Epidemic spreading with information-driven vaccination

Zhongyuan Ruan, $¹$ Ming Tang, $²$ and Zonghua Liu^{1,*}</sup></sup>

¹*Department of Physics, East China Normal University, Shanghai 200062, People's Republic of China*

²*Web Science Center, University of Electronic Science and Technology of China, Chengdu 611731, People's Republic of China* (Received 12 April 2012; revised manuscript received 8 September 2012; published 27 September 2012)

Epidemic spreading has been well studied in the past decade, where the main concentration is focused on the influence of network topology but little attention is paid to the individual's crisis awareness. We here study how the crisis awareness, i.e., personal self-protection, influences the epidemic spreading by presenting a susceptible-infected-recovered model with information-driven vaccination. We introduce two parameters to quantitatively characterize the crisis awareness. One is the information creation rate *λ* and the other is the information sensitivity *η*. We find that the epidemic spreading can be significantly suppressed in both the homogeneous and heterogeneous networks when both *λ* and *η* are relatively large. More interesting is that the needed vaccine will be significantly reduced when the information is well spread, which is a good news for the poor countries and regions with limited resources.

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I. INTRODUCTION

We live in an information era filled with all kinds of information, such as the news, advertisements, rumors, etc. These messages play a more and more important role in our decision makings, ranging from purchasing stocks in a financial market to shopping at a supermarket. Thus, most of our activities will be seriously influenced by the information that we can obtain. On the other hand, it has been pointed out that human behaviors may also have influence on some dynamic processes, such as cooperation [\[1–4\]](#page-4-0) and epidemic spreading [\[5–11\]](#page-4-0) on complex networks. For example, when an epidemic breaks out, the authorities will most probably close the schools or some other public places to inhibit disease prevalence, and the individuals will take variant measures to avoid being infected, such as staying at home, wearing face masks, taking vaccinations, etc. [\[5\]](#page-4-0). A recent report shows that the self-initiated behavioral changes will also influence the mobility patterns of individuals $[12]$.

An effective approach to suppress epidemic spreading is the vaccination. This topic has attracted great interest and is mainly focused on the targeted immunization [\[13–15\]](#page-4-0), the acquaintance immunization $[16,17]$, etc. These strategies can theoretically prevent the prevalence of an epidemic and thus bring us a huge hope of controlling an epidemic. However, they are difficult to apply to our real life directly as they are "compulsory," i.e., they overlook the voluntariness of the individuals. In fact, the factors that affect a person's decision to vaccinate are very complicated, including self-interest, religious beliefs, and altruism $[18]$. A feasible way to deal with this situation is to employ game theory. Through introducing payoffs for each individual, people can weigh up the pros and cons of vaccinating [\[19,20\]](#page-4-0). These investigations have made big progress in studying epidemic spreading in real situations, but there are still many unrealistic aspects. For instance, they all assume that an individual will contact (get information) all of his neighbors simultaneously $[21-23]$ $[21-23]$. Obviously, this is not true. The truth is that we can contact only some of our

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neighbors at a time and the contacted neighbors change with time.

It is well known that individuals have crisis awareness, e.g., if one realizes that there are infected people around him, he will spontaneously take some measures to protect himself so that the risk will be reduced at most. We think that except for the above mentioned vaccinating strategies, the crisis awareness also takes a key role in epidemic spreading. Thus, in this paper, we study how the crisis awareness influences epidemic spreading. We argue that the crisis awareness can be represented by the information-driven vaccination. We introduce two parameters to quantitatively characterize the information-driven vaccination. One is the information creation rate *λ*, which represents the information amount obtained by the individuals. The other is the information sensitivity *η*. The higher the sensitivity is, the larger possibility for the individuals to take vaccination. Our numerical simulations show that the epidemic spreading can be significantly suppressed in both the homogeneous and heterogeneous networks when both *λ* and *η* are relatively large.

The paper is organized as follows. In Sec. II., we present the susceptible-infected-recovered (SIR) model with information-driven vaccination. In Sec. [III,](#page-1-0) we present the numerical simulations on a homogenous network. In Sec. [IV,](#page-3-0) we present the numerical simulations on a heterogenous network. Finally, some discussions and conclusions are given in Sec. [V.](#page-3-0)

II. A SIR MODEL WITH INFORMATION-DRIVEN VACCINATION

Based on the observation that the crisis awareness can suppress the epidemic spreading and the information around us can influence the crisis awareness, we conclude that the two processes of epidemic spreading and information diffusion are closely related. Thus, we here link the epidemiological SIR model [\[24\]](#page-5-0) with the information transmission model together to form a new model, called a *SIR model with informationdriven vaccination*. Generally speaking, the two dynamics processes perform on different networks, i.e., the epidemic spreading is based on a contacting network of the individuals while the information diffusion is based on a medium network,

^{*}zhliu@phy.ecnu.edu.cn

such as the Internet. The two related networks thus form an overlay network $[25,26]$. In this paper, for simplicity, we assume that the two networks have the identical topology. Therefore, the two dynamical processes can be considered as proceeding on one network. In this framework, each node in the SIR model with information-driven vaccination has two functions, transferring both epidemic and information.

The standard SIR model consists of three elements, the susceptible *S*, infected *I* , and recovered *R*. Each susceptible node can be infected with a probability $β$ at each time step if it is connected to one infected node. At the same time, the infected nodes may be self-recovery or become refractory with a probability μ . When the information-driven vaccination is considered, a fourth element, vaccination V , should be introduced where a susceptible will have a probability $\kappa(t)$ to become vaccinated. The *V* status is the same as the *R* status in the sense that both will not be infected again. But they are different in the aspect that the *R* status refers to the individuals who recover without vaccination while the *V* status refers to the individuals who take vaccinations. Thus, the SIR model with information-driven vaccination can be represented as follows:

$$
S + I \xrightarrow{\beta} 2I,
$$

\n
$$
I \xrightarrow{\mu} R,
$$

\n
$$
S \xrightarrow{\kappa(t)} V.
$$

\n(1)

Information transmission can be described by packets delivery. There is an information transmission between two neighboring nodes A and B if there is at least one packet passing from A to B or the inverse. The continuous delivery of packets will produce an information flow. We assume that at each time step, there are λN new packets to be created in the network with randomly chosen origins and destinations, where *λ* represents the information creation rate. The packets are considered to be noninteracting; thus there are no queues in the network. The routing of the packets follows the shortest-path algorithm [\[27–29\]](#page-5-0). That is, each packet will be forwarded one step along its shortest path at each time step. When a packet arrives its destination, it will be removed from the system. According to this algorithm, there will be a large number of packets in the network when λ is large and only a small number of packets when *λ* is small. For the case of small *λ*, only part of all the nodes of the network will be occupied by the packets at each time step and the occupied part will increase with *λ*.

At each time step, a susceptible individual will make a decision on its vaccination by collecting information from its neighbors. The collected information depends on two factors. Suppose that at time *t*, a node will receive information packets from its several neighbors and $m_I(t)$ of them are infected nodes. $m_I(t)$ will increase with λ and is the first factor. The second factor is how the individuals are sensitive to the collected information. Let η be the sensitivity to information. We suppose that the possibility for a susceptible individual to take a vaccination will be proportional to both $m_I(t)$ and η and can be expressed as

$$
\kappa(t) = 1 - e^{-\eta \frac{m_I(t)}{k}},
$$
 (2)

where k is the number of neighbors. When a node receives packets from all its neighbors and $\eta = 1$, the possibility to vaccinate will be the infected fraction of neighbors [\[30\]](#page-5-0). Specifically, the model (1) will return to the standard SIR model when $\eta = 0$. When $\eta \to \infty$, a susceptible will be extremely sensitive to the collected information. Once an infected neighbor is found, the susceptible will choose to take a vaccination immediately. Notice that for a fixed *η*, Eq. (2) will gradually approach a saturation as m_I increases. The reason to choose this form is based on the observation that the decision for a susceptible to take a vaccination is largely stimulated by the infected neighbors found at first. The later discovered infected neighbors will not help much to increase the probability $\kappa(t)$ as the susceptible may probably have been vaccinated by that time.

III. CASE OF HOMOGENOUS NETWORK

In this section, we study how the information-driven vaccination influences epidemic spreading on a homogenous network. Without loss of generality, we consider an Erdos-Renyi (ER) network with size $N = 2000$ and average degree $\langle k \rangle = 6$, which can be constructed by the algorithms given in Refs. [\[31,32\]](#page-5-0). We fix the parameters $\beta = 0.06$ and $\mu = 0.1$ in this paper. Initially, we randomly choose 1% of the total nodes to be infected. To measure the effect of epidemic spreading, we let ρ_X ($X = S, I, R, V$) represent the fraction of the *X* component in the total nodes. All the results will be obtained by taking the average on 20 different realizations.

We first study the influence of the information creation rate *λ* on epidemic spreading by fixing the information sensitivity $\eta = 0.2$. We find that for a fixed evolution time, both the infected density ρ_I and the refractory density ρ_R will decrease with the increase of λ . Figure 1 shows the evolution of $\rho_I(t)$ and $\rho_R(t)$ for three typical $\lambda = 0.2, 0.5$, and 1, respectively. From Fig. $1(a)$ we see that the peak of $\rho_I(t)$ decreases with the increase of λ . From Fig. 1(b) we see that when the epidemic spreading is ended, the final refractory density ρ_R will seriously depend on λ . Thus, the information-driven vaccination is a good way to control the epidemic spreading,

FIG. 1. (Color online) Evolution of $\rho_I(t)$ and $\rho_R(t)$ for $\eta = 0.2$, where the "solid-black," "dashed-red," and "dotted-green" lines represent the cases of $\lambda = 0.2, 0.5$, and 1, respectively.

FIG. 2. (Color online) Evolution of $\rho_I(t)$ and $\rho_R(t)$ for $\lambda = 0.2$, where the solid-black, dashed-red, and dotted-green lines represent the cases of $\eta = 0.2, 0.8,$ and 2, respectively.

i.e., the more information, the better the effect. This result can be qualitatively explained as follows: For larger *λ*, there will be more information packets in the network and thus results in more nodes covered by the packets. In this case, it is very easy for a node to find its infected neighbors and then take a vaccination. When more and more nodes are vaccinated, the epidemic spreading will be naturally suppressed.

Then, we study the influence of the information sensitivity *η* on epidemic spreading by fixing the information creation rate $\lambda = 0.2$. Figure 2 shows the evolution of $\rho_I(t)$ and $\rho_R(t)$ for three typical $\eta = 0.2, 0.8,$ and 2, respectively. It is easy to see that Fig. 2 is similar to Fig. [1,](#page-1-0) indicating that both $\rho_I(t)$ and $\rho_R(t)$ will also decrease with the increase of *η*. We can similarly explain this result as follows: For a larger *η*, there is more possibility for a node to take a vaccination once it finds some infected neighbors, and thus reduces the epidemic spreading. In sum, from the case of both small *λ* and small *η* we see that refusing vaccinations for religious, cultural, or other reasons will result in a higher peak in the ρ_I curve and a larger asymptotic value of ρ_R , indicating that refusing vaccinations will result in more people being influenced by the diseases.

To illustrate the influence of *λ* and *η* on epidemic spreading in detail, we calculate the final refractory density $\rho_R(\infty)$ when $\rho_I(\infty) = 0$. Figure 3 shows how $\rho_R(\infty)$ changes with λ and *η*. We see that $\rho_R(\infty)$ has larger values when either λ or *η* is small and then gradually decreases to a small value when both *λ* and *η* are relatively large. More interesting is that $ρ_R(∞)$ will not change for a further increase of *λ* and *η*, indicating an effect of saturation. This phenomenon can be understood as follows. When there are too many packets in the network, there will be two or more packets passing through one link at each time step, causing the information redundancy and thus resulting in a saturated $\rho_R(\infty)$.

It is generally believed that the more people are vaccinated, the less will be the final refractory density. Thus, a key question is how to make the final vaccinated density $\rho_V(\infty)$ maximum, based on the voluntariness of the individuals. Intuitively, it is believed that both the larger information creation rate *λ* and the larger sensitivity η are beneficial for the increasing of $\rho_V(\infty)$. To confirm it, Fig. 4(a) shows how the values of

FIG. 3. (Color online) The final refractory density $\rho_R(\infty)$ versus the parameters *λ* and *η*.

λ influence the evolution of $ρ_V(t)$ for fixed $η = 3$, where the solid-black, dashed-red, and dotted-green lines represent the cases of $\lambda = 0.2, 0.5$, and 1, respectively. It is noticed that in the initial process of $t < 10$, the increasing rate of $\rho_V(t)$ for $\lambda =$ 0.2 is smaller than that of $\lambda = 0.5$ and consecutively smaller than that of $\lambda = 1$, indicating the proportional relationship between $\rho_V(t)$ and λ . However, the final steady $\rho_V(t)$ for $t \geq$ 100 does not keep this proportional relationship but instead by changing the positions between the largest one and the smallest one. We have observed the similar results for varying *η*; see Fig. $4(b)$ for fixed $\lambda = 2$ where the solid-black, dashed-red, and dotted-green lines represent the cases of $\lambda = 0.2, 0.8$, and 2, respectively.

To understand the strange behavior observed in Fig. 4, Fig. [5](#page-3-0) shows how the final $\rho_V(\infty)$ depends on the parameters λ and

FIG. 4. (Color online) (a) Evolution of $\rho_V(t)$ for fixed $\eta = 3$ where the solid-black, dashed-red, and dotted-green lines represent the cases of $\lambda = 0.2, 0.5$, and 1, respectively. (b) Evolution of $\rho_V(t)$ for fixed $\lambda = 2$ where the solid-black, dashed-red, and dotted-green lines represent the cases of $\eta = 0.2, 0.8$, and 2, respectively.

FIG. 5. (Color online) The final vaccinated density $\rho_V(\infty)$ versus the parameters *λ* and *η*.

η. It is easy to see that in the parameter plane of λ and *η*, there is an optimal area of $\rho_V(\infty)$. $\rho_V(\infty)$ will become small when *λ* and *η* are out of the optimal region. When both *λ* and *η* are relatively small, it is the normal case where $\rho_V(\infty)$ increases with both *λ* and *η*. While in the case of relatively larger *λ* and *η*, $\rho_V(\infty)$ decreases with the increase of λ and *η*, which is counterintuitive. The reason is that $\rho_V(t)$ is decided not only by λ and *η* but also depends on the infected density $ρ_I(t)$. When λ and η are large, $\rho_V(t)$ will increase in the beginning of evolution much faster than in the case of small *λ* and *η*, thus suppressing the spreading of $\rho_I(t)$ and resulting in less final $\rho_V(\infty)$. Combining with Fig. [3,](#page-2-0) we can easily find that both $\rho_R(\infty)$ and $\rho_V(\infty)$ are small when λ and η are large. This finding tells us that when the information is fully spread out and the people are willing to accept it, the information

FIG. 6. (Color online) The final refractory density $\rho_R(\infty)$ versus the parameters *λ* and *η*.

FIG. 7. (Color online) Evolution of $\rho_I(t)$ and $\rho_R(t)$ for $\lambda = 0.5$ and $\eta = 0.8$, where the solid-black and dashed-red lines represent the cases of a ER network and a BA network, respectively.

can not only control the epidemic spreading but also reduce the required vaccine. This is especially meaningful to the poor countries and regions where the supplied vaccine is limited.

IV. CASE OF HETEROGENOUS NETWORK

Many realistic networks exhibit scale-free properties, such as the Internet and World-Wide Web (WWW), as well as the food webs and collaboration networks [\[31\]](#page-5-0). To study the effect of information-driven vaccination on a scale-free network, we perform simulations on an Albert-Barabasi (BA) network [\[31\]](#page-5-0) with size $N = 2000$ and average degree $\langle k \rangle = 6$. We still keep $\beta = 0.06$ and $\mu = 0.1$. Figure 6 shows the result. We can easily see that it is similar to Fig. [3](#page-2-0) of the ER network but with some differences. For heterogenous networks, there are high degree nodes which make information packets delivery more efficient, i.e., the packets can go through less steps to their destination. This causes the total number of packets in a BA network to be less than in a ER network if we keep the packets creation rate to be the same. Thus, the effect of information-driven vaccination is not so good for a BA network. Figure 7 shows the evolutions of ρ_I and ρ_R in the BA network and ER network for $\lambda = 0.5$ and $\eta = 0.8$, respectively. The peak in Fig. 7(a) is much higher in the BA network than in the ER network, implying that the effect of information-driven vaccination is worse in a BA network than in a ER network. Figure $7(b)$ also confirms this point where the stabilized ρ_R is larger in the BA network than in the ER network.

V. DISCUSSIONS AND CONCLUSIONS

The main feature of this work is that it is based on the voluntariness of individuals, in contrast to the previous compulsory immunization. In the modern society with fast information spreading, the human rights become more and more important. Thus, the approach of information-driven vaccination becomes more realistic. In this situation, it is necessary to study the effect of crisis awareness, even if its effect may be not so good as the previous compulsory immunizations. Of course, it will be better if we can compare

FIG. 8. (Color online) $\rho_R(\infty)$ versus $\rho_V(\infty)$ for different immunization strategies where the parameters are $N = 2000$, $\langle k \rangle =$ 6, $\beta = 0.06$, and $\mu = 0.1$, and the symbols circles, up-triangles, down-triangles, and diamonds represent the cases of random, degreebased, betweenness-based, and information-driven immunizations, respectively.

the approach of information-driven vaccination with other immunization strategies [\[33–36\]](#page-5-0).

To solve this problem, we first consider the case of random immunization. Different from the varying $\rho_V(t)$ in Sec. [III,](#page-1-0) ρ_V will be fixed as a constant in the case of random immunization and we thus have $\rho_V = \rho_V(\infty)$. For the convenience of comparison, we choose the same ER network and the same parameters, i.e., $N = 2000$, $\langle k \rangle = 6$, $\beta = 0.06$, and $\mu = 0.1$, as in Sec. [III](#page-1-0) except the different ρ_V . In numerical simulations, we randomly choose $\rho_V(\infty)N$ nodes to be immunized at the beginning. We find that the final $\rho_R(\infty)$ decreases with the increase of $\rho_V(\infty)$. The curve with "circles" in Fig. 8 shows the result. Then we consider the case of degree-based immunization by choosing the $\rho_V(\infty)N$ nodes with the largest degrees to be immunized [\[33\]](#page-5-0). The curve with "up-triangles" in Fig. 8 shows the result. Similarly, the curve with "down-triangles" in Fig. 8 shows the result

of betweenness-based immunization [\[33\]](#page-5-0). It is easy to see that both the curves with up-triangles and down-triangles are much lower than that of circles, confirming that the degree-based and betweenness-based immunizations are much better than the random immunization.

For the case of information-driven vaccination, we fix $\eta = 3$ and change the value of $\rho_V(\infty)$ by increasing λ . We find that when λ is small, $\rho_V(\infty)$ increases with λ . However, when *λ* is further increased, $ρ_V(∞)$ will decrease with *λ*. This phenomenon has been explained in Sec.[III.](#page-1-0) It is interesting that $\rho_R(\infty)$ does not show this transition but keep monotonously decreasing with the further increasing of *λ*. The curve with "diamonds" in Fig. 8 shows the result. This result shows that the information-driven vaccination is fundamentally different from the above three compulsory immunizations. When both *λ* and *η* are large enough, a small $ρ_V(∞)$ can suppress the value of $\rho_R(\infty)$, indicating that the needed vaccine to control the epidemic will be significantly reduced.

In conclusion, based on the observation that information also influences the vaccination, we present a SIR model with information-driven vaccination to study epidemic spreading. This model combines the vaccination into the standard SIR model and thus makes it become a four-status SIRV model. We find that there are two factors which influence the epidemic spreading, i.e., the information creation rate *λ* and the sensitivity η , and the epidemic spreading can be suppressed by them. More interestingly, we reveal that when both *λ* and *η* are large, the final refractory density will be reduced, which provides an efficient approach to control epidemic spreading. These results may provide a deep insight for health authorities to cope with disease spreading in the future.

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