Circular ratchets as transducers of vertical vibrations into rotations

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Granular ratchets are well-known devices that when driven vertically produce a counterintuitive horizontal transport of particles. Here we report the experimental observation of a complementary effect: the striking ability of *circular ratchets* to convert their vertical vibration into their own rotation. The average revolution speed shows a maximum value for an optimal tooth height. With no special effort the rotation speed could be maintained steady during several hours. Unexpected random arrests and reversals of the velocity were also observed abundantly.

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I. INTRODUCTION

A number of recent publications reported remarkable phenomena related with certain devices cleverly designed to convert stochastic energy into directed transport of particles in the absence of any net bias force [1–10]. The interest in such devices can be traced back to a famous century-old work by Smoluchowski [11] who considered a gedanken experiment involving two basic elements: a miniature ratchet, consisting of a toothed wheel and pawl, connected to an axis with four vanes immersed in a molecular gas. The interest then was connected with the possibility of using the composed device containing the toothed wheel and pawl to bypass the second law of thermodynamics. Much later, Feynman showed that such composed device does not produce work at *equilibrium* conditions [12,13]. However, directed transport of particles and energy is clearly possible in the presence of *nonequilibrium* fluctuations [14–19]. For instance, an ingenious experimental realization of the aforementioned Smoluchowski-Feynman type of device, called a *granular rotor*, was reported recently [8] where the out-of-equilibrium fluctuations are provided by inelastically colliding grains combined with a crucial symmetry breaking application of a coating to one side of each vane. More recently, the kinetic properties of granular rotors were studied theoretically in the presence of dry (Coulomb) friction [4].

Transport of particles in the absence of any net bias force can be produced by shaking linear ratchets, i.e., sawtooth-shaped surfaces, and is known nowadays to be present abundantly in many areas of scientific and technological interest such as, e.g., molecular motors in biology, particle separation on nano- and microscales, and several others (for detailed reviews see, e.g., Refs. [14–19]). The understanding of the transport produced by directed motion produced by shaking linear ratchets is particularly interesting in biology because, as emphasized by Lipowsky and Harms [20], cells and subcellular organelles undergo directed motion during a plethora of biological processes such as cell locomotion, intracellular transport, or cell division. All such motions are powered by molecular motors [15,16] which are able to transduce the free energy released from chemical reactions directly into mechanical work. Several aspects about how this conversion occurs are still far from being understood [21].

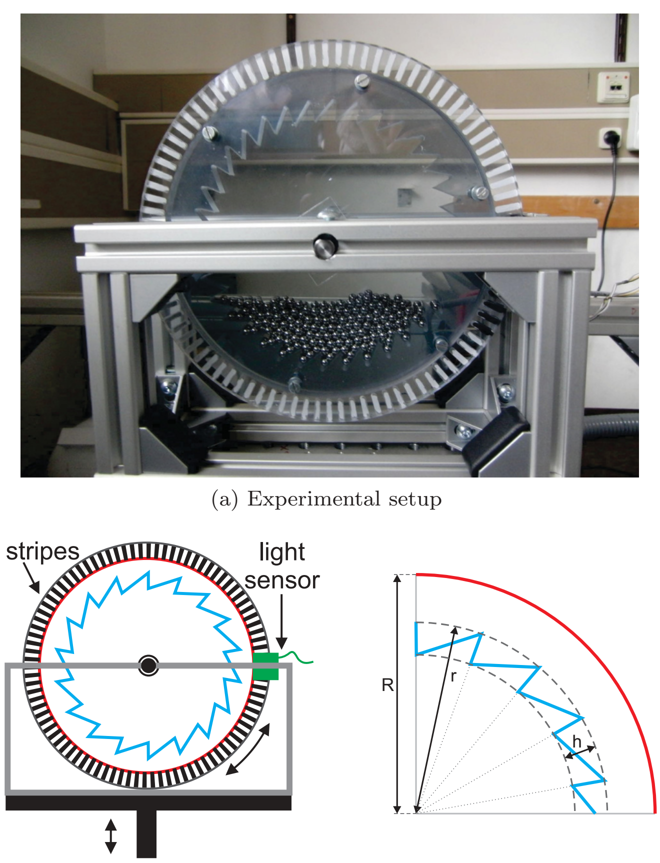
The problem of horizontal transport in a granular layer which is vibrated vertically by an asymmetric base was also considered, both experimentally [22,23] and numerically [24,25]. These so-called "granular ratchets" were shown to exhibit interesting dynamical features akin to those characterizing Brownian-motor models. For instance, Levanon and Rapaport [24] have shown that a layer of granular material placed on a vertically vibrating sawtooth-shaped base exhibits horizontal flow whose speed and direction depends in a complex manner on the parameters specifying the system. Discrete-particle simulations revealed that the induced flow rate varies with height within the granular layer and that oppositely directed flows can occur at different levels.

More recently, Costantini *et al.* [19] have shown by numerical simulations that a nonrotationally symmetric body, whose orientation is fixed and whose center of mass can slide along a rectilinear guide under the effect of inelastic collisions with a surrounding gas of particles, displays directed motion. These authors presented very interesting models showing how the lack of time reversal induced by the inelasticity of collisions can be exploited to generate a steady average drift. In the limit of a heavy device, they derived an effective Langevin equation whose parameters depend on the microscopic properties of the system and found a fairly good quantitative agreement between the theoretical predictions and simulations concerning effective friction, diffusivity, and average velocity. The control parameter space of a discrete proxy of a linear ratchet was recently shown to contain certain isoperiodic stable structures both in the classical current [5] and in its quantum counterpart [1].

As mentioned, the literature contains references to "granular rotors" which, however, describe rather distinct phenomena than the one reported here. For instance, Cleuren and Eichhorn [18] investigate the rotational motion of an arbitrarily shaped tracer object (which they name granular rotor) around a fixed axis during incessant dissipative collisions with a surrounding gas. A granular rotor was realized experimentally by Eshuis *et al.* [8] and subsequently investigated theoretically by Talbot *et al.* [4]. Note, however, that such devices *do not involve shaking any granular ratchet* but, instead, are realizations of the Smoluchowski-Feynman type gedanken experiment [13–16], operating with four coated vanes driven by a vibrofluidized low-density granular gas.

All previous investigations about ratchet transport of granular particles that we know of dealt exclusively with horizontal flows driven by sawtooth-shaped surfaces fixed (stationary) on vertically shaking platforms, i.e., by linear ratchets [22,23]. In this context it is natural to ask what would happen when grains are shaken in asymmetric surfaces having a geometrical shape distinct from a straight line. In this paper, we report the experimental observation of a complementary effect to the aforementioned horizontal transport of grains by linear ratchets: the striking ability of a novel device, namely a *circular ratchet*, to convert vertical vibrations into rotations. More specifically, we report the observation of rotations generated by a circular ratchet constructed by suitably bending the familiar sawtooth-shaped linear ratchet to form a circle *with the teeth in its inner part* as illustrated in Fig. 1. Depending on the teeth orientation, the circular ratchet may rotate either clockwise or anticlockwise. The rotations produced by circular ratchets display a number of interesting characteristics which are discussed below.

In the next section we describe our circular ratchet. The basic characteristics of the circular ratchet are reported in Sec. III. Finally, Sec. IV summarizes our main conclusions and mentions briefly some interesting open problems.



(a) Experimental setup

(b) Circular ratchet (c) Teeth geometry

FIG. 1. (Color online) (a) Photograph of the circular ratchet loaded with stainless steel spheres. (b) Sketch indicating the light sensor which detects transitions between black and white stripes located on the rim of the ratchet. (c) Ratchet geometry: baseline radius r, teeth-height h, and the rim $R - r$ containing the alternating stripes used to record the speed of rotation (see text). For a movie illustrating the rotating ratchet, see the Supplemental Material [26].

II. CIRCULAR RATCHETS

The circular ratchets that we investigated [Figs. 1(a) and 1(b)] are made of a polycarbonate (Lexan) and are confined between walls made of the same material. The inner core is fixed to the outer walls by six screws, equally spaced to avoid mass unbalance. The inner sawtooth-shaped core of the circular ratchet is 8.2 mm thick and has the profile shown in Fig. 1(c). The inner radius r defining the teeth baseline is 60 mm. The confining walls are electrically conducting in order to avoid spurious effects from static electricity. We studied ratchets containing 36 teeth oriented radially to the center and with height $h \pm 0.1$ mm where $h \in \{0,1.1,2.3,3.5,4.7,5.9,7.1,8.3\}$. The height h could be adjusted by replacing the inner Lexan profile. To do so we used computer-aided design (CAD) and a computerized numerical controlled (CNC) mill to avoid noticeable unbalances of the construction. After that, the profiles were polished to remove all unintentional remains of the milling. The granular material used to drive the ratchet consists of 188 stainless steel spheres of 4 mm in diameter and weighting 267 ± 0.1 mg each (total mass of 50.2 g).

As illustrated in Fig. 1(a), the circular ratchet is supported by a needle bearing and steel axis fixed to a metal frame mounted on an electromagnetic shaker (TIRA shaker TV 5220-120). The shaker is capable of providing vertical sinusoidal driving forces up to 2 kN and up to several kHz. The outer rim $R - r$ in the front wall of the ratchet [see Fig. 1(c)] contains an alternating pattern made by 87 equally spaced black-and-white stripes which are used to determine the position and the rotation speed of the ratchet. Such transitions are recorded with a double light sensor (light barriers) using standard X4 encoding for a quadrature encoder. In this way, one revolution of the ratchet is divided into 348 identical steps. A small electronic board counts the impulses from the light barriers. A computer then checks several times per second for the actual value of steps marking the sense of rotation and saves the results (together with the appropriate time) into a file. In this way we are able to record position and time in a computer and derive the rotation speed.

It is important to emphasize that, of course, both the linear and the circular ratchets need to vibrate vertically in order to properly operate. However, while linear ratchets normally stay fixed to their vibrating platform, the circular ratchet introduced here rotates, i.e., moves with respect to the vibrating platform. Thus, circular ratchets have the interesting ability of converting their vertical vibration into their own rotation. The rotation arises as a combination of two steps: (a) the circular ratchet acts as a shaken container providing motion to the grains, (b) the motion of the grains (which is random because of collisions) brings directed circular motion to the ratchet because of the asymmetry.

III. ROTATION BY SHAKING

The shaking parameters are controlled by a frequency generator. The vibration amplitude is measured by a magnetic stripe and a Hall sensor as described in [27]. If not stated otherwise, the shaking parameters are fixed to a frequency $f = 16$ Hz and an amplitude A of (6.0 ± 0.1) mm. Experimentation

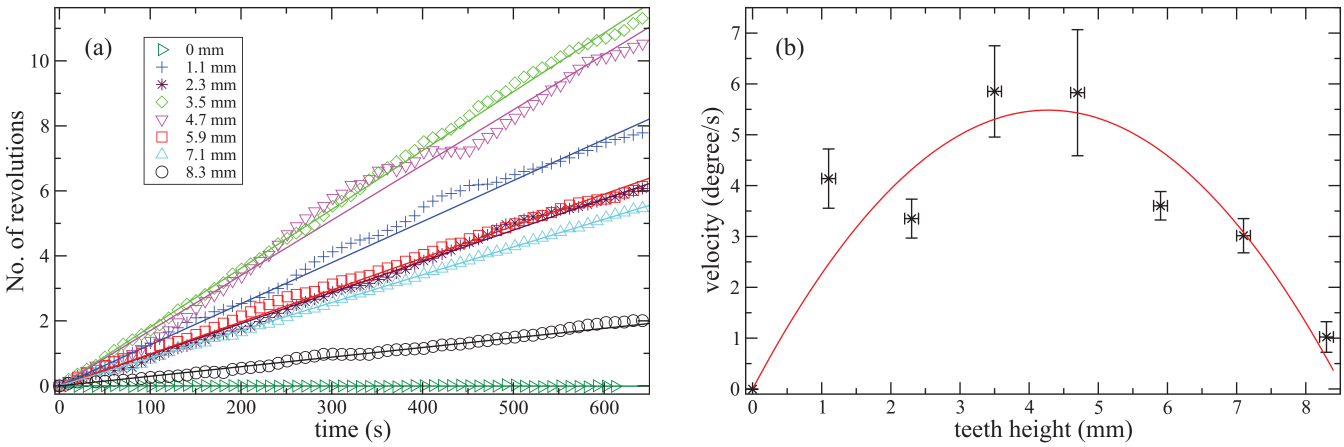


FIG. 2. (Color online) (a) Typical revolution speeds measured for different teeth heights h. The lines are linear fits to the data. (b) Average revolution speed as a function of the height h, showing the existence of an optimal height for maximal speed. Here y-error bars result from different measurements for the same parameters while x-error bars correspond to small construction imperfections of the teeth heights (± 0.1 mm). The solid line shows a quadratic fit of the points.

reveals that heavier particles give better quality measurements (i.e., less fluctuations) because of the mass balance between the ratchet and the particles, since particles need to overcome inertia and the resistance of the bearing of the ratchet. For this reason we study steel spheres.

Figure 2(a) presents a few examples of the revolution speeds which are typically measured for different teeth sizes h when shaking at 16 Hz and 6 mm amplitude. As the figure shows, the ratchet rotates fastest for moderately big teeth sizes, being slower for both larger or smaller teeth. Despite some noise, no rotation is observed when no teeth ($h = 0$) are present. As shown by simulations [28,29], the movement of a single particle in a ratchet potential can alternate repeatedly between chaos and a periodic regime. Simulations also show that the speed of the transport current is sensitive to the parameters. All this implies that it is quite natural to observe certain fluctuations in the measurements. Figure 2(b) shows the average velocity measured for different teeth heights where one sees a clear maximum at about $h = 4$ mm. A quadratic fit of the data points has been added to guide the eye to the almost parabolic shaped velocity distribution. The error bars were determined calculating the average velocity and standard deviation of single runs [see Fig. 2(a)] and repeating this measurement cycle several times independently and calculating the average error.

In Fig. 3 we present the behavior of the average velocity for a given teeth height of 8.3 mm and several shaking frequencies f as a function of the shaking amplitude A. These values fix the maximum acceleration $\Gamma = 4\pi^2 A f^2/g$, where g is the acceleration due to gravity. The average velocity is obtained from a linear fit of the data collected during one measurement cycle which typically lasts about 10 min. Figure 3 shows a general trend of the rotation speed to increase with increasing shaking power. Additionally, the measurements show a nonlinear dependence of the rotation speed with the shaking frequency. Unfortunately, all such dependencies are quite hard to characterize in detail because of the chaotic behavior and inherent fluctuations of the measurements as mentioned above [28].

Figure 4 shows the evolution of the rotation speed for the largest teeth, $h = 8.3$ mm, during our standard interval of measurement ($\approx$600 s). The figure also displays typical rotation speed measured during much longer time intervals, namely 8000 s. Series of longer measurements were performed to check the robustness of the rotation during long runs. As it is also common in noncircular ratchets [28], when observed over long time intervals the dynamics may jump between several coexisting (multistable) states (see curve "forward 2" in Fig. 4). This means that the rotation speed is sensitive to the whole shaking procedure and may fluctuate between distinct values. This sensitivity prevents a complete reproducibility of individual runs of the experiment. The (absolute) speed of the ratchet remains essentially the same even after repeatedly disassembling and reassembling the ratchet in order to either

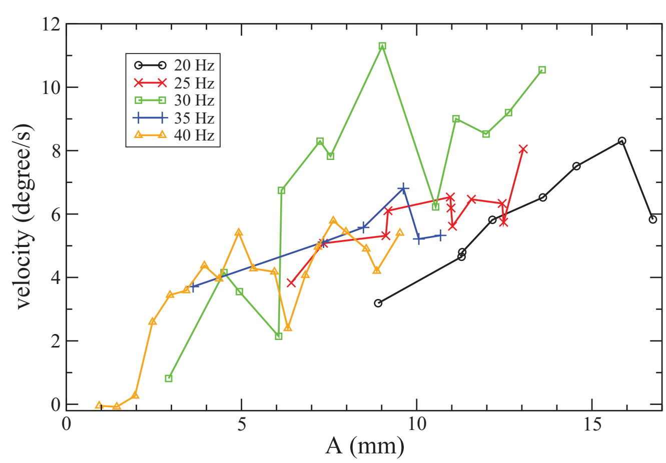


FIG. 3. (Color online) The average angular velocity measured as a function of the amplitude A for different shaking frequencies and for $h = 8.3$ mm. There is a clear tendency for the velocity to increase with A which, however, is affected by the known fact that ratchets may jump between several distinct multistable states [28]. To guide the eye, we connected points obtained for equal frequencies with lines.

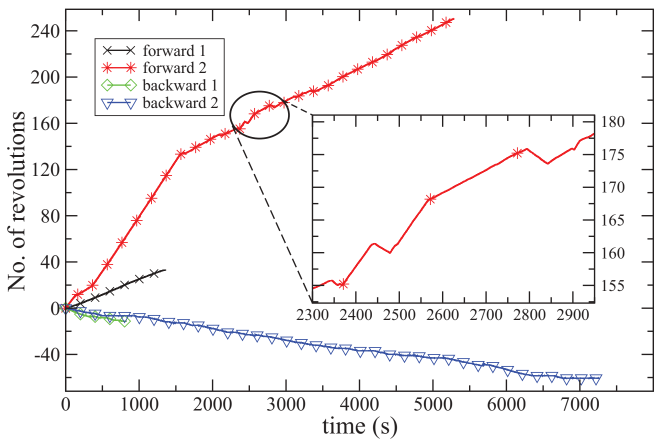


FIG. 4. (Color online) Long-time evolution of the rotation speed for the largest teeth, $h = 8.3$ mm, frequency $f = 30$ Hz, and amplitude $A = 7.2$ mm. Negative speeds result from reassembling the ratchet with the teeth tilted in the opposite direction. 8000 s $\simeq$ 2.2 h. The inset shows reversals of the rotation in the circular ratchet. In each curve, we plot one symbol after every 20×10^3 experimental data points.

replace teeth profiles or to reverse the teeth (to reverse the sense of rotations). The slight difference in the absolute velocity of the backward and the forward rotation can be attributed to the multistable states and minor imperfections in the experiment. Figure 4 shows just a few typical speeds in order not to overload the figure. The inset in the figure shows reversals of the ratchet rotation, an effect analogous to the reversal of the direction of transport known for linear ratchets [23]. As for linear ratchets, such reversals can have an essentially arbitrary duration. Note that our experimental setup does not allow the radius of the circular ratchet to grow indefinitely. This restricts the maximum possible amplitude of vibration: if the amplitude increases too much, the spheres begin to collide with the upper part of the circular ratchet and more complex phenomena ensue. Our experimental setup allows us to investigate several but not all possible aspects of the phenomena at hand. In particular, our figures show rotation reversals to occur in individual runs and we see no reason for this behavior not to be reproduced by averaged velocities, should one need them.

In addition to the sawtooth wheel seen in Fig. 1, we have also experimented with a few different symmetry-breaking shapes like, for example, a binnedlike structure capable of storing several particles inside each bin, smoothly curved teeth, and a few others. With all shapes tried it was always possible to find conversion of the vertical shaking into rotation, even though not always steady over long periods and containing much more noise. Different geometries of the teeth shape can be an interesting theme for future experiments and/or simulations.

IV. CONCLUSIONS AND OUTLOOK

We reported experiments showing that circular ratchets filled with granular material may be used to convert vertical vibrations into rotations. In such experiments the ratchet itself vibrates and rotates, simultaneously. In contrast with linear granular ratchets, in circular ratchets the granular particles stay spatially confined, i.e., they do not spread across the whole ratchet.

The possibility of converting vertical vibrations into rotations complements previous applications of granular ratchets, which so far were restricted to the horizontal transport of particles. For simplicity, in this work we limited ourselves to presenting a proof of principle and focusing on the impact of the teeth height h and shaking parameters on the rotation speed. The efficiency of the conversion of vibrations into rotations is however influenced by a large set of parameters that also need to be investigated individually. An additional open task now is to perform a search of control parameters that optimize the operation of this promising method to produce rotations using out of nonequilibrium fluctuations generated by mechanical vibrations.

Our experiment involves essentially a two-dimensional setup. It would be also desirable to explore more complex "bulky" circular ratchets, investigating three-dimensional sets of spheres and, more importantly, allowing one to probe the role of the friction introduced by the lateral walls confining the spheres. By construction, the granular material is constrained to remain always inside the circular ratchet. This interesting characteristic suggests two situations worth exploring, namely (i) the rotational behavior in regimes where the particles can collide both with the lower and upper part of the ratchet, and (ii) the dynamics of the circular ratchet in the limit of vanishing gravitational acceleration.

In conclusion, we find circular ratchets to be a potentially fruitful addition to the arsenal of experiments allowing one to prospect the physical properties of granular matter and hope that they will motivate further investigations.

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