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Transmission of electromagnetic waves through a two-layer plasma structure with spatially nonuniform electron density

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Transmission of a p-polarized electromagnetic wave through a two-layer plasma structure with spatially nonuniform distributions of electron density in the layers is studied. The case, when the electromagnetic wave is obliquely incident on the structure and is evanescent in both plasma layers, is considered. The conditions for total transparency of the two-layer structure are found for the thin slab case and when the plasma inhomogeneity is weak. It is shown that the transmission coefficient of the p-polarized wave can be about unity, even if the plasma inhomogeneity is large. The effects of plasma inhomogeneity on transparency of the structure are more important if the slabs are thick, comparing with the case of thin layers.

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I. INTRODUCTION

Interaction of electromagnetic waves with overcritical density plasmas with $\omega < \omega_{pe}$, where ω and ω_{pe} are the wave and plasma frequencies, respectively, has been intensively studied for many years [1,2]. The problem is of interest to plasma diagnostics, plasma heating, radio communications, radar applications, and photonic technologies such as plasmonic devices, etc. [1,3–8].

Electromagnetic waves propagation in different plasma and plasmalike structures, in particular, transmission through nonuniform plasma have been studied by numerous research groups [1,9–16] by different methods. In particular, it was shown that the process of the Raman scattering of a pump electromagnetic wave on Langmuir oscillations produced by the signal wave can be effective as a mechanism of information transfer through a nonuniform plasma layer [17]. In [2,14,18,19], to study propagation of electromagnetic waves along the gradients of plasma and dielectric permittivities, exact analytical solutions of the Maxwell equations were used. Taking into account the cubic nonlinearity, it was shown that, during the reflectionless transmission of a transverse electromagnetic wave through an inhomogeneous plasma containing large-amplitude, small-scale (subwave-length) structures (in particular, opaque regions), strong wave field splashes can occur in certain plasma sublayers [18]. In [20,21], the possibility of reflectionless tunneling of a transverse electromagnetic wave through a specially shaped barrier in the case, when the dielectric permittivity of the barrier is positive, and the barrier is caused by the inhomogeneous dielectric profile, was studied.

It was shown previously that total transmission of electromagnetic waves through a slab of dense plasma with negative permittivity ($\varepsilon < 0$) can be achieved by including on each side or from one side of the overcritical density plasma a boundary layer with positive permittivity $0 < \varepsilon < 1$ [22–25]. In these configurations, the total transmission of p-polarized electromagnetic waves through a slab of dense plasma is possible due to resonant excitation of surface waves at plasma boundaries. Effects of electron temperature and an external magnetic field on transparency of the overcritical density layer due to excitation of evanescent waves were studied in Refs. [24,26]. However, in most of the studies on transmission of p-polarized electromagnetic waves through nonuniform plasma, single-layer plasma structures were considered. The effects of plasma nonuniformity on transmission of p-polarized electromagnetic waves through two-layer and three-layer structures were studied by Ramazashvili [23]. In [23], the case when the boundary-layer plasma with $0 < \varepsilon < 1$ has the linear dependence on the x coordinate was considered. It was noticed that a slight inhomogeneity of a boundary slab does not prevent reflectionless transmission of p-polarized waves through a dense plasma [23]. However, for arbitrary electron-density distributions in the plasma slabs, including the case of the nonuniform dense-plasma layer, the effects of plasma nonuniformity on transparency of the layered structures are not well studied.

In this paper, we study the propagation of a p-polarized electromagnetic wave through a two-layer plasma structure with rather arbitrary spatially nonuniform distributions of electron density in the layers. The study is carried out both analytically and numerically. The conditions for total transparency of the structure are obtained for the thin slab case and when the plasma inhomogeneity is weak. Studying the weak inhomogeneity case, we use the WKB (Wentzel-Kramers-Brillouin) method [1]. In our numerical study, the cosinelike distributions of the electron density, which are typical for most of the laboratory plasmas [27–29], are considered and the reflection coefficients are calculated for different widths of the plasma slabs. The analytical results are obtained for arbitrary spatial distributions of electron density.

II. BASIC EQUATIONS

Let us consider a two-layer plasma structure immersed in vacuum (air) (see Fig. 1). It is assumed that the spatially averaged plasma density of the first layer Pl1 is small ($0 < \overline{\varepsilon}_1 < 1$, where $\overline{\varepsilon}_1$ is the spatially averaged dielectric permittivity of the layer), while the second slab Pl2 is dense with $\overline{\varepsilon}_2 < 0$ (here, $\overline{\varepsilon}_2$ is the spatially averaged dielectric permittivity of the second layer).

An electromagnetic wave is obliquely incident from a semi-infinite vacuum (air) region V1 at the plasma layer Pl1 on the left, and the transmitted wave propagates into a semi-infinite vacuum (air) region V2 on the right. We

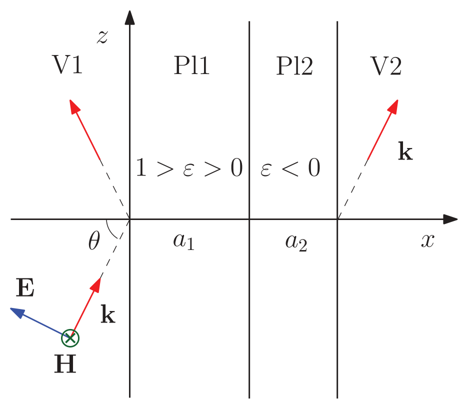

FIG. 1. (Color online) Schematic representation of the propagation of an electromagnetic wave through the two layers of nonuniform plasma.

assume that the wave is p polarized with the wave vector $\mathbf{k} = k_x\mathbf{e_x} + k_z\mathbf{e_z}$ (i.e., the electric field vector $\mathbf{E}$ is in the incidence xz plane). There are incident (with $k_x > 0$) and reflected (with $k_x < 0$) waves in region V1, excepting for the case of total transmission. There is no reflected wave in region V2. In the total transmission case, there are no reflected waves in a vacuum region. The plasma regions Pl1 and Pl2 have widths a_1 and a_2, correspondingly, and the waves are evanescent (with $\mathrm{Re}\, k_x = 0$) there. It is assumed that the plasma slabs are uniform in y and z directions and can be nonuniform in the x direction. We neglect the nonlinear effects [i.e., it is supposed that the electron oscillatory velocity $e|E|/(m_e\omega)$ is smaller than the wave phase velocity ω/k, where E is the intensity of the electric wave field, e and m_e are the electron charge and mass, respectively].

We assume that ions are immobile in a plasma layer, the phase velocity of the electromagnetic wave is larger than the electron thermal velocity, and the electron collision frequency is smaller than the wave frequency.

The electromagnetic field of the p-polarized wave is represented by

$$\mathbf{E} = (E_x(x),0,E_z(x))\exp(ik_z z - i\omega t),$$
$$\mathbf{H} = (0,H_y(x),0)\exp(ik_z z - i\omega t).$$

The field components E_x, E_z, and H_y, as functions of plasma permittivity, can be found from the Maxwell's equations:

$$\mathrm{rot}\mathbf{E} = ik\mathbf{H}, \tag{1}$$

$$\mathrm{rot}\mathbf{H} = -ik\varepsilon\mathbf{E}, \tag{2}$$

where $\varepsilon = 1 - \omega_{pe}^2(x)/\omega^2$ is the plasma permittivity, $\omega_{pe}(x)$ is the plasma frequency, $k = \omega/c$, c is the speed of light.

Using Eqs. (1) and (2), one finds the wave field components E_z and H_y in the vacuum region V1:

$$E_z = \frac{k_x A_v}{k}[-\exp(ik_x x) + \Gamma_v \exp(-ik_x x)], \tag{3}$$

$$H_y = A_v[\exp(ik_x x) + \Gamma_v \exp(-ik_x x)], \tag{4}$$

where the first terms in the brackets of the expressions (3) and (4) account for the incident wave and the second terms account for the reflected wave. Γ_v is the reflection coefficient, and A_v is the amplitude of the incident wave.

The following introduces the function $u(x)$, characterizing the local wave impedance $Z(x)$:

$$u(x) = -ikZ(x) = -ik\frac{E_z(x)}{H_y(x)}. \tag{5}$$

Then, the function $u(x)$ for the electromagnetic wave field in region V1 is

$$u_{v1}(x) = ik_x\frac{\exp(ik_x x) - \Gamma_v \exp(-ik_x x)}{\exp(ik_x x) + \Gamma_v \exp(-ik_x x)}. \tag{6}$$

In the vacuum region V2, the function is equal to

$$u_{v2}(x) = ik_x. \tag{7}$$

The dependence of magnetic field component H_y on the x coordinate in a nonuniform plasma region can be found from the equation, following from Maxwell's Eqs. (1) and (2) [30]:

$$\varepsilon(x)\frac{d}{dx}\left[\frac{1}{\varepsilon(x)}\frac{dH_y}{dx}\right] - \kappa^2(x)H_y = 0, \tag{8}$$

where $\kappa = \sqrt{k_3^2 - k^2\varepsilon}$ is the local reverse skin depth of the electromagnetic wave.

The component of electric field E_z is connected with the magnetic field component by the following expression:

$$E_z = \frac{i}{k\varepsilon(x)}\frac{dH_y}{dx}. \tag{9}$$

Equation (8) cannot be solved analytically for arbitrary dielectric permittivity distributions. However, an approximate analytical solution of the equation can be found in some limiting cases, for example, in the case of thin plasma slabs ($\kappa a \ll 1$) or when plasma nonuniformity is weak.

III. THE CASE OF THIN PLASMA SLABS

If the plasma slabs are thin, one can neglect the second term on the right-hand side of Eq. (8), and the equation can be represented in the simplified form:

$$\varepsilon(x)\frac{d}{dx}\left[\frac{1}{\varepsilon(x)}\frac{dH_y}{dx}\right] \approx 0. \tag{10}$$

Equation (10) has the following solution:

$$H_y(x) \approx A\int_{x_0}^{x}\varepsilon(x)dx + B, \tag{11}$$

where x_0 is a coordinate, and A and B are constants.

Using Eqs. (9) and (11), one obtains the expression for the z component of the electric field,

$$E_z(x) \approx iA/k, \tag{12}$$

and, as a result, the expression for the function u, characterizing the local wave impedance in the plasma regions:

$$u(x) \approx \frac{A}{A\int_{x_0}^{x}\varepsilon(x)dx + B}. \tag{13}$$

Since the tangential electric and magnetic field components are continuous at interfaces, the function u is also continuous. Matching the functions, characterizing the local wave impedances, at each interface, one gets the following system of equations:

$$ik_x \frac{1-\Gamma_v}{1+\Gamma_v} = \frac{A_1}{B_1}, \tag{14}$$

$$\frac{A_1}{\int_0^{a_1} A_1\varepsilon_1(x)dx + B_1} = \frac{A_2}{B_2}, \tag{15}$$

$$\frac{A_2}{\int_{a_1}^{a_1+a_2} A_2\varepsilon_2(x)dx + B_2} = ik_x, \tag{16}$$

where the indexes 1 and 2 are for the regions Pl1 and Pl2, respectively.

The system of Eqs. (14)–(16) is obtained, assuming that $x_0 = 0$ and $x_0 = a_1$ in Eq. (11) for the regions Pl1 and Pl2, correspondingly.

Solving Eqs. (14)–(16), one finds the reflection coefficient:

$$\Gamma_v = \frac{a_1\bar{\varepsilon}_1 + a_2\bar{\varepsilon}_2}{a_1\bar{\varepsilon}_1 + a_2\bar{\varepsilon}_2 + 2i/k_x}, \tag{17}$$

where $\bar{\varepsilon}_1 = \int_0^{a_1} \varepsilon_1(x)dx/a_1$, $\bar{\varepsilon}_2 = \int_{a_1}^{a_1+a_2} \varepsilon_2(x)dx/a_2$ are the average dielectric permittivities of the slabs Pl1 and Pl2, respectively.

Using Eqs. (13)–(16), one also gets the dependencies on the x coordinate for the wave impedance in the plasma layers Pl1 and Pl2, correspondingly:

$$Z_1(x) = iu_1(x)/k = \frac{i/k}{\int_0^x \varepsilon_1(x)dx - a_1\bar{\varepsilon}_1 - a_2\bar{\varepsilon}_2 - i/k_x},$$

$$Z_2(x) = iu_2(x)/k = \frac{i/k}{\int_{a_1}^x \varepsilon_2(x)dx - a_2\bar{\varepsilon}_2 - i/k_x}.$$

As seen from Eq. (17), the total transparency of the two-layer structure is achieved ($\Gamma_v = 0$), when the following condition is satisfied:

$$a_1\bar{\varepsilon}_1 + a_2\bar{\varepsilon}_2 = 0. \tag{18}$$

The condition of total transparency (18) is similar to that obtained for the case when the plasma slabs are uniform. However, in the former case there are the averaged dielectric permittivities $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ instead of the permittivities ε_{10} and ε_{20}, where ε_{10} and ε_{20} are the permittivities of the uniform plasma slabs Pl1 and Pl2, respectively. Note that in the case of uniform plasma slabs the wave number k_z has also to satisfy the following condition [31]:

$$k_z = \pm k\sqrt{\frac{\varepsilon_{10}\varepsilon_{20}}{\varepsilon_{10}+\varepsilon_{20}}}, \tag{19}$$

while in the case considered here an additional condition for k_z is absent, because we neglected in Eq. (8) the term, accounting for the dependence of the wave filed on the wave number.

The conditions for k_z, required for total transparency of the structure in the thin slab case, can be obtained, if one accounts for the second term on the left-hand side of Eq. (8). In this case, Eq. (8) can be solved using the method of successive iterations.

Applying this method, we assume that the magnetic filed component can be presented in the form $H_y \approx H_0 + H_1$, where $|H_1| \ll |H_0|$ and H_0 is the solution of Eq. (10) described by expression (11). In this case, one gets the following equation for H_1:

$$\varepsilon(x)\frac{d}{dx}\left[\frac{1}{\varepsilon(x)}\frac{dH_1}{dx}\right] - \kappa^2 H_0 = 0. \tag{20}$$

From Eq. (20) one obtains the expression for H_1 and, consequently, the expression for the y component of magnetic field in a plasma region:

$$H_y \approx H_0 + H_1 = A\int_{x_0}^x \varepsilon(x)dx + B + \int_{x_0}^x \varepsilon(x)\left[\int_{x_0}^x \frac{\kappa^2(x)}{\varepsilon(x)}\left(A\int_{x_0}^x \varepsilon(x)dx + B\right)dx\right]dx. \tag{21}$$

Using Eqs. (9) and (21), one gets the following expression for the z component of electric field in a plasma layer:

$$E_z = \frac{i}{k}\left[A + \int_{x_0}^x \frac{\kappa^2(x)}{\varepsilon(x)}\left(A\int_{x_0}^x \varepsilon(x)dx + B\right)dx\right]. \tag{22}$$

Then, the function $u(x)$, characterizing the wave impedance in a plasma layer, is

$$u(x) = -ik\frac{E_z}{H_y} = \frac{1 + \int_{x_0}^x \frac{\kappa^2(x)}{\varepsilon(x)}\left(\int_{x_0}^x \varepsilon(x)dx + C\right)dx}{C + \int_{x_0}^x \varepsilon(x)dx + \int_{x_0}^x \varepsilon(x)\left[\int_{x_0}^x \frac{\kappa^2(x)}{\varepsilon(x)}\left(\int_{x_0}^x \varepsilon(x)dx + C\right)dx\right]dx}, \tag{23}$$

where $C = B/A$.

For this particular case, it is convenient to locate the beginning of the coordinate system ($x = 0$) at the Pl1-Pl2 interface and take $x_0 = 0$ in Eq. (23). Then, from the boundary condition $u_1(0) = u_2(0)$ one gets that $C_1 = C_2$.

Accounting for the continuity of the function $u(x)$ at plasma-vacuum boundaries $x = -a_1$ and $x = a_2$, and assuming that the transparency of the slabs is total ($\Gamma_v = 0$), one obtains the following system of equations:

$$u_1(-a_1) = ik_x, \tag{24}$$

$$u_2(a_2) = ik_x, \tag{25}$$

where the functions u_1 and u_2 are determined by Eq. (23) with $x_0 = 0$.

It follows from Eqs. (24) and (25) that in the case of total transparency,

$$u_1(-a_1) = u_2(a_2). \tag{26}$$

In the general case, there is not an analytical solution of Eqs. (24)–(26). Meanwhile, Eq. (26) will be approximately fulfilled if

$$\int_0^{-a_1} \varepsilon_1(x)dx \approx \int_0^{a_2} \varepsilon_2(x)dx, \tag{27}$$

$$\int_0^{-a_1} \frac{\kappa_1^2(x)}{\varepsilon_1(x)}dx \approx \int_0^{a_2} \frac{\kappa_2^2(x)}{\varepsilon_2(x)}dx, \tag{28}$$

$$\int_0^{-a_1} \varepsilon_1(x)\left(\int_0^x \frac{\kappa_1^2(x)}{\varepsilon_1(x)}dx\right)dx \approx \int_0^{a_2} \varepsilon_2(x)\left(\int_0^x \frac{\kappa_2^2(x)}{\varepsilon_2(x)}dx\right)dx, \tag{29}$$

$$\int_0^{-a_1} \varepsilon_1(x)\left[\int_0^x \frac{\kappa_1^2(x)}{\varepsilon_1(x)}\left(\int_0^x \varepsilon_1(x)dx\right)dx\right]dx \approx \int_0^{a_2} \varepsilon_2(x)\left[\int_0^x \frac{\kappa_2^2(x)}{\varepsilon_2(x)}\left(\int_0^x \varepsilon_2(x)dx\right)dx\right]dx, \tag{30}$$

$$\int_0^{-a_1} \frac{\kappa_1^2(x)}{\varepsilon_1(x)}\left(\int_0^x \varepsilon_1(x)dx\right)dx \approx \int_0^{a_2} \frac{\kappa_2^2(x)}{\varepsilon_2(x)}\left(\int_0^x \varepsilon_2(x)dx\right)dx. \tag{31}$$

Note that the condition for total transparency (27), obtained accounting for the second term on the right-hand side of Eq. (8), coincides with the condition (18) obtained without taking into account the term. Therefore, the condition (27) is more important than the conditions (28)–(31). If the plasma slabs are uniform, then one gets from Eqs. (28)–(31) the relation,

$$\kappa_1/\varepsilon_1 + \kappa_2/\varepsilon_2 = 0.$$

The wave number, satisfying this equation, coincides with that described by expression (19). Therefore, in the case of thin nonuniform slabs one can expect that the resonant wave number approximately equals to

$$k_{z0} = k\sqrt{\bar{\varepsilon}_1\bar{\varepsilon}_2/(\bar{\varepsilon}_1 + \bar{\varepsilon}_2)}. \tag{32}$$

IV. THE CASE OF WEAK NONUNIFORMITY

An approximate solution of Eqs. (1) and (2) can also be found in the case, when plasma nonuniformity is weak (i.e., when the dielectric permittivity varies slightly at the distance which is about the skin depth κ^{-1}). To get a solution in this case, it is convenient to use the equation for the x component of electric field, following from Maxwell's Eqs. (1) and (2) [1]:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)E_x + \frac{\omega^2}{c^2}\varepsilon(x)E_x + \frac{\partial}{\partial x}\left(E_x\frac{\partial}{\partial x}\ln\varepsilon(x)\right) = 0. \tag{33}$$

Let us assume that the x component of electric field can be presented in the form,

$$E_x(x,z) = \alpha(x)F(x)\exp[ik_\varepsilon(x)\alpha(x)z], \tag{34}$$

where $k_\varepsilon(x) = \frac{\omega}{c}\sqrt{\varepsilon(x)}$, and $\alpha = \sin\theta(x)$, $\theta(x)$ is the angle between the wave vector and the x axis.

Substituting (34) into (33), one gets that [1]

$$k_\varepsilon(x)\alpha(x) = k_\varepsilon(0)\alpha(0) = \text{const}, \tag{35}$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\omega^2}{c^2}\varepsilon_{\text{eff}}(x)F = 0, \tag{36}$$

where $\alpha(0) = \sin\theta(0) = k_z/k$,

$$\varepsilon_{\text{eff}}(x) = \varepsilon(x) - \sin^2\theta_0 + \frac{c^2}{\omega^2}\left[\frac{1}{2\varepsilon}\frac{\partial^2\varepsilon}{\partial x^2} - \frac{3}{4\varepsilon^2}\left(\frac{\partial\varepsilon}{\partial x}\right)^2\right].$$

Here, it is assumed that the point $x = 0$ is located at the V1-Pl1 boundary.

If the nonuniformity of plasma is weak, the following inequalities are fulfilled:

$$|\varepsilon(x) - \sin^2\theta_0| \gg \left|\frac{c^2}{\omega^2}\frac{1}{2\varepsilon}\frac{\partial^2\varepsilon}{\partial x^2}\right|, \tag{37}$$

$$|\varepsilon(x) - \sin^2\theta_0| \gg \left|\frac{c^2}{\omega^2}\frac{3}{4\varepsilon^2}\left(\frac{\partial\varepsilon}{\partial x}\right)^2\right|, \tag{38}$$

and

$$\varepsilon_{\text{eff}}(x) \approx \varepsilon(x) - \sin^2\theta_0. \tag{39}$$

Taking into account that $\sin\theta_0 = k_z/k$ and $\kappa = \sqrt{k_3^2 - k^2\varepsilon}$, the conditions (37) and (38) can be rewritten in the form:

$$\left|\frac{\partial^2\varepsilon}{\partial x^2}\right| \ll |2\varepsilon|\kappa^2, \tag{40}$$

$$\left|\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial x}\right| \ll \frac{2\kappa}{\sqrt{3}}. \tag{41}$$

If $|\partial\varepsilon/\partial x| \ll 2\kappa^3/k^2$ and the dielectric permittivity is described by Eq. (39), then the approximate expression for the dependence of the x component of electric field on the x coordinate is

$$E_x(x) \approx \frac{k_z}{k\sqrt{\kappa(x)\varepsilon(x)}}\left[C\exp\left(\int_{x_0}^x \kappa dx\right) + D\exp\left(-\int_{x_0}^x \kappa dx\right)\right], \tag{42}$$

where x_0 is a coordinate, and C and D are constants.

Taking into account that $H_y = (k\varepsilon/k_z)E_x$ and using the expression for E_x (42), one gets the approximate expression for the component of the magnetic field:

$$H_y(x) \approx \frac{\varepsilon(x)}{\sqrt{\kappa(x)\varepsilon(x)}}\left[C\exp\left(\int_{x_0}^x \kappa dx\right) + D\exp\left(-\int_{x_0}^x \kappa dx\right)\right]. \tag{43}$$

Since $E_z = (i/k\varepsilon)\partial H_y/\partial x$, in the case of weak plasma nonuniformity the expression for the z component of electric field is

$$E_z(x) \approx \frac{i\kappa(x)}{k\sqrt{\kappa(x)\varepsilon(x)}}\left[C\exp\left(\int_{x_0}^x \kappa dx\right) - D\exp\left(-\int_{x_0}^x \kappa dx\right)\right]. \tag{44}$$

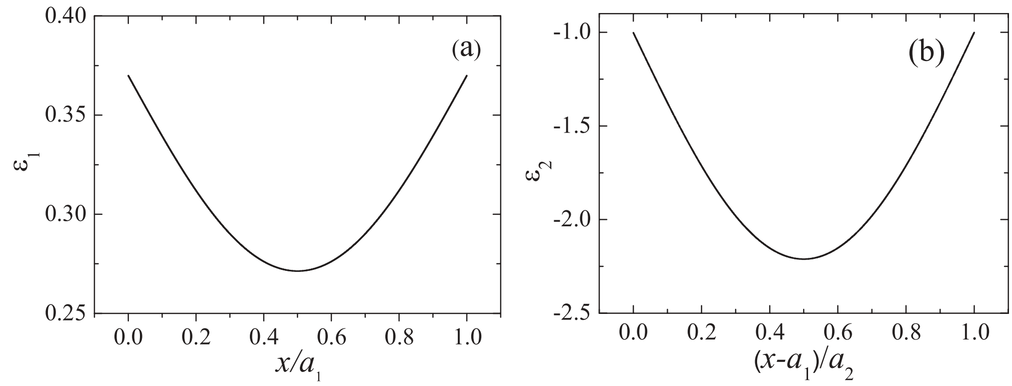

FIG. 2. The dielectric permittivity profiles in the first (a) and second (b) plasma slabs considered in the numerical analysis.

Using the expressions (43) and (44), one obtains the following expression for $u(x)$, characterizing the wave impedance,

$$u(x) \approx \frac{\kappa(x)}{\varepsilon(x)} \frac{C \exp\left(\int_{x_0}^{x} \kappa dx\right) - D \exp\left(-\int_{x_0}^{x} \kappa dx\right)}{C \exp\left(\int_{x_0}^{x} \kappa dx\right) + D \exp\left(-\int_{x_0}^{x} \kappa dx\right)}. \quad (45)$$

Thus, the functions $u_1(x)$ and $u_2(x)$, characterizing the wave impedance in the regions Pl1 and Pl2, are, correspondingly,

$$u_1(x) \approx \frac{\kappa_1(x)}{\varepsilon_1(x)} \frac{\exp\left(\int_0^x \kappa_1 dx\right) - A \exp\left(-\int_0^x \kappa_1 dx\right)}{\exp\left(\int_0^x \kappa_1 dx\right) + A \exp\left(-\int_0^x \kappa_1 dx\right)}. \quad (46)$$

$$u_2(x) \approx \frac{\kappa_2(x)}{\varepsilon_2(x)} \frac{\exp\left(\int_{a_1}^x \kappa_2 dx\right) - B \exp\left(-\int_{a_1}^x \kappa_2 dx\right)}{\exp\left(\int_{a_1}^x \kappa_2 dx\right) + B \exp\left(-\int_{a_1}^x \kappa_2 dx\right)}, \quad (47)$$

where A and B are constants. $x_0 = 0$ and $x_0 = a_1$ for the regions Pl1 and Pl2, respectively.

Remember that in the vacuum regions V1 and V2, the $u(x)$ dependence is determined by Eqs. (6) and (7), respectively.

Assuming that the wave impedance is continuous at boundaries $x = 0$, $x = a_1$, and $x = a_1 + a_2$, one gets the following system of equations:

$$\frac{ik_x(1-\Gamma_v)}{1+\Gamma_v} = \frac{\kappa_1(0)}{\varepsilon_1(0)} \frac{1-A}{1+A},$$
$$\frac{\kappa_1(a_1)}{\varepsilon_1(a_1)} \frac{\exp\left(\int_0^{a_1} \kappa_1 dx\right) - A\exp\left(-\int_0^{a_1} \kappa_1 dx\right)}{\exp\left(\int_0^{a_1} \kappa_1 dx\right) + A\exp\left(-\int_0^{a_1} \kappa_1 dx\right)} = \frac{\kappa_2(a_1)}{\varepsilon_2(a_1)} \frac{1-B}{1+B},$$
$$\frac{\kappa_2(a_1+a_2)}{\varepsilon_2(a_1+a_2)} \frac{\exp\left(\int_{a_1}^{a_1+a_2} \kappa_2 dx\right) - B\exp\left(-\int_{a_1}^{a_1+a_2} \kappa_2 dx\right)}{\exp\left(\int_{a_1}^{a_1+a_2} \kappa_2 dx\right) + B\exp\left(-\int_{a_1}^{a_1+a_2} \kappa_2 dx\right)} = ik_x. \quad (48)$$

From the system (48) one can find the reflection coefficient Γ_v and the transparency coefficient $T = \sqrt{1 - |\Gamma_v|^2}$. We do not present the expressions for Γ_v and T here, because they are very big and uninformative.

Instead of that, consider the case of total transparency ($\Gamma_v = 0$). In this case, it follows from the first equation of the system (48) that

$$A = \left(\frac{\kappa_1(0)}{\varepsilon_1(0)} - ik_x\right) \Big/ \left(\frac{\kappa_1(0)}{\varepsilon_1(0)} + ik_x\right). \quad (49)$$

The constant B can be found from the third equation of the system (48):

$$B = \exp\left(2 \int_{a_1}^{a_1+a_2} \kappa_2(x) dx\right) \times \left(\frac{\kappa_2(a_1+a_2)}{\varepsilon_2(a_1+a_2)} - ik_x\right) \Big/ \left(\frac{\kappa_2(a_1+a_2)}{\varepsilon_2(a_1+a_2)} + ik_x\right). \quad (50)$$

Thus, the second equation of the system (48) with the constants A and B described be Eqs. (49) and (50) determines the conditions for total transparency of the two-layer structure in the case of weak nonuniformity.

If

$$\frac{\kappa_1(a_1)}{\varepsilon_1(a_1)} = -\frac{\kappa_2(a_1)}{\varepsilon_2(a_1)}, \quad (51)$$

and

$$\frac{\kappa_2(a_1+a_2)}{\varepsilon_2(a_1+a_2)} = -\frac{\kappa_1(0)}{\varepsilon_1(0)}, \quad (52)$$

from the second equation of the system (48) one gets that the following condition should be also satisfied:

$$\int_0^{a_1} \kappa_1(x) dx = \int_{a_1}^{a_1+a_2} \kappa_2(x) dx. \quad (53)$$

If the slabs are spatially uniform, the conditions (51)–(53) coincide with the conditions of total transparency presented in Ref. [23].

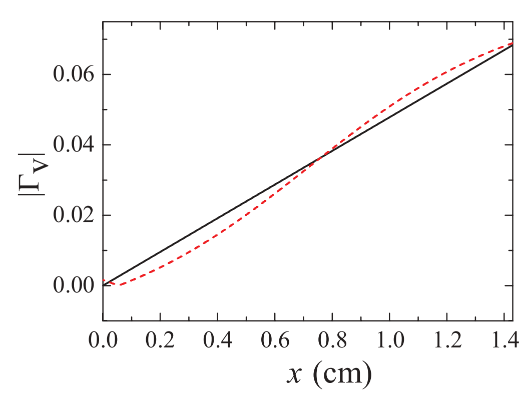

FIG. 3. (Color online) The absolute value of the reflection coefficient for $k_z = k_{z0} = 1.148$ cm^{-1}, $a_2 = 0.25$ cm in the cases of uniform (solid curve) and nonuniform (dashed curve) plasma slabs.

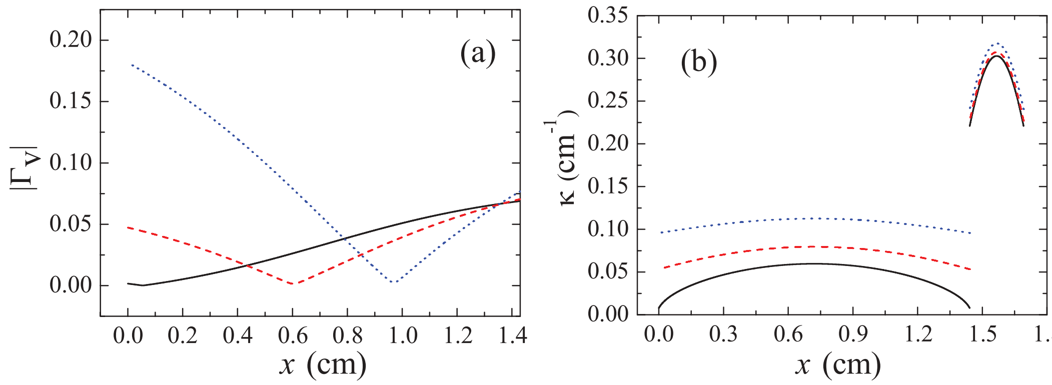

FIG. 4. (Color online) The absolute value of the reflection coefficient (a) and the reverse skin depth (b) for $k_z = k_{z0}$ (solid curve), $1.1k_{z0}$ (dashed curve), and $1.3k_{z0}$ (dotted curve) in the nonuniform plasma case. The other parameters are the same as in Fig. 3.

V. NUMERICAL RESULTS

In Secs. III and IV, the conditions for total transparency of the two-layer structure were obtained for the case of thin plasma slabs and when plasma nonuniformity is weak. In the general case, transmission of an electromagnetic wave through a two-layer nonuniform-plasma structure can be studied only numerically. To carry out the numerical study, it is convenient to use the equation for $u(x)$, describing the wave impedance and following from Maxwell's Eqs. (1) and (2) [30]:

$$\frac{\partial u}{\partial x} = \frac{1}{\varepsilon(x)}[\kappa^2(x) - \varepsilon^2(x)u^2]. \quad (54)$$

Assuming that $u = u_1 + iu_2$, where u_1 and u_2 are real functions, one gets from the previous equation the system,

$$\frac{\partial u_1}{\partial x} = \frac{1}{\varepsilon(x)}\left[\kappa^2(x) - \varepsilon^2(x)\left(u_1^2(x) - u_2^2(x)\right)\right], \quad (55)$$

$$\frac{\partial u_2}{\partial x} = -2\varepsilon(x)u_1(x)u_2(x). \quad (56)$$

The system of Eqs. (55) and (56) should be accompanied by boundary conditions. Since at plasma-vacuum boundary Pl2-V2 $u(a_1 + a_2) = ik_x$, then $u_1(a_1 + a_2) = 0$ and $u_2(a_1 + a_2) = k_x$. It is obvious that u_1 and u_2 are also continuous functions.

In our numerical model, we assume that the boundary V1-Pl1 can be located at different $x(\leqslant a_1)$, and the reflection coefficient can be presented in the form $\Gamma_v = \Gamma_1 + i\Gamma_2$, where Γ_1 and Γ_2 are real functions. If the boundary is located at $x = 0$, then the first slab size equals to a_1. For the case of location of the boundary at $x > 0$ the width of slab Pl1 is $a_1 - x$ and is smaller than a_1.

Taking into account the continuity of the functions u_1 and u_2 at the boundary V1-Pl1, one gets from Eq. (6) the following expression for absolute value of the reflection coefficient:

$$|\Gamma_v(x)| = \sqrt{\Gamma_1(x)^2 + \Gamma_2(x)^2} = \frac{\sqrt{\left(u_1(x)^2 + u_2(x)^2 - k_x^2\right)^2 + (2u_1(x)k_x)^2}}{u_1(x)^2 + (u_2(x) + k_x)^2}, \quad (57)$$

where $u_1(x)$ and $u_2(x)$ are the functions, characterizing the wave impedance at the Pl1-V1 interface, which can be obtained numerically from Eqs. (55) and (56). The system of ordinary differential Eqs. (55) and (56) is solved numerically by a fourth-order Runge-Kutta method.

The reflection coefficient was calculated assuming that the dielectric permittivities in the plasma slabs are cosinelike functions (see Fig. 2). It was supposed that the x dependencies of dielectric permittivities on the x coordinate have the following form: $\varepsilon_1 = \varepsilon_{1\max} - A_m \cos(\pi(x - a_1/2)/a_1)$ for $x < a_1$; $\varepsilon_2 = B_m \cos(\pi(x - (2.0a_1 + a_2)/2)/a_2) + \varepsilon_{2\min}$ for $x > a_1$, where $\varepsilon_{1\max} = 0.37$, $\varepsilon_{2\min} = -1$, $A_m = \pi(\varepsilon_{1\max} - \bar{\varepsilon}_1)/2$, $B_m = \pi(\bar{\varepsilon}_2 - \varepsilon_{2\min})/2$. The calculations were carried out for

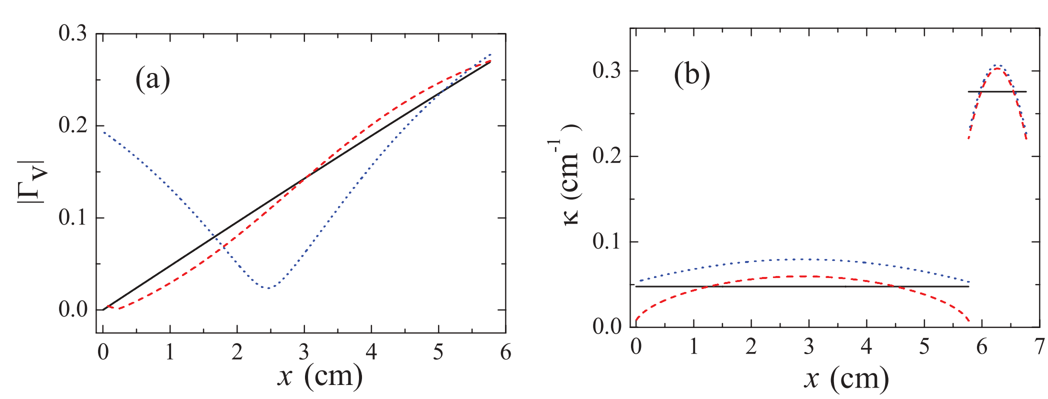

FIG. 5. (Color online) The same as in Fig. 4 at $a_2 = 1$ cm for the uniform plasma case and $k_z = k_{z0} = 1.148$ cm^{-1} (solid curve), as well as for the nonuniform plasma case at $k_z = k_{z0}$ (dashed curve) and $k_z = 1.1k_{z0}$ (dotted curve).

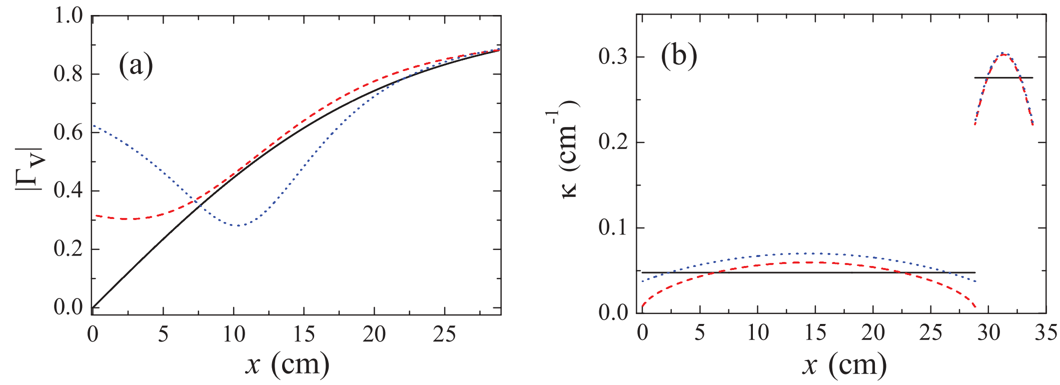

FIG. 6. (Color online) The same as in Fig. 5 at $a_2 = 5$ cm for the uniform plasma case and $k_z = k_{z0}$ (solid curve), as well as for the nonuniform plasma case at $k_z = k_{z0}$ (dashed curve) and $k_z = 1.05k_{z0}$ (dotted curve).

$\omega = 2\pi \times 9 \times 10^8\ \mathrm{s}^{-1}$ and different widths of the second plasma slab Pl2: $a_2 = 0.25$ cm, 1 cm, and 5 cm. The resonant width of the first plasma layer Pl1 was obtained from the relation $a_1 = -a_2\bar{\varepsilon}_2/\bar{\varepsilon}_1$, which coincides with one of the conditions of total transparency in the uniform plasma case [29]. We considered the cases when the wave number k_z is the same as the resonant wave number in the uniform plasma case [$k_z = k_{z0}$, determined by Eq. (19)] and is different from k_{z0}.

In Fig. 3, the dependence $|\Gamma_v(x)|$ is shown for the nonuniform plasma case and compared with that obtained for the case when the plasma slabs are spatially uniform. Both curves in Fig. 3 are obtained for $k_z = k_{z0} = 1.148\ \mathrm{cm}^{-1}$. It is seen from Fig. 3 that the absolute value of the reflection coefficient $|\Gamma_v(x)|$ at $x = 0$ in the nonuniform plasma case is very small ($\sim 2 \times 10^{-3}$) (i.e., transparency of the system, consisting of nonuniform plasma layers, is nearly total), similarly to the uniform plasma case.

If the wave number k_z is larger than k_{z0}, then the reflection coefficient at $x = 0$ is also larger than obtained at $k_z = k_{z0}$ (see Fig. 4). It means that transparency of the two-layer structure depends on the wave number k_z, in agreement with the analytical results presented in Sec. III for the thin slab case [see Eqs. (28)–(31)]. Increasing the wave number k_z, one also increases the reverse skin depth κ, and the condition of the thin plasma slab ($\kappa_1 a_1 \ll 1$) may become not applicable for large k_z.

In Fig. 5(a), the dependencies $|\Gamma_v(x)|$ are shown for $a_2 = 1$ cm in the following cases: uniform plasma slabs and $k_z = k_{z0}$ (solid curve); nonuniform plasma slabs and $k_z = k_{z0}$ (dashed curve); nonuniform plasma slabs and $k_z = 1.1k_{z0}$ (dotted curve). In the case of nonuniform plasma and $k_z = k_{z0}$, the absolute value of the reflection coefficient $|\Gamma_v|$ at $x = 0$ is very small ($\sim$0.006). If the plasma slabs are nonuniform and $k_z = 1.1k_{z0}$, the absolute value of the reflection coefficient is about 0.2 at $x = 0$. For this case, the function $|\Gamma_v(x)|$ has a minimum at $x \approx 2.5$ cm, and $\kappa_1 a_1$ is larger than 0.4 at some x (i.e., the thin slab approximation is not applicable if $a_2 = 1$ cm).

We also considered the case when $\overline{\kappa_1} a_1 > 1$, where $\overline{\kappa}_1$ is the average reverse skin depth for the first plasma slab. To study the case of thick plasma slabs, it was assumed that $a_2 = 5$ cm. If both plasma slabs are nonuniform and thick, then transparency of the two-layer structure is smaller than that in the thin slab case (see Fig. 6). An increase of k_z with respect to k_{z0} is accompanied by an enlargement of $|\Gamma_v(0)|$.

Note that $\kappa_2 > \kappa_1$, and nonuniformity of the first slab affects more strongly on transparency of the structure than that of the second layer [see Fig. (7)].

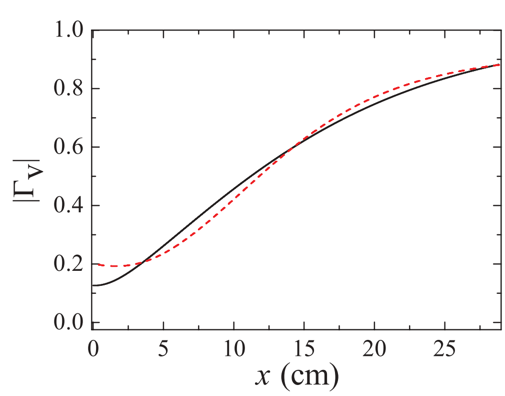

FIG. 7. (Color online) The absolute value of the transparency coefficient obtained at $k_z = k_{z0}$ with uniform first layer and nonuniform second layer (solid curve) and with nonuniform first layer and uniform second layer (dashed curve). The other parameters are the same as in Fig. 6.

VI. CONCLUSIONS

We have studied analytically and numerically the transparency of two-layer plasma structures with nonuniform spatial distributions of dielectric permittivity in the layers. The case of the p-polarized electromagnetic wave obliquely incident on this structure has been considered. The electromagnetic wave field has been assumed to be evanescent in both layers. Conditions (27)–(31) and (51)–(53) for total transparency of the two-layer structure have been found analytically in the thin plasma slab case as well as when the plasma nonuniformity is weak. In our numerical study, we have considered both cases of thin and thick plasma slabs.

For thin plasma slabs, it has been found that the condition of total transparency depends on the average dielectric permittivities (plasma densities) in the slabs and the widths of the slabs [see Eq. (27)]. There is a dependence of the transparency coefficient on the wave number, but it is small, that is, transmission of waves with different wave lengths and small reflection coefficients through the thin plasma slabs is possible (see Fig. 4).

If the plasma slabs are thick ($\kappa a \geqslant 1$), plasma nonuniformity affects essentially transparency of the two-layer structure. Moreover, in this case, a small deviation of the wave number from the resonant number ($k_{z0} = k\sqrt{\bar{\varepsilon}_1\bar{\varepsilon}_2/(\bar{\varepsilon}_1 + \bar{\varepsilon}_2)}$) may be accompanied by an essential increase in the reflection coefficient [see Fig. 6(a)]. Nonuniformity of the first slab with $0 < \bar{\varepsilon}_1 < 1$ affects more strongly the wave transmission than that of the second layer. This is explained by the fact that the conditions of weak nonuniformity (40) and (41) depend on the skin depth, and the skin depth in the first layer is larger than that in the second slab.

The analysis of transmission of the obliquely incident p-polarized waves through an inhomogeneous plasma layer is more complicated than that for the waves propagating in the direction of the density gradient and the case of the oblique s-polarized waves [1,2,18]. In the case considered here, the effective dielectric permittivity ε_{eff} depends not only on the dielectric permittivity and the angle of wave incidence, but ε_{eff} is also a function of the derivatives $d\varepsilon/dx$ and $d^2\varepsilon/dx^2$ (see Sec. IV). To our knowledge, due to the complexity in the expression for the effective dielectric permittivity exact analytical solutions describing transmission of the obliquely incident p-polarized waves through an inhomogeneous plasma layer are not available at present.

Note that some simplifications have been used in our study. In particular, the electron temperature and collisions effects have been neglected. These effects can influence the transmission of electromagnetic waves through the structure and loss of wave electromagnetic energy, and were studied by previous authors [24,32]. We also limited our study by the case when the local plasma frequencies $\omega_{pe}(x)$ in the dense-plasma layer are larger than the wave frequency. Meantime, in most of the laboratory plasmas there are the resonance points near plasma boundaries, where the plasma frequency is approximately equal to the wave frequency. In these points, dissipation of wave electromagnetic energy takes place [30,33], and it may be accompanied by a decrease of the transmission coefficient. However, for the high-frequency limit, considered here, we expect that the effect of the resonance points on the energy dissipation and the transmission coefficient is small. Plasma inhomogeneity may also affect propagation and excitation of waves corresponding to the low-frequency range [34]. We also neglected nonlinear effects, which can influence the transmission of electromagnetic waves through the structure, even increasing it at certain conditions [14,35–38]. Moreover, our study has been limited only by the case of the two-layer structure with the evanescent waves in each plasma layer. In the structures with a larger number of layers, it was shown recently that total transparency can be achieved without surface mode excitation, for example, exploiting the standing wave resonances [39].

In conclusion, we have shown that the transmission coefficient of the p-polarized waves obliquely incident on a two-slab structure can be about unity, even if the plasma nonuniformity is large. The nonuniformity effects are more important if the slabs are thick, compared with the case of thin layers. The results obtained in this paper can be useful for laboratory plasma experiments, as well as in communications and plasmonic applications.

ACKNOWLEDGMENTS

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[1] V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon, New York, 1970).
[2] A. B. Shvartsburg, Phys. Usp. **50**, 37 (2007).
[3] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[4] R. Zia, J. A. Schuller, A. Chandran, and M. L. Brongersma, Mater. Today **9**, 20 (2006).
[5] K. Y. Bliokh, Y. P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori, Rev. Mod. Phys. **80**, 1201 (2008).
[6] E. Fourkal, I. Velchev, and A. Smolyakov, Phys. Rev. A **79**, 033846 (2009).
[7] M. Keidar, M. Kim, and I. D. Boyd, J. Spacecr. Rockets **45**, 445 (2008).
[8] Y. A. Akimov and H. S. Chu, Phys. Rev. B **83**, 165412 (2011).
[9] E. A. Gibson, M. Pennybacker, A. I. Maimistov, I. R. Gabitov, and N. M. Litchinitser, J. Opt. **13**, 024013 (2011).
[10] C. A. M. Butler, I. R. Hooper, A. P. Hibbins, J. R. Sambles, and P. A. Hobson, J. Appl. Phys. **109**, 013104 (2011).
[11] W. L. Barnes, W. A. Murray, J. Dintinger, E. Devaux, and T. W. Ebbesen, Phys. Rev. Lett. **92**, 107401 (2004).
[12] M. M. Sigalas, C. M. Soukoulis, C.-T. Chan, and D. Turner, Phys. Rev. B **53**, 8340 (1996).
[13] D. L. Tang, A. P. Sun, X. M. Qiu, and P. K. Chu, IEEE Trans. Plasma Sci. **31**, 405 (2003).
[14] N. S. Erokhin and V. E. Zakharov, Doklady Physics **56**, 362 (2011).
[15] C. S. Gurel and E. Oncu, Prog. Electromagn. Res. B **21**, 385 (2010).
[16] L. X. Ma, H. Zhang, and C. X. Zhang, J. Electromagnet. Wave. **22**, 2285 (2008).
[17] S. V. Nazarenko, A. C. Newell, and V. E. Zakharov, Phys. Plasmas **1**, 2827 (1994).
[18] N. S. Erokhin and V. E. Zakharov, Plasma Phys. Rep. **37**, 762 (2011).
[19] N. S. Erokhin and V. E. Zakharov, Doklady Physics **52**, 485 (2007).
[20] A. B. Shvartsburg, M. Marklund, G. Brodin, and L. Stenflo, Phys. Rev. E **78**, 016601 (2008).
[21] A. B. Shvartsburg and G. Petite, Opt. Lett. **31**, 1127 (2006).

[22] R. Dragila, B. Luther-Davies, and S. Vukovic, Phys. Rev. Lett. **55**, 1117 (1985).
[23] R. R. Ramazashvili, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 235 (1986).
[24] E. Fourkal, I. Velchev, C. M. Ma, and A. Smolyakov, Phys. Plasmas **13**, 092113 (2006).
[25] A. I. Smolyakov, E. A. Fourkal, S. I. Krasheninnikov, and N. Sternberg, Prog. Electromagn. Res. **107**, 293 (2010).
[26] S. Ivko, A. Smolyakov, I. Denysenko, and N. A. Azarenkov, Phys. Rev. E **84**, 016407 (2011).
[27] M. A. Lieberman and A. J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing* (Wiley, New York, 1994).
[28] N. A. Azarenkov, I. B. Denysenko, A. V. Gapon, and T. W. Johnston, Phys. Plasmas **8**, 1467 (2001).
[29] I. B. Denysenko, A. V. Gapon, N. A. Azarenkov, K. N. Ostrikov, and M. Y. Yu, Phys. Rev. E **65**, 046419 (2002).
[30] A. N. Kondratenko, *Surface and Volume Waves in Bounded Plasmas* (Energoatomizdat, Moscow, 1985).
[31] K. N. Ostrikov, M. Y. Yu, and L. Stenflo, Phys. Rev. E **61**, 782 (2000).
[32] N. A. Azarenkov, I. B. Denisenko, and K. N. Ostrikov, J. Phys. D: Appl. Phys. **28**, 2465 (1995).
[33] N. A. Azarenkov, A. V. Gapon, I. B. Denisenko, and A. I. Smolyakov, Phys. Scr. **59**, 298 (1999).
[34] S. I. Popel, S. V. Vladimirov, and V. N. Tsytovich, Phys. Rep. **259**, 327 (1995).
[35] O. M. Gradov and L. Stenflo, Phys. Lett. A **83 A**, 257 (1981).
[36] O. M. Gradov, R. R. Ramazashvili, and L. Stenflo, Plasma Phys. **24**, 1101 (1982).
[37] S. I. Popel, S. I. Kopnin, I. N. Kosarev, and M. Y. Yu, Adv. Space Res. **37**, 414 (2006).
[38] K. N. Ostrikov, M. Y. Yu, and N. A. Azarenkov, J. Appl. Phys. **84**, 4176 (1998).
[39] N. Sternberg and A. I. Smolyakov, IEEE Trans. Plasma Sci. **37**, 1251 (2009).