} includes the estimated evacuation times of these agents. Notice that between $t-1$ and t, which we call a time step, there is only one agent updating its strategy once. During a simulation round all n agents update their strategies once, so that a simulation round consists of n time steps.

When computing the equilibrium configurations, we update the strategies of the agents with the shuffle update rule [40]. With the rule, on each simulation round, the order in which the agents update their strategies is randomized. We consider the random updating order to describe the behavior of evacuees more realistically than a fixed order. When we implement the best-response updating scheme in continuous-time simulations of moving agents (see Sec. VI), we set each agent to update its strategy frequently as a Poisson process.

V. EQUILIBRIUM CONFIGURATIONS

In this section, we study the spatial equilibria of the above presented game. We consider a static situation, that is, a snapshot or an instant of an evacuation situation, and calculate the equilibrium of the game at that moment.

The equilibria are calculated using the best-response dynamics, described in Sec. IV, which quickly converges to an equilibrium. When updating their strategies, the agents monitor their neighbors' strategies on the previous time step and select their best-response strategy. In this section, the best-response dynamics is only used to achieve the equilibrium configuration in a given static situation, and thus, also any other method of calculating the equilibrium could be used as well. In Sec. VI, we use similar best-response dynamics to model agents' decision making in a dynamically changing environment during an evacuation.

A. Equilibria in discrete grids

We study the game in a cellular automata environment with 40×40-cm square cells that can be occupied by one agent. In evacuations, people tend to form a half-circle-shaped crowd in front of an exit. To simulate this situation, we study the game in a grid where all cells within a given radius from the exit are occupied by agents. Each agent plays the game against its Moore neighborhood, that is, the agents in the surrounding eight cells. Note that the agents on the sides of the crowd have empty cells in their neighborhood and, thus, fewer than eight neighbors.

The equilibrium configurations and the convergence of the best-response dynamics in the spatial HD game has previously been studied by Sysi-Aho *et al.* [25]. In their simulations, they used a square lattice with periodic boundary conditions. In contrast to our model, where the parameter of the HD game depends on the agents' locations, they used a constant value for the whole grid. Figure 2 illustrates the proportion of hawks (impatient agents) as a function of the game parameter in the setting of Sysi-Aho *et al.* The figure shows that the fraction of hawks depends stepwise on the game parameter $\Delta u(T_{ij})$ defined by Eq. (9). To be precise, there are eight different levels the proportion of hawks can have. The reason for this stepwise dependence is the Moore neighborhood, in which the game is played, which implies just eight possible proportions. Each agent plays against eight other agents and the agents' best responses depend on the strategies of the others. Hence, for the equilibrium state, for each parameter value the number of doves (patient agents) in each hawk's neighborhood and the number of doves in each dove's neighborhood can be determined. These numbers directly determine the proportion of hawks and doves. When using the von Neumann neighborhood, where each agent has four neighbors, the proportion of hawks can only have four different levels. For a more detailed analysis of the topic, see [25].

In our game model, the parameter $C/\Delta u(T_{ij})$ of the HD game changes as a function of the distance to the exit. The result is a polymorphic population, where the crowd is divided into areas with different proportions of impatient agents. In front of the exit, where the value of the parameter is larger, there are fewer impatient agents and when moving further back

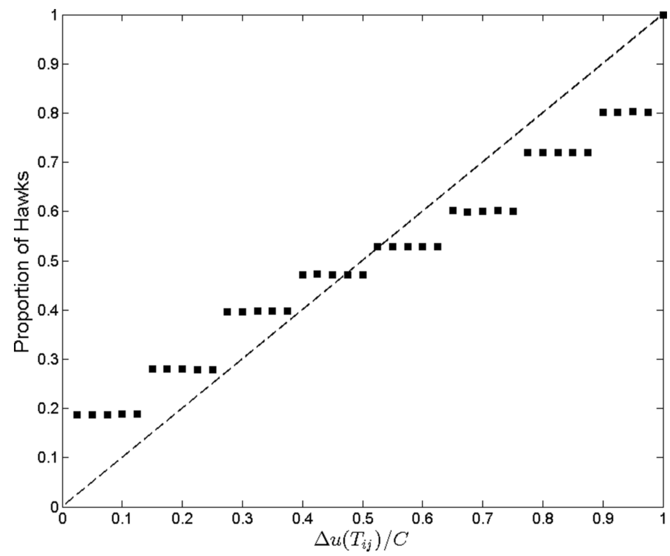


FIG. 2. The black squares show the average fraction of hawks (impatient agents) in equilibrium configurations as a function of the parameter $\Delta u(T_{ij})/C$. The values are averages of 20 simulations in a 50×50-cell lattice with periodic boundary conditions. The dashed line describes the fraction of hawks in the game's mixed strategy equilibrium, which is achieved in a nonspatial well-mixed setting.

in the crowd the proportion increases. When the crowd is large enough, the maximum number of eight different areas occur, as shown in Fig. 3. Right in front of the exit, it is profitable to be impatient only if there are no other impatient agents in the neighborhood. A little further away, the best response is to be impatient even if there is another impatient agent in the neighborhood. Finally, in the very back row, the best response is to be impatient even if all neighbors are impatient. In this area, the game is no longer a HD, but turns into a PD, as the value of $C/\Delta u(T_{ij})$ falls below one. In smaller crowds, not all of the eight different areas can be found.

Figure 4 illustrates the effect of the parameters on the equilibrium configurations. In each case there are 628 agents, but the parameters T_{ASET} and T_0, which determine the cost function $u(T_i)$, are varied. In Figs. 4(a)–4(d), $T_{\mathrm{ASET}} = T_0$ and the cost function has a value of zero at the exit and starts to

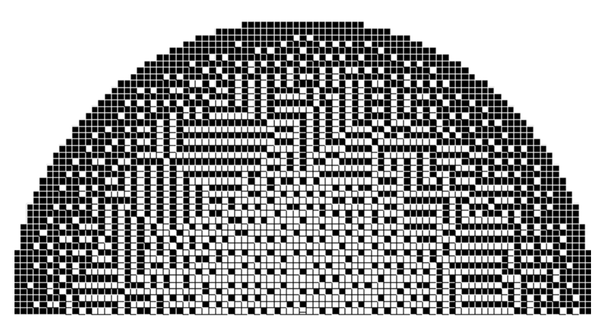
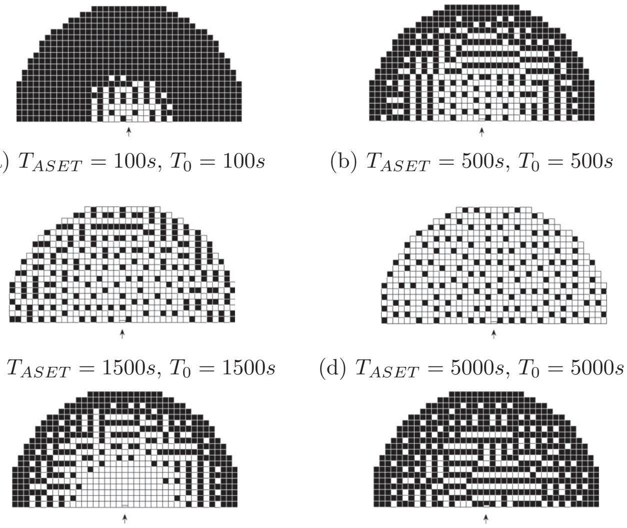


FIG. 3. An equilibrium configuration for 3180 agents with parameter values $T_{\mathrm{ASET}} = T_0 = 2800$ s. Black squares represent impatient agents and white squares represent patient agents. The value of the parameter $C/\Delta u(T_{ij})$ depends on the distance to the exit and the areas with different black-white patterns correspond to the eight different levels of hawks in Fig 2.



(a) $T_{ASET} = 100s$, $T_0 = 100s$ (b) $T_{ASET} = 500s$, $T_0 = 500s$

(c) $T_{ASET} = 1500s$, $T_0 = 1500s$ (d) $T_{ASET} = 5000s$, $T_0 = 5000s$

(e) $T_{ASET} = 500s$, $T_0 = 400s$ (f) $T_{ASET} = 500s$, $T_0 = 1000s$

FIG. 4. Equilibrium configurations with 628 agents for different values of the parameters T_{ASET} and T_0. The black squares represent impatient agents and the white squares represent patient agents.

increase quadratically as a function of T_i. When the available safe egress time is small, there are patient agents only in a small area in front of the exit and a vast majority of the agents are impatient. As the value of T_{ASET} increases, the number of patient agents starts to increase. Finally, in the equilibrium of Fig. 4(d), all agents play the HD game with such parameter values that the best response of an agent is to be impatient only when all of its neighbors are patient. Hence, there are no two impatient agents as neighbors in the whole crowd.

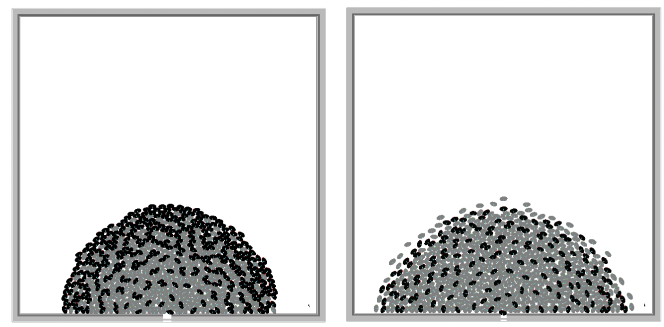
In Fig. 4(e), T_{ASET} is larger than T_0 and, hence, the agents right in front of the exit do not play the game. The cost function starts to increase quadratically when T_i exceeds 100. In Fig. 4(f), T_{ASET} is smaller than T_0 and the derivative of the cost function is already quite large in front of the exit. Hence, also in front of the exit, there is a larger proportion of impatient agents than in the other figures.

The convergence of the best-response algorithm is quite fast. Regardless of the number of agents, the equilibrium was found in fewer than ten iteration rounds, where all agents updated their strategies. The fast convergence is important for the applicability of the model in a simulation environment. If an equilibrium is found with a few iterations of a very simple updating scheme, it is conceivable that similar patterns occur also in real-life situations. Naturally, in addition to the fast convergence, also the model needs to be realistic in order to achieve realistic patterns.

B. Equilibria in continuous space

In order to apply the game to a continuous space, we define the neighborhood of each agent as an area around them. The agents play the game against the other agents within a skin-to-skin distance of less than 40 cm. This means that the number of neighbors is not constant as in the discrete grid.

Figure 5 illustrates equilibrium configurations for the continuous model. The parameters used in Fig. 5(a) are similar to those used in Fig. 4(b) in the discrete grid, and Fig. 5(b) is similar to 4(c). The occurring patterns are very similar to the discrete equilibria. Nevertheless, because the number of neighbors is not constant as in the grid model, also the division of the crowd into different areas is not as clear.



(a) $T_{ASET} = 500s$, $T_0 = 500s$ (b) $T_{ASET} = 1500s$, $T_0 = 1500s$

FIG. 5. (Color online) Equilibrium configurations in a continuous setting with 628 agents. The black agents are impatient and the gray ones are patient.

VI. EFFECT OF STRATEGY CHOICE ON EGRESS FLOWS

The behavior of impatient agents is different from the patient ones and the differences need to be considered when simulating pedestrian flows. We have implemented the game model with the best-response dynamics to the FDS + EVAC simulation software [41]. FDS + EVAC uses the social-force model of Helbing *et al.* [5,42,43] to model the movement of each agent. Each agent follows its own equation of motion:

$$m_i \frac{d^2\mathbf{x}_i(t)}{dt^2} = \mathbf{f}_i(t) + \boldsymbol{\xi}_i(t), \tag{11}$$

where $\mathbf{x}_i(t)$ is the position of agent i at time t, $\mathbf{f}_i(t)$ is the force exerted on agent i by the surroundings, m_i is the mass of agent i, and the last term, $\boldsymbol{\xi}_i(t)$, is a small random fluctuation force. The velocity of agent i is given by $\mathbf{v}_i(t) = d\mathbf{x}_i/dt$.

The force on the agent i has many components:

$$\mathbf{f}_i = \frac{m_i}{\tau_i}\left(\mathbf{v}_i^0 - \mathbf{v}_i\right) + \sum_{j\neq i}\left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} +\right) + \sum_{w}\left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right), \tag{12}$$

where the first term on the right-hand side describes the motive force on the evacuating agent. Each agent tries to walk with its own specific walking speed, $v_i^0 = |\mathbf{v}_i^0|$, towards an exit or some other target, whose direction is given by $\mathbf{v}_i^0$. The relaxation time parameter τ_i sets the strength of the motive force, which makes an agent accelerate towards the preferred walking speed. The first sum describes agent-agent interactions and the sum over w describes agent-wall interactions.

The agent-agent and agent-wall interaction forces consist of two parts. The physical contact forces are denoted by $\mathbf{f}_{ij}^c$ and $\mathbf{f}_{iw}^c$. Pedestrians' psychological tendency to keep a certain distance to other pedestrians and walls is described by the psychological social forces $\mathbf{f}_{ij}^{soc}$ and $\mathbf{f}_{iw}^{soc}$.

The cross section of a human body is elliptical. This is taken into account in the model by describing each agent with three overlapping circles and adding rotational equations similar to (12). For simplicity, these equations are not given in this paper.

For a more detailed description of the movement model, see Refs. [5,41–45].

We describe the difference between the patient and impatient agents in the model by altering three parameters of the social-force model.

(i) *Impatient agents do not avoid contacts with other agents as much.* This is reflected by a smaller magnitude of the social force $\mathbf{f}_{ij}^{soc}$ for the impatient agents.

(ii) *Impatient agents accelerate faster to their target velocity.* This is implemented by decreasing the individual relaxation time τ_i for the impatient agents.

(iii) *Impatient agents move more nervously.* This is implemented by increasing the individual random fluctuation force $\boldsymbol{\xi}_i(t)$ for the impatient agents.

The stronger fluctuation makes an impatient agent push around more in the tight crowd, which can be considered characteristic for impatient behavior. Increasing the target velocity for the impatient agents would have quite similar effect as the decreasing of the relaxation time τ_i. In this model, we decided to alter the value of τ_i, which describes the pace in which the agent accelerates to its target velocity. Altering also the target velocity would not improve the model, but it would add an extra parameter.

Some aspects should be emphasized about the coupling of the behavioral game model and the social-force egress flow model. The agents are assumed to monitor their neighbors and, based on their behavior, conclude whether they are patient or impatient. The behavioral game model determines which strategy, patient or impatient, each agent selects in a given situation. In this section, we assume that the agents decide their strategies (patient/impatient) based on the game model described in this paper and adjust their behavior to the given strategy by changing their individual parameters of the social-force model. Hence, the simulation results of this section answer the following question: What happens in a crowd if the agents select their strategies based on the game model of Secs. II and III and adjust their behavior to their strategies by changing their individual movement parameters as described?

A. Simulations with fixed strategies

We ran test simulations to study the outcomes of the interaction between impatient and patient agents with the above defined parameter changes for the impatient agents. In the simulations described in this section, the strategies of the agents are fixed; that is, the agents do not update their strategies during the simulations. This enables us to better study the performance of the impatient and patient agents on individual level as well as the effect that different proportions of impatient and patient agents have on the egress flow on the crowd level.

We ran simulations with 50 impatient and 50 patient agents randomly located in a 7×7-m room with one 1.2-m-wide exit. The strategies of the agents were fixed and the egress of the crowd through the exit was simulated for 60 s. The simulation was repeated 40 times with different random initial locations. Figure 6 displays the average number of patient and impatient agents inside the room over time and shows that the impatient agents are able to leave the room faster. The result is in line with the assumption of game (5) that impatient agents can overtake their patient neighbors. Hence, when we use these parameter values for the patient and impatient agents, the outcome of the simulation corresponds approximately to the agents' assumption that impatient agents overtake their patient neighbors.

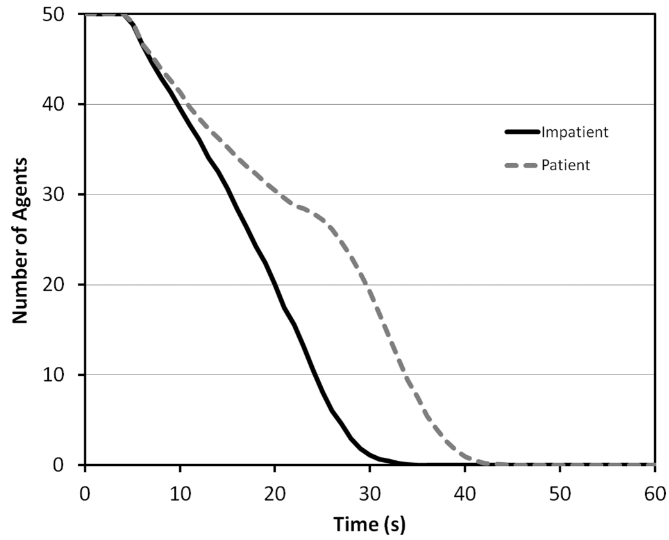


FIG. 6. The egress of 50 patient and 50 impatient agents with fixed strategies from a room with one 1.2-m-wide exit. The graph displays the average number of patient and impatient agents inside the room over time.

In the social-force model used here, we do not model the possible injuries that would result from conflicts between impatient agents. However, the threat of these injuries is considered by the agents in the form of the parameter *cost of conflict C*, when they select their strategies. A possible way of modeling injuries would be to make an impatient agent fall down with some probability if there are other impatient agents in the neighborhood. This would be a topic for future modeling and would require developing the social-force model to enable agents falling down. In this study, we assume that the threat of injury is considered when selecting strategies, but the actual injuries do not occur during the simulations.

It should also be noted that the actual exit capacity β may depend on whether there is congestion in front of the exit. However, these changes in the capacity are very difficult to notice by an individual in the middle of the crowd. Hence, in this model we assume that the agents estimate the capacity only based on the width of the exit.

The proportion of impatient agents in the population affects egress flows through exits. We studied this effect with simulations, where 200 agents evacuated through a 0.8-m-wide exit. At the start of the simulation, the exit door was shut and the agents formed a half circle in front of the door. In the static situation with the door shut, the agents updated their strategies until an equilibrium was reached. Then, the strategy of each agent was fixed to the equilibrium strategy for the rest of the simulation and the exit door was opened. The proportion of impatient agents in the simulations was altered by using different values of T_{ASET} when calculating the equilibrium.

We used 11 different values of T_{ASET} and ran 100 simulations for each value. Figure 7 illustrates the average and median flows through the exit as a function of the

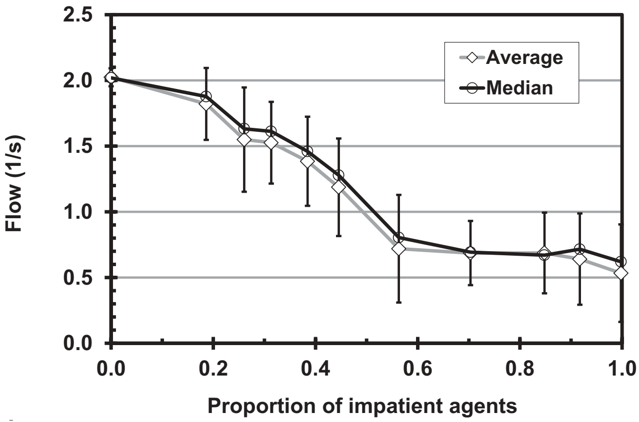


FIG. 7. Average flow for 200 agents through a 0.8-m-wide exit with different proportions of impatient agents in the population.

proportion of impatient agents. Fastest flows are achieved when the population consists of only patient agents and the flow decreases significantly as the number of impatient agents increases. This outcome is closely related to the well-known faster-is-slower effect, where individuals pushing harder towards an exit lead to jams and reduced flows.

B. Simulations with adaptive strategies

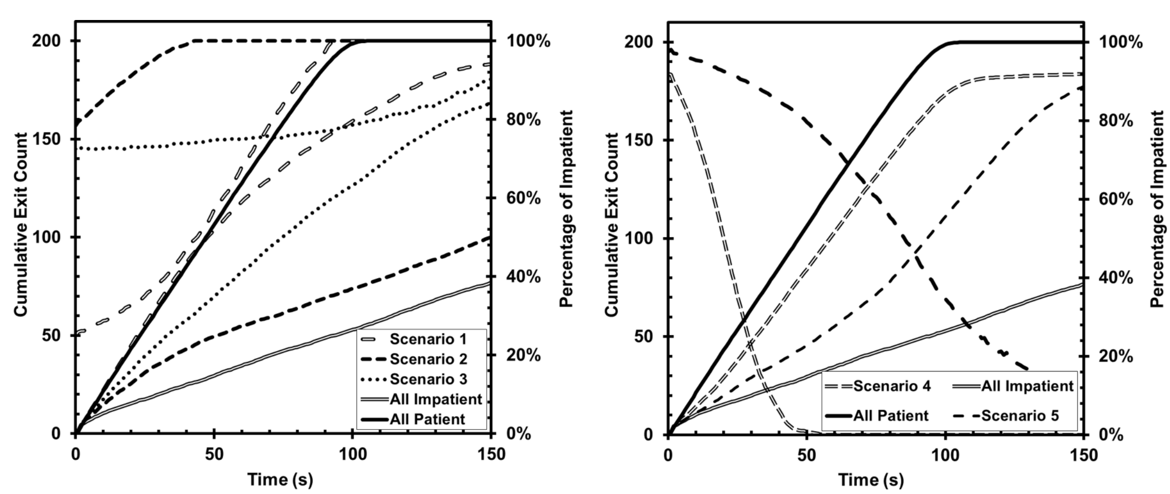
In evacuation situations, the conditions may change dramatically over time. A harmless looking situation may quickly turn very dangerous. Also a threatening situation may calm down for example by the interference of fire fighters. In a simulation model, the agents should be able to react to such changes in conditions. We ran simulations with the model in scenarios where the available safe egress time T_{ASET} changed over time and the agents interacted continuously updating their strategies frequently throughout the simulation. In the

TABLE I. The scenarios used in the simulations of Fig. 8. T_{ASET} and T_0 define the cost function at the start of the simulation. ΔT_{ASET} describes the linear change in the value of T_{ASET} in 1 s.

Scenario	T_{ASET} (s)	T_0 (s)	ΔT_{ASET}
1	200	100	−2
2	100	100	−2
3	100	100	−0.5
4	10	100	+4
5	10	100	+1

scenarios, the value of T_{ASET} was set to linearly increase or decrease over time. In the simulations, each agent was set to update its strategy several times each second. Because the best-response dynamics converges to an equilibrium in a few iteration rounds, the crowd can be considered to be close to the equilibrium of the current "snapshot" situation. This holds all the time. It should be noted that such equilibrium, although computed in a dynamically changing environment, is myopic in the sense that it does not take into account the agents' forthcoming moves or other such issues. This assumption may hold well for agents in a rapidly changing and/or uncertain environment. The assumption of several updates per second for each agent is not realistic for a real crowd. Estimating the update frequency and simulating such situations would be an interesting topic for future research. Table I presents the studied scenarios and Fig. 8 shows the results of the simulations. The results show that the fraction of patient and impatient agents quickly reacts to the changing conditions, which, in turn, affects the flow through the exit.

Figure 9 shows the cumulative exit counts of each of the 50 simulation runs for scenario 1 and scenario 5 of Table I. In scenario 1, the egress proceeds steadily and almost similarly in all simulations for the first 50 s. At that point the increased



(a) (b)

FIG. 8. Results of simulations with a time-dependent available safe egress time T_{ASET}. Two hundred agents were set to evacuate through a 0.8-m-wide exit and, in different scenarios, the value of T_{ASET} changed over time. The results for each scenario are averages over 50 simulations. The figures display the cumulative number of evacuated agents (the curves starting from the origin) and the percentage of impatient agents among the agents inside the room. Figure 8(a) shows three scenarios where the value of T_{ASET} linearly decreases over time. Figure 8(b) shows two scenarios with linearly increasing T_{ASET}. The scenarios are presented in Table I. For reference, both figures display the average cumulative exit counts with populations consisting of only impatient and only patient agents.

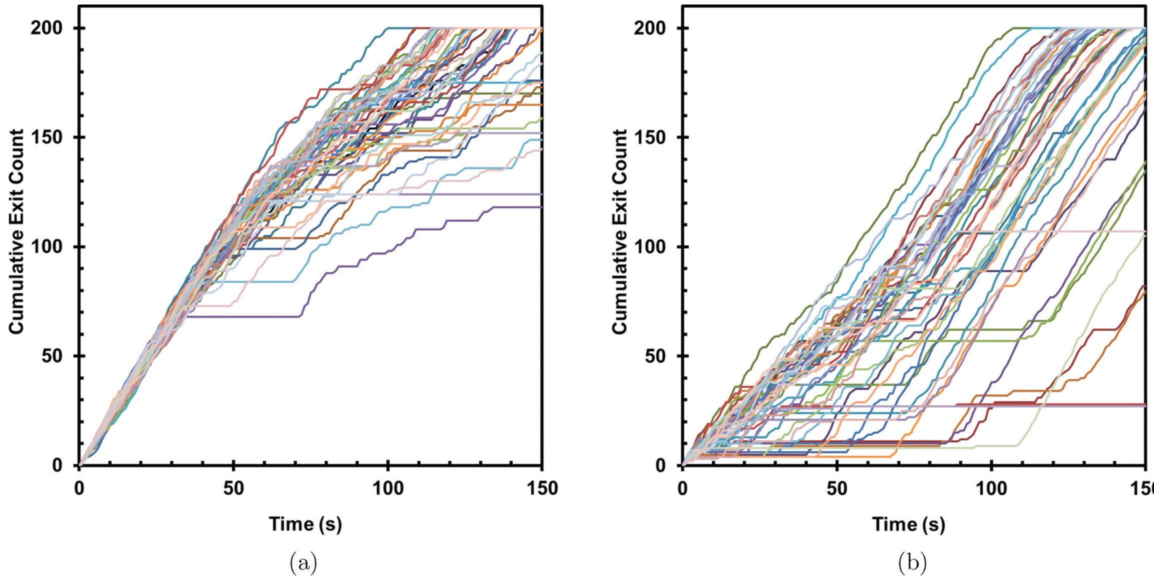


FIG. 9. (Color online) The cumulative exit counts for each of the 50 simulations of (a) scenario 1 and (b) scenario 5.

threat has caused so many agents to turn impatient that they start to cause clogging at the exit. The clogging is a stochastic phenomenon and, thus, causes high variation between the different simulation runs. This is even more apparent in the simulations of scenario 5: In some runs clogging blocks the exit for long periods, while in some other runs of the same scenario, the egress proceeds steadily throughout the simulation without any disturbances. Hence, multiple simulations are needed to get a picture of the safety of a given scenario.

VII. SUMMARY

Starting from intuitive assumptions regarding the interaction of myopic agents in an evacuating crowd under congestion, we have derived a spatial game model. The model describes the behavior of pedestrians in situations where a bottleneck along the egress route slows down the pedestrian flow and the time available for evacuation is a limited resource. Each agent can adopt one of two different strategies: impatient and patient behavior. The agents play the game against their neighbors in the crowd. The model derived from simple assumptions turns out to be a hawk-dove game or a prisoner's dilemma, where the parameter of the game varies depending on the agents' locations in the crowd.

Equilibria of the game are calculated with different parameter values in a spatial setting, where the agents form a half circle in front of an exit. The equilibrium configurations are polymorphic in the sense that, depending on the location in the crowd, the proportions and patterns of patient and impatient agents vary.

The game is coupled with the widely used social-force egress simulation model. The agents select their strategies based on the presented game model by making the individual movement parameters dependent on the strategy. The impatient agents are set to push harder towards the exit. The agents update their strategies frequently, based on a myopic best-response learning scheme, as they observe the changes in the situation and the actions of their neighbors.

The jams created at bottlenecks along the exit route are often considered to be caused by irrational behavior, a state of psychological panic. However, this study shows that, under threatening conditions, clogging may be caused by crowd members who act rationally according to simple and intuitive assumptions. The rationality of the agents is implemented through a best-response reaction rule, which optimizes the behavior given the actual local conditions. Still, as the hawk-dove game constitutes a social dilemma, something like a "tragedy of the commons" may result from the rational individual behavior, as the evacuation takes longer for everyone.

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APPENDIX

A 2×2 game, where the players maximize their payoffs,

$$\begin{array}{c|c|c|} & \text{D} & \text{C} \\ \hline \text{D} & a,a & r,s \\ \hline \text{C} & s,r & b,b \\ \hline \end{array} \tag{A1}$$

is the PD game if $r > b > a > s$. Note that if the matrix elements are interpreted as costs to be minimized, which is the case in the PD game (5), the condition is $s > a > b > r$. Irrespective of the choice of the other player, playing D is the rational choice that always yields a higher payoff than playing C. The strategy pair (D,D) is also the unique Nash equilibrium of the game. Nevertheless, if both players would play C, they would get higher payoffs than when both play D, but (C,C) is not a Nash equilibrium. So there is an obvious paradox hidden in this game. Due to this nature of the game, one can interpret strategy C as cooperation and strategy D as defection from cooperation. PD is the most well-known 2×2 game in economics and game theory literature.

Game (A1) is a HD game if $r > b > s > a$ ($a > s > b > r$ in the case of cost minimization). A HD game is the basic game in evolutionary game theory [26]. The game has two Nash equilibria: (C,D) and (D,C) [or (D,H) and (H,D) as they are denoted in the context of HD games]. The HD game also has one NE in mixed strategies, where both players play D with probability $p = (r - b)/(r + s - a - b)$, and C with probability $1 - p$. In PD the NE pair (D,D) is an ESS, while in HD only the mixed strategy Nash equilibrium is an ESS.

Evolutionarily stable strategy is a strategy which, if adopted by all members of a population, cannot be invaded by any alternative (mutant) strategy through the operation of natural selection. Mathematically, ESS means that the corresponding strategy pair is the stable point of the dynamical system describing an infinite population of players repeatedly pairing at random to play the game and reproducing their kind in numbers proportional to their performance.

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