Electron acoustic shock waves in a collisional plasma

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(Received 21 July 2012; published 18 December 2012)

A nonlinear analysis for the finite amplitude electron acoustic wave (EAW) is considered in a collisional plasma. The fluid model is used to describe the two-temperature electron species in a fixed ion background. In general, in electron-ion plasma, the presence of wave nonlinearity, dispersion, and dissipation (arising from fluid viscosity) give rise to the Korteweg–de Vries Burgers (KdVB) equation which exhibits shock wave. In this work, it is shown that the dissipation due to the collision between electron and ion in the presence of collective phenomena (plasma current) can also introduce an anomalous dissipation that causes the Burgers term and thus leads to the generation of electron acoustic shock wave. Both analytical and numerical analysis show the formation of transient shock wave. Relevance of the results are discussed in the context of space plasma.

DOI: 10.1103/PhysRevE.86.066408 PACS number(s): 52.35.Fp, 52.35.Tc, 94.20.Ac

I. INTRODUCTION

Study of the electron acoustic wave (EAW) retained its interest due to its presence in laboratory experiments and numerical simulations [1–5] as well as in space plasma environment [6]. These waves exist in a two-temperature electrons and stationary ion plasma [7,8]. The cold electron fluid provides the inertia and the hot electron fluid provides necessary restoring force whereas the massive ions serve as an unperturbed charge neutral background for the generation of EAW [8]. The phase velocity (v_{ph}) of the EAW lies between thermal speed of cold ($v_{tc} = \sqrt{T_c/m}$) and hot electron ($v_{th} = \sqrt{T_h/m}$) species ($v_{tc} < v_{ph} < v_{th}$), where $T_{h(c)}$ is the temperature of the hot (cold) electron and m is the electron mass. Therefore, in these time scales ions can be considered as a stationary charge neutral agent. The existence of EAWs requires the density of cold population to be small compared to the density of hot electron species. Thereby, the EAW speed is $c_{se} = \sqrt{(T_h/m)(n_{c0}/n_{h0})}$, where $n_{h(c)0}$ is the equilibrium densities of hot (cold) electron species. The linear mode analysis reveals that the linear dispersion relation of EAWs is given by [7,9,10] $\omega^2 = k^2c_{se}^2(1 + 3k^2\lambda_{Dc}^2)/(1 + k^2\lambda_{Dh}^2)$, where k is the wave number, $\lambda_{Dh(c)} = \sqrt{T_{h(c)}/(4\pi n_{h(c)0}e^2)}$ is the hot (cold) electron Debye length. In the absence of the pressure of cold electrons (compared to hot electrons) and in the long wavelength limit ($k^2\lambda_{Dh}^2 \ll 1$), one get $\omega \simeq kc_{se}$. In contrast to the ion acoustic wave, this EAW usually suffers stronger damping because of the easier mobility of the cold electrons than ions. However, this wave is less affected by Landau damping if $n_{h0} \gg n_{c0}$ and $T_c \ll T_h$ [8]. The physical reason is that for sufficiently low cold electron density (compared to hot electron density), the damping of EAWs is strongly reduced while the cold electron component allows the wave to propagate.

The Fast Auroral SnapshoT (FAST) observations in the intermediate (altitude < 4000 km) auroral region, geotail, and the polar observations at higher altitude (between $\sim 2R_E$ and $8R_E$, R_E being earth's radius) auroral region confirm the existence of EAWs in several parts of magnetosphere [7,11]. Most of the electrostatic high frequency noises excited in the auroral plasma are due to the EAW [12]. In the case of strong excitation, the EAW readily evolves into nonlinear stage and forms several nonlinear structures like solitons, double layers, turbulence, wave modulations (envelope solitons), shocks, electron holes, etc. in many regions of earth's magnetosphere: preferentially in polar magnetosphere and in different auroral regions [13–23]. Electron acoustic wave is also observed in a laboratory [24]. Most of these nonlinear structures of EAWs, observed by the several satellites, are related to the parallel electric field fluctuations [25–27].

All the earlier investigations are in the collisionless regime. However, collisions always take place in the plasma transport processes both in laboratory as well as space plasma environment (e.g., at the auroral ionosphere altitudes, collisions generally may not be neglected [28]). Thus it is pertinent to investigate the propagation characteristics of the nonlinear EAW in the presence of dissipation due to collision.

In this paper, we investigate the propagation characteristics of finite amplitude nonlinear EAWs in the presence of electron-ion direct collisions. In electron-ion collisional plasma, the dynamics of the finite amplitude nonlinear wave is governed by a Korteweg–de Vries equation with a linear damping term that arises due to the direct collision between the different species of plasma constituents [29]. In this case, due to the weak dissipation, the nonlinear wave retains its solitonic structures with diminishing amplitude and increasing width of the solitons [29]. On the other hand, due to the collision between same species particles (viscosity), the dynamics of the nonlinear wave is usually governed by the Korteweg–de Vries

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Burgers (KdVB) equation that exhibits shock wave [30]. In this case, the Burgers term is responsible for the shock wave arising due to the viscosity. However, one of our main findings in this work is that here, we have shown that the electron-ion direct collision (collision between different species) in the presence of plasma current (collective phenomena) can also introduce an anomalous dissipation that causes the Burgers term and thus is responsible for the generation of electron acoustic shock wave. The dynamics of the nonlinear wave is shown to be governed by the usual KdVB equation. The shock is generated due to the balance between the nonlinearity and the combined action of dispersion and dissipation due to collision. The results of the present investigation could explain shock wave generation and the particle acceleration mechanism in space plasma [13,31].

The paper is organized in the following manner. The model and the basic equations are discussed in Sec. II. The KdVB equation describing the propagation of the nonlinear EAW is derived in Sec. III. In Sec. IV, we present the analytical and numerical solution of the derived KdVB equation. Finally, the results are discussed in the context of space plasmas in Sec. V.

II. MODEL EQUATIONS

The plasma considered here is unbounded, homogeneous, and unmagnetized consisting of electrons (cold and hot) and ions. The collisions are taken between cold electron and stationary ions. The plasma is quasineutral: $n_{h0} + n_{c0} = n_0$. As mentioned before, the dynamics of the EAW is mainly related to the dynamics of the cold electrons and it is a relatively low-frequency wave with phase velocity lying in the range, $v_{tc} \ll \omega/k \ll v_{th}$. The oscillation time scale of this EAW is typically $\sim\omega_{pc}^{-1}$ ($\omega_{pc} = \sqrt{4\pi n_{c0}e^2/m}$ is the plasma frequency of cold electron species), much larger than the oscillation time scale of hot electron $\sim \omega_{ph}^{-1}$ ($\omega_{ph} = \sqrt{4\pi n_{h0}e^2/m}$ is the plasma frequency of hot electron species) due to the fact that for the existence of the EAW, $n_{h0} \gg n_{c0}$ [8]. In this slow time scale, hot electrons move so fast relative to these waves that they have sufficient time to maintain the thermodynamic equilibrium (in analogy to the ion acoustic wave in electron-ion two component plasma [32]) and therefore with respect to this low-frequency wave, one can assume that hot electron density follows Boltzmann distribution [23],

$$n_h = n_{h0} \exp\left(\frac{e\varphi}{T_h}\right), \tag{1}$$

where φ is the electrostatic potential. Note that there could arise situations where the ensemble of two groups of electrons are not always in thermodynamic equilibrium [33].

In equation of motion for the cold electrons, we neglect the pressure term as the cold electron temperature $T_c \ll T_h$, hot electron temperature (typically in the auroral region $T_h \sim$ (200–500) eV and $T_c \sim$ (1–10) eV [10]). We take all dependent variables as functions of coordinate variable x and time variable t. Generalization to other space variables is simple and straightforward. The momentum equation for cold electrons is given by

$$mn_c\left(\frac{\partial}{\partial t} + v_c\frac{\partial}{\partial x}\right)v_c = -n_c eE - mn_c\nu_c v_c, \tag{2}$$

where e is the magnitude of the electronic charge, v_c is the cold electron fluid velocity, ν_c is the collision frequency between cold electrons and stationary background ions (large mass), and E is the electric field in the x direction. The continuity equation for cold electrons is given by

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c v_c) = 0. \tag{3}$$

To close these equations, we need to consider equations for the electric field E from Maxwell's equations,

$$\frac{\partial E}{\partial x} = 4\pi e(n_0 - n_c - n_h), \quad \frac{\partial E}{\partial t} = 4\pi e n_c v_c. \tag{4}$$

Since most of the observations are related to the parallel electric field fluctuations [25–27], the plasma is unmagnetized $\nabla \times \mathbf{B} = 0$ and therefore, in the above, the equation for $\partial E/\partial t$ is the balance between displacement current and particle current. Furthermore, since hot electrons are Boltzmann distributed, they do not have directed resultant velocity to contribute to the current and therefore, the current is carried only by the cold electron species [34]. Using Eq. (4) in Eq. (2) with $E = -\partial\varphi/\partial x$, we have

$$\left(\frac{\partial}{\partial t} + v_c\frac{\partial}{\partial x}\right)v_c = \frac{e}{m}\frac{\partial\varphi}{\partial x} + \frac{\nu_c}{4\pi e n_c}\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial x}\right). \tag{5}$$

To investigate the dynamics of the nonlinear EAW, it is convenient to use dimensionless variables for the above equations and therefore, we define $\hat{x} = x/\lambda_{Dh}, \hat{t} = \omega_{pc}t, \hat{n}_c = n_c/n_{c0}$, $\hat{n}_h = n_h/n_{h0}$, $\hat{\phi} = e\varphi/T_h$, and $\hat{v} = v_c/v_{th}$. The above Eqs. (3)–(5) can be recast as in the following dimensionless form:

$$\frac{\partial\hat{n}_c}{\partial\hat{t}} + \frac{\partial}{\partial\hat{x}}(\hat{n}_c\hat{v}_c) = 0, \tag{6}$$

$$\frac{\partial^2\hat{\phi}}{\partial\hat{x}^2} = (e^{\hat{\phi}} + \alpha\hat{n}_c - \beta), \tag{7}$$

and

$$\left(\frac{\partial}{\partial\hat{t}} + \hat{v}_c\frac{\partial}{\partial\hat{x}}\right)\hat{v}_c = \frac{\partial\hat{\phi}}{\partial\hat{x}} + \left(\frac{\nu_c}{\omega_{pc}}\right)\frac{1}{\hat{n}_c}\frac{\partial}{\partial\hat{t}}\left(\frac{\partial\hat{\phi}}{\partial\hat{x}}\right), \tag{8}$$

where $\alpha = n_{c0}/n_{h0}$ and $\beta = n_0/n_{h0}$. Equations (6)–(8) are the basic equations that describe the model. Hereafter, we remove the notation hat from the variables for simplicity and work with normalized variables.

III. REPRESENTATION OF KORTEWEG–DE VRIES BURGERS EQUATION

To study the nonlinear propagation characteristics of the EAW in a collisional plasma, the reductive perturbation technique has been employed and the following stretched coordinate has been introduced:

$$\xi = \sqrt{\epsilon}\,(x - Mt); \quad \text{and} \quad \tau = \epsilon^{3/2}t, \tag{9}$$

where M is the phase velocity of the mode normalized by the thermal speed v_{th} and ϵ is a dimensionless parameter that measures the order of smallness of the perturbations. The dynamical variables n_c, v_c, and ϕ are expanded in power series of ϵ as

$$f = f^{(0)} + \sum_{i=1}^{\infty} \epsilon^i f^{(i)}, \tag{10}$$

where $f = n_c, n_h, v_c, \phi$, $f^{(0)} = 1$ for n_c, n_h and $f^{(0)} = 0$ for $f = v_c, \phi$. To introduce the effects of finite electron-ion collision (under the assumption that ν_c/ω_{pc} is small but finite) and also to make the nonlinear perturbation consistent, we introduce the following scaling which is compatible to the physical assumption:

$$\frac{\nu_c}{\omega_{pc}} = \nu\sqrt{\epsilon}, \tag{11}$$

where $\nu \sim O(1)$, i.e., of the order of unity. Note that if ν_c/ω_{pc} is not so small, one can still use the same substitution but now ν should be large. This does not present any hurdle to the subsequent theoretical analysis [35]. The only difference, as we shall see later, is that the numerical results will exhibit a sharply rising shock front (monotonic shock) for large ν $[\nu_c/\omega_{pc} \sim O(1)]$ as compared to oscillatory shock for finite $\nu \sim O(1)$ (ν_c/ω_{pc} is small).

Finally, substitution of (9) and (10) into the dynamical equations (6)–(8) leads to the following relations in lowest powers of ϵ:

$$\phi^{(1)} + \alpha n_c^{(1)} = 0, \quad -Mv^{(1)} = \phi^{(1)}, \quad Mn_c^{(1)} = v^{(1)}. \tag{12}$$

The equations in the above give rise to the dispersion relation

$$M^2 = \alpha \Rightarrow \omega^2 = k^2 c_{se}^2 \text{ (dimensional form).} \tag{13}$$

Next, the dynamical equations in the next higher powers of ϵ are obtained as

$$\begin{gathered}
\frac{\partial^2\phi^{(1)}}{\partial\xi^2} - \frac{\phi^{(1)2}}{2} = \phi^{(2)} + \alpha n_c^{(2)}, \\
\frac{\partial v^{(1)}}{\partial\tau} + \frac{1}{2}\frac{\partial v^{(1)2}}{\partial\xi} + \nu M\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = \frac{\partial}{\partial\xi}(\phi^{(2)} + Mv^{(2)}), \\
\frac{\partial n_c^{(1)}}{\partial\tau} + \frac{\partial}{\partial\xi}\left(v^{(1)}n_c^{(1)}\right) = \frac{\partial}{\partial\xi}\left(Mn_c^{(2)} - v^{(2)}\right).
\end{gathered} \tag{14}$$

Finally, elimination of the second order terms from Eq. (14) together with relation (12) yields the following Korteweg–de Vries Burgers (KdVB) equation for finite amplitude nonlinear EAW for density $N \equiv (3+2\alpha)n_c^{(1)}/\sqrt{\alpha}$ and $\bar{\tau} \equiv \alpha\tau/2$:

$$\frac{\partial N}{\partial\bar{\tau}} + N\frac{\partial N}{\partial\xi} + \frac{1}{\sqrt{\alpha}}\frac{\partial^3 N}{\partial\xi^3} = \nu\frac{\partial^2 N}{\partial\xi^2}. \tag{15}$$

The dissipative term in the right-hand side of Eq. (15) represents the Burgers' term that is proportional to ν, arising due to the collision between the cold electrons and ions. In the absence of collision there is no Burgers term in Eq. (15) and the equation reduces to the KdV equation for nonlinear EAWs. Thus the Burgers term present here due to the electron-ion collision brings the physics of shock solution similar to those obtained from viscosity [30].

IV. ELECTRON ACOUSTIC SHOCK STRUCTURES

The Burgers term in Eq. (15) implies the possibility of the existence of a shock structure. This one-dimensional KdVB equation [Eq. (15)] is not a completely integrable Hamiltonian system. In other words, the energy of the system is not conserved and hence, exact analytical solution of the one-dimensional KdVB is not possible [36]. One can obtain an approximate solution by the perturbation analysis [36]. However, analytically, we can study the nature of the solution of Eq. (15) by the traveling wave solution technique [37]. To apply this, we transform Eq. (15) into the traveling wave (stationary wave) frame $\zeta = \xi - U\bar{\tau}$ with U, the constant velocity in the stationary frame. We prefer to write the KdVB equation in terms of scaled potential fluctuation $\psi \equiv (3 + 2\alpha)\phi^{(1)}/\alpha^{3/2}$ since most of the observations are in electrostatic potential fluctuations [25–27]. The equation of $\psi(\bar{\tau},\xi)$ is given by

$$\frac{\partial\psi}{\partial\bar{\tau}} - \psi\frac{\partial\psi}{\partial\xi} + \frac{1}{\sqrt{\alpha}}\frac{\partial^3\psi}{\partial\xi^3} = \nu\frac{\partial^2\psi}{\partial\xi^2}. \tag{16}$$

We integrate the transformed equation with respect to ζ subject to the boundary conditions ψ, $d\psi/d\zeta$, and $d^2\psi/d\zeta^2 \to 0$ as $|\zeta| \to \infty$. This finally leads to the following equation:

$$\frac{d^2\psi}{d\zeta^2} = \sqrt{\alpha}\left[U\psi + \frac{1}{2}\psi^2 + \nu\frac{d\psi}{d\zeta}\right]. \tag{17}$$

In the $(\psi,\ d\psi/d\zeta)$ plane, Eq. (17) has two singular points $(\psi = 0,\ d\psi/d\zeta = 0)$ and $(\psi = -2U,\ d\psi/d\zeta = 0)$. The former equation corresponds to the equilibrium downstream state and the latter corresponds to the upstream state. The singular point (0,0) is always a saddle point while the nature of the second one can be determined from the asymptotic behavior of the solution of the form $\sim\exp(p\zeta)$ [30] of the

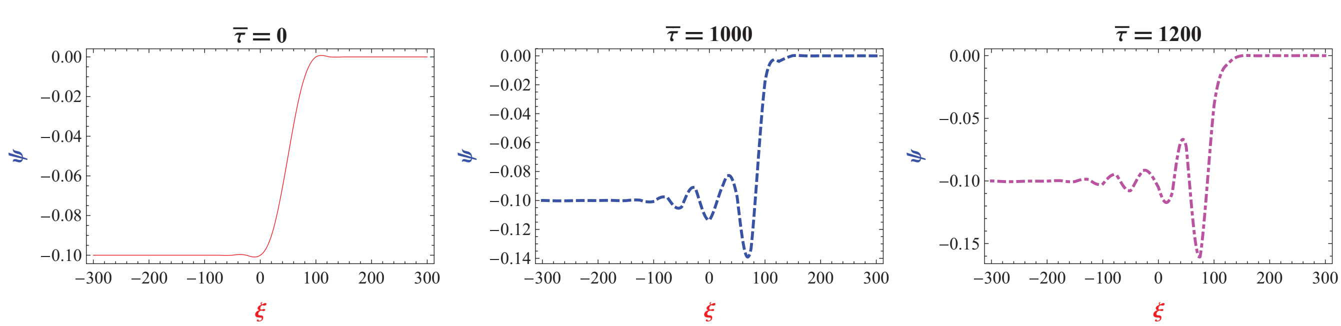

FIG. 1. (Color online) Evaluation of an oscillatory shock. The upstream value is $U = 0.05$. The plasma parameters are $\alpha = 0.2$ and $\nu = 0.1$. At $\bar{\tau} = 0, 1000, 1200$. An oscillatory shock is fully developed at $\bar{\tau} = 1200$. Here we measure ξ in units of λ_{Dh} and time in units of ω_{pc}^{-1}.

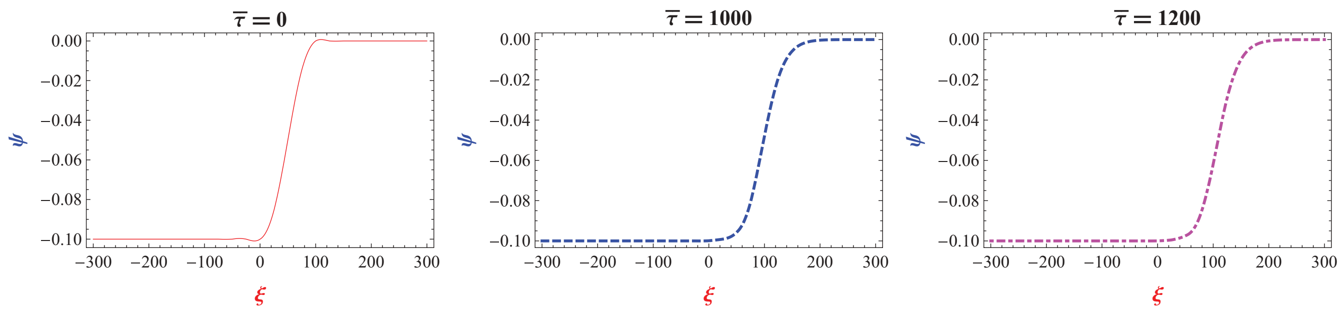

FIG. 2. (Color online) Evaluation of monotonic shock for $\nu = 1$. The other parameters are the same as in Fig. 1.

linearized Eq. (17). This yields

$$p = \frac{\nu\sqrt{\alpha}}{2}\left[1 \pm \sqrt{\left(1 - \frac{4U}{\nu^2\sqrt{\alpha}}\right)}\right].$$

It follows from this equation that the singular point $(-2U,0)$ is a stable focus or stable node according as $\nu^2 \lessgtr 4U/\sqrt{\alpha}$. The stable focus always corresponds to the oscillatory nature, whereas the stable node corresponds to the monotonic nature of the solution. Actually, if one assumes that for $\zeta = \infty (\xi = \infty)$, the particle was located at $\psi = 0$, then at $\zeta = -\infty$ $(\xi = -\infty)$ it appears at the point $\psi = -2U$. Thus the solution describes a shocklike structure. The shock is oscillatory or monotonic in nature according to

$$\mathcal{M} \gtrless 1 + \frac{1}{4}\left(\frac{\nu_c}{\omega_{pc}}\right)^2,$$

where $\mathcal{M} = 1 + \epsilon(U/M)$ is the Mach number.

Moreover, we solve the KdVB equation (16) numerically using the MATHEMATICA based finite difference scheme with the following initial steplike wave form [38]:

$$\psi(\xi,0) = \begin{cases} A & \text{for } \xi \leqslant 0, \\ \frac{A}{2}(1+\cos k\xi) & \text{for } 0 < \xi < \pi/k, \\ 0 & \text{for } \xi \geqslant \pi/k, \end{cases}$$

where A and k are the initial amplitude and wave number. The value of k determines the temporal evolution for fixed A. The wave evolves quickly (slowly) for large (small) value of k. Equation (16) is solved within the spatial interval $\xi \in [-L,L]$ with the above initial condition and the boundary conditions: $\psi(-L,\tau) = A$, $\psi(L,\tau) = 0$, and $\psi_\xi(-L,\tau) = 0 = \psi_\xi(L,\tau)$. To obtain adequate results for the computation, we take $L = 300$, $A = 2U = 0.1$, and $k = \pi/100$. The plasma parameter is $\alpha = n_{c0}/n_{h0} = 0.2$ ($\ll 1$, a necessary condition for the existence of EAW) relevant to space plasmas. To estimate the Burgers term we consider two different cases: one is weak dissipation and the other is strong dissipation represented by the numerical values $\nu = 0.1$ and $\nu = 1$, respectively. These numerical estimations confirm the existence of both oscillatory shock for weak dissipation and monotonic shock for strong dissipation, as shown in Figs. 1 and 2. The comparison between the three curves in Fig. 1 shows that the oscillatory shock is fully developed at $\bar{\tau} = 1200$. A similar conclusion holds for monotonic shock in Fig. 2. The transition from the upstream to the far downstream state changes from being of oscillatory to monotonic nature as dissipation ν increases from 0.1 to 1. The shock strength (related to the extreme upstream and downstream values) is given by $\psi(-\infty) - \psi(+\infty) = -2U$. Thus, the time-dependent numerical solutions, as shown in Figs. 1 and 2, agree well with the prediction made by the preceding time-independent analysis. The numerical solutions are obtained for the normalized electrostatic potential fluctuation ψ. Both figures show that $\psi < 0$. The relation between the potential and cold electron density [Eq. (12)] reveals that $n_c^{(1)} > 0$ implying the compressive nature of the shock.

V. DISCUSSIONS

We investigate the propagation characteristics of nonlinear EAWs in the presence of electron-ion collision. The dynamics of the nonlinear wave is shown to be governed by the Korteweg–de Vries Burgers' equation due to the collision induced dissipation. In the present work, we have shown that the dissipation arises due to that electron-ion collision through the collective phenomena is responsible for the Burgers' term. This bring the physics of shock wave and plays the similar role of viscosity. The numerical solution (depending on the strength of the dissipation) confirms the existence of both oscillatory and monotonic shock structures. The observed shock is compressive in nature with sufficient cold electron density enhancement in the upstream side of the shock.

The EAWs exist in polar magnetosphere [11] and the electrostatic shock waves are observed in this region [13]. Thus the results of the present investigation could be useful for understanding the physics of shock waves in the polar magnetosphere. Moreover, the variation of potential resulting from propagation of compressional shock waves is of significant importance in the auroral plasma region. It is well known that the charged particle acceleration and electromagnetic waves emission occur in the auroral acceleration region [31]. The electrons accelerate both in the upward and downward direction along the geomagnetic field in the auroral region. The particle energization due to the conversion of wave energy into particle kinetic energy is thought to drive particle acceleration in the auroral region and the particle density in the acceleration region is low [31]. The results of the present investigation reveal that the negatively charged cold electrons with energies less than

$$\mathcal{E} = -e\phi = T_h\left(\frac{2(\mathcal{M}-1)\alpha^2}{3+2\alpha}\right)$$

are reflected by the shock wave due to the negative potential. As a consequence of this particle reflection, the electrons are energized due to the passing of the shock wave. This initiates the particle acceleration mechanism in the auroral region. According to the previous investigations [14,17,18], it is believed that double layers are the physical mechanism for the particle acceleration in auroral plasma. Present investigations reveal that the shock wave generated due to the collisions between cold electron and stationary ion could also be a viable physical mechanism for the auroral particle acceleration.

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