Efficiencies and coefficients of performance of heat engines, refrigerators, and heat pumps with friction: A universal limiting behavior

João P. S. Bizarro[*] and Paulo Rodrigues[†]
Associação Euratom–IST, Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal

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For work-producing heat engines, or work-consuming refrigerators and heat pumps, the percentage decrease caused by friction in their efficiencies, or coefficients of performance (COP's), is approximately given by the ratio $W_{\rm fric}/W$ between the work spent against friction forces and the work performed by, or delivered to, the working fluid. This universal scaling, which applies in the limit of small friction ($W_{\rm fric}/W \lesssim 20\%$) and when the engine's figures of merit (FOM's, either efficiencies or COP's) do not come too close to unity (no higher than, say, 0.5 in the case of heat-engine efficiencies), allows a simple and quick estimate of the impact that friction losses can have on the FOM's of thermal engines and plants, or of the level of those losses from the observed and predicted FOM's. In the case of refrigerators and heat pumps, if $W_{\rm fric}/W \lesssim 20\%$ is not ensured (actually a condition that can be largely relaxed for heat engines), the COP percentage decrease due to friction approaches asymptotically $(W_{\rm fric}/W)/(1 + W_{\rm fric}/W)$ instead of $W_{\rm fric}/W$. Estimates for the level of frictional losses using the Carnot (or, for heat engines and power plants only, the Curzon-Ahlborn) predictions and observed FOM's of real power plants, heat engines, refrigerators, and heat pumps show that they usually operate in domains where these behaviors are valid.

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I. INTRODUCTION

Born as the modern science it is today from the need to understand how thermal energy can be transformed into mechanical work and vice versa, thermodynamics has always been concerned at its heart with the efficiency and performance of cyclic thermal engines [1–4]. An unavoidable feature in every thermal engine is frictional dissipation, which corresponds to that part of the work delivered by the fluid (in a work-producing heat engine) or the surroundings (in a refrigerator or in a heat pump) that is spent against friction forces. Friction in thermal engines has been treated occasionally within finite-time thermodynamics, but always in a particular manner, as found best suited for the problem of interest [5–9]. Recently, a more systematic approach to incorporate friction explicitly in thermodynamics has been proposed [10,11], which has found support in kinetic theory [12], and has been used to address the efficiency of heat engines when friction is present [13]. The distinctive feature in the framework developed to account for friction in thermodynamics is that the work transfers as perceived by the system and the surroundings are no longer symmetric [10,11,13], contrary to the usual assumption found in textbooks [1–4]. The same framework is used in this paper to show that the figures of merit (FOM's) [efficiencies or coefficients of performance (COP's)] for thermal engines (heat engines, refrigerators, or heat pumps) with friction follow an approximate scaling which depends on the FOM's for the frictionless engines and on the fraction of frictional losses [14]. That the thermodynamics of heat engines, refrigerators (or air conditioners), and heat pumps is not old-fashioned physics can be attested to by the fact that the subject continues to attract interest and to keep researchers active, not only in the classical but in the quantum realm as well [15–26].

In essence, and from a practical point of view, this paper addresses a plain, yet important, question: What is the impact of a given (actual or guessed) percentage of frictional losses on the known efficiency or COP of a thermal engine when it has been calculated, as is often done, with no account for friction? An answer to this question is here found which is not only direct and the same for all types of engines but, as illustrated below, allows also one to solve the inverse problem of how to estimate the level of work done against friction from the actual FOM and some frictionless prediction of the latter. A general expression is indeed obtained which yields a simple and straightforward relation between frictional losses and thermal-engine efficiency or performance, with no need to know in detail the thermodynamic processes undergone by the working fluid. Besides the fundamental interest always present in unveiling a universal behavior, such a result has important practical consequences as well. For instance, when optimizing the efficiency or performance of a thermal engine or plant, it can be of great help in judging how much effort will be worthwhile spending in reducing friction (by immediate evaluation of its impact on the respective FOM), as compared to other sources of inefficiency such as heat leak, thermal resistance, or other finite-time effects [5–9]. Being understood that what is presented below is not ground-breaking physics, as it is issued from good old, solid, and simple thermodynamic analysis (using the first and second laws only), it should be nonetheless easy to verify that it constitutes a number of useful physical results.

II. ANALYSIS AND DISCUSSION

A. FOM's of heat engines, refrigerators, and heat pumps with friction: Definitions and general behavior

The schematic diagrams for a heat engine and for a refrigerator or a heat pump operating with friction between a hot and a cold reservoir (whose temperatures are $T_{\rm h}$ and

[*]bizarro@ipfn.ist.utl.pt
[†]par@ipfn.ist.utl.pt

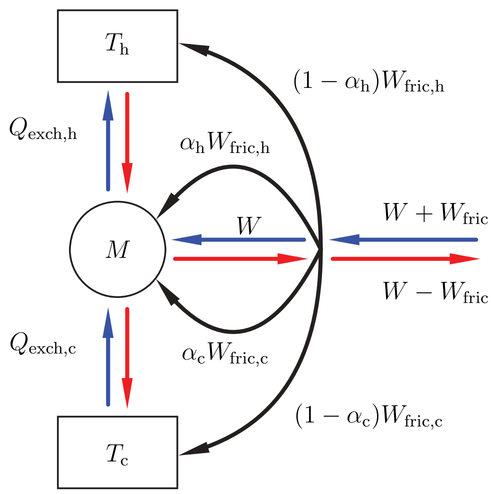

FIG. 1. (Color) Schematic diagram of a thermal engine with friction: red (blue) arrows indicate the energy transfers occurring in a heat engine (refrigerator or heat pump) used for producing work (for cooling or heating), and black arrows indicate the energy transfers caused by frictional dissipation.

T_c, respectively) are shown in Fig. 1: they are identical to the standard flow diagrams found in textbooks [2–4], with the exception of additional work and heat flows related to frictional losses [13]. Hence, to the usual energy transfers between the hot or cold reservoirs and the working fluid ($Q_{exch,h}$ and $Q_{exch,c}$, respectively) and to the work performed by, or delivered to, the latter (W), one must add the work done against friction forces (W_{fric} for the total of the cycle, and $W_{fric,h}$ and $W_{fric,c}$ for the halves of the cycle where the hot or cold reservoirs are present, respectively) and specify which fractions are dissipated in the fluid (α_h and α_c) and which are dissipated in the reservoirs ($1-\alpha_h$ and $1-\alpha_c$) [27]. To be noted is that, because of friction, the useful work extracted from the fluid in a heat engine (or the work needed to operate a refrigerator or a heat pump) is not W but $W - W_{fric}$ (or $W + W_{fric}$). To define appropriate FOM's for a given type of engine when friction is present, one must only consider that a FOM is always the ratio between "what one gets out of the engine" and "what one puts into it" [13].

Hence, the work that is ultimately extracted out of the heat engine of Fig. 1 is $W - W_{fric}$ and, to produce this result, the fluid receives (when interacting with the hot reservoir) the energy $Q_h \equiv Q_{exch,h} + \alpha_h W_{fric,h}$. However, the actual energy one must put into the engine (to be spent by the hot reservoir) is $Q_h - W_{fric,h}$, less than Q_h because the energies $\alpha_h W_{fric,h}$ and $(1-\alpha_h)W_{fric,h}$ are recirculated back to the engine (the former to the fluid, the latter to the reservoir). The FOM for a heat engine with friction is thus defined as the efficiency [11,13]:

$$\eta_{fric} \equiv \frac{W - W_{fric}}{Q_h - W_{fric,h}} = \eta \frac{1 - W_{fric}/W}{1 - \eta W_{fric,h}/W}, \tag{1}$$

where $\eta \equiv W/Q_h$ is the efficiency for the frictionless engine. Resorting once again to Fig. 1 to define the FOM or, more precisely, the COP for a refrigerator (or air conditioner), one has to put into the engine the work $W + W_{fric}$ (to have W delivered to the fluid). During heat extraction, the fluid receives the energy $Q_c \equiv Q_{exch,c} + \alpha_c W_{fric,c}$, which is not entirely from the cold reservoir. The energy effectively extracted from this reservoir is $Q_c - W_{fric,c}$, not only because the energy $(1-\alpha_c)W_{fric,c}$ is dissipated back into it, but also because the energy $\alpha_c W_{fric,c}$ dissipated in the fluid does not come from it. Therefore, the COP for a refrigerator is

$$\varepsilon_{fric} \equiv \frac{Q_c - W_{fric,c}}{W + W_{fric}} = \varepsilon \frac{1 - \varepsilon^{-1} W_{fric,c}/W}{1 + W_{fric}/W}, \tag{2}$$

where $\varepsilon \equiv Q_c/W$ is the frictionless COP. In a similar manner, the COP for a heat pump with friction is

$$\epsilon_{fric} \equiv \frac{Q_h + W_{fric,h}}{W + W_{fric}} = \epsilon \frac{1 + \epsilon^{-1} W_{fric,h}/W}{1 + W_{fric}/W}, \tag{3}$$

with $\epsilon \equiv Q_h/W$ its frictionless counterpart and, in this case, $Q_h \equiv Q_{exch,h} - \alpha_h W_{fric,h}$ [28]. Equations (1)–(3) generalize the usual definitions for the FOM's of thermal engines to situations where these operate with friction, and constitute a first result of this paper.

To dissipate any doubts that may arise regarding the definitions (1)–(3) for the FOM's with friction, obtained above following a practical, engineeringlike approach, it may be useful to rederive them in a more formal manner, starting from the Clausius inequality (or the second law) with friction [11]. Recalling that the heat engine depicted in Fig. 1 is cyclic, so the fluid's entropy is the same after one cycle, the total entropy change comes only from the reservoirs and reads

$$\frac{Q_{exch,c} + (1-\alpha_c)W_{fric,c}}{T_c} - \frac{Q_{exch,h} - (1-\alpha_h)W_{fric,h}}{T_h}$$
$$= \frac{Q_h - W_{fric,h} - W + W_{fric}}{T_c} - \frac{Q_h - W_{fric,h}}{T_h} \geqslant 0, \tag{4}$$

where the equality has been established with recourse to energy conservation (or the first law). Equation (4) is a form of the Clausius inequality and yields

$$\eta_{fric} = \frac{W - W_{fric}}{Q_h - W_{fric,h}} \leqslant \frac{T_h - T_c}{T_h} \equiv \eta_{Carnot}, \tag{5}$$

which is a statement of Carnot's theorem on maximum efficiency [1–4], and thus also a confirmation of the appropriateness of definition (1) for η_{fric} [11,13]. Going back to Fig. 1, but focusing instead on the mode of operation for a refrigerator or a heat pump, one has for the entropy change

$$\frac{Q_{exch,h} + (1-\alpha_h)W_{fric,h}}{T_h} - \frac{Q_{exch,c} - (1-\alpha_c)W_{fric,c}}{T_c}$$
$$= \frac{Q_c - W_{fric,c} + W + W_{fric}}{T_h} - \frac{Q_c - W_{fric,c}}{T_c}$$
$$= \frac{Q_h + W_{fric,h}}{T_h} - \frac{Q_h + W_{fric,h} - W - W_{fric}}{T_c} \geqslant 0, \tag{6}$$

whence

$$\varepsilon_{fric} = \frac{Q_c - W_{fric,c}}{W + W_{fric}} \leqslant \frac{T_c}{T_h - T_c} \equiv \varepsilon_{Carnot} \tag{7}$$

and

$$\epsilon_{fric} = \frac{Q_h + W_{fric,h}}{W + W_{fric}} \leqslant \frac{T_h}{T_h - T_c} \equiv \epsilon_{Carnot}. \tag{8}$$

Therefore, (7) and (8) justify once more the choices made in (2) and (3), as they show that $\varepsilon_{\text{fric}}$ and ϵ_{fric} are bounded from above by the well-known COP's derived for reversible refrigerators or heat pumps (which are those operating according to a reverse Carnot cycle) [1,3,4].

It is a simple exercise to check that (as it should) the FOM's for thermal engines with friction (η_{fric}, $\varepsilon_{\text{fric}}$, and ϵ_{fric}) are never larger than their respective frictionless equivalents (η, ε, and ϵ). The former quantities become functions of the latter and of W_{fric}/W, which can be interpreted as the level of friction losses, if either $W_{\text{fric,h}}$ or $W_{\text{fric,c}}$ is fixed (it suffices to fix one of the two because $W_{\text{fric,h}} + W_{\text{fric,c}} = W_{\text{fric}}$). The restrictions $0 \leqslant \eta < 1$ and $0 \leqslant W_{\text{fric}}/W \leqslant 1$ apply for heat engines, whereas for refrigerators there are no constraints (apart from the obvious positiveness conditions $\varepsilon \geqslant 0$ and $W_{\text{fric}}/W \geqslant 0$), and for heat pumps the only restriction is $\epsilon \geqslant 1$. However, it is expected that the domain of operation for economically viable refrigerators and heat pumps be such as to also have $0 \leqslant W_{\text{fric}}/W \leqslant 1$, and still that $\varepsilon \geqslant 1$ be the case for good refrigerators. Still concerning domains of operation, access to regions where $\eta \to 1$ for heat engines, and where $\varepsilon,\epsilon \to \infty$ (while $W_{\text{fric}}/W \to \infty$ because $W \to 0$) for refrigerators and heat pumps, is severely hindered by the second law. Hence, when analyzing the impact of frictional losses on the efficiency and performance of thermal engines, the main region of interest will be $0 \leqslant W_{\text{fric}}/W < 1$ and $\eta^{-1},\varepsilon,\epsilon > 1$. Assuming that $W_{\text{fric,h}} = W_{\text{fric,c}} = W_{\text{fric}}/2$, contour plots for η_{fric}/η, $\varepsilon_{\text{fric}}/\varepsilon$, and $\epsilon_{\text{fric}}/\epsilon$ as functions of, respectively, η, ε, and ϵ and of W_{fric}/W are given in Fig. 2. It is clear in all three cases that, in the region of interest identified above (more precisely, in the limit $W_{\text{fric}}/W \to 0$ and $\eta^{-1},\varepsilon,\epsilon \to \infty$), the contour levels become close to equally spaced horizontal lines, which means that η_{fric}, $\varepsilon_{\text{fric}}$, and ϵ_{fric} become proportional to η, ε, and ϵ, respectively, and decrease approximately linearly with increasing W_{fric}/W.

B. FOM's of heat engines, refrigerators, and heat pumps with friction: Universal asymptotic behavior

The behavior identified in Fig. 2 can be retrieved analytically by expanding (1)–(3) as

$$\frac{\eta_{\text{fric}}}{\eta} = \left(1 - \frac{W_{\text{fric}}}{W}\right)\left[1 + \frac{W_{\text{fric,h}}}{W}\eta + \left(\frac{W_{\text{fric,h}}}{W}\eta\right)^2 + \cdots\right], \tag{9}$$

$$\frac{\varepsilon_{\text{fric}}}{\varepsilon} = \left[1 - \frac{W_{\text{fric}}}{W} + \left(\frac{W_{\text{fric}}}{W}\right)^2 + \cdots\right]\left(1 - \frac{W_{\text{fric,c}}}{W}\varepsilon^{-1}\right), \tag{10}$$

and

$$\frac{\epsilon_{\text{fric}}}{\epsilon} = \left[1 - \frac{W_{\text{fric}}}{W} + \left(\frac{W_{\text{fric}}}{W}\right)^2 + \cdots\right]\left(1 + \frac{W_{\text{fric,h}}}{W}\epsilon^{-1}\right). \tag{11}$$

Hence, defining $\Delta\eta \equiv \eta - \eta_{\text{fric}}$, $\Delta\varepsilon \equiv \varepsilon - \varepsilon_{\text{fric}}$, and $\Delta\epsilon \equiv \epsilon - \epsilon_{\text{fric}}$, and if $W_{\text{fric}}/W \ll \eta^{-1},\varepsilon,\epsilon$ (note that $W_{\text{fric,h}}, W_{\text{fric,c}} \leqslant W_{\text{fric}}$) and $W_{\text{fric}}/W \ll 1$, retaining the leading term in (9)–(11) yields

$$\frac{\Delta\eta}{\eta} \approx \frac{\Delta\varepsilon}{\varepsilon} \approx \frac{\Delta\epsilon}{\epsilon} \approx \frac{W_{\text{fric}}}{W}, \tag{12}$$

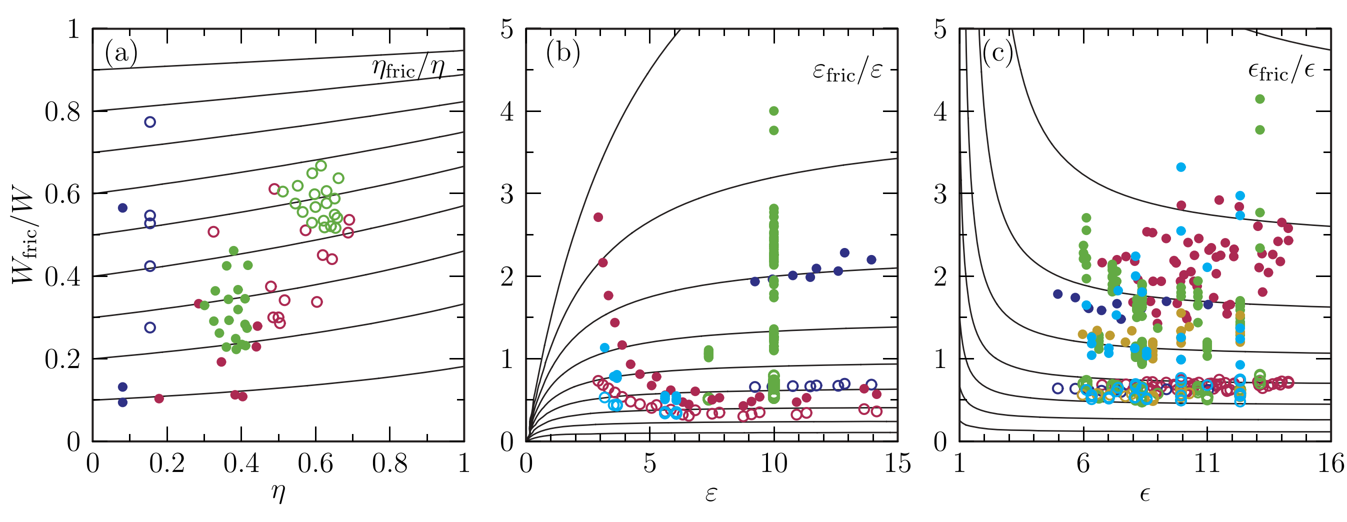

FIG. 2. (Color) Contour plots for the ratio between the friction and frictionless FOM's (efficiencies or COP's) of (a) heat engines (η_{fric}/η), (b) refrigerators ($\varepsilon_{\text{fric}}/\varepsilon$), and (c) heat pumps ($\epsilon_{\text{fric}}/\epsilon$) as functions of the frictionless FOM's and of the level of frictional losses (W_{fric}/W): contour levels start at 1.0 for the line $W_{\text{fric}}/W = 0$ and decrease upward by jumps of 0.1. Also shown as open (or full) circles are estimates of W_{fric}/W as given by (12) [or (13) for refrigerators and heat pumps], where observed efficiencies and measured COP's have been used for η_{fric}, $\varepsilon_{\text{fric}}$, and ϵ_{fric} and the theoretical bounds η_{Carnot} (or $\eta_{\text{Curzon-Ahlborn}}$ for heat engines), $\varepsilon_{\text{Carnot}}$, and ϵ_{Carnot} for η, ε, and ϵ, respectively. In (a) data are shown for several thermal power plants (in red) [24], a micrometer-sized stochastic engine (in blue) [25], and a high-performance thermoacoustic engine (in green) [30]; in (b) data are shown for a high-temperature refrigerator (in red) [26], an ejector-compression refrigeration system (in blue) [31], various commercial chillers and air conditioners (in green) [32], and various compressors used in supermarket refrigeration systems (in cyan) [33]; in (c) data are shown for two moderately high-temperature heat pumps (in red) [34], an advanced air-source heat pump (in blue) [35], various commercial heat pumps (in green) [32], several residential heat pumps (in cyan) [36], and heat-pumping equipment with compact heat exchangers (in orange) [37].

which means that the relative decreases induced by friction in the FOM's of heat engines, refrigerators, and heat pumps have the same limiting behavior. Note that, for large COP values but arbitrary friction levels, (2) and (3), or (10) and (11), show that COP degradation due to friction already exhibits a same asymptotic behavior for both refrigerators and heat pumps, namely,

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{\Delta\epsilon}{\epsilon} \approx \frac{W_{\text{fric}}/W}{1+W_{\text{fric}}/W}, \tag{13}$$

which further reduces to (12) in the limit of small W_{fric}/W. It must be noted that the scaling (12), which reads as the main contribution in this paper, is independent [as well as, by that matter, (13)] of the assumption $W_{\text{fric,h}} = W_{\text{fric,c}}$, which was used to generate the plots in Fig. 2 [29].

Because it needs not all the friction details, (12) is particularly useful as a simple, straightforward, and fast tool to estimate either the level of frictional losses or the impact they have on the FOM's of thermal engines and power (or refrigeration) plants. For instance, taking for η, ε, and ϵ the theoretical Carnot figures η_{Carnot}, $\varepsilon_{\text{Carnot}}$, and ϵ_{Carnot} of, respectively, (5), (7), and (8) (which are upper bounds for the frictionless efficiencies and COP's [1–4]) and for η_{fric}, $\varepsilon_{\text{fric}}$, and ϵ_{fric} the actual observed FOM's (which amounts to assume that all losses come from friction), (12) yields W_{fric}/W as an upper estimate for the percentage of frictional losses. The maximum impact of friction on the efficiency and performance of thermal engines has been thus estimated for a wide variety of devices (and of their operation conditions): for several existing power plants [24,38], for a micrometer-sized stochastic engine [25,39], for a high-performance thermoacoustic engine [30,40], for a high-temperature refrigerator [26,41], for an ejector-compression refrigeration system [31,42], for several commercial chillers and air conditioners [32,43], for different compressors used in supermarket refrigeration systems [33,44], for two moderately high-temperature heat pumps [34,45], for an advanced air-source heat pump [35,46], for various types of commercial heat pumps [32,43], for different residential heat pumps [36,47], and for heat-pumping equipment with compact heat exchangers [37,48]. In the case of refrigerators and heat pumps, estimates have been obtained using the intermediate formula (13) as well. The outcome of such an exercise is given in Fig. 2 also, which shows that a large majority of points does fall within the region where either (12) or (13) is already a good approximation (where contour levels become closer and closer to equally spaced parallel lines or, at least, to parallel lines for refrigerators and heat pumps) [49]. It must not be forgotten that these results overestimate W_{fric}/W (even with, in many cases, $W_{\text{fric}}/W \gtrsim 50\%$), not only because the maximum possible theoretical (and actually unattainable) values have been used for η, ε, and ϵ, but the assumption has also been made that efficiency degradation comes entirely from friction. Most likely, with more realistic predictions for the frictionless FOM's [other than the Carnot formulas (5), (7), and (8)], the points in Fig. 2 would come down to populate more densely the region where (12) is comfortably valid.

To see that such would indeed be the case, at least for heat engines and power plants, the Carnot efficiencies used for η have been replaced by the more realistic estimates provided by the Curzon-Ahlborn efficiencies at maximum power of finite-time heat engines [4,50]:

$$\eta_{\text{Curzon-Ahlborn}} = 1 - \sqrt{\frac{T_{\text{c}}}{T_{\text{h}}}}. \tag{14}$$

The Curzon-Ahlborn expression (14) is a finite-time result, allowing thus for losses other than frictional, and generally assumes that dissipation vanishes in the limit of an infinite-time (hence quasistatic) thermodynamic cycle [4,15,18–22,24,25,50]. This means, in particular, that $\eta_{\text{Curzon-Ahlborn}}$ does not necessarily account for sliding friction, a paradigm for quasistatic (yet irreversible) processes [10,11,13,51,52], and so it can be taken as a frictionless estimate [to plug in (9) or (12)] with no risk of having friction twice accounted for (both in η, set equal to $\eta_{\text{Curzon-Ahlborn}}$, and in η_{fric}). So, in Fig. 2 are shown new estimates of W_{fric}/W for the same power plants and engines as before [24,25,30,53], but now as yielded by (12) when observed efficiencies and expression (14) are used for η_{fric} and η, respectively. As expected, the points calculated in this manner appear in Fig. 2 displaced downward, towards levels of frictional losses such that $W_{\text{fric}}/W \lesssim 50\%$ (and well below this value in most cases), which are more reasonable and realistic.

Despite the striking simplicity of the linear formula in (12), note that retrieving the actual $\Delta\eta$, $\Delta\varepsilon$, or $\Delta\epsilon$ caused by friction in a real engine still requires knowledge of W_{fric}, hence of how work is done against frictional forces. However, and assuming that the frictionless quantities W and η, ε, or ϵ are already known from previous investigations, it is certainly easier to calculate W_{fric} alone than to go through the entire thermodynamic cycle again (computing, in addition to W_{fric}, all the other work and heat flows), the separate treatment of frictional losses being generally possible because current friction models are additive [5–9,13]. Equation (12) is specially valuable, as a tool to be used in the project of thermal engines, if and when the need arises to assess how the level of frictional losses can affect the efficiency, or performance, of a given design whose frictionless FOM and output has already been worked out.

The universal asymptotic behavior seen in Fig. 2 can be better grasped by changing or renaming variables according to $\xi \equiv \eta^{-1}$, ε, ϵ and $\delta \equiv W_{\text{fric}}/W$, and rewriting (1)–(3) as (keeping $W_{\text{fric,h}}/W_{\text{fric}} = W_{\text{fric,c}}/W_{\text{fric}} = 1/2$)

$$\frac{\eta_{\text{fric}}}{\eta}(\xi,\delta) = \frac{1-\delta}{1-\xi^{-1}\delta/2}, \tag{15}$$

$$\frac{\varepsilon_{\text{fric}}}{\varepsilon}(\xi,\delta) = \frac{1-\xi^{-1}\delta/2}{1+\delta}, \tag{16}$$

and

$$\frac{\epsilon_{\text{fric}}}{\epsilon}(\xi,\delta) = \frac{1+\xi^{-1}\delta/2}{1+\delta}, \tag{17}$$

so (12) and (13) become

$$\frac{\Delta\eta}{\eta}(\xi,\delta) \approx \frac{\Delta\varepsilon}{\varepsilon}(\xi,\delta) \approx \frac{\Delta\epsilon}{\epsilon}(\xi,\delta) \approx \delta \tag{18}$$

and

$$\frac{\Delta\varepsilon}{\varepsilon}(\xi,\delta) \approx \frac{\Delta\epsilon}{\epsilon}(\xi,\delta) \approx \frac{\delta}{1+\delta}, \tag{19}$$

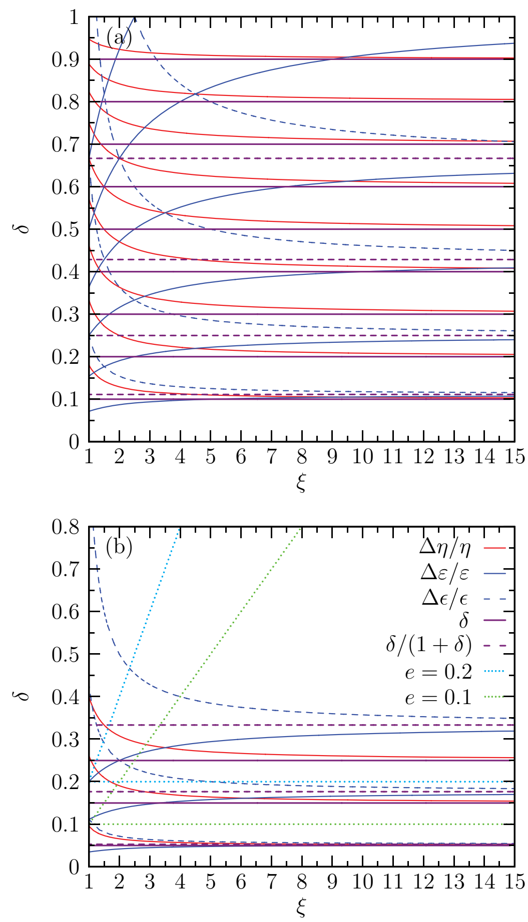

FIG. 3. (Color) Contour levels [starting at 0.0 for the line $\delta = 0$ and increasing upward by jumps of 0.1 in (a), and for 0.05, 0.15, and 0.25 in (b)] for the relative decrease in efficiencies ($\Delta\eta/\eta$) and COP's ($\Delta\varepsilon/\varepsilon$ and $\Delta\epsilon/\epsilon$) due to friction [and for the asymptotes δ and $\delta/(1+\delta)$] as functions of a same set of variables (ξ and δ). Also shown in (b) are the upper bounds imposed by two different values of the truncation criteria $\xi^{-1}\delta < e$ and $\delta < e$.

respectively. The truncation error one is willing to accept in going from (15)–(17) to (18) can be monitored via a small number e such that $\xi^{-1}\delta < e$ and $\delta < e$.

Equations (15)–(19) are plotted in Fig. 3, together with two different choices of e, showing that the asymptotic expressions (12) or (18) are very good approximations for every type of engine when $W_{\rm fric}/W \lesssim 20\%$ [54]. For economically viable engines, this value seems a reasonable upper bound for the percentage of frictional losses, and can even be relaxed in the case of heat engines. For the latter, the quality of the approximations (12) or (18) extends actually to the entire domain of operation, except where $\eta \to 1$ [13], as can be checked in Figs. 2 and 3. This difference in behavior between heat engines, on the one hand, and refrigerators and heat pumps, on the other, can be understood by noting not only that going from (9) to (12) involves one single truncation error, whereas going from (10) or (11) to (12) involves two, but also that the only criterion ($\xi^{-1}\delta < e$) involved in approximating $\Delta\eta/\eta$ is much less restrictive than the additional condition ($\delta < e$) needed to approximate $\Delta\varepsilon/\varepsilon$ and $\Delta\epsilon/\epsilon$. In any case, as far as refrigerators and heat pumps are the only concerned, (13) or (19) already yield a good intermediate asymptotic approximation for their COP's with friction, which does not need the more severe restriction $\delta < e$ and which, covering the range of present-day COP's ($\varepsilon,\epsilon \lesssim 15$ according to Fig. 2), can be safely extended up to $W_{\rm fric}/W \lesssim 50\%$ [as inferred from Fig. 3(a)].

As far as efficiencies are concerned, a final word may be appropriate about the relevance of the results presented above, which basically show that the FOM's of heat engines, refrigerators, and heat pumps alike exhibit the same dependence on frictional losses when the latter are modest ($W_{\rm fric}/W = \delta \ll 1$) and the former do not come close to unity ($\eta^{-1},\varepsilon,\epsilon = \xi \gg 1$). Whereas, from a practical point of view, it is highly desirable to have COP's significantly higher than unity, heat-engine efficiencies should thrive in order to approach this value. Whence the question of what's the interest of an approximation which degrades at high, close-to-one efficiencies? The answer is to be found in the fact that, although sought after, unit efficiencies are severely hindered by the second law of thermodynamics, and so the right question should be how relevant is the approximation given in (12) or (18) for modern heat engines and power plants? Looking at Fig. 2(a), one may say that approximating $\eta_{\rm fric}/\eta$ by equally spaced horizontal parallel lines is more or less acceptable when $\eta \lesssim 50\%$ (when $\xi \gtrsim 2$ in Fig. 3 for $\Delta\eta/\eta$), thus encompassing the realistic Curzon-Ahlborn data points (and leaving aside the estimates derived from the theoretical Carnot upper bounds), such points being representative of a wide variety of engines and plants [24,25,30]. Considering additionally that the range $\eta \lesssim 50\%$ also covers most of the present and near-future efficiency figures one can find in various technical reports, for instance, on vehicle combustion engines [55], or on different types of power plants [56–61], the relevance of (12) or (18) becomes established, moreover so because promising directions towards improving engine efficiency will likely include technologies to reduce friction (such as friction-reduced lubricating oils [55]), thus pushing data further downward in Figs. 2 and 3. All this having been said, even if only as a first, crude estimate before embarking on a fully detailed calculation, (12) or (18) will certainly be of use in many practical situations.

III. CONCLUSION

In summary, this paper shows that, when there is friction, the efficiencies of heat engines and the COP's of refrigerators or heat pumps (here generalized to include frictional dissipation explicitly) follow a same limiting behavior, if operation is not pushed into regions forbidden by the second law (unit efficiencies or arbitrarily large COP's, the latter because of vanishingly small input work) and if frictional losses remain within acceptable limits (which can be quantified for a given accuracy in the approximation). In such a case, the percentage decrease in the efficiencies, or COP's, is equal to the ratio between the work spent against friction forces and the work coming out, or going into, the fluid (which ratio measures the level of losses due to friction). Besides its intrinsic fundamental interest, this result provides a useful, simple, and fast means to quantify the decrease in a thermal engine's FOM caused by friction or, inversely, to assess the level of frictional losses from its observed FOM and respective frictionless estimate. A further advantage is that the expression obtained is universal: it does not depend on engine details and does not need the recalculation of heat and work transfers for a thermodynamic cycle with friction (assuming they have already been computed for the frictionless case). In fact, it suffices to know the frictionless FOM and the level of frictional losses expected to predict the efficiency or COP with friction, or to know the observed efficiency or COP to predict the level of losses. Note that it is not an immediate purpose of this paper to seek explicit paths to improve the efficiencies and performances of thermal engines, but rather to put forth a straightforward and uncomplicated manner to assess friction effects which, in turn, may indeed lead to the identification of the best strategies for engine improvement. Upper bounds for the level of frictional losses estimated from the Carnot and observed FOM's of a wide variety of real power plants, heat engines, refrigerators, and heat pumps seem to indicate that the approximation is valid in most of the domains where actual thermal engines usually operate. This has been confirmed with more realistic friction estimates obtained using the Curzon-Ahlborn efficiencies for heat engines and power plants. Since the requirement on the smallness of friction becomes more stringent when approximating the COP's of refrigerators and heat pumps than the efficiencies of heat engines, an intermediate asymptotic regime (which is also a function only of frictional losses but allows for higher levels of the latter) has been derived, as an alternative, for the percentage decrease in COP's caused by friction.

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[40] Measured efficiencies are taken from the η plot of Fig. 6 of Ref. [30], while Carnot efficiencies are calculated using Eq. (7) therein for η_C, with T_h retrieved from the same Fig. 6 and T_a from the reported fact that, for $T_h = 580\,^\circ\mathrm{C}$, $\eta = 32\%$ and $\eta/\eta_C = 49\%$.
[41] Carnot and experimental COP's are taken from Table 1 of Ref. [26], where they appear under ε_C and $\varepsilon_{\mathrm{exp}}$, respectively.
[42] Measured and Carnot COP's are retrieved from Fig. 5 of Ref. [31] where, for the purposes of calculating the latter COP's, the heat source and sink temperatures are taken as the temperatures of the cold and hot reservoirs, respectively.
[43] Measured COP's are taken as the energy efficiency ratios (EER's) and COP's specified for cooling and heating, respectively, in the tables with the technical characteristics of the various commercial devices in Ref. [32], and respective Carnot COP's are calculated from the temperatures defining the standard characterization conditions of the cooling and heating modes. Whenever two temperatures are quoted for the cold (hot) end of the device (for instance, inlet and outlet water temperatures in the condenser or evaporator), the lowest (highest) value is retained as the temperature of the cold (hot) reservoir.
[44] Measured COP's are taken from the "System2-phase1" tables in Appendix A of Ref. [33] with the characteristics of dual and medium temperature compressors, after conversion of the quoted EER's (given in $\mathrm{Btu\,h^{-1}/W}$) according to $\mathrm{COP} = \mathrm{EER}/3.412$, whereas Carnot COP's are calculated from the saturated suction and condensing temperatures therein, assumed as the temperatures of the cold and hot reservoirs, respectively.
[45] Measured and Carnot COP's are retrieved from Figs. 5 and 8 and Table 5 of Ref. [34], with the water temperatures at the evaporator inlet and condenser outlet taken as the temperatures of the cold and hot reservoirs, respectively.
[46] Measured and Carnot COP's are retrieved from Figs. 11 and 12 of Ref. [35], the air set temperatures and the heated-water temperatures being used as the cold- and hot-reservoir temperatures, respectively.
[47] Measured COP's are given in Tables 8, 10, 12, 15, and 24 of Ref. [36], with Carnot COP's calculated taking as temperatures for the cold and hot reservoirs the (ground) source or outside air temperatures and the supply temperatures, respectively.
[48] Measured COP's are taken from Tables 4 and 5 of Ref. [37], as well as the temperatures of the cold and hot reservoirs needed for computing Carnot COP's, given there in the columns where the heat-pumping systems are identified by the types of their heat sources and sinks and respective temperatures.
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