Amplitude modulation of hydromagnetic waves and associated rogue waves in magnetoplasmas

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It is shown that the dynamics of amplitude-modulated compressional dispersive Alfvénic (CDA) waves in a collisional megnetoplasma is governed by a complex Ginzburg-Landau (CGL) equation. The nonlinear dispersion relation for the modulational instability of the CDA waves is derived and investigated numerically. It is found that the growth rate of the modulational instability decreases (increases) with the increase of the normalized electron-ion collision frequency α (the plasma β). The modulational instability criterion for the CGL equation is defined precisely and investigated numerically. The region of the modulational instability becomes narrower with the increase of α and β, indicating that the system dissipates the wave energy by collisions, and a stable CDA wave envelope packet in the form of a hole will be a dominant localized pulse. For a collisionless plasma, i.e., $\alpha = 0$, the CGL equation reduces to the standard nonlinear Schrödinger (NLS) equation. The latter is used to investigate the modulational (in)stability region for the CDA waves in a collisionless magnetoplasma. It is shown that, within unstable regions, a random set of nonlinearly interacting CDA perturbations leads to the formation of CDA rogue waves. In order to demonstrate that the characteristics of the CDA rogue waves are influenced by the plasma β, the relevant numerical analysis of the appropriate nonlinear solution of the NLS equation is presented. The application of our investigation to space and laboratory magnetoplasmas is discussed.

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I. INTRODUCTION

In a cold magnetized plasma two different electromagnetic waves can propagate [1] for a specified frequency. In general, one wave propagates with a faster speed than the other, and thus a reasonable nomenclature for them would be "fast mode" and "slow mode." Unfortunately, depending on the specific frequency band, different research communities use a different terminology to distinguish between these two modes. This practice can be confusing when comparing results across different research areas, especially when more complicating features are considered, such as finite ion and electron temperatures and multiple ion species. In a sense, the present study falls into this category. Thus, it is useful to define, at the outset, the wave modes that are being investigated. For a single ion species, and in the frequency band near the ion cyclotron frequency, the two plasma electromagnetic modes are most commonly known as "Alfvén waves." In the laboratory-oriented community, the slower mode is frequently described as the "shear" Alfvén wave, and the faster mode as the "compressional" Alfvén wave. The present study focuses entirely on the compressional mode [2]. The latter was reported first by Adlam and Allen [3] for large-amplitude waves that travel into a cold collisionless electron-ion plasma containing a magnetic field when it is rapidly compressed. Later, extensive work was done to examine the properties of this mode; see, e.g., Refs. [4–8]. Some of these works incorporated the effect of electron-ion collisions, so a compressional dispersive Alfvén (CDA) wave exists. The appropriate dynamical equation for studying the CDA waves is a complex Ginzburg-Landau (CGL) equation. The CGL equation is one of the most-studied nonlinear equations in the physics community. It describes on a qualitative and often even on a quantitative level a vast variety of phenomena from nonlinear waves to second-order phase transitions and from superconductivity, superfluidity, and Bose-Einstein condensation to liquid crystals, strings in field theory, and plasmas [9–13]. On the other hand, the CGL equation describes the slow modulation of a periodic pattern in space and time near the threshold of an instability, where a band of modes become unstable. The CGL equation admits localized soliton solutions, which have the general name "dissipative solitons" [14]. However, when the dissipation is neglected (in our case the electron-ion collision vanishes) then the CGL equation reduces to the usual nonlinear Schrödinger (NLS) equation. The latter predicts the regions of the modulational instability of the compressional Alfvén waves, when the electron-ion collision is neglected. Within the modulational unstable envelope pulse region, it is possible for a random perturbation of the amplitude to grow and this may thus lead to the creation of Alfvénic rogue waves. Recently, the rogue waves in a multicomponent plasma have been experimentally observed [15], while the theoretical precursors of the rogue waves in plasmas were reported by many authors

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(see e.g. Refs. [16–19]). Indeed, rogue waves have been studied in many different systems including nonlinear fiber optics [20], parametrically driven capillary waves [21], Bose-Einstein condensates [22], superfluids [23], optical cavities [24], plasmonics [25], and narrow-band directional ocean waves [26].

In this paper, we study the dynamics of amplitude-modulated CDA waves across an external magnetic field in a collisional magnetoplasma. In Sec. II, the basic set of fluid equations for the nonlinear CDA waves are presented and the CGL equation is derived. In Sec. III, the stability analysis and the propagation properties of the amplitude-modulated CDA waves are discussed. Furthermore, when the electron-ion collision vanishes the CGL equation reduces to the NLS equation. By using the latter, the nonlinear evolution of the modulationally unstable CDA rogue waves has been investigated numerically. Finally, a summary of our results and their applications appear in Sec. IV.

II. GOVERNING EQUATIONS AND THE DERIVATION OF THE CGL EQUATION

We consider the nonlinear propagation of CDA waves across a uniform magnetic field ($B_0\hat{z}$) in an electron-ion plasma, where $\hat{z}$ is the unit vector along the z axis in a Cartesian coordinate system and B_0 is the strength of the magnetic field. The restoring force on the CDA waves comes from the magnetic pressure, whereas the ion mass provides the inertia to sustain the CDA waves. The CDA wave dispersion is due to the electron polarization drift in the wave electric field $E_\perp = E_x\hat{x} + E_y\hat{y}$, where $\hat{x}$ and $\hat{y}$ are the unit vectors along the x and y axes, respectively. The CDA wave magnetic field is aligned along the z axis. In a quasineutral plasma with $n_e = n_i \equiv n$, where n_e and n_i are the electron and ion number densities, respectively, the x components of the electron and ion fluid velocities are equal (i.e., $u_{ex} = u_{ix} \equiv u$), whereas the x and y components of the electron fluid velocities differ owing to the electron polarization drift. The electrons carry currents only along the y direction. The CDA waves compress the magnetic-field lines without bending them, and they are accompanied by density perturbations.

We are interested in examining the nonlinear propagation of one-dimensional CDA waves along the x axis in a quasineutral collisional magnetoplasma. The nonlinear dynamics of the CDA waves in our plasma is governed by [8]

$$\frac{Dn}{Dt} + n\frac{\partial u}{\partial x} = 0, \tag{1}$$

$$\frac{Du}{Dt} + \frac{1}{2n}\frac{\partial B^2}{\partial x} + \frac{\beta}{n}\frac{\partial n^3}{\partial x} = 0, \tag{2}$$

and

$$\frac{\partial B}{\partial t} + \frac{\partial (uB)}{\partial x} - \frac{\partial}{\partial x}\left[\frac{D}{Dt}\left(\frac{1}{n}\frac{\partial B}{\partial x}\right)\right] - \alpha\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B}{\partial x}\right) = 0, \tag{3}$$

where $\beta = 4\pi n_0 k_B T/B_0^2$, $\alpha = \nu_{ei}/\omega_{LH}$, $D/Dt = (\partial/\partial t) + u\partial/\partial x$, ν_{ei} is the constant electron-ion collision frequency, $B(= B_0 + B_1)$ the sum of the ambient and wave magnetic fields, m_e and m_i are the electron and ion masses, respectively, $T(= T_e + T_i)$ is the sum of the electron and ion temperatures, n_0 is the equilibrium plasma number density, e is the magnitude of the electron charge, k_B is the Boltzmann constant, $\omega_{LH}(= \sqrt{\omega_{ce}\omega_{ci}})$ is the lower-hybrid resonance frequency, $\omega_{ce}(= eB_0/m_e c)$ and $\omega_{ci}(= eB_0/m_i c)$ are the electron and ion gyrofrequencies, respectively, and c is the speed of light in vacuum. Here, n is normalized by n_0, B by B_0, u by the Alfvén speed $V_A = B_0/\sqrt{4\pi n_0 m_i}$, t by ω_{LH}^{-1}, and x by the electron skin depth $\lambda_e(= c/\omega_{pe} \equiv C_A/\omega_{LH})$, with $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ the electron plasma frequency.

To examine the nonlinear wave modulation of the CDA waves, the independent variables are stretched as [27–30]

$$\xi = \epsilon(x - v_g t) \quad \text{and} \quad \tau = \epsilon^2 t, \tag{4}$$

where $0 < \epsilon < 1$ is a small (real) parameter and v_g is the envelope group velocity to be determined later. The dependent variables are expanded as

$$\mathbf{A}(x,t) = \mathbf{A}_0 + \sum_{m=1}^{\infty}\epsilon^m \sum_{L=-m}^{m} \mathbf{A}_L^{(m)}(\xi,\tau)\exp(iL\Theta), \tag{5}$$

where

$$\mathbf{A}_L^{(m)} = \left[n_L^{(m)} \quad u_L^{(m)} \quad B_L^{(m)}\right]^T,$$
$$\mathbf{A}_L^{(0)} = [1 \quad 0 \quad 1]^T, \text{and} \quad \Theta = kx - \omega t.$$

Here k and ω are real variables representing the fundamental (carrier) wave number and the angular frequency, respectively. All elements of $\mathbf{A}_L^{(m)}$ satisfy the reality condition $\mathbf{A}_{-L}^{(m)} = \mathbf{A}_L^{*(m)}$, where the asterisk denotes the complex conjugate.

By substituting Eqs. (4) and (5) into Eqs. (1)–(3) and collecting terms of the same powers of ϵ, the first-order ($m = 1$) equations with $L = 1$ gives

$$n_1^{(1)} = \frac{k^2}{\omega^2 - 3k^2\beta}B_1^{(1)}, \; u_1^{(1)} = \frac{\omega k}{\omega^2 - 3k^2\beta}B_1^{(1)}, \tag{6}$$

with ω satisfying the relation

$$\omega^3 + \left(\frac{ik^2\alpha}{1+k^2}\right)\omega^2 - \left(\frac{k^2 + 3k^2\beta(1+k^2)}{1+k^2}\right)\omega - \frac{3ik^4\alpha\beta}{1+k^2} = 0. \tag{7}$$

Clearly, the linear dispersion relation (7) has three complex roots for real values of k. The imaginary (ω_i) and the real (ω_r) parts of ω are numerically investigated in Figs. 1 and 2. The real frequency ω_r represents the CDA wave frequency, while the imaginary frequency ω_i indicates the damping rate. It is clear that increasing α, for $k > 0.2$, decreases the damping rate, as shown in Fig. 1(a), while the increase of α does not affect the CDA wave frequency, as depicted in Fig. 1(b). The effect of the plasma β on ω_i and ω_r is depicted in Fig. 2. It is found that, for large k, the increase of β would lead to an increase in the damping rate. Also, it is seen that increasing β makes the CDA wave frequency become higher. It is worth mentioning that only one root of the complex dispersion relation (7) is plotted in Figs. 1 and 2. The numerical investigation for the second root shows that the increase of both α and β would lead to enhance the damping rate, but the CDA wave frequency does not change. The third root yields a negative CDA wave frequency, which is not physically acceptable.

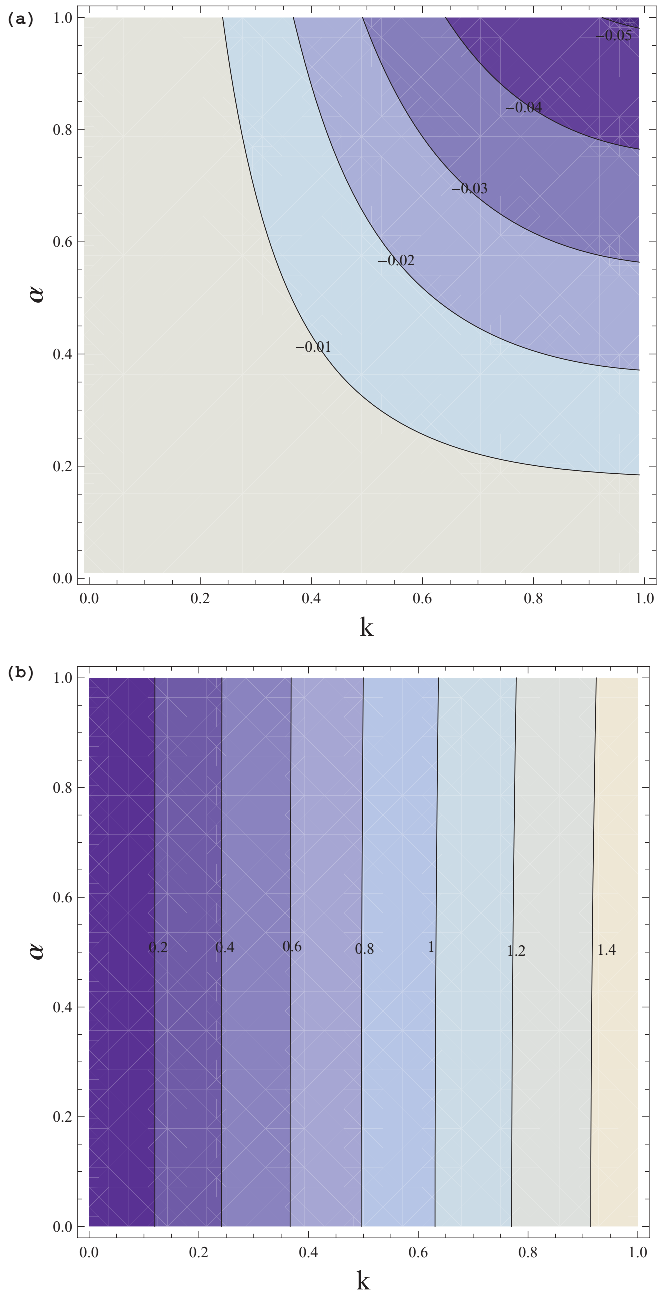

FIG. 1. (Color online) The variation of (a) the imaginary frequency ω_i and (b) the real frequency ω_r in the k-α plane for $\beta = 0.6$.

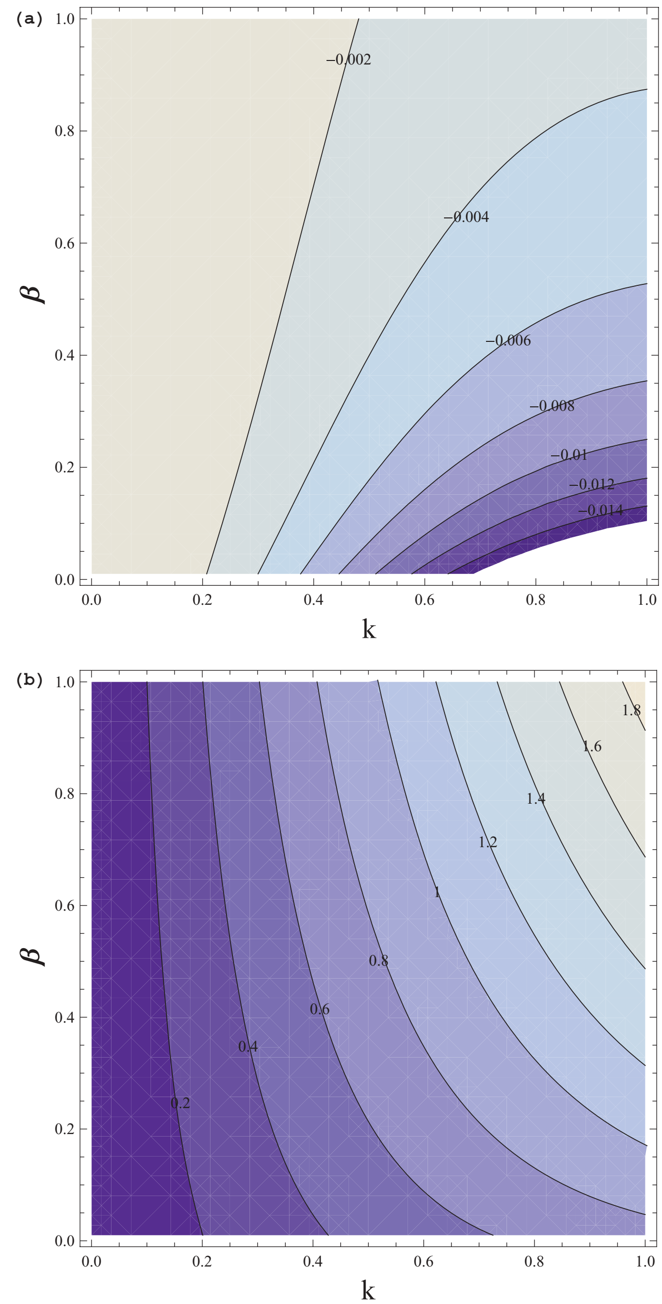

FIG. 2. (Color online) The variation of (a) the imaginary frequency ω_i and (b) the real frequency ω_r in the k-β plane for $\alpha = 0.1$.

The second-order ($m = 2$) reduced equations with $L = 1$ yield

$$n_1^{(2)} = \frac{k}{(\omega^2 - 3k^2\beta)^2}\left[k(\omega^2 - 3k^2\beta)B_1^{(2)} - 2i\omega(\omega - kv_g)\frac{\partial B_1^{(1)}}{\partial \xi}\right], \tag{8}$$

$$u_1^{(2)} = \frac{1}{(\omega^2 - 3k^2\beta)}\left[k\omega B_1^{(2)} - i(\omega - kv_g)\frac{(\omega^2 + 3k^2\beta)}{(\omega^2 - 3k^2\beta)}\frac{\partial B_1^{(1)}}{\partial \xi}\right], \tag{9}$$

with the compatibility condition

$$v_g = \frac{\partial \omega}{\partial k} = -\frac{k}{\delta}\left[2i\alpha - \frac{2\omega(\omega^2 - (\omega^2 - 3k^2\beta)^2)}{(\omega^2 - 3k^2\beta)^2}\right], \tag{10}$$

where

$$\delta = 1 + k^2\left[1 + \frac{(\omega^2 + 3k^2\beta)}{(\omega^2 - 3k^2\beta)^2}\right].$$

The compatibility condition (10) represents the group velocity of the CDA waves. It is seen that the group velocity is composed of real (v_{gr}) and imaginary (v_{gi}) parts. The complex group velocity is common in absorbing and active media, yet its precise physical meaning is unclear. While in the

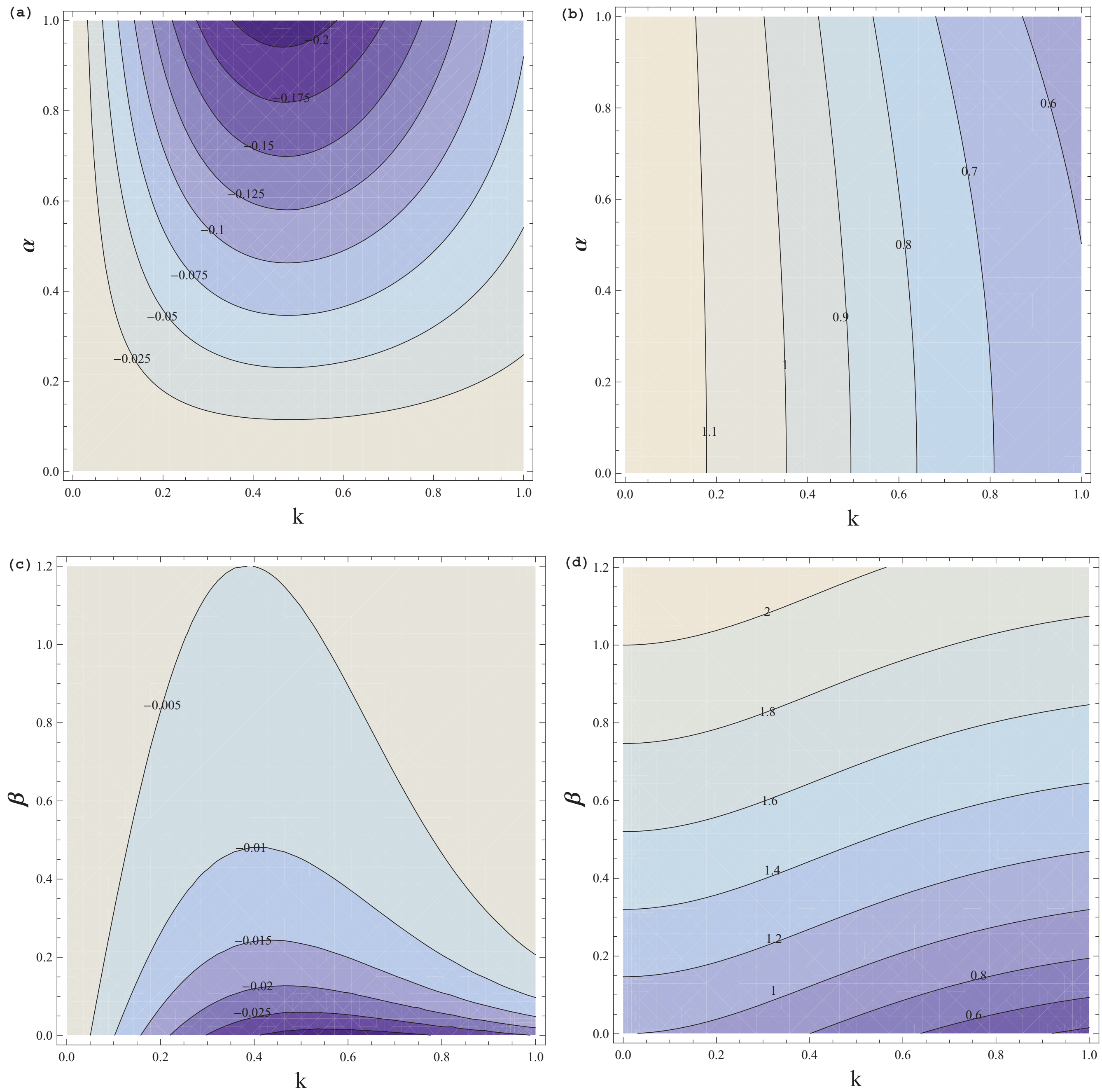

FIG. 3. (Color online) The profile of the complex group velocity v_{gi} (a) in the k-α plane for $\beta = 0.1$ and (b) in the k-β plane for $\alpha = 0.1$. The profile of the real group velocity v_{gr} (c) in the k-α plane for $\beta = 0.1$ and (d) in the k-β plane for $\alpha = 0.1$.

case of a nondissipative medium the group velocity of the propagating waves is exactly equal to the observable energy velocity defined as the ratio between the energy flux and the total energy density, in a dissipative medium it is in general a complex quantity which cannot be associated with the velocity of energy transport. Nevertheless, we find that the complex group velocity may contain information about the wave energy absorption in the medium [31]. The effects of the plasma parameters α and β on v_{gi} and v_{gr} are shown in Fig. 3. It is found that the increase of α and β would lead to a decrease in v_{gi}, as illustrated in Figs. 3(a) and 3(b), respectively. We speculate that due to such behavior the system dissipates wave energy for higher collision frequency and weaker magnetic field. However, the increase of α and β causes the real group velocity v_{gr} to increase, as depicted in Figs. 3(c) and 3(d), respectively.

The second-harmonic modes ($m = L = 2$) arising from the nonlinear self-interaction of the carrier waves are obtained in terms of $(B_1^{(1)})^2$ as

$$n_2^{(2)} = c_1^{(22)}\left(B_1^{(1)}\right)^2, \tag{11}$$

$$u_2^{(2)} = c_2^{(22)}\left(B_1^{(1)}\right)^2, \tag{12}$$

and

$$B_2^{(2)} = c_3^{(22)}\left(B_1^{(1)}\right)^2, \tag{13}$$

where $c_1^{(22)}$, $c_2^{(22)}$, and $c_3^{(22)}$ are given in the Appendix.

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth-order harmonic. Its strength is analytically determined by taking the $L = 0$ component of the third-order reduced equations, which can be expressed as

$$n_0^{(2)} = c_1^{(20)}\left|B_1^{(1)}\right|^2, \tag{14}$$

$$u_0^{(2)} = c_2^{(20)}\left|B_1^{(1)}\right|^2, \tag{15}$$

and

$$B_0^{(2)} = c_3^{(20)}\left|B_1^{(1)}\right|^2, \tag{16}$$

where $c_1^{(20)}$, $c_2^{(20)}$, and $c_3^{(20)}$ are given in the Appendix.

Finally, the third-harmonic modes ($m = 3$ and $L = 1$), with the aid of Eqs. (11)–(16), give a set of equations. The compatibility condition for these equations yields the CGL equation

$$i\frac{\partial \Psi}{\partial \tau} + P\frac{\partial^2 \Psi}{\partial \xi^2} + Q|\Psi|^2\Psi = 0, \tag{17}$$

where $B_1^{(1)} \equiv \Psi$ for simplicity. The dispersion coefficient $P(= P_r + iP_i)$ is

$$P = \frac{L_1}{(\omega^2 - 3k^2\beta)^3}\left[1 + k^2\left(1 + \frac{\omega^2 + 3k^2\beta}{(\omega^2 - 3k^2\beta)^2}\right)\right]^{-1}, \tag{18}$$

where L_1 is

$$\begin{aligned} L_1 = {}& \omega^3(9k^2\beta + \omega^2) - i(\alpha - i\omega)[\omega^6 - 9k^2\beta\omega^4 - 27k^4\beta^2(k^2\beta - \omega^2)] \\ & + kv_g[54k^4\beta^2(k^2\beta - \omega^2) - 18k^2\beta\omega^2(1 - \omega^2) - 2\omega^4(1 + \omega^2) + k\omega v_g(9k^2\beta + \omega^2)], \end{aligned}$$

and the nonlinear coefficient $Q(= Q_r + iQ_i)$ is

$$Q = \frac{L_2}{(\omega^2 - 3k^2\beta)^4}\left[1 + k^2\left(1 + \frac{\omega^2 + 3k^2\beta}{(\omega^2 - 3k^2\beta)^2}\right)\right]^{-1}, \tag{19}$$

where

$$L_2 = k\left(\begin{aligned} & k^5[9k^4\beta^2(i\alpha + 3\omega) + \omega^3(-1 + i\alpha\omega + 3\omega^2) - 3k^2\beta\omega(1 + 2i\alpha\omega + 6\omega^2)] \\ & \quad -2c_3^{(20)}k\omega(\omega^2 - 3k^2\beta)^3 - 2c_3^{(22)}k(\omega^2 - 3k^2\beta)^3[\omega + k^2(i\alpha + 3\omega)] \\ & \quad +c_2^{(22)}(\omega^2 - 3k^2\beta)^3[3k^4\beta - \omega^2 + k^2(1 + 3\beta - \omega^2)] \\ & \quad -c_2^{(20)}(\omega^2 - 3k^2\beta)^2[9k^6\beta^2 + \omega^4 + 3k^4\beta(1 + 3\beta - 2\omega^2) + k^2\omega^2(1 - 6\beta + \omega^2)] \\ & \quad -c_1^{(20)}k(\omega^2 - 3k^2\beta)^2\{9k^4\beta^2(\omega + i\alpha) + \omega^3[\omega(\omega + i\alpha) - 1] - 3k^2\beta\omega[2\omega(\omega + i\alpha) - 3]\} \\ & \quad +c_1^{(22)}k(\omega^2 - 3k^2\beta)^2\{9k^4\beta^2(\omega + i\alpha) + \omega^3[\omega(\omega + i\alpha) - 1] - 3k^2\beta\omega[2\omega(\omega + i\alpha) + 3]\} \end{aligned}\right).$$

III. STABILITY ANALYSIS AND DISCUSSION

We study the stability of the CDA waves in a warm collisional plasma across a uniform magnetic field, when the modulation of the wave packet amplitude takes place in the direction of the carrier wave propagation. We consider the dynamic solution of the CGL equation (17). Accordingly, we separate the amplitude Ψ into two parts:

$$\Psi = [\Psi_0 + \delta\Psi(\xi,\tau)]\exp(iQ|\Psi_0|^2\tau), \tag{20}$$

where Ψ_0 is the constant-amplitude perturbation, $\delta\Psi(\delta\Psi \ll \Psi_0)$ is the small-amplitude perturbation, and the nonlinear frequency shift is $(-Q|\Psi_0|^2)$. After substituting Eq. (20) into Eq. (17) and linearizing the result with respect to $\delta\Psi(\xi,\tau)$, the evolution equation for the perturbation will be

$$i\frac{\partial}{\partial \tau}\delta\Psi + P\frac{\partial^2}{\partial \xi^2}\delta\Psi + Q|\Psi_0|^2(\delta\Psi + \delta\Psi^*) = 0, \tag{21}$$

where $\delta\Psi^*$ is the complex conjugate of $\delta\Psi$. We introduce

$$\delta\Psi(\xi,\tau) = U\exp[i(K\xi - \Omega\tau)] + V\exp[-i(K\xi - \Omega^*\tau)], \tag{22}$$

where U and V are complex constant amplitudes, with $(K\xi - \Omega\tau)$ as the modulation phase, and where $K(|K| \ll k)$ and $\Omega(\Omega \ll \omega)$ are the wave number and the frequency of the modulated waves, respectively. Using Eq. (22) into Eq. (21) gives a linear homogeneous system of equations for U and V:

$$(\Omega + PK^2 - Q|\Psi_0|^2)U - Q|\Psi_0|^2V = 0 \tag{23}$$

and

$$Q^*|\Psi_0|^2U + (\Omega - P^*K^2 + Q^*|\Psi_0|^2)V = 0. \tag{24}$$

The coupled system of Eqs. (23) and (24) gives the following nonlinear dispersion relation for the CDA wave:

$$\begin{aligned} & \Omega^2 + 2i(K^2P_i - Q_i|\Psi_0|^2)\Omega - K^2\left[K^2\left(P_i^2 + Q_i^2\right)\right. \\ & \quad \left. - 2(P_iQ_i + P_rQ_r)|\Psi_0|^2\right] = 0. \end{aligned} \tag{25}$$

Solving Eq. (25), we obtain

$$\Omega_{\pm} = -i(K^2 P_i - Q_i|\Psi_0|^2) \pm \sqrt{P_r K^2(P_r K^2 - 2Q_r|\Psi_0|^2) - Q_i^2|\Psi_0|^4}. \quad (26)$$

Substituting Eq. (26) into Eq. (22) and after mathematical manipulation we arrive at the modulational instability criterion for the CDA waves as

$$P_i Q_i + P_r Q_r > 0. \quad (27)$$

The inequality (27) fulfils the well-known Lange and Newell's criterion [32]. Consequently, the CDA wave is stable for $P_i Q_i + P_r Q_r < 0$ and it is unstable for $P_i Q_i + P_r Q_r > 0$.

Based upon the above finding, we determine in various regimes the modulational instability of the CDA waves that are based on the inequality (27). The latter is investigated in Figs. 4 and 5. It is obvious that there are two domains for the unstable pulses and one for stable pulses. Increasing α would lead to a decrease of the instability domain and an increase of the stable regime, as illustrated in Figs. 4, for low k. The instability region shifts to higher k with the increase of α. This behavior supports our result that is depicted in Fig. 1(a), where the modulational instability growth rate decreases with the increase of electron-ion collisions. The effect of the plasma β on the modulational instability of the CDA wave packets is investigated in Fig. 5. It is seen that the increase of the plasma β (i.e., the magnetic field decreases or the ion temperature increases) would lead to a shrinkage of the stability region. On the other hand, the instability region becomes wider for large k.

When electron-ion collision is neglected, i.e., $\alpha = P_i = Q_i = 0$, then the CGL equation (17) reduces to the standard NLS equation. Hence, the nonlinear dispersion relation (26) for the amplitude modulation reduces to

$$\Omega^2 = (P_r K^2)^2 \left(1 - \frac{(2Q_r/P_r)|\Psi_0|^2}{K^2}\right). \quad (28)$$

One immediately sees that, if $P_r Q_r > 0$, the amplitude-modulated envelope is unstable for $K < K_c = \sqrt{2Q_r/P_r}|\Psi_0|$, i.e., for perturbation wavelengths larger than a critical value $2\pi/K_c$ (and stable for shorter wavelengths). The maximum instability growth rate occurs at $K_m = K_c/\sqrt{2}$, where the local-instability growth rate is given by

$$\Gamma = \mathrm{Im}(\Omega) = \left[(P_r K^2)^2 \left(\frac{K_c^2}{K^2} - 1\right)\right]^{1/2}. \quad (29)$$

However, if $P_r Q_r < 0$, the CDA will be stable against external perturbations. On the other hand, if $P_r Q_r < 0$, the amplitude-modulated envelope will be stable against external perturbations and the CDA carrier wave will be modulationally "stable" and may propagate in the form of "dark" ("black" or "gray") envelope wavepackets, i.e., a propagating localized "hole" (a "void") amidst a uniform wave energy region. However, for "positive" $P_r Q_r$, the carrier wave is modulationally "unstable"; it may either "collapse," due to (possibly random) external perturbations, or lead to the formation of "bright" envelope-modulated wavepackets, i.e., localized envelope "pulses" confining the carrier wave [33,34].

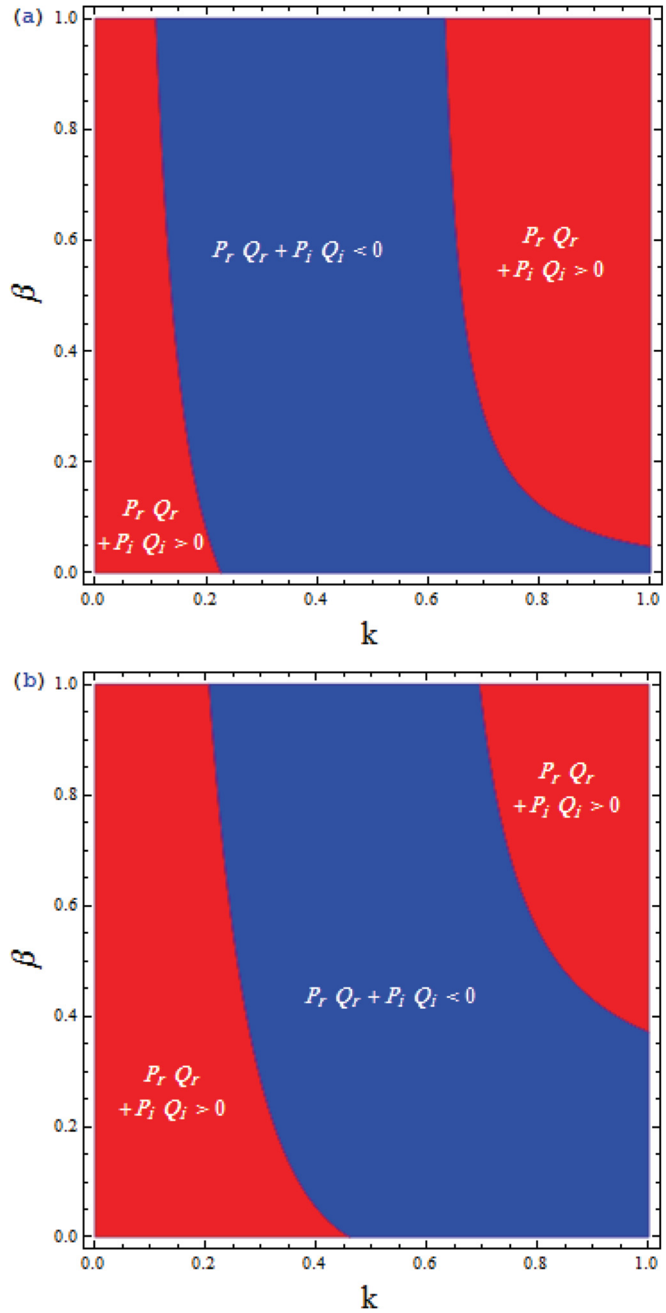

FIG. 4. (Color online) The stability criteria $P_i Q_i + P_r Q_r < 0$ and the instability condition $P_i Q_i + P_r Q_r > 0$ determined in the k-β plane for (a) $\alpha = 0.3$ and (b) $\alpha = 0.6$.

To investigate the role of the plasma β on the CDA waves, we have plotted the variation of the product $P_r Q_r$ against K for different values of β, as depicted in Fig. 6(a). It is found that increasing β would make the stable regime (i.e., $P_r Q_r < 0$) wider, while the unstable region becomes narrower (i.e., $P_r Q_r > 0$). Furthermore, the increase of β decreases the critical wave number at which the instability sets in. This behavior is confirmed in Fig. 6(b), where the modulational instability growth rate Γ decreases with the increase of the plasma β. Furthermore, the increase of K would lead to an increase in the growth rate, for small wave numbers, till Γ reached a critical value, $\Gamma \equiv \Gamma_c$, then the growth rate would

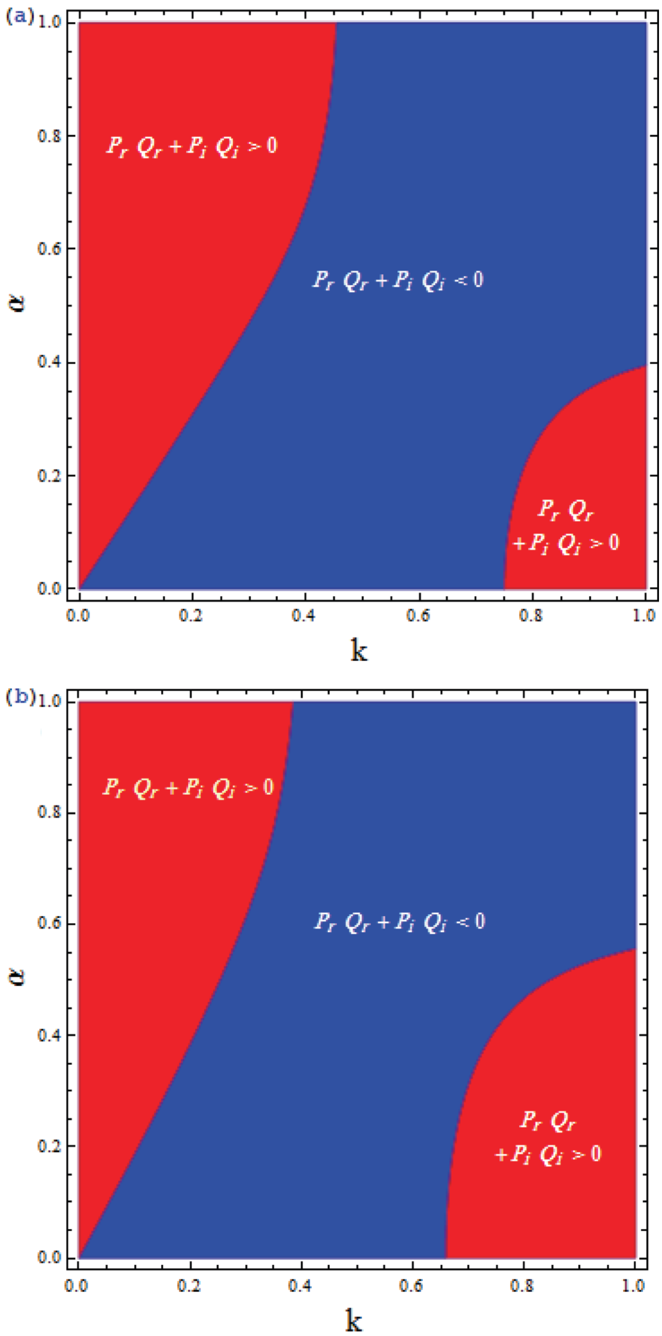

FIG. 5. (Color online) The stability criteria $P_iQ_i + P_rQ_r < 0$ and the instability condition $P_iQ_i + P_rQ_r > 0$ determined in the k-α plane for (a) $\beta = 0.1$ and (b) $\beta = 0.3$.

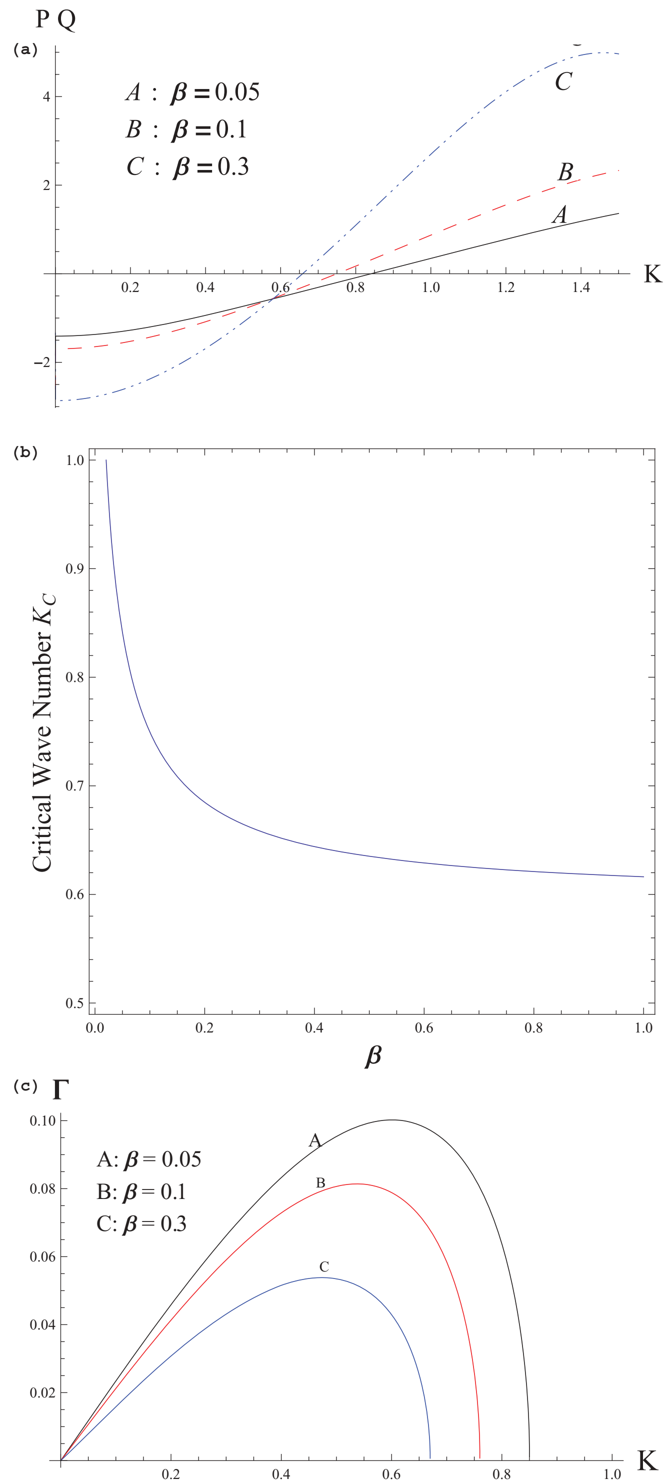

FIG. 6. (Color online) The effect of the plasma β on (a) the product P_rQ_r against K, (b) the critical wave number K_c, and (c) the growth rate Γ against K.

decrease again at higher K. Such a behavior illustrates that the compressional Alfvén waves become much more unstable for small K, but the instability decreases for large wave numbers.

Now, we have determined precisely the regions of a special modulational unstable solution, i.e., for $P_rQ_r > 0$, which is local in both space and time. Within the modulational instability region, a random perturbation of the amplitude grows and thus creates a CDA rogue wave. Equation (17), with real P_r and Q_r, has a rational solution that is located on a nonzero background and localized both in τ and ξ directions [19,35]:

$$\Psi(\xi,\tau) = \sqrt{\frac{P_r}{Q_r}}\left[\frac{4(1+2iP_r\tau)}{1+4P_r^2\tau^2+4\xi^2} - 1\right]\exp(iP_r\tau). \quad (30)$$

The solution (30) predicts that the CDA wave energy is concentrated into a small region due to the nonlinear properties of the

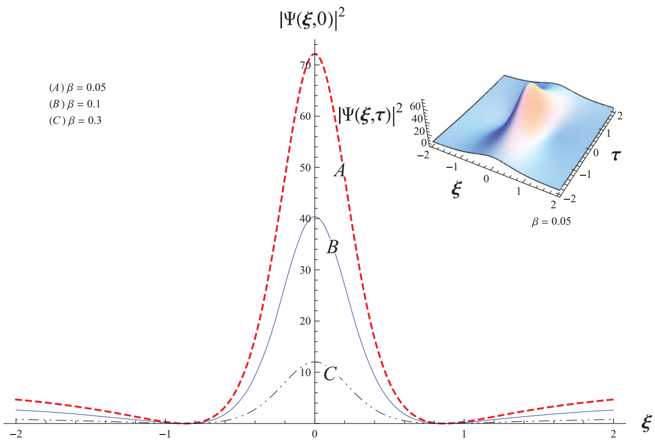

FIG. 7. (Color online) The absolute value of the magnetic field envelope B represented by Eq. (30) for different values of the plasma β.

medium. Therefore, the CDA rogue waves carry a significant amount of the wave energy into a relatively small area in space. It is worthwhile explaining that rogue wave creation is still unsolved question. On the other hand, theories vary depending on the conditions under which these waves appear [36]. One remarkable feature of rogue waves is that they appear visibly from nowhere and disappear without a trace [37]. Nonlinear dynamics is one of the approaches that has been successful in predicting the basic features of rogue waves [38]. So, the spontaneous development of rogue waves is still under investigation by many authors, using studies of optics [39], plasmas [17], water-wave tank experiments [40], etc. However, one of the prototypes suggested to model rogue waves is the so-called Peregrine soliton [41]. This solution describes the growing evolution of a small, localized perturbation of a plane wave with the subsequent peak amplification above the plane wave. The large-amplitude peak appears just once in evolution (being doubly localized rather than periodic in space and time). The present work predicts and defines the possible existence region for propagating Alfvénic rogue waves and the effect of the plasma β on the wave amplitude. Therefore, the present nonlinear approach is fruitful in a description of Alfvénic rogue waves. The profile of the CDA rogue wave and its dependence on the plasma β is depicted in Fig. 7. It is seen that increasing the plasma β causes the CDA rogue wave amplitude to become shorter. We speculate that this behavior could be explained as follows: the stronger magnetic field increases the nonlinearity of the system, and therefore the rogue wave amplitude becomes greater.

IV. SUMMARY

Summing up, we have investigated the modulational instability of the CDA waves that are propagating across a uniform magnetic field in a warm electron-ion plasma. The dynamics of the modulated CDA wave packets is governed by the CGL equation, with real and complex coefficients of the group dispersion and the nonlinearities. The CDA wave group velocity has real and imaginary components. The imaginary part of the group velocity decreases with the increase of α and β, while the real group velocity exhibits an opposite behavior. Furthermore, the region of the modulational instability of the CDA waves becomes narrower with the increase of α and β, which indicates that the system dissipates the wave energy via electron-ion collisions and hence a stable envelope wave packet in the form of a hole will be a dominant feature of an amplitude-modulated CDA pulse. For a collisionless magnetoplasma, i.e., $\alpha = 0$, our CGL equation reduces to the NLS equation with real group dispersion and nonlinear coefficients. A modulational instability analysis of the NLS equation has been carried out. It has been shown that, within the modulational unstable envelope pulse region, a random set of nonlinearly interacting amplitude-modulated CDA perturbations would form CDA rogue waves. The dependence of the rogue wave profile on the plasma β is investigated. Our results reveal that the stronger magnetic fields increase the nonlinearity of the system, and therefore the rogue wave amplitude becomes greater. In conclusion, we stress that the results presented here can have applications in many branches of space and laboratory magnetoplasmas. Specifically, the nonlinear CDA waves could be associated with the localized short-scale (of width the order of several electron skin depths) Alfvénic wave packets that are observed [42,43] in the magnetized plasma of the Earth's magnetosphere and the solar wind. The results should also be helpful in understanding the nonlinear propagation of dispersive Alfvén waves in laboratory collisional magnetoplasmas that have been used for the fundamental study of nonlinear Alfvénic activities in the Earth's ionospheric plasmas.

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APPENDIX: COEFFICIENTS OF EQS. (11)–(16)

The coefficients used in Eqs. (11)–(16) are as follows:

$$c_1^{(22)} = -\frac{k^2\begin{pmatrix}\omega^5 + 6k^6\beta[i\alpha(1+3\beta)+\omega+6\beta\omega] + 2k^2\omega^3(2-3\beta+i\alpha\omega+2\omega^2)\\ +k^4\omega[-2+9\beta^2-6i\alpha(-1+2\beta)\omega+2(7-12\beta)\omega^2]\end{pmatrix}}{2(-3k^2\beta+\omega^2)^2[-\omega^3+6k^4\beta(i\alpha+2\omega)+k^2\omega(1+3\beta-2i\alpha\omega-4\omega^2)]},$$

$$c_2^{(22)} = -\frac{k\omega\begin{pmatrix}\omega^5 + 6k^6\beta[3i\alpha(1+\beta) + (5+6\beta)\omega] + 2k^2\omega^3(1-3\beta+i\alpha\omega+2\omega^2)\\ +k^4\omega[3\beta(2+3\beta) - 2i\alpha(-1+6\beta)\omega + 6(1-4\beta)\omega^2]\end{pmatrix}}{2(-3k^2\beta+\omega^2)^2[-\omega^3 + 6k^4\beta(i\alpha+2\omega) + k^2\omega(1+3\beta-2i\alpha\omega-4\omega^2)]},$$

$$c_3^{(22)} = -\frac{k^2\{\,3\omega^5 + 18k^6\beta^2(i\alpha+3\omega) + 3k^4\beta\omega(4+9\beta-4i\alpha\omega-12\omega^2) + 2k^2\omega^3[-9\beta+\omega(i\alpha+3\omega)]\}}{2(-3k^2\beta+\omega^2)^2[-\omega^3 + 6k^4\beta(i\alpha+2\omega) + k^2\omega(1+3\beta-2i\alpha\omega-4\omega^2)]},$$

$$c_1^{(20)} = \frac{\{2k^3[1+3(1+k^2)\beta]\omega - 2k(1+k^2)\omega^3 - [3k^4\beta(2+3\beta) + 2k^2(1-3\beta)\omega^2 + \omega^4]v_g\}}{(-3k^2\beta+\omega^2)^2 v_g\left(1+3\beta-v_g^2\right)},$$

$$c_2^{(20)} = \frac{\left\{6k^5\beta\omega - 2k(1+k^2)\omega^3 - [3k^4\beta(2+3\beta) + 2k^2(1-3\beta)\omega^2 + \omega^4]v_g + 2k^3\omega v_g^2\right\}}{(-3k^2\beta+\omega^2)^2\left(1+3\beta-v_g^2\right)},$$

and

$$c_3^{(20)} = -\frac{\begin{pmatrix}6k^3\beta[1+3(1+k^2)\beta]\omega - 6(k+k^3)\beta\omega^3\\ +v_g(3k^4\beta(2+3\beta) + 2k^2(1-3\beta)\omega^2 + \omega^4 + 2k\omega\{\omega^2 + k^2[-1-3(1+k^2)\beta+\omega^2]\}v_g)\end{pmatrix}}{(-3k^2\beta+\omega^2)^2 v_g\left(1+3\beta-v_g^2\right)}.$$

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