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Effects of reservoir squeezing on quantum systems and work extraction

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We establish a quantum Otto engine cycle in which the working substance contacts with squeezed reservoirs during the two quantum isochoric processes. We consider two working substances: (1) a qubit and (2) two coupled qubits. Due to the effects of squeezing, the working substance can be heated to a higher effective temperature, which leads to many interesting features different from the ordinary ones, such as (1) for the qubit as working substance, if we choose the squeezed parameters properly, the positive work can be exported even when $T_H < T_L$, where T_H and T_L are the temperatures of the hot and cool reservoirs, respectively; (2) the efficiency can be higher than classical Carnot efficiency. These results do not violate the second law of thermodynamics and it can be understood as quantum fuel is more efficient than the classical one.

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I. INTRODUCTION

The subject of quantum heat engine [1–5] has attracted increasing attention since it was first reported by Scovil and Schultz-Dubois [6]. The essential difference between classical and quantum heat engine is that the working substance of quantum heat engine is quantum systems, e.g., spin or coupled spins [7–11], harmonic-oscillator systems [12], multilevel system [13], or cavity quantum electrodynamical systems [14]. The main interests of these studies focus on whether it can improve the efficiency of quantum heat engine beyond the classical limit [15], how to better the work extraction [16,17], and under which condition the positive work can be extracted [9,17]. Among all of these studies, Scully and his collaborators proposed a quantum heat engine based on cavity quantum electrodynamical system and claimed that the work can be extracted from a single heat bath via vanishing quantum coherent [15]. This result shows that quantum coherent as a quantum resource can better the work extraction.

In this paper, we consider another quantum resource, squeezed reservoir. We construct a quantum Otto cycle based on squeezed reservoir. We consider a single qubit and coupled qubits as working substance, respectively. In the thermodynamical cycle, the squeezed reservoir alters the steady state of the working substance. As a result, some interesting features appear; for example, the efficiency can be higher than Carnot efficiency and even when $T_H < T_L$, positive work can be done. Furthermore, we notice that entanglement is a quantum resource, and quantum engine with entangled system as working substance is an interesting topic [18–20]. Hence, it is of interest to discuss the effects of entanglements in the two steady states on the basic thermodynamical quantities during the cycle, as we shall show the dependence of thermodynamics quantities on the entanglement are changed strikingly by the squeezing.

The paper is organized as follows. In Sec. II, we present a general description of the cycle. In Secs. III and IV we consider two cases, i.e., a qubit and two interacting qubits, as working substance and discuss the effects of squeeze on the steady states of working substance and thermodynamics. Conclusions are given in Sec. V.

II. GENERAL DESCRIPTION OF THE CYCLE

In our discussion, we consider a four-stroke quantum Otto heat engine, which includes two quantum isochoric processes (stages 1 and 3) and two quantum adiabatic processes (stages 2 and 4). Detailed performance of the cycle is described as follows.

(i) Stage 1: The working substance with certain probability P_{i0} in each energy level contacts with a hot squeezed reservoir. The squeezed reservoir can be described by a unitary squeezed operators (with squeezed parameters r_1 and ϕ_1) acting on a thermal equilibrium state at temperature T_H. The working substance under consideration can be a qubit with transition frequency ω_1 (see Sec. III) or two identical qubits with transition frequency ω and coupling constant J_1 (see Sec. IV). During this stage, only heat is transferred due to the change in occupation probabilities. Hence, this stage is a quantum isochoric process [16,17,21]. The dynamics of the working substance can be described by the Markovian master equation in Lindblad form [22–24]. After enough time, the system will fall into a steady state with the occupation probability P_{i1} of each energy level.

(ii) Stage 2: The system is isolated from the reservoir and then undergoes a quantum adiabatic expansion process, in which the transition frequency changes from ω_1 to ω_2 (for one qubit) and coupling constant changes from J_1 to J_2 (for two qubits). As a result, the energy structure is varied from E_{i1} to E_{i2}. We assume that the expansion is slow enough so that the probability in each eigenstate keeps unchanged according to the quantum adiabatic theorem. No heat is transferred and an amount of work is done by the system in this stage.

(iii) Stage 3: This stage is almost an inverse process of Stage 1. The working substance is coupled to a cold squeezed reservoir at temperature T_L and squeezed parameters r_2 and ϕ_2 while the energy structure is kept fixed. During this isochoric process, the population of each energy level changes from P_{i1} to P_{i2}, and some heat is transferred but no work is done in this stage.

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(iv) Stage 4: The system is removed from the cold squeezed reservoir and undergoes a quantum adiabatic contraction, in which the transition frequency changes from ω_2 to ω_1 (for one qubit) and the coupling constant changes from J_2 to J_1 (for two qubits). Probability of each eigenstate P_{i2} is maintained. Accordingly, the energy structure is varied from E_{i2} to E_{i1}, and an amount of work is done during this stage on the system.

A condition $P_{i2} = P_{i0}$ is needed for a complete cycle. During the cycle, the heat transferred and the work done can be obtained according to the quantum version of the first law of thermodynamics [16,17,21]. In this interpretation, the heat transfer is the change of occupations in the energy levels and the work is the shift of the energy levels. As a result, the heat absorbed, Q_1, and released, Q_2, and the net work in the whole cycle are

$$Q_1 = \sum_i E_{i1}(P_{i1} - P_{i2}), \tag{1}$$

$$Q_2 = \sum_i E_{i2}(P_{i2} - P_{i1}), \tag{2}$$

and

$$W = Q_1 + Q_2 = \sum_i (E_{i1} - E_{i2})(P_{i1} - P_{i2}). \tag{3}$$

Here, $Q > 0$ and $Q < 0$ correspond to absorption and release of heat from and to the reservoir while $W > 0$ and $W < 0$ correspond to work performed by and on the quantum heat engine. Generally speaking, for the mentioned quantum Otto cycle, a restriction about the temperature $T_H > T_L$ is necessary to obtain positive work. Moreover, for a two-level system (qubit) as working substance and the reservoir at thermal equilibrium state, a more rigorous condition $T_H > T_L \frac{\omega_1}{\omega_2}$ is needed [17] in order to have positive work output, i.e., $W > 0$. However, in the following discussion, we can see even when $T_H < T_L$, positive work may be done by the system with appropriate condition.

III. EXAMPLE 1: QUBIT SYSTEM

In this section, we consider a two-level system or qubit as working substance. The Hamiltonian of a qubit is $H_1 = \frac{1}{2}\hbar\omega\sigma^z$. We first consider the qubit interacts with the squeezed reservoir, i.e., Stages 1 and 3 in the cycle. Assuming the squeezed reservoir at temperature T and two squeezed parameters are r and ϕ, and the transition frequency is ω for the qubit, the master equation of the qubit can be written as [22] (in the interaction picture)

$$\begin{aligned}\frac{\partial}{\partial t}\rho = {} & \gamma(N+1)\left(\sigma^-\rho\sigma^+ - \frac{1}{2}\sigma^+\sigma^-\rho - \frac{1}{2}\rho\sigma^+\sigma^-\right) \\ & + \gamma N\left(\sigma^+\rho\sigma^- - \frac{1}{2}\sigma^-\sigma^+\rho - \frac{1}{2}\rho\sigma^-\sigma^+\right) \\ & - \gamma M\sigma^+\rho\sigma^+ - \gamma M^*\sigma^-\rho\sigma^-,\end{aligned} \tag{4}$$

where $N = n\cosh 2r + \sinh^2 r$, $M = -\frac{1}{2}\sinh 2r e^{i\phi}(2n+1)$. Here, $n = [\exp(\frac{\hbar\omega}{kT}) - 1]^{-1}$ is the Planck distribution giving the number of thermal photons at temperature T with frequency ω, γ denotes the coupled strength, and k is the Boltzmann constant. After some simple calculations, we will find that the squeezed phase ϕ does not affect the steady state of the system and the steady state of the system can be obtained as

$$\rho = \frac{1}{(2n+1)\cosh 2r} \times \begin{pmatrix} n\cosh 2r + \sinh^2 r & 0 \\ 0 & n\cosh 2r + \cosh^2 r \end{pmatrix}. \tag{5}$$

Now we return to our quantum Otto cycle. According to Eqs. (1), (2), and (3), we have

$$Q_1 = \hbar\omega_1 \Delta P_e, \tag{6}$$

$$Q_2 = -\hbar\omega_2 \Delta P_e, \tag{7}$$

$$W = Q_1 + Q_2 = \hbar(\omega_1 - \omega_2)\Delta P_e, \tag{8}$$

where

$$\begin{aligned}\Delta P_e = {} & \frac{\sinh\left[\frac{1}{2}\left(\frac{\hbar\omega_2}{kT_2} - \frac{\hbar\omega_1}{kT_1}\right)\right]}{2\cosh\frac{\hbar\omega_1}{2kT_1}\cosh\frac{\hbar\omega_2}{2kT_2}} + \frac{\sinh^2 r_1}{\cosh 2r_1}\tanh\frac{\hbar\omega_1}{2kT_1} \\ & - \frac{\sinh^2 r_2}{\cosh 2r_2}\tanh\frac{\hbar\omega_2}{2kT_2}.\end{aligned} \tag{9}$$

The first term on the right-hand side of Eq. (9) coincides with the usual results while the last two terms include the effects of reservoir squeezing. Figure 1 exhibits the numerical example for our quantum Otto heat engine. We plot the net work output for such a cycle as functions of T_L and T_H when $r_1 = 1, r_2 = 0$, and $\frac{\omega_1}{\omega_2} = 2$. We focus on the isoline map (right figure). In this isoline map, the $T_L - T_H$ plane can be divided into three parts by two lines: $T_H = T_L$ and $T_H = \frac{\omega_1}{\omega_2}T_L$. Below $T_L = T_H$ is the classical forbidden region. In this region, positive work cannot be exported by both classical and quantum heat engine. Between $T_L = T_H$ and $T_H = \frac{\omega_1}{\omega_2}T_L$ is the quantum forbidden region. In this region, positive work can be done by classical heat engine but cannot by the quantum one. In Fig. 1, however, we can see that in both of these two regions positive work can be done due to the effect of squeezing.

We should emphasize that this interesting phenomenon does not violate the second law of thermodynamics. This can be understood as follows: for the state at the end of each quantum isochoric process, i.e., Eq. (5), when $r = 0$ we can recover the usual result $\rho_{\text{th}} = \frac{1}{\mathcal{Z}_1}\exp(-\beta H_1)$ with $\mathcal{Z}_1 = \operatorname{Tr}\exp(-\beta H_1)$, i.e., a two-level system at thermal equilibrium with temperature T and transition frequency ω. However, when $r \neq 0$, the steady state can also be seen as a thermal equilibrium

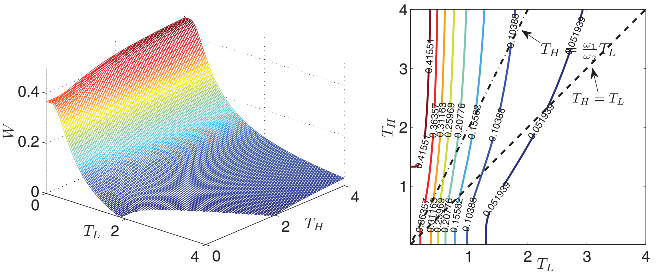

FIG. 1. (Color online) The work done by the two-level system (in units of $\hbar\omega_2$) as functions of T_H and T_L (in units of $\frac{\hbar\omega_2}{k}$). Other parameters are chosen as $r_1 = 1$, $r_2 = 0$, and $\frac{\omega_1}{\omega_2} = 2$.

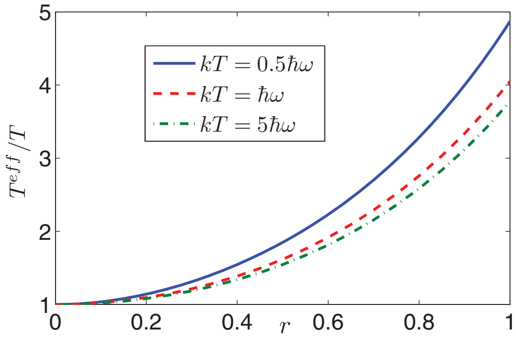

FIG. 2. (Color online) T^{eff}/T as a function of squeezed parameter r at different temperature.

state but at effective temperature T^{eff}, which depends on the environment temperature T and squeezed parameter r as

$$kT^{\text{eff}} = \frac{\hbar\omega}{\ln\left[\frac{\tanh^2 r + \exp(\frac{\hbar\omega}{kT})}{1+\tanh^2 r \exp(\frac{\hbar\omega}{kT})}\right]}. \tag{10}$$

T^{eff} is always higher than T (see Fig. 2). Hence, at the end of the quantum isochoric process, the effective temperature of the working substance is higher than the reservoir. As a consequence, when $T_H < \frac{\omega_2}{\omega_1}T_L$, T_H^{eff} may be higher than $\frac{\omega_2}{\omega_1}T_L$ and positive work can be done. This means that quantum fuel is more efficient than classical one. Figure 2 shows the dependence of T^{eff} on the squeezed parameter r at different temperatures. Two features can be observed from the figure. First, squeezed parameters can always increase the effective temperature of the qubit. Second, a lower reservoir temperature is more efficient in increasing the effective temperature of the qubit at the end of the quantum isochoric process. When the reservoir temperature is large enough (compared to $\hbar\omega$), T^{eff}/T tends to $\cosh 2r$, which is independent of the reservoir temperature.

Due to the advantage of quantum fuel, the positive work condition for our quantum Otto cycle is

$$\frac{\tanh\frac{\hbar\omega_1}{2kT_1}}{\cosh 2r_1} < \frac{\tanh\frac{\hbar\omega_2}{2kT_2}}{\cosh 2r_2}, \tag{11}$$

which is equivalent to $T_H^{\text{eff}} > \frac{\omega_1}{\omega_2}T_L^{\text{eff}}$, and the efficiency is

$$\eta_q = \frac{W}{Q_1} = 1 - \frac{\omega_2}{\omega_1}. \tag{12}$$

This result is the same as the one obtained in Ref. [17], in which the working substance is brought into some kind of contact with two equilibrium heat baths, and it seems that squeezing cannot improve the heat engine efficiency. However, in Ref. [17], the positive work condition $T_H > \frac{\omega_1}{\omega_2}T_C$ is needed, which leads to η_q lower than the Carnot efficiency $\eta_c = 1 - \frac{T_L}{T_H}$. In the present cycle, the positive condition is $T_H^{\text{eff}} > \frac{\omega_1}{\omega_2}T_L^{\text{eff}}$. Hence, when $r_1 > r_2$, η_q may be higher than η_c, i.e., squeezed reservoir improves the efficiency. Examples can also be found in Sec. IV.

IV. EXAMPLE 2: TWO-QUBIT SYSTEM WITH DIPOLE INTERACTION

In this section we consider two interacting qubits as working substance. The Hamiltonian of the system reads

$$H_2 = \tfrac{1}{2}\hbar\omega\left(\sigma_1^z + \sigma_2^z\right) + J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+). \tag{13}$$

Here, J is the exchange constant, $J > 0$ and $J < 0$ correspond to the antiferromagnetic and the ferromagnetic cases, respectively. In this paper, we only consider the antiferromagnetic case, i.e., $J > 0$. The four eigenvectors and corresponding eigenvalues for this Hamiltonian can be easily obtained as

$$\begin{aligned}
|s_1\rangle &= |00\rangle, \quad E_1 = -\hbar\omega,\\
|s_2\rangle &= |11\rangle, \quad E_2 = \hbar\omega,\\
|s_3\rangle &= \frac{\sqrt{2}}{2}(|10\rangle + |01\rangle), \quad E_3 = J,\\
|s_4\rangle &= \frac{\sqrt{2}}{2}(|10\rangle - |01\rangle), \quad E_4 = -J.
\end{aligned} \tag{14}$$

Similar to Sec. III, we first consider the steady state of the system induced by squeezed reservoir at temperature T and squeezed parameters r and ϕ. In the interaction picture, the master equation for the system reads

$$\begin{aligned}
\frac{\partial}{\partial t}\rho = \gamma\sum_{i=1,2}\Bigg[&(N_i + 1)\left(X_i^-\rho X_i^+ - \frac{1}{2}X_i^+X_i^-\rho - \frac{1}{2}\rho X_i^+X_i^-\right)\\
&+ N_i\left(X_i^+\rho X_i^- - \frac{1}{2}X_i^-X_i^+\rho - \frac{1}{2}\rho X_i^-X_i^+\right)\\
&- \gamma M_i X_i^+\rho X_i^+ - \gamma M_i^* X_i^-\rho X_i^-\Bigg],
\end{aligned} \tag{15}$$

where the two eigenoperators and the corresponding eigenfrequencies

$$\begin{aligned}
X_1^- &= \frac{\sqrt{2}}{2}(|s_1\rangle\langle s_3| + |s_4\rangle\langle s_2|), \quad \hbar\omega_1 = \hbar\omega + J,\\
X_2^- &= \frac{\sqrt{2}}{2}(|s_3\rangle\langle s_2| - |s_1\rangle\langle s_4|), \quad \hbar\omega_2 = \hbar\omega - J,
\end{aligned} \tag{16}$$

satisfy $[H_2, X_i^\pm] = \pm\omega_i X_i^\pm$ and the coefficients are

$$\begin{aligned}
N_i &= n(\omega_i)\cosh 2r + \sinh^2 r,\\
M_i &= -\tfrac{1}{2}\sinh 2r e^{i\phi}[2n(\omega_i) + 1].
\end{aligned} \tag{17}$$

Here, $n(\omega_i) = [\exp(\frac{\hbar\omega_i}{kT}) - 1]^{-1}$. The derivation about this equation is given in Appendix A. We can obtain the initial-state independent steady state after some calculation as

$$\rho = \sum_i P_i|s_i\rangle\langle s_i| = \begin{pmatrix} P_2 & 0 & 0 & 0\\ 0 & \frac{1}{2}(P_3 + P_4) & \frac{1}{2}(P_3 - P_4) & 0\\ 0 & \frac{1}{2}(P_3 - P_4) & \frac{1}{2}(P_3 + P_4) & 0\\ 0 & 0 & 0 & P_1 \end{pmatrix}, \tag{18}$$

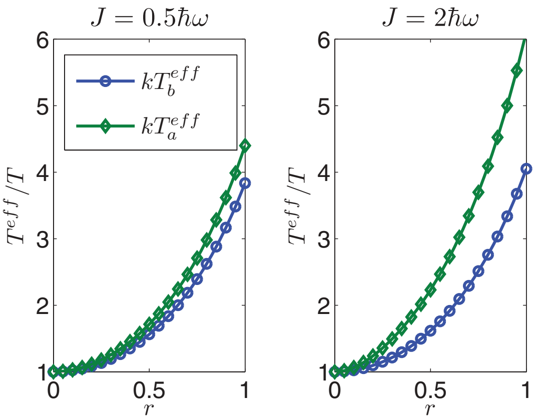

FIG. 3. (Color online) Effective temperature $kT_{ij}^{\rm eff}$ as a function of squeezed parameter r at different temperatures. Here we have set $kT = \hbar\omega$.

where

$$\begin{aligned} P_1 &= \frac{(N_1+1)(N_2+1)}{(2N_1+1)(2N_2+1)},\\ P_2 &= \frac{N_1N_2}{(2N_1+1)(2N_2+1)},\\ P_3 &= \frac{N_1(N_2+1)}{(2N_1+1)(2N_2+1)},\\ P_4 &= \frac{N_2(N_1+1)}{(2N_1+1)(2N_2+1)}, \end{aligned} \tag{19}$$

are occupation probabilities of the system. We can see these probabilities are also ϕ independent. We should note that when $r = 0$, the steady state Eq. (18) is equivalent to the thermal equilibrium state $\rho_{\rm th} = \frac{1}{\mathcal{Z}_2}\exp(-\beta H_2)$, where $\mathcal{Z}_2 = {\rm Tr}\exp(-\beta H_2)$. However, different from Sec. III, when $r \neq 0$, Eq. (18) is not a thermal equilibrium state. It is a nonequilibrium steady state [25] and we cannot define a unique effective temperature for this state [21]. But any two energy levels $|s_i\rangle$ and $|s_j\rangle$ can have an effective temperature defined as

$$kT_{ij}^{\rm eff} = \frac{E_i - E_j}{\ln P_j - \ln P_i}. \tag{20}$$

From the structure of Eq. (15), we have $T_{23}^{\rm eff} = T_{41}^{\rm eff} \equiv T_b^{\rm eff}$, $T_{24}^{\rm eff} = T_{31}^{\rm eff} \equiv T_a^{\rm eff}$. Figure 3 shows the dependence of $T_a^{\rm eff}$ and $T_b^{\rm eff}$ on the squeezed parameter r in both weak and strong coupling cases. We can see that when $r = 0$, the steady state of the system is the thermal equilibrium state since $T_a^{\rm eff} = T_b^{\rm eff} = T$, which confirms our foregoing analysis. When $r \neq 0$, reservoir squeezing increases the effective temperatures; $T_a^{\rm eff}$ and $T_b^{\rm eff}$ are no longer equal. Moreover, in strong coupling region ($J = 2\hbar\omega$), the effective temperatures are higher than the one in weak coupling case. We discuss two thermodynamic quantities for the steady state, the specific heat at constant squeezing $C_r = (\frac{\partial U}{\partial T})_r$, where $U = \langle H\rangle = \sum_i P_iE_i$ is the energy of the system, and von Neumann entropy $S = -\rho\ln\rho$, which is an extension of the Gibbs entropy to the quantum case. The von Neumann entropy is proportional to the thermodynamical entropy. The numerical results are shown in Fig. 4. We can see from the figure that for fixed r, the specific heat is a nonmonotone function of T, it first increases as T increases and then decreases, and it approaches zero when $T\to\infty$. For fixed T, specific heat decreases as r increases monotonously. The results for the entropy are different, i.e., S increases as both T and r increase.

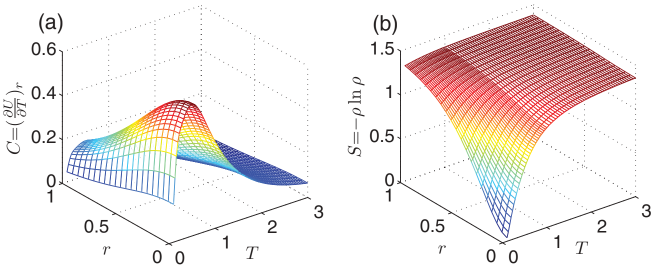

FIG. 4. (Color online) The specific heat at constant squeeze and von Neumann entropy as functions of r and T (in units of $\hbar\omega/k$). We have chosen $J = 0.5\hbar\omega$.

Another aspect of the system is the entanglement in the steady state. We take concurrence [26] as the measure of the entanglement. For the state given in Eq. (18), the concurrence can be written as $C = 2\max\{\frac{1}{2}|P_3 - P_4| - \sqrt{P_1P_2},0\}$. It is of interest to study effects of entanglement in the two steady states on the basic thermodynamics quantities in the cycle. According to the expression for the entanglement, we can solve the relation between the exchange constant J and concurrence C. Based on this relation, we can obtain the dependence of the thermodynamics quantities such as Q_1, Q_2, W, and η on the two concurrences in the two steady states. The analytical expressions for these dependences are too complicated even in the case of $\hbar\omega = 0$. As a result, only the numerical results are given in the paper. These can be found in Figs. 5, 6, and 7. Here we have set $r_2 = 0$ in all these figures, i.e., working substance interacts with a thermal reservoir in Stage 3. Several features can be seen from the figures. (1) When $\hbar\omega = 0$, the isoline of efficiency is open curve. This means the dependence of η on c_1 and c_2 is monotonic. In detail, the efficiency increases monotonically with c_1 for fixed c_2 and decreases monotonically with c_2 for fixed c_1. When $\hbar\omega \neq 0$, the isoline of efficiency becomes a quasiloop. This means the

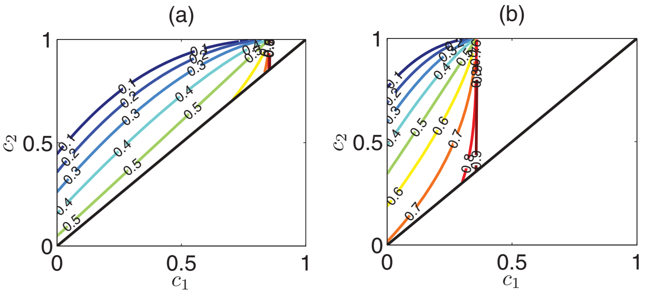

FIG. 5. (Color online) Variation of the efficiency η with the entanglement of the two steady states at the end of stages 1 and 3 in an isoline map of efficiency for $kT_1 = 2kT_2, \hbar\omega = 0, r_2 = 0$, and (a) $r_1 = 0.2$, (b) $r_1 = 0.5$.

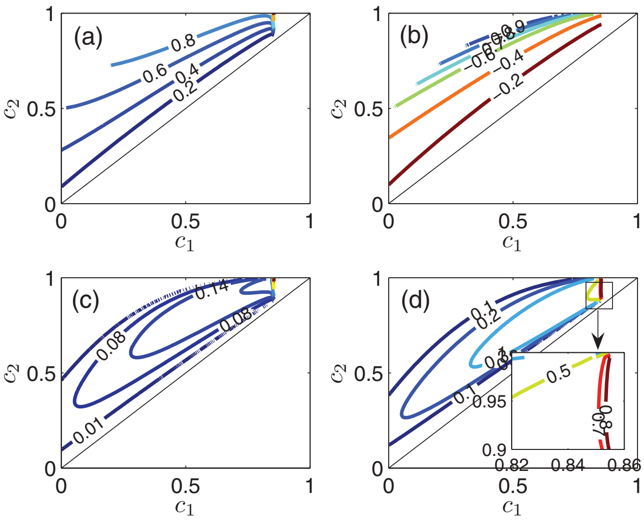

FIG. 6. (Color online) Variations of (a) Q_1, (b) Q_2, (c) W, and (d) η with c_1 and c_2 in isoline maps of efficiency for $kT_1 = 2kT_2$, $r_1 = 0.2$, $r_2 = 0$, and $\hbar\omega = kT_2$.

dependence of η on c_1 and c_2 is no longer monotonic. When c_1 is fixed, the efficiency η increases first as c_2 increase and then decreases, which indicates that there exists an optimal c_2, which leads to maximum value efficiency for fixed c_1. The situation is the same when c_2 is fixed. (2) Due to the effects of reservoir squeezing, the entanglements in the steady states in Stage 1 have an upper bound and this upper bound decreases as squeezed parameter r increases. This can be seen from Fig. 5, where the possible region of c_1 is smaller when r is larger. (3) The acceptable ranges for positive work output always $c_2 > c_1$, even when $\hbar\omega$ is larger. This is quiet different from previous results. Hence, $c_2 > c_1$ is a necessary condition for positive work output. Moreover, we can see from the figure that due to the squeezed reservoir in Stage 1, the efficiency can reach about 0.8, which exceeds the Carnot efficiency 0.5 for the parameters $kT_1 = 2kT_2$. We should also emphasize that it does not violate the second law of thermodynamics because squeezed reservoir can heat the working substance to a higher effective temperature although the effective temperatures between different energy levels are not equal (see Fig. 3). We can also see from the figures that when the squeezed parameter r_1 is small (for example $r_1 = 0.2$) the phenomenon that the efficiency exceeds the Carnot efficiency can only appear in a very small range of region (see the inset of Figs. 6 and 7). However, when the squeezed parameter r_1 is larger (for example $r = 0.5$) the region in which the efficiency exceed Carnot efficiency becomes larger. This is another evidence that squeezed reservoir improves the efficiency.

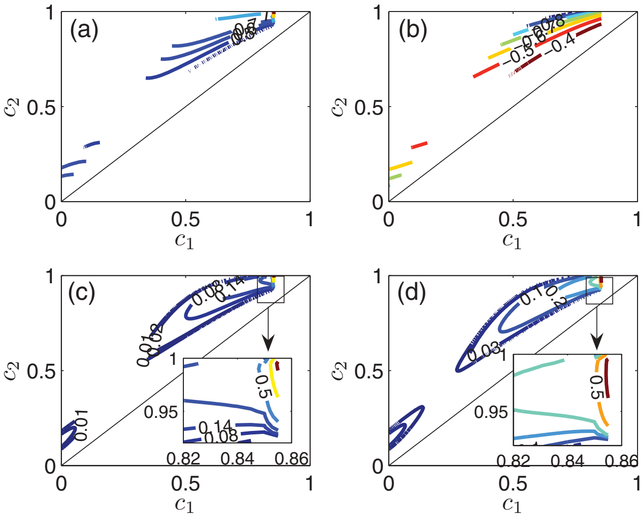

FIG. 7. (Color online) Variations of (a) Q_1, (b) Q_2, (c) W, and (d) η with c_1 and c_2 in isoline maps of efficiency for $kT_1 = 2kT_2$, $r_1 = 0.2$, $r_2 = 0$, and $\hbar\omega = 5kT_2$.

V. SUMMARY

In this paper, we have studied the effects of reservoir squeezing on the steady states of systems and constructed a quantum Otto engine based on squeezed reservoirs. For one qubit system, the steady state of the system is a thermal equilibrium state with effective temperature higher than the reservoir temperature. For multiqubit system, the steady state is not a thermal equilibrium state because the effective temperatures between any two energy levels are not equal. However, all of these effective temperatures are higher than the reservoir. As a result, the quantum Otto cycle may exhibit some new features: positive work can be done even when $T_H < T_L$ and the efficiency can be higher than the Carnot one. These interesting features do not violate the second law of thermodynamics. It can be understood as quantum resources such as coherent and squeezing can be used to extract work more efficiently. The effects of entanglement for two qubits quantum Otto heat engine are also discussed.

ACKNOWLEDGMENTS

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APPENDIX: THE DERIVATION FOR THE MASTER EQUATION

The derivation for the master Eq. (4) can be found in many textbooks such as Refs. [22,24]. Here we only give a simple introduction about the derivation of Eq. (15). Equation (4) can also be obtained in a similar manner.

The interaction between the system and the environment can be written as

$$V = \sigma_1^{+} B + \sigma_1^{-} B^{\dagger}. \tag{A1}$$

Here we have assumed that only qubit 1 is interacting with the reservoir. We can obtain the similar conclusion for qubit 2 or both qubits interacting with the reservoir. $B = \sum_k g_k b_k$ is the operator of the reservoir. Defining the eigenoperators as given in Eq. (16) and their corresponding eigenfrequencies satisfying $[H_2, X_i^{\pm}] = \pm\hbar\omega_i X_i^{\pm}$. According to the Hamiltonian of the system and reservoir, the interaction V can be transformed into an interaction picture, called $V(t)$. The Redfield equation in the interaction picture [27]

$$\frac{\partial}{\partial t}\rho = -\mathrm{Tr}_E \int_0^t \{V(t),[V(\tau),\rho_S \otimes \rho_E]\}d\tau \tag{A2}$$

can be used, and after some calculations we have

$$\begin{aligned}\frac{\partial}{\partial t}\rho = \sum_{i=1,2}\Bigg[&K_i\left(X_i^-\rho X_i^+ - \frac{1}{2}X_i^+X_i^-\rho - \frac{1}{2}\rho X_i^+X_i^-\right)\\ &+ G_i\left(X_i^+\rho X_i^- - \frac{1}{2}X_i^-X_i^+\rho - \frac{1}{2}\rho X_i^-X_i^+\right)\\ &- P_iX_i^+\rho X_i^+ - P_i^*X_i^-\rho X_i^-\Bigg].\end{aligned} \tag{A3}$$

In the derivation of the above equation, we have used the rotating wave approximation and the relation $(X_i^+)^2 = (X_i^-)^2 = 0$. The coefficients in the equation are defined as

$$\begin{aligned}K_i &= 2\int_0^t \langle B(t)B^\dagger(\tau)\rangle e^{i\omega_i(t-\tau)}d\tau\\ G_i &= 2\int_0^t \langle B^\dagger(\tau)B(t)\rangle e^{i\omega_i(t-\tau)}d\tau\\ P_i &= 2\int_0^t \langle B(t)B(\tau)\rangle e^{i\omega_i(t+\tau)}d\tau,\end{aligned} \tag{A4}$$

where $\langle\cdots\rangle$ denotes the average over the reservoir state ρ_E. For the squeezed reservoir

$$\rho_E = \prod_{\otimes k} S_k \rho_{E\mathrm{th}} S_k^\dagger, \tag{A5}$$

where $\rho_{E\mathrm{th}}$ is the thermal equilibrium state operator for the reservoir at temperature T and $S_k = \exp(\frac{1}{2}\xi^* b_k^2 - \frac{1}{2}\xi b_k^{\dagger 2})$ is the squeezed operator. Here, $\xi = re^{i\phi}$. After the standard process for the calculation of Eq. (A4) [22,24,27], where the Born-Markov approximation and the Weisskopf-Winger approximation are used, we obtain

$$K_i = \gamma(N_i + 1), \quad G_i = \gamma N_i, \quad P_i = \gamma M_i, \tag{A6}$$

and then we reach Eq. (15).

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