Comment on "Plasma oscillations and nonextensive statistics"

Xiao-Chang Chen*
School of Materials Science and Engineering, Nanchang University, Nanchang 330031, China

Xiao-Qing Li†
Department of Physics, Nanjing Normal University, Nanjing 210097, China
(Received 7 June 2012; published 27 December 2012)

The paper authored by Lima *et al.* [Phys. Rev. E **61**, 3260 (2000)] has discussed the dispersion relation and Landau damping of a Langmuir wave in the context of the nonextensive statistics proposed by Tsallis. However, the results obtained in this paper are not appropriate. In this comment on the paper we shall derive the correct analytic formulas for both the dispersion relation and Landau damping in the Tsallis formalism. We hope that this comment will be useful in providing the correct results.

DOI: 10.1103/PhysRevE.86.068401 PACS number(s): 52.35.Fp, 05.45.−a, 05.20.−y, 05.90.+m

I. INTRODUCTION

Over the last few years, it has been proven that systems which present long-range interactions, long-time memory, fractality of the corresponding space time, or intrinsic inhomogeneity are intractable within the conventional B-G statistics [1,2]. So there has been an increasing focus on the new statistical approach, i.e., nonextensive statistical mechanics (NSM), in recent years. For $q \neq 1$, it gives a power law distribution and only when the parameter $q \to 1$ Maxwellian distribution is recovered [3]. NSM has been successfully applied to stellar polytropes [4], two-dimensional (2D) Euler and drift turbulence in a pure electron plasma column [5], as well as to the peculiar velocity function of galaxy clusters [6]. In particular, Liu *et al.* [7] showed a reasonable indication for the non-Maxwellian velocity distribution from plasma experiments.

Dispersion relations are fundamental and important for studying the wave in the plasma. According to the dispersion relations, we can study the problem of instability, propagation, refraction, and absorption of the plasma wave. While power law tail distributions have often been discussed in plasma physics cases such as the experimental results and Coulomb Fokker Planck model analysis of Liu *et al.* [7], and while such distributions are also a feature of Tsallis-type analysis, the particular paper that has made this connection is that of Lima *et al.* [8]. The paper authored by Lima *et al.* [8] studied the dispersion relation of a Langmuir wave based on nonextensive distribution; the results show that a nonextensive formalism presents a good fit to the experimental data, while the standard Maxwellian distribution provides only a crude description. However, the results of this paper are fundamentally flawed because they are obtained by using only a one-dimensional (1D) equilibrium distribution, a distribution which should be what is called in Tsallis theory the marginal distribution i.e., the one obtained by integrating over the momenta perpendicular to the direction of interest. If this one dimensionality implies that this behavior does not apply in other directions, however, that implies that the nonextensive distribution involves dynamics only in this direction, a situation which is difficult to obtain in any physically realistic situation and certainly not in that of Ref. [7]. In the experimental part of Ref. [7], the data referred to in the isotropic three-dimensional (3D) non-Maxwellian distribution function arises in the Coulomb Fokker Planck equation because of the interplay between the energy deposition, which is 3D isotropic in velocity and mostly into the lower energy electrons with higher Coulomb cross section, and the Fokker Planck evolution is manifest in the distribution of this change towards higher energy. The distribution of energy is slower and slower and over a much longer range as the energy increases. This is a typical Tsallis scenario as the scale length increases and the dynamics become less and less localized. It should be noted that these are not actual equilibria but a kind of quasi-equilibria, between which thermodynamic transfer or equilibration is far from evident. In any case it is clear that for Ref. [7], since the deposition and transport mechanisms are 3D isotropic, so must the actual distribution be 3D isotropic. In these circumstances the 1D marginal distribution should be obtained by integration over the momenta perpendicular to the chosen direction (i.e., that of a wave vector) as done in this work. In this comment (really a correction) on the paper by Lima *et al.* [8] we will rework the concept by beginning with the appropriately 3D symmetric distribution projected (integrated over p_y, p_z) down to the correct 1D marginal distribution result. From this we will obtain the resulting correct analytic formulas for both the plasma wave dispersion relation and Landau damping in detail according to the Tsallis formalism, and for $q \neq 1$ (at $q = 1$, both are Maxwellian) the results will differ in important details. It is our hope that the discussion here will be useful in the field of plasma physics.

The paper is organized as follows. In Sec. II we briefly introduce the nonextensive distribution function. The generalized dispersion relation and Landau damping for Langmuir wave are obtained in Sec. III. Finally, the summary is given in Sec. IV.

II. NONEXTENSIVE DISTRIBUTION FUNCTION

First, let us recall some basic facts about Tsallis statistics. In Tsallis statistics, the entropy has the form [3] of

$$S_q = k_B \frac{1 - \sum_i p_i^q}{q - 1}, \tag{1}$$

*xcchen1985@sina.com
†njxqli2010@163.com

where k_B is the Boltzmann constant, q is a parameter quantifying the degree of nonextensivity, and p_i is the probability of the ith microstate. The B-G entropy is recovered in the limit $q \to 1$. The basic property of Tsallis entropy is the nonadditivity or nonextensivity for $q \neq 1$. For example, for two systems A and B, the rule of composition [3] reads

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \quad (2)$$

In the nonextensive description, the 3D equilibrium distribution function can be written as [9]

$$f_q(\mathbf{p}) = A_q\left[1-(q-1)\frac{\mathbf{p}^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} = A_q\left[1+(1-q)\frac{\mathbf{p}^2}{2m^2v_T^2}\right]^{-\frac{1}{1-q}}, \quad (3)$$

according to the normalizing condition

$$\int f_q(\mathbf{p})\,d\mathbf{p}\frac{1}{(2\pi)^3} = n_0, \quad (4)$$

and the normalization constant reads

$$A_q = L_q\frac{(\sqrt{2\pi})^3}{(mv_T)^3}n_0, \quad (5)$$

in which

$$L_q = \frac{\Gamma\left(\frac{1}{1-q}\right)}{\left(\frac{1}{1-q}\right)^{3/2}\Gamma\left(\frac{1}{1-q}-\frac{3}{2}\right)}, \quad \frac{1}{3} < q \leqslant 1 \quad (6)$$

and

$$L_q = \frac{3q-1}{2}\frac{\left(\frac{1}{q-1}\right)^{-3/2}\Gamma\left(\frac{1}{q-1}+\frac{3}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}, \quad q \geqslant 1, \quad (7)$$

where $\mathbf{p}$, $v_T = \sqrt{k_BT/m}$, k_B, T, m, and n_0 denote, respectively, the momentum of particles, thermal speed, Boltzmann constant, temperature of particles, mass of particles, and particle number density. As one may check, for $q < 1/3$, the q distribution is unnormalizable. For $1/3 < q \leqslant 1$, the momentum of the particles can take any value. For $q \geqslant 1$, the distribution function [Eq. (3)] exhibits a cutoff on the maximum value allowed for the momentum of the particles, which is given by

$$p_{\max} = \sqrt{2/(q-1)}mv_T. \quad (8)$$

We see that in the limit $q \to 1$, $p_{\max}$ goes to infinity, and Eq. (3) reduces to the Boltzmann distribution function

$$f_{q=1}(\mathbf{p}) = \frac{(\sqrt{2\pi})^3}{(mv_T)^3}n_0\exp\left(-\frac{\mathbf{p}^2}{2m^2v_T^2}\right). \quad (9)$$

In order to define the temperature of the system, which is described by the nonextensive distribution, we will calculate the average kinetic energy below. For $1/3 < q \leqslant 1$,

$$\langle E_q\rangle = \left\langle\frac{\mathbf{p}^2}{2m}\right\rangle = \int\frac{\mathbf{p}^2}{2m}f_q(\mathbf{p})\,d\mathbf{p}\frac{1}{(2\pi)^3} = \frac{L_q}{(2\pi)^3}\frac{(\sqrt{2\pi})^3}{(mv_T)^3}n_0,$$

$$\int_0^\infty\frac{p^2}{2m}\left[1-(q-1)\frac{p^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}}4\pi p^2\,dp = \frac{L_q}{\sqrt{2\pi}}\frac{1}{(mv_T)^3}\frac{n_0}{m}\int_0^\infty p^4\left[1+\frac{1-q}{2m^2v_T^2}p^2\right]^{-\frac{1}{1-q}}dp = \frac{2}{5q-3}\frac{3}{2}n_0mv_T^2 = \frac{2}{5q-3}\frac{3}{2}n_0k_BT, \quad (10)$$

where Eq. (10) has been calculated using the integral formula [10, p. 325], that is,

$$\int_0^\infty x^{\mu-1}(1+\beta x^p)^{-\nu}dx = \frac{1}{p}\beta^{-\frac{\mu}{p}}B\left(\frac{\mu}{p},\nu-\frac{\mu}{p}\right) \quad (11)$$

with $|\arg\beta| < \pi, p > 0, 0 < \mathrm{Re}\mu < p\mathrm{Re}\nu$, $q > 3/5$ is required in Eq. (10) on the basis of Eq. (11). B is the β function, and the relation of the β function and γ function is [10, p. 909]

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}. \quad (12)$$

For $q \geqslant 1$,

$$\langle E_q\rangle = \left\langle\frac{\mathbf{p}^2}{2m}\right\rangle = \int\frac{\mathbf{p}^2}{2m}f_q(\mathbf{p})d\mathbf{p}\frac{1}{(2\pi)^3} = \frac{L_q}{(2\pi)^3}\frac{(\sqrt{2\pi})^3}{(mv_T)^3}n_0,$$

$$\int_0^{p_{\max}}\frac{p^2}{2m}\left[1-(q-1)\frac{p^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}}4\pi p^2\,dp = \frac{L_q}{\sqrt{2\pi}}\frac{1}{(mv_T)^3}\frac{n_0}{m}\int_0^{p_{\max}}p^4\left[1-(q-1)\frac{p^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}}dp = \frac{L_qn_0}{\sqrt{\pi}}\frac{2mv_T^2}{(q-1)^{5/2}}\int_0^1 t^{3/2}(1-t)^{\frac{1}{q-1}}dt = \frac{2}{5q-3}\frac{3}{2}n_0mv_T^2 = \frac{2}{5q-3}\frac{3}{2}n_0k_BT, \quad (13)$$

where Eq. (13) has been calculated using the transformation $t = (q-1)\,p^2/2m^2v_T^2$ and integral formula [10, p. 324], that is,

$$\int_0^1 x^{\mu-1}(1-x^\lambda)^{\nu-1}dx = \frac{1}{\lambda}B\left(\frac{\mu}{\lambda},\nu\right) \quad (14)$$

with $\mathrm{Re}\mu > 0$, $\mathrm{Re}\nu > 0, \lambda > 0$. So the average kinetic energy can be expressed as

$$\langle E_q\rangle = \left\langle\frac{\mathbf{p}^2}{2m}\right\rangle = \frac{2}{5q-3}\frac{3}{2}n_0k_BT = \frac{3}{2}n_0k_BT_q, \quad (15)$$

where $T_q = 2T/(5q-3)$ is the physical temperature of the nonextensive system. We see that in the limit $q \to 1$, $T_{q=1} = T$ and the average kinetic energy reduces to $\langle E_{q=1}\rangle = 3n_0k_BT/2$, which is the standard result in B-G statistics.

III. THE GENERALIZED DISPERSION RELATION AND LANDAU DAMPING

For the longitudinal wave propagating in an unmagnetized, collisionless, isotropic plasma, the longitudinal dielectric function of electron can be written as [11,12]

$$\varepsilon_k^l = 1 + \frac{4\pi e^2}{k^2}\int\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i\delta}\left[\mathbf{k}\cdot\frac{\partial f_q(\mathbf{p})}{\partial\mathbf{p}}\right]\frac{d\mathbf{p}}{(2\pi)^3}, \quad (16)$$

where we consider the direction of wave vector $\mathbf{k}$ to be along the x axis, and Eq. (16) becomes

$$\varepsilon_k^l = 1 + \frac{4\pi e^2}{k^2}\int \frac{dp_x}{2\pi}\frac{k\frac{\partial}{\partial p_x}}{\omega - kv_x + i\delta}\int f_q(\mathbf{p})\frac{dp_y\,dp_z}{(2\pi)^2} = 1 + \frac{4\pi e^2}{k^2}\int \frac{k}{\omega - kv_x + i\delta}\frac{\partial f_q(p_x)}{\partial p_x}\frac{dp_x}{2\pi}, \tag{17}$$

where e is the electron charge and $i\delta$ comes from Landau rules ($\delta \to 0^+$) [12]. Note that $f_q(p_x)$ is the marginal distribution, which is given by [see (29.9) and (29.10) of Ref. [12]]

$$f_q(p_x) = \int f_q(\mathbf{p})\frac{dp_y\,dp_z}{(2\pi)^2}. \tag{18}$$

Next, we will derive the expression of the marginal distribution in the nonextensive framework. Substituting Eq. (3) into Eq. (18), for $3/5 < q \leqslant 1$, we obtain

$$f_q(p_x) = \frac{4L_q}{(mv_T)^3}\frac{n_0}{\sqrt{2\pi}}, \quad \int_0^\infty dp_z \int_0^\infty \left[1 - (q-1)\frac{p_x^2 + p_y^2 + p_z^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} dp_y; \tag{19}$$

then the integral in Eq. (19) over p_y is

$$\int_0^\infty \left[1 - (q-1)\frac{p_x^2 + p_y^2 + p_z^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} dp_y = \int_0^\infty \left[\frac{2m^2v_T^2 + (1-q)\left(p_x^2 + p_z^2\right)}{2m^2v_T^2} + \frac{(1-q)}{2m^2v_T^2}p_y^2\right]^{-\frac{1}{1-q}} dp_y$$

$$= \left\{\frac{2m^2v_T^2 + (1-q)\left(p_x^2 + p_z^2\right)}{2m^2v_T^2}\right\}^{-\frac{1}{1-q}},$$

$$\int_0^\infty \left\{1 + \frac{(1-q)}{2m^2v_T^2 + (1-q)\left(p_x^2 + p_y^2\right)}p_y^2\right\}^{-\frac{1}{1-q}} dp_y = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)\left(\frac{1}{1-q}\right)^{1/2}}{\Gamma\left(\frac{1}{1-q}\right)}\left(2m^2v_T^2\right)^{\frac{1}{1-q}}, \tag{20}$$

$$\left\{2m^2v_T^2 + (1-q)\left(p_x^2 + p_z^2\right)\right\}^{-\frac{1}{1-q} + \frac{1}{2}},$$

where Eq. (20) has been calculated using the integral formula (11), and substituting Eq. (20) into Eq. (19), according to the same method, we can calculate the integral over p_z. Finally Eq. (19) becomes

$$f_q(p_x) = \frac{L_q}{q}\frac{\sqrt{2\pi}n_0}{mv_T}\left[1 - (q-1)\frac{p_x^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}+1}. \tag{21}$$

For $q \geqslant 1$, substituting Eq. (3) into Eq. (18), we obtain

$$f_q(p_x) = \frac{4L_q}{(mv_T)^3}\frac{n_0}{\sqrt{2\pi}}, \quad \int_0^{p_{z\max}} dp_z \int_0^{p_{y\max}} \left[1 - (q-1)\frac{p_x^2 + p_y^2 + p_z^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} dp_y; \tag{22}$$

then the integral in Eq. (22) over p_y becomes

$$\int_0^{p_{y\max}} \left[1 - (q-1)\frac{p_x^2 + p_y^2 + p_z^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} dp_y = \int_0^{p_{y\max}} \left[\frac{2m^2v_T^2 - (q-1)\left(p_x^2 + p_z^2\right)}{2m^2v_T^2} - \frac{(q-1)}{2m^2v_T^2}p_y^2\right]^{\frac{1}{q-1}} dp_y$$

$$= \left[\frac{2m^2v_T^2 - (q-1)\left(p_x^2 + p_z^2\right)}{2m^2v_T^2}\right]^{\frac{1}{q-1}},$$

$$\int_0^{p_{y\max}} \left[1 - \frac{(q-1)}{2m^2v_T^2 - (q-1)\left(p_x^2 + p_y^2\right)}p_y^2\right]^{\frac{1}{q-1}} dp_y = \frac{\left[2m^2v_T^2 - (q-1)\left(p_x^2 + p_z^2\right)\right]^{\frac{1}{q-1}+\frac{1}{2}}}{2\sqrt{q-1}\left(2m^2v_T^2\right)^{\frac{1}{q-1}}}.$$

$$\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{1}{q-1}}\,dt = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{1}{q-1}\right)}{(q-1)^{\frac{3}{2}}\Gamma\left(\frac{1}{q-1} + \frac{3}{2}\right)}, \quad \frac{\left[2m^2v_T^2 - (q-1)\left(p_x^2 + p_z^2\right)\right]^{\frac{1}{q-1}+\frac{1}{2}}}{\left(2m^2v_T^2\right)^{\frac{1}{q-1}}}, \tag{23}$$

where Eq. (23) has been calculated using the transformation $t = (q-1)p_y^2/[2m^2v_T^2 - (q-1)(p_x^2+p_y^2)]$ and integral formula (14); then substituting Eq. (23) into Eq. (22), according to the same method, we can calculate the integral over p_z. Finally Eq. (22) becomes

$$f_q(p_x) = \frac{L_q}{q}\frac{\sqrt{2\pi}n_0}{mv_T}\left[1-(q-1)\frac{p_x^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}+1}. \quad (24)$$

Obviously, the marginal distribution [Eqs. (21) and (24)] is different from the 1D distribution in the context of nonextensive statistics [9]

$$f_q(p_x) = B_q\frac{\sqrt{2\pi}n_0}{mv_T}\left[1-(q-1)\frac{p_x^2}{2m^2v_T^2}\right]^{\frac{1}{q-1}} \quad (25)$$

in which

$$B_q = \frac{\Gamma\left(\frac{1}{1-q}\right)}{\left(\frac{1}{1-q}\right)^{1/2}\Gamma\left(\frac{1}{1-q}-\frac{1}{2}\right)}, \quad -1 < q \leqslant 1, \quad (26)$$

and

$$B_q = \frac{1+q}{2}\frac{\left(\frac{1}{q-1}\right)^{-1/2}\Gamma\left(\frac{1}{q-1}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}, \quad q \geqslant 1, \quad (27)$$

unlike the classical B-G statistics. On physical grounds, obviously, using the marginal distribution (18) to calculate the permittivity [formula (17)] accords with the demands of plasma physics and the results are valid; otherwise the calculations will be problematic. It is the reason why the results obtained by Lima *et al.* are not appropriate.

Substituting the marginal distribution Eqs. (21) and (24) into the dielectric function Eq. (17), we obtain

$$\varepsilon_k^l = 1 + \frac{\omega_{pe}^2}{k^2v_T^2}\left[\frac{3q-1}{2} - Z_q(x)\right], \quad (28)$$

where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency and x is the dimensionless parameter, namely, $x = \omega/\sqrt{2}kv_T$. $Z_q(x)$ is the generalized plasma dispersion function in the context of Tsallis statistics,

$$Z_q(x) = L_q\frac{x}{\sqrt{\pi}}\int\frac{1}{x-\xi+i\delta}[1-(q-1)\xi^2]^{\frac{1}{q-1}}d\xi, \quad (29)$$

where $\xi = v_x/\sqrt{2}v_T$; in the limit $q \to 1$, it is reduced to the standard form in B-G statistics [11]

$$Z_{q=1}(x) = \frac{x}{\sqrt{\pi}}\int\frac{1}{x-\xi+i\delta}\exp(-\xi^2)d\xi. \quad (30)$$

Using the Plemelj formula [11],

$$\frac{1}{z\pm i0} = \wp\frac{1}{z} \mp i\pi\delta(z), \quad (31)$$

where $\wp$ denotes the principal value, then the generalized plasma dispersion function [Eq. (29)] can be written as

$$Z_q(x) = L_q\frac{x}{\sqrt{\pi}}\wp\int\frac{1}{x-\xi}\left[1-(q-1)\xi^2\right]^{\frac{1}{q-1}}d\xi - iL_q\sqrt{\pi}x[1-(q-1)x^2]^{\frac{1}{q-1}}. \quad (32)$$

When $\omega \gg kv_T$, namely, $x \gg 1$, the real part of Eq. (32) becomes

$$L_q\frac{x}{\sqrt{\pi}}\wp\int\frac{1}{x-\xi}[1-(q-1)\xi^2]^{\frac{1}{q-1}}d\xi = \frac{L_q}{\sqrt{\pi}}\wp\int[1-(q-1)\xi^2]^{\frac{1}{q-1}}\left(1+\frac{\xi}{x}+\frac{\xi^2}{x^2}+\cdots\right)d\xi, \quad (33)$$

thus Eq. (32) can be expressed as

$$Z_q(x) \approx \frac{3q-1}{2} + \frac{1}{2x^2} + \frac{2}{5q-3}\frac{3}{4x^4} - iL_q\sqrt{\pi}x[1-(q-1)x^2]^{\frac{1}{q-1}}. \quad (34)$$

When $\omega \ll kv_T$, namely, $x \ll 1$, introducing the transformation $\xi = \eta + x$, then the real part of Eq. (32) can be written as

$$L_q\frac{x}{\sqrt{\pi}}\wp\int\frac{1}{x-\xi}[1-(q-1)\xi^2]^{\frac{1}{q-1}}d\xi, = L_q\frac{x}{\sqrt{\pi}}\wp\int[1-(q-1)(\eta^2+2\eta x+x^2)]^{\frac{1}{q-1}}\frac{d\eta}{-\eta} \approx -L_q\frac{x}{\sqrt{\pi}}\wp\int[1-(q-1)\eta^2]^{\frac{1}{q-1}}\frac{d\eta}{\eta} = 0; \quad (35)$$

then Eq. (32) can be expressed as

$$Z_q(x) \approx -iL_q\sqrt{\pi}x[1-(q-1)x^2]^{\frac{1}{q-1}}, \quad (36)$$

which can be used in investigating the low-frequency wave, such as the ion acoustic waves. It should be noted that the process is not pinpoint in Eq. (35); the real part should be a very small quantity, which may be obtained by numerical method. However, the small quantity can be neglected when Eq. (32) is substituted into the dielectric function Eq. (28).

Substituting Eq. (34) into the dielectric function Eq. (28), according to the longitudinal dispersion relation $\mathrm{Re}\varepsilon_k^l = 0$, thus the generalized dispersion relation of Langmuir wave is obtained,

$$\omega^2 = \omega_{pe}^2 + \frac{2}{5q-3}3k^2v_T^2 = \omega_{pe}^2 + 3k^2v_{Tq}^2, \quad (37)$$

where $v_{Tq} = \sqrt{k_BT_q/m}$ is the physical thermal speed and T_q is the physical temperature defined in Sec. II. As expected, in the limit $q \to 1$, Eq. (37) reduces to

$$\omega^2 = \omega_{pe}^2 + 3k^2v_T^2, \quad (38)$$

being the standard result in B-G statistics [11]. Thus the dispersion relation of a Langmuir wave obtained by Lima *et al.* [8],

$$\omega^2 = \omega_{pe}^2 + \frac{2}{3q-1}3k^2v_T^2,$$

is inappropriate. According to Fig. 1 in their paper, we can see that the dispersion relation for a Tsallis formalism presents a good fit to the experimental data of Ref. [7] when $0.7 < q < 0.85$; obviously, it should be $0.82 < q < 0.91$ based on the correct result.

Next we will derive the expression of Landau damping. The Landau damping rate can be written as [11]

$$\gamma_k^l = -\frac{\mathrm{Im}\varepsilon_k^l}{\frac{\partial}{\partial\omega}\mathrm{Re}\varepsilon_k^l}\bigg|_{\omega=\omega^l}; \tag{39}$$

according to Eqs. (37), (28), and (34), we have that $\omega \approx \omega_{pe}$, $\mathrm{Re}\varepsilon_k^l \approx 1 - \omega_{pe}^2/\omega^2$, $(\partial/\partial\omega)\mathrm{Re}\varepsilon_k^l = 2/\omega_{pe}$, $\mathrm{Im}\varepsilon_k^l = L_q\sqrt{\pi/2}\,(\omega\omega_{pe}^2)/(kv_T)^3 \cdot [1-(q-1)\omega^2/(2k^2v_T^2)]^{1/(q-1)}$. Combined with Eq. (39), we obtain the generalized Landau damping as

$$\gamma_k^l = -L_q\sqrt{\frac{\pi}{8}}\omega_{pe}\left(\frac{k_d}{k}\right)^3, \quad \left[1-(q-1)\left(\frac{k_d^2}{2k^2}+\frac{3}{5q-3}\right)\right]^{\frac{1}{q-1}}, \tag{40}$$

where $k_d = \omega_{pe}/v_T$ is the electronic Debye wave number. In the limit $q \to 1$, Eq. (40) reduces to

$$\gamma_k^l = -\sqrt{\frac{\pi}{8}}\omega_{pe}\left(\frac{k_d}{k}\right)^3 \exp\left(-\frac{k_d^2}{2k^2}-\frac{3}{2}\right), \tag{41}$$

which is the classical Landau expression for the damping decrement in the framework of B-G statistics [11].

IV. SUMMARY

In this comment, we have discussed the dispersion property and Landau damping of a Langmuir wave in an unmagnetized, collisionless, 3D isotropic plasma with the nonextensive distribution in Tsallis statistics. The correct generalized dispersion relation and Landau damping are obtained. In the limiting case ($q \to 1$) the classical results based on the B-G statistics are recovered. It is our hope that the discussion here will serve as a useful introduction to the field of plasma physics.

ACKNOWLEDGMENTS

The work was supported by the National Natural Science Foundation of China under Grant No. 10963002, the International S&T Cooperation Program of China (2009DFA02320), and Jiangxi Province, Program for Innovative Research Team in Nanchang University, and the National Basic Research Program of China (973 Program) (No. 2010CB635112). Finally, we would like to thank the anonymous referee for reading and commenting on the manuscript and for important suggestions on the work.

[1] C. Tsallis, in *New Trends in Magnetism, Magnetic Materials and Their Applications*, edited by J. L. Moran-Lopez and J. M. Sanchez (Plenum Press, New York, 1994).
[2] C. Tsallis, Chaos, Solitons Fractals **6**, 539 (1995).
[3] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[4] A. R. Plastino and A. Plastino, Phys. Lett. A **177**, 177 (1993).
[5] B. M. Boghosian, Phys. Rev. E **53**, 4754 (1996).
[6] A. Lavagno, G. Kaniadakis, M. Rego-Monteiro, P. Quarati, and C. Tsallis, Astrophys. Lett. and Comm. **35**, 449 (1998).
[7] J. M. Liu, J. S. De Groot, J. P. Matte, T. W. Johnston, and R. P. Drake, Phys. Rev. Lett. **72**, 2717 (1994).
[8] J. A. S. Lima, R. Silva, Jr., and Janilo Santos, Phys. Rev. E **61**, 3260 (2000).
[9] R. Silva, Jr., A. R. Plastino, and J. A. S. Lima, Phys. Lett. A **249**, 401 (1998).
[10] I. S. Gradshteyn and I. M. Ryzhik, in *Table of Integrals, Series, and Products*, edited by A. Jeffrey and D. Zwillinger (Elsevier Academic Press, New York, 2007).
[11] X. Q. Li, *Collapsing Dynamics of Plasmons* (Chinese Science and Technology Press, Beijing, 2004).
[12] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, translated by J. B. Sykes and R. N. Franklin (Pergamon Press, Oxford, 1981).