Multifractality in domain wall dynamics of a ferromagnetic film

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We investigate the multifractal properties in the dynamics of domain walls of a ferromagnetic film. We apply the Multifractal Detrended Fluctuation Analysis method in experimental Barkhausen noise time series measured in a 1000-nm-thick Permalloy film under different driving magnetic field frequencies, and calculate the fluctuation function $F_q(s)$, generalized Hurst exponent $h(q)$, multifractal scaling exponent $\tau(q)$, and the multifractal spectrum $f(\alpha)$. Based on this procedure, we provide experimental evidence of multifractality in the dynamics of domain walls in ferromagnetic films and identify a rich and strong multifractal behavior, revealed by the changes of the scaling properties of over the entire Barkhausen noise signal, independently of the driving magnetic field rate employed in the experiment.

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I. INTRODUCTION

Since the discovery of the effect [1], Barkhausen noise (BN) has been employed as a tool to investigate ferromagnetic materials [2–4]. Due to its stochastic character, BN is analyzed using statistical treatments on the experimental time series. Traditionally, the BN times series statistical analysis considers the distribution of amplitudes, distributions of jump sizes and jump durations, power spectrum, and average size of an avalanche as a function of its duration [3,4]. All of these statistical functions are, in general, well described by cutoff-limited power-laws, which are understood as the fingerprint of a critical dynamics [5]. BN can be understood as a result of the complex microscopic magnetization process and irregular motion of domain walls in ferromagnetic materials and, for this reason, recently, it has attracted growing interest as an example of response of a disordered system exhibiting crackling noise, becoming an excellent candidate for investigating scaling phenomena [4–7]. From this new point of view, the study of BN becomes valuable since several systems in many situations, remarkably, present response signals, or time series, that usually share common characteristic features. It is the case of BN in ferromagnetic materials, the seismic activity in earthquakes, the dynamics of vortices in superconductors, the fluctuations in the stock market, the acoustic emission in microfractures processes, and the shear response of a granular media [4]. In all these systems, the dynamics and the signal statistical properties seem to be independent of the microscopic and macroscopic details of the samples, being, however, controlled just by a few general properties, such as the system dimensionality and the range of the relevant interactions [8].

Many of the time series from these systems exhibit self-similarity, which is the signature of a fractal nature in the system [9]. In this case, the definition of fractal is associated to the ability of a system to present similar features with a change of scale, reflected in the response signal which has similar and reproducible statistical features.

Particularly, monofractal systems are homogeneous, in the sense their time series have the same scaling properties throughout the entire signal and can be indexed by a single global exponent, called Hurst exponent H [10].

On the other hand, multifractal systems can be understood in a complementary concept, as a composition of interlaced fractals, each one with its own fractal dimension. In this sense, a multifractal system is a generalization of a fractal system, in which a single exponent is not enough to describe its dynamics, but a continuous spectrum of exponents is required [11].

Multifractal time series are characterized by a hierarchy of exponents that describe the scale behavior of various subsets of the respective studied time series [9]. There are several methods to analyze multifractality in time series, such as the partition function formalism [9,12], and the Wavelet Transform Modulus Maxima (WTMM) [13–15] method, that provide good outcomes for nonstationary time series affected by trends. In another approach, an important alternative for multifractal analysis corresponds to the implementation of the Multifractal Detrended Fluctuation Analysis (MF-DFA) method [16,17].

In recent decades, multifractal analysis has been widely performed in time series originated by several natural complex systems, including heartbeat dynamics [18], earthquakes [19], electrostatic plasma turbulence in tokamak [15], dynamics of turbulent flows [20], geophysical time series [21], turbulence in fluids [22], fluctuations in financial markets [23], and complex networks [24]. In all of them, theory or simulations and experimental results seem to agree well and the analysis evidences the existence of multifractality in all these signals.

In this paper, we report an experimental evidence for multifractality in the domain wall dynamics of ferromagnetic films, by analysis Barkhausen noise time series. We investigate the multifractal properties in ferromagnetic systems by using the algorithm called Multifractal Detrended Fluctuation Analysis [16] and, for experimental time series, we calculate the fluctuation function $F_q(s)$, generalized Hurst exponent $h(q)$, multifractal scaling exponent $\tau(q)$, and the multifractal spectrum $f(\alpha)$ [11]. We apply the multifractal analysis to Barkhausen noise and identify the multifractal behavior in the time series, revealed by the changes of the scaling properties

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of over the entire Barkhausen signal. Moreover, we show that multifractality in the dynamics of ferromagnetic domain walls is rich and strong, independently of the driving magnetic field rate employed in the experiment.

The paper is organized as follows. In Sec. II we present the Barkhausen noise experiment and discuss the traditional statistical analysis obtained for the studied sample. In Sec. III we consider the multifractal concept and describe the MF-DFA method employed in this work. In Sec. IV, we present the results of the multifractal analysis and discuss the multifractal characteristics in Barkhausen noise and in domain wall dynamics of a ferromagnetic film. Finally, the last section is devoted to conclusions and discussion of open problems.

II. EXPERIMENT

The experimental results analyzed in this work consist of experimental Barkhausen noise time series measured in a ferromagnetic film under different driving magnetic field frequencies.

The Barkhausen noise in ferromagnetic materials corresponds to the time series of voltage pulses detected by a sensing coil wound around a ferromagnetic material submitted to a variable magnetic field [1–4]. The noise is produced by sudden and irreversible changes of magnetization, mainly due to the irregular motion of the domain walls (DW) in a disordered magnetic material, a result of the interactions between the DWs and pinning centers, such as defects, impurities, dislocations, and grain boundaries [2,3,25–27].

For this study, we perform Barkhausen noise measurements in a ferromagnetic film with nominal composition $Ni_{81}Fe_{19}$ (Permalloy) and thickness of 1000 nm, produced by magnetron sputtering. X-ray diffraction results verify the polycrytalline structural character of the film, as well as quasistatic magnetization curves indicate isotropic in-plane magnetic properties with an out-of-plane anisotropy contribution. Detailed information on the film production and structural and magnetic characterization of the sample employed in this work can be found in Ref. [28].

We record Barkhausen noise time series using the traditional inductive technique in an open magnetic circuit. The studied sample has dimensions 10 mm × 4 mm × 1000 nm. Sample and pickup coils are inserted in a long solenoid with compensation for border effects, to ensure an homogeneous applied magnetic field on the sample. The sample is driven by a triangular magnetic field, applied along the main axis, with an amplitude high enough to saturate magnetically the film. The driving field frequency is varied in the range 0.03–0.4 Hz. BN is detected by a sensing coil (400 turns, 3.5 mm long, 4.5 mm wide, 1.25 MHz resonance frequency) wound around the central part of the sample. A second pickup coil, with the same cross section and number of turns, is adapted to compensate the signal induced by the magnetizing field. The Barkhausen signal is then amplified and filtered using a 100 kHz low-pass preamplifier filter, and finally digitalized by an analog-to-digital converter board (PCI-DAS4020/12 board—Measurement Computing) with sampling rate of 4×10^6 sample per second. BN measurements for all driving field frequencies are performed under similar experimental conditions. The time series are acquired just around the central part of the hysteresis loop, near the coercive field, where the domain wall motion is the main magnetization mechanism [2,27,29] and the noise achieves the condition of stationarity [30]. In particular, for each driving field frequency, the statistical scaling properties are obtained from 150 experimental Barkhausen noise time series.

The universality class of the Barkhausen noise in a sample is identified by measuring the distributions of avalanche sizes and durations, the power spectrum, and the average size vs. duration, usually described by cutoff-limited power-laws and related to the critical exponents τ, α, ϑ, and $1/\sigma\nu z$, respectively [3]. Through this traditional BN statistical analysis, for the ferromagnetic film employed in this work, two of us have previously measured values $\tau \sim 1.5$, $\alpha \sim 2.0$, and $\vartheta \sim 1/\sigma\nu z \sim 2.0$ for the exponents, a result obtained for the smallest magnetic field rate. Besides, we verified rate-dependent τ and α, while $1/\sigma\nu z$ and ϑ are constant critical exponents. Thus, by considering this wide statistical analysis, the agreement between experimental results and well-known predictions for bulk polycrystalline magnets [3,4,31] indicates that Barkhausen noise measured in our sample and dynamics of domain walls are described by mean-field theory [28,29,31], since this film presents a typical three-dimensional magnetic behavior, with predominant strong long-range dipolar interactions governing the domain wall dynamics.

III. MULTIFRACTAL ANALYSIS

In this work, the multifractal analysis is based on the Multifractal Detrended Fluctuation Analysis method [16,17].

MF-DFA basically consists of five steps. First of all, we assume that x_k is an experimental Barkhausen noise time series, of length N. Thus, in the first step, the accumulated profile from a time series, so-called random walk like time series [17], is determined by the following equation:

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \quad i = 1, \ldots, N, \tag{1}$$

where $\langle x \rangle$ denotes the mean of the time series x_k.

In step two, for a given time scale s, the accumulated profile from Eq. (1) is divided into $N_s \equiv \mathrm{int}(N/s)$ integer disjoint segments of equal length s. In step three, for each one of the N_s segments, the local trend is determined by a polynomial fitting of the data, and then the variance for each segment $\nu = 1, \ldots, N_s$ is estimated through

$$F^2(\nu,s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu - 1) + i] - y_\nu(i)\}^2, \tag{2}$$

where $y_\nu(i)$ is the polynomial fitting for the segment ν.

In step four, the average of the variances over all segments is computed to obtain the qth-order fluctuation function $F_q(s)$. In this case, for $q \neq 0$, the fluctuation function is given by

$$F_q(s) \equiv \left\{ \frac{1}{N_s} \sum_{\nu=1}^{N_s} [F^2(\nu,s)]^{q/2} \right\}^{1/q}, \tag{3}$$

while for $q = 0$,

$$F_0(s) \equiv \exp\left\{\frac{1}{N_s}\sum_{\nu=1}^{N_s} ln[F^2(\nu,s)]\right\}. \tag{4}$$

Here, we are concerned with how q dependent fluctuation function $F_q(s)$ depends on the time scale s, for some values of q. For this purpose, steps two to four must be repeated for different values of time scale s. According to Ref. [16], for very large scales $s \geqslant N/4$, the employed procedure becomes statistically unreliable, due to the fact that the number of segments N_s averaging in Eqs. (3) and (4) is very small. Thus, we take for our Barkhausen signal analysis the maximum scale value of $N/10$ and the minimum scale value with nearly 200 points.

Finally, in the last step, the scaling behavior of the fluctuation functions $F_q(s)$ is determined by estimating the slope of the plot of $\log_{10}[F_q(s)]$ vs. $\log_{10}[s]$, for a range of q values. In particular, we use q between -10 to 10. Thus, if the experimental Barkhausen noise time series present a long-range power-law correlation, then $F_q(s)$ increases for sufficiently large values of s, following the power-law scaling given by

$$F_q(s) \approx s^{h(q)}, \tag{5}$$

where $h(q)$ is the so-called generalized Hurst exponent.

To estimate the $h(q)$ values for distinct values of q, we regress $h(q)$ on $F_q(s)$, Eq. (5). Thereby strengthening this idea, for monofractal time series, the $h(q)$ is independent of q, since the behavior of the scale of the variances $F^2(\nu,s)$ is identical for all segments ν, resulting in $h(q) = H$. So, it will be observed any considerable dependence of $h(q)$ over q, just in case of small and large fluctuations differ, characterizing a multifractal behavior evidenced in the time series.

From this point, the multifractal scaling exponent $\tau(q)$ can be determined from $h(q)$ by the relation

$$\tau(q) = qh(q) - 1. \tag{6}$$

In this case, if there is a linear dependence of the spectrum $\tau(q)$ with q, the time series is considered monofractal, otherwise it is multifractal.

Furthermore, it is possible to characterize the multifractality of time series by considering the multifractal spectrum $f(\alpha)$, where α is the Hölder exponent.[1] The multifractal spectrum $f(\alpha)$ is related to $\tau(q)$ via a Legendre transform [9,32]

$$\alpha = \tau'(q), \tag{7}$$

and

$$f(\alpha) = q\alpha - \tau(q). \tag{8}$$

The magnitude of multifractality in time series can be determined by the width of the spectrum $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$. Thus, intuitively, the wider is the multifractal spectrum, the richer and stronger is the multifractality of the time series.

IV. RESULTS AND DISCUSSION

By applying the MF-DFA method described in the last section, we analyze the multifractal properties in experimental Barkhausen noise time series measured in a 1000-nm-thick Permalloy film under different driving magnetic field frequencies.

Figure 1 shows one of our Barkhausen noise times series and the associated random walk like time series, obtained by using Eq. (1). As can be seen, experimental Barkhausen noise is composed of a series of intermittent voltage pulses, due to avalanches in the magnetization, combined with background instrumental noise. Since we employ the traditional inductive technique to measure the BN in films, the corresponding voltage signal is weaker than the usually obtained for ferromagnetic bulk samples [31].

By considering the BN time series, at a first glance, it is possible to obtain some interesting information. First of all, an important point resides in the fact that the time series presents local fluctuations with both large and small magnitudes, a feature related to a possible known fractal and possible multifractal behavior. Previous reports have indicated self-similarity properties in the Barkhausen noise, at sufficiently low domain wall velocity, as well as have suggested the critical exponent obtained for the distribution of avalanche sizes as an indirect measurement of the fractal dimension of the pinning field [33].

Another point that can be considered here is the self-similarity reflected in the random walk like time series. In this case, it is easy to verify that by zooming over different scales results in similar behavior, indicating fractal properties as well.

Besides, as a third point, the shape of the associated random walk reveals some persistent structure of the BN time series. Thus, in order to verify that the measured accumulated profile presented in Fig. 1 is strictly related to the magnetic signal from the sample, we also analyze a background instrumental noise time series, measuring the instrumental response without the sample. It must be pointed out that the background noise has a behavior similar to the one verified to a white-noise-like time series, as reported in Ref. [17]. In this case, the result given by Eq. (1) corresponds to an accumulated profile with absence of tendencies or persistent structures, maintaining the profile close to the average, in a normal distribution, with features completely different from that observed to multifractal time series [17] and to our BN signal.

The multifractal character of the Barkhausen noise can be initially identified from the plot of the fluctuation function with respect to the time scale. Figure 2 presents log-log plot of the fluctuation function $F_q(s)$, given by Eq. (3), with respect to the time scale s, for selected q values, for an experimental BN time series and for the background instrumental noise.

In Fig. 2(a), for the BN signal, it is verified that the slopes of $F_q(s)$ are q dependent. In this case, for small segment sizes (small scales), it is possible to note a considerable difference in $F_q(s = \text{small})$ results for distinct q values. The small segments are able to distinguish between local periods

[1]To avoid a misunderstanding with the used notation, it is important to keep in mind that, in the traditional statistical analysis performed for Barkhausen noise measurements [28,31], we assume τ and α as the critical exponents measured from the distributions of jump sizes and jump durations, respectively. On the other hand, in the context of the multifractal analysis, the similar symbols $\tau(q)$ and α present distinct meanings, corresponding to the multifractal scaling exponent and Hölder exponent, respectively.

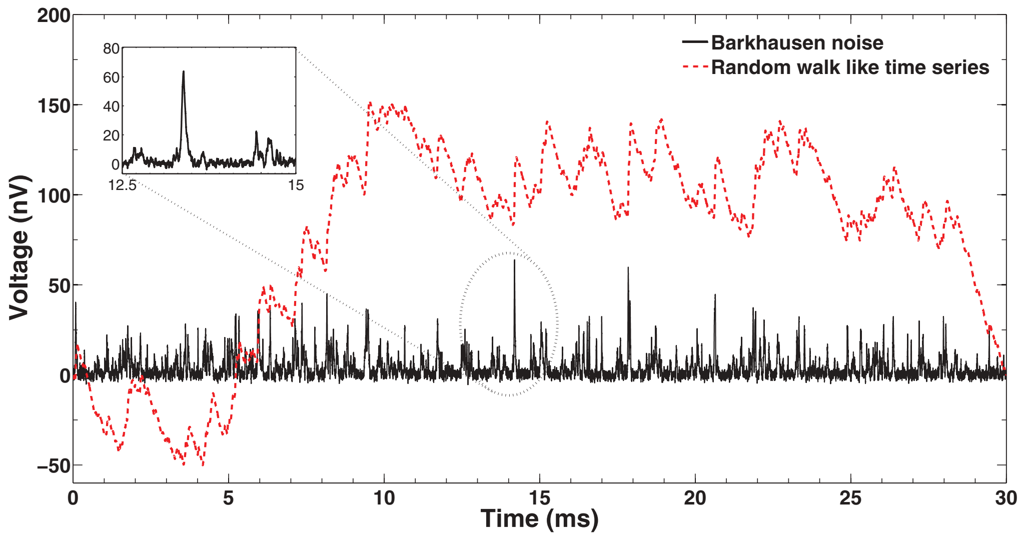

FIG. 1. (Color online) Experimental Barkhausen noise time series measured in our 1000-nm-thick Permalloy film (solid black line) and the random walk like time series (dashed red line). BN is converted to random walk by using Eq. (1). The inset shows a zoom over a small scale, evidencing the intermittent pulses combined with background instrumental noise, as well as self-similarity properties.

with large and small fluctuations in the BN time series. On the other side, large segment sizes (large scales) cross diverse local periods of the analyzed time series with small and large fluctuations and will average out their differences in magnitude [16,17], leading to a close result of $F_q(s = \text{large})$ for distinct q values. These characteristics infer that the experimental Barkhausen noise signal has a multifractal temporal structure.

In contrast to the BN signal, in Fig. 2(b), for the background instrumental noise, periods with small and large fluctuations are not observed and, for this reason, it is verified the same difference between the $F_q(s)$ results for distinct q values, independently of the segment sizes, indicating that background instrumental noise has none multifractal features, leading to the result $h(q) = H$.

Remarkable points of the multifractal behavior in the dynamics of ferromagnetic domain walls can be also verified through the generalized Hurst exponent $h(q)$, multifractal scaling exponent $\tau(q)$, and multifractal spectrum $f(\alpha)$, respectively obtained through Eqs. (5), (6), and (8). Since the non-multi-fractal behavior for a white-noise-like time series is well known, its analysis can be used for comparison to the results obtained for our experimental times series. Thus, Fig. 3 shows $h(q)$, $\tau(q)$, and $f(\alpha)$ for an experimental BN time series, a background instrumental signal and for a simulated white noise times series.

From Fig. 3, for the background instrumental signal, as well as for the simulated white noise, approximately constant $h(q)$ values are observed for all the q interval, between -10 and 10. This relation of $h(q)$ with q leads to a linear dependence of

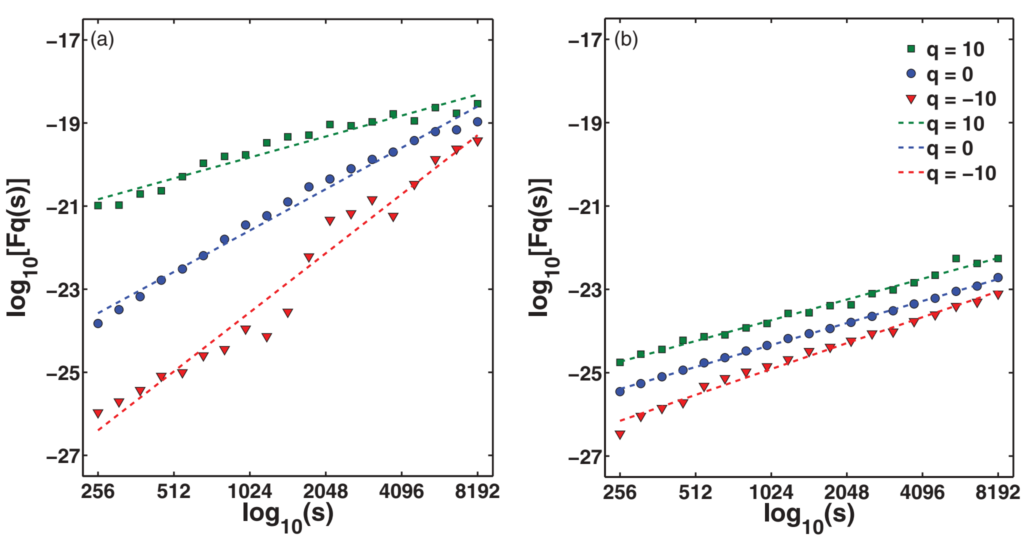

FIG. 2. (Color online) (a) Log-log plot of the fluctuation function $F_q(s)$, given by Eq. (3), with respect to the time scale s for selected q values, obtained for a selected experimental Barkhausen noise time series measured with driving magnetic field frequency of 0.08 Hz (symbols), together with the corresponding regression slopes (dashed lines). (b) Similar plot for a background instrumental noise time series.

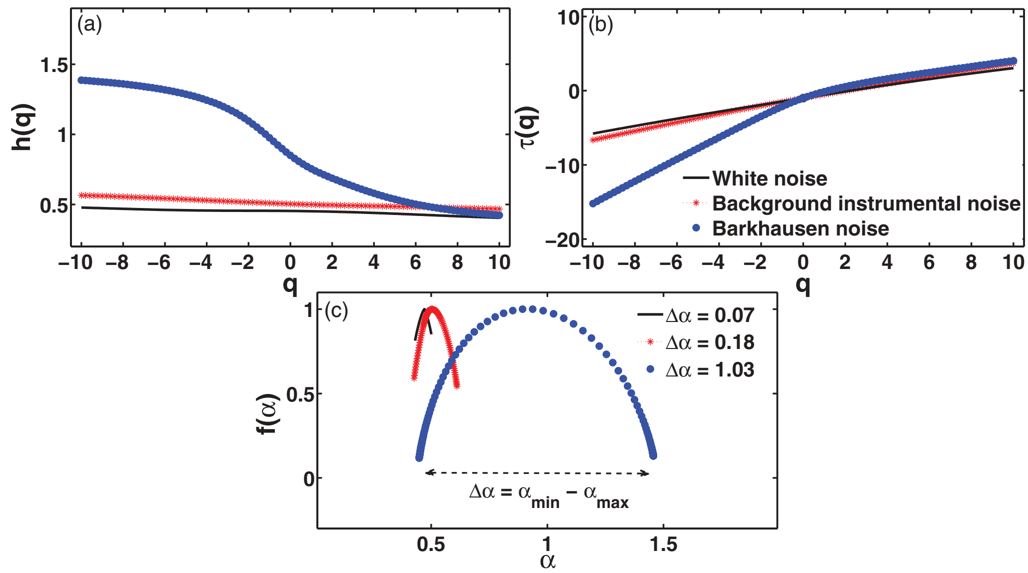

FIG. 3. (Color online) (a) Generalized Hurst exponent $h(q)$ for the same selected Barkhausen noise-noise time series measured with driving magnetic field frequency of 0.08 Hz (blue circles) of Fig. 2, background instrumental signal (red asterisks), and for a simulated white noise signal (solid black line). (b) Similar plot for the multifractal scaling exponent $\tau(q)$. (c) Multifractal spectrum $f(\alpha)$ for the very same signals. Here, $h(q)$, $\tau(q)$, and $f(\alpha)$ are obtained, respectively, through Eqs. (5), (6), and (8), and $\Delta\alpha$ corresponds to the width of the multifractal spectrum.

$\tau(q)$ with q for both signals. Moreover, the linear dependence gives rise to a multifractal spectrum $f(\alpha)$ given by a small arc, with small width.

The absolute value for the width of the multifractal spectrum $\Delta\alpha$, that corresponds to the difference between the maximum and minimum α, as well as the shape of the multifractal spectrum, are related to the temporal variation of the generalized Hurst exponent $h(q)$ [17]. In this sense, the wider $\Delta\alpha$ is, the richer and stronger the multifractality of the analyzed time series is. For the considered white noise and background instrumental noise, we find $\Delta\alpha$ equals to 0.07 and 0.18, respectively. Thus, from these results, the $h(q)$ and $\tau(q)$ behaviors and the obtained $\Delta\alpha$ values can be understood as evidences of non-multifractality for both signals.

On the other hand, when the experimental Barkhausen noise time series is considered, the results are completely distinct. Here, $h(q)$ presents a decrease as the q value is increased, leading to a nonlinear $\tau(q)$ dependence with q. Consequently, the resulting multifractal spectrum $f(\alpha)$ is a wide arc, or a

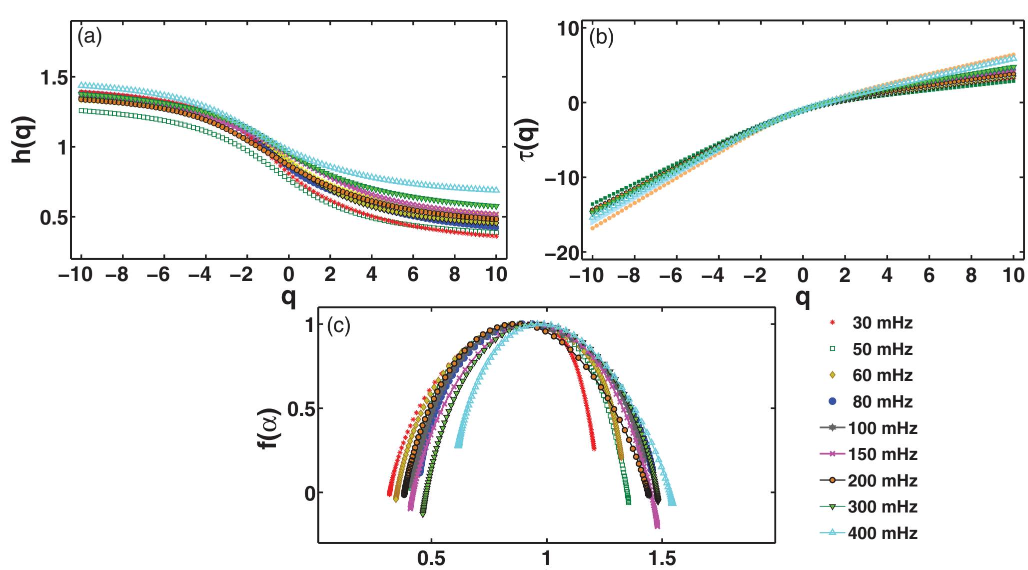

FIG. 4. (Color online) Mean behavior of the (a) generalized Hurst exponent $h(q)$, (b) multifractal scaling exponent $\tau(q)$, and (c) multifractal spectrum $f(\alpha)$ for Barkhausen noise time series measured with selected driving magnetic field frequencies, varied in the range 0.03–0.4 Hz. Here, the mean behavior is obtained from the multifractal analysis of 150 BN acquisitions measured for each frequency.

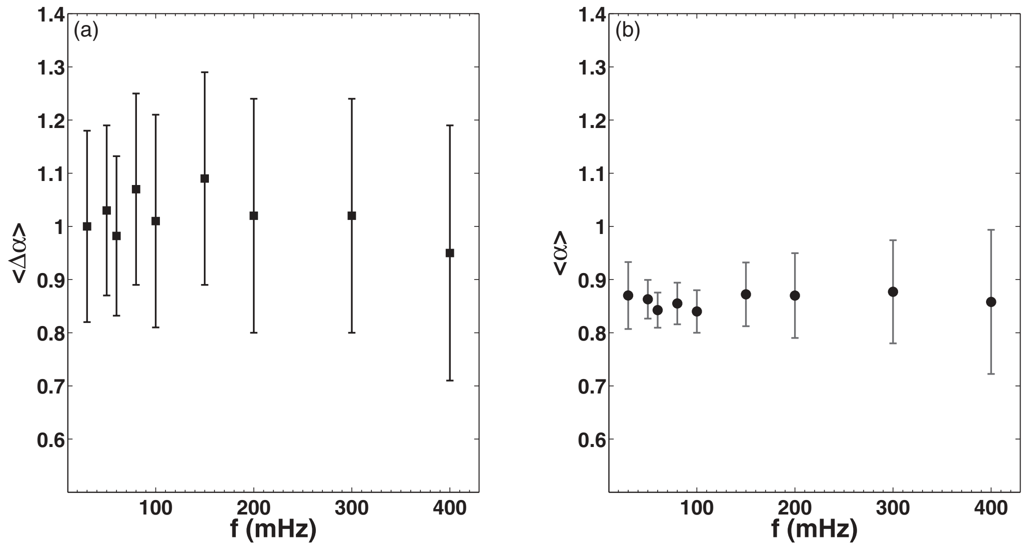

FIG. 5. (a) Mean value for the width of the multifractal spectrum, and (b) mean value for the position of the multifractal spectrum peak as a function of the driving field magnetic frequency. The error bars corresponds to the estimated standard deviation of the mean.

wide inverted parabola. In this considered case, the multifractal spectrum width is found to be $\Delta\alpha = 1.03$.

Combining the behaviours observed for $h(q)$ and $\tau(q)$ with and the obtained $\Delta\alpha$ value, these results correspond to clear signatures of the existence of a multifractal behavior in the BN times series, and multifractality in the domain wall dynamics of ferromagnetic films, as also identified in a wide variety of systems [15,18–24].

The most striking finding in the multifractal behavior in the Barkhausen noise time series measured in our ferromagnetic film resides in the fact that multifractal critical features continue to appear even when the driving magnetic field rate in the experiment is changed. From the multifractal analysis of 150 acquisitions recorded for each frequency, Fig. 4 presents the mean behavior of $h(q)$, $\tau(q)$, and $f(\alpha)$ for Barkhausen noise time series measured with selected driving magnetic field frequencies, varied in the range 0.03–0.4 Hz. In this case, it is interesting to note that the decrease of $h(q)$ with an increasing q and the nonlinear behavior of $\tau(q)$ with q are observed for all frequencies. Furthermore, the degree of multifractality is given by the calculus of the absolute value of the spectrum width. Here, the obtained mean width $\langle\Delta\alpha\rangle$ values are between 0.9 and 1.1, a signature of a rich and strong multifractal behavior, irrespective of the measurement.

Figure 5 presents the mean value for the width of the multifractal spectrum, and the mean value for the position of the multifractal spectrum peak, both as a function of the driving field magnetic frequency. Despite the small fluctuations and within the experimental error ranges, the mean width $\langle\Delta\alpha\rangle$ values are close to $\sim$1, as well as the peak position ones are $\langle\alpha\rangle \sim 0.85$, for all the time series measured with different driving field frequencies. Thus, through the systematic comparison of our results for distinct frequencies, the multifractality in the dynamics of domain walls of ferromagnetic films seems not to present any clear dependence on the driving magnetic field rate employed in the experiment.

Considering the traditional statistical analysis performed for Barkhausen noise, it is well known that a difference may exist among experimentally obtained critical exponents τ, α, ϑ, and $1/\sigma\nu z$, due to a different magnetization reversal mechanism, different domain type, or different driving magnetic field rate [29]. As already cited, for the very same BN measurements analyzed in this work, we found rate-dependent τ and α, while ϑ and $1/\sigma\nu z$ constant critical exponents [28], in concordance with theoretical predictions for the universality class which our film is included. However, surprisingly, with the multifractal analysis, the multifractality and $h(q)$, $\tau(q)$, and $f(\alpha)$ behaviors seem to be not affected by the driving magnetic field rate.

V. CONCLUSION

In summary, in this paper we investigate multifractal properties in the domain wall dynamics of a ferromagnetic film. By applying the Multifractal Detrended Fluctuation Analysis method, we analyze the multifractality of experimental Barkhausen noise time series measured in 1000-nm-thick Permalloy film under different driving magnetic field frequencies. In the procedure, we calculate the fluctuation function $F_q(s)$, generalized Hurst exponent $h(q)$, multifractal scaling exponent $\tau(q)$, and the multifractal spectrum $f(\alpha)$. Using this approach, we compare the Barkhausen noise results with the ones obtained for background instrumental signal and simulated white noise time series.

Through the results obtained with the multifractal analysis, we provide experimental evidence of multifractality in domain wall dynamics of ferromagnetic films. The multifractal behavior is revealed by the changes of the scaling properties of over the entire Barkhausen signal. The multifractality of experimental Barkhausen noise times series enables us to quantify the great complexity of the dynamics of domain walls of a ferromagnetic film. Moreover, we emphasize that all our experimental results directly confirm a rich and strong multifractality in the BN times series, independently of the driving magnetic field rate employed in the experiment.

Considering this fact, we show that the employed MF-DFA method is sensitive to the intrinsic and general features of the signal and of the physical process that generates it, and not to the driving field rate employed in the experiment. Thus, an interesting study is the application of the multifractal analysis in BN experimental time series measured in films with distinct thicknesses and structural character, in order to test the universality of the multifractal behavior and its relation with general properties, such as the system dimensionality and the range of the relevant interactions governing the domain wall dynamics. These experiments and analyses are currently in progress.

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