Propagation speed of a starting wave in a queue of pedestrians

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(Received 14 May 2012; published 19 September 2012)

The propagation speed of a *starting wave*, which is a wave of people's successive reactions in the relaxation process of a queue, has an essential role for pedestrians and vehicles to achieve smooth movement. For example, a queue of vehicles with appropriate headway (or density) alleviates traffic jams since the delay of reaction to start is minimized. In this paper, we have investigated the fundamental relation between the propagation speed of a starting wave and the initial density by both our mathematical model built on the stochastic cellular automata and experimental measurements. Analysis of our mathematical model implies that the relation is characterized by the power law $\alpha\rho^{-\beta}$ ($\beta \neq 1$), and the experimental results verify this feature. Moreover, when the starting wave is characterized by the power law ($\beta > 1$), we have revealed the existence of optimal density, where the required time, i.e., the sum of the waiting time until the starting wave reaches the last pedestrian in a queue and his/her travel time to pass the head position of the initial queue, is minimized. This optimal density inevitably plays a significant role in achieving a smooth movement of crowds and vehicles in a queue.

DOI: 10.1103/PhysRevE.86.036113 PACS number(s): 89.40.−a, 89.20.−a, 45.70.Vn, 05.90.+m

I. INTRODUCTION

Various kinds of self-driven many-particle (SDP) systems, such as the evacuation dynamics of pedestrians and vehicular traffic, have attracted a great deal of attention in a wide range of fields during the last few decades [1–3]. Most of these complex systems are interesting not only from the point of view of natural sciences for fundamental understanding of how nature works but also from the points of view of applied sciences and engineering for the potential practical use of the results of the investigations. Especially, interdisciplinary investigations for the dynamics of jamming phenomena in SDP systems, so-called *Jamology*, have progressed by developing sophisticated mathematical models considered as a system of interacting particles driven far from equilibrium with a central focus on the jamming phenomena in traffic flow [4–25].

Some of these contributions to analyze the mechanism of jamming formation indicate that one of the most important factors to cause the jamming phenomena is a *sensitivity* which indicates the time delay of the reaction of the particles to the stimulus. As an example, if the reactions of drivers are extremely sensitive, the drivers can avoid the traffic jam by adjusting their behavior immediately to the movement of their lead car [9,11]. The reaction time of pedestrians is similarly important for smooth movement of crowds. For example, it is a serious issue for organizers to achieve smooth movement of a teeming number of athletes at the start of a marathon since the athletes located at the rearward position have an unavoidable delay in the queue, and this delay raises the possibility of a traffic disturbance in the surrounding area. We would like to point out that the wave of successive reactions in a relaxation process in a queue, the so-called *starting wave*, plays a significant role for the waiting time in a queuing system of pedestrians and vehicles since a quick start in walking and driving accomplishes the smoothest movement of crowds and vehicles in a queue. In order to resolve a queue of pedestrians and vehicles, the departure rate from the cluster should become larger than the arrival rate in principle. From this point of view, it is important to investigate the propagation speed of the starting wave since the fast propagation speed of the starting wave eventually actualizes a high departure rate.

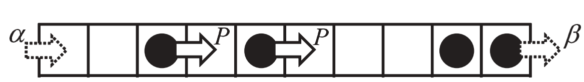
In this paper we have investigated the propagation speed of a starting wave of pedestrians and have found that the fundamental relation between the velocity of the starting wave and the initial density of people standing in the queue is characterized by the power law $\alpha\rho^{-\beta}$ ($\beta \neq 1$), using numerical simulations based on our mathematical model and experimental measurements. Moreover, we have also revealed the existence of the optimal density, where the required time of the last pedestrians in a queue is minimized. The required time is the sum of the waiting time until the starting wave reaches the last pedestrian in a queue and his/her travel time to pass the head position of the initial queue.

This paper is organized as follows. In Sec. II, we propose the mathematical model for pedestrians walking built on the stochastic cellular automaton in analogy with the mathematical models for vehicular traffic. The fundamental relation between the velocity of the starting wave and the initial density is investigated by both numerical simulations and analytical calculations for our model in Sec. III. Then, the existence of optimal density, which minimizes the required time in a queue, is shown in Sec. IV. In Sec. V, the results obtained from our mathematical model are verified by experimental measurements of real pedestrians. Finally, Sec. VI is devoted to the concluding discussions.

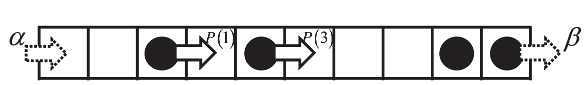
II. MATHEMATICAL MODEL FOR PEDESTRIANS WALKING IN A QUEUE

In this section, we explain in detail our mathematical model built on the stochastic cellular automaton models, such as Asymmetric Simple Exclusion Process (ASEP) [26–28] and Zero Range Process (ZRP) [29–31], which can capture the fundamental features of jamming phenomena in various collective dynamical systems. Here, let us briefly introduce a discrete-time version of the stochastic cellular automaton models; that is, the entry rate α, hopping rate p, and exit rates β are replaced by probabilities at each time step. ASEP is considered as a model of interacting random walks which consists of a large number of walkers on a one-dimensional lattice. These walkers move with probability p to the right and with probability q to the left. Especially, the motion of the walkers is allowed only in one direction in the case of $q = 0$, so-called Totally Asymmetric Simple Exclusion Process (TASEP), which is the paradigmatic model of all transport processes [see Fig. 1(a)]. ZRP is also considered as a mass transport model after appropriate exact mapping to an asymmetric exclusion process; however, the hopping probability depends on the distance to the next particle in front [see Fig. 1(b)]. If the hopping probability in ZRP does not depend on the distance to the next particle in front, ZRP reduces to TASEP. Particularly, these cellular automaton models are more suitable for describing the queuing system for pedestrians than the mathematical model based on the traditional queuing theory [32], such as a queuing process M/M/1. The first and second "M" represents Poisson arrivals and exponential service times, respectively, and the last "1" indicates that the number of service windows. The M/M/1 does not include the effect of spatial structure; however, cellular automaton models can consider both spatial structure and the excluded-volume effect explicitly. In fact, an extension of the M/M/1 queuing process with a spatial structure and excluded-volume effect was recently introduced [33–35], and its dynamical features are analyzed [36] as the ASEP on a semi-infinite chain with an open boundary.

Now, we explain our model for pedestrians walking in a queue. Let us imagine that the passage is partitioned into L identical cells and that each cell can accommodate at most one particle (pedestrian) at a time, enforcing the excluded-volume effect. Note that, in the following, we refer to "particle" as a representation of a pedestrian in our model and "pedestrian" as a person. The length of each cell corresponds to 0.5 m by considering the reasonable volume-exclusion effect of



(a) TASEP with open boundary condition.



(b) ZRP with open boundary condition.

FIG. 1. The schematic view of two mass transport models with discrete time. Particles are inserted with probability α at the left boundary if the left cell is empty. Particles are removed from the right cell with probability β.

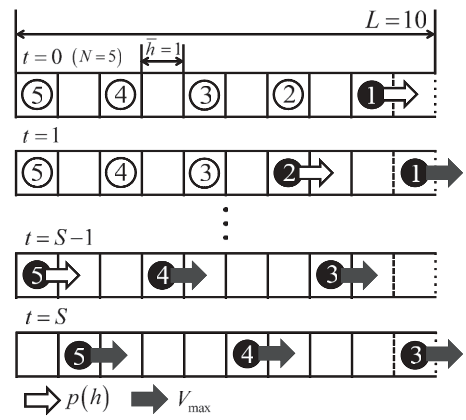


FIG. 2. Schematic view of the time development of our mathematical model for the parameters $(N,L,\bar{h}) = (5,10,1)$ in the case of $V_{\max} = 2$. Each number indicates the particle label. Only black particles can move forward with $p(h)$ (white arrow) or $V_{\max}$ (gray arrow). A white arrow indicates a particle that moves forward for the first time with probability $p(h)$ in this dynamics, whereas a gray arrow indicates a particle that moves forward with maximum velocity $V_{\max}$ after the particle has moved once. The dotted line at the right boundary corresponds to the open boundary condition; that is, particles leave from the system via the right boundary. The dashed line next to the dotted line indicates the starting line; that is, the propagation speed of the starting wave is calculated by the length from this starting line to the left boundary.

pedestrians, as noted in Sec. III. N $(2 \leqslant N \leqslant L)$ indicates the total number of particles whose initial interval distances are equally set as $\bar{h}$ cell. We mention the case $N = 1$ briefly here. Our model can naturally consider the case $N = 1$; however, $N = 1$ is omitted in our model since we focus on the propagation dynamics of the starting wave; that is, the requisite number of particles is more than 1. The parameters $(N,L,\bar{h})$ satisfy the equally spaced condition as follows:

$$\bar{h} = \frac{L}{N} - 1. \tag{1}$$

We impose the semiopen boundary condition, where particles walk away from the right boundary. The update rules of our cellular automaton model are applied in *parallel* to all particles as follows (also see Fig. 2): First of all $(t = 0)$, only the particle at the head of a queue (particle 1) moves forward with hopping probability $p(h)$, which depends on its current headway distance h, defined by the vacant cells in front of it. The function form of $p(h)$ is defined by the experimental data as we describe later. In the case of particle 1 we assume the hopping probability as $p(\infty)$ since there is no predecessor. Note that, in order to investigate the propagation speed of successive reactions in this study, none of the following particles can move forward before the starting wave reaches them. Then $(t = 1)$ the following particle (the second particle in the queue, i.e., particle 2) can move forward with probability $p(h)$. In the case of Fig. 2, this probability $p(h)$ at time $t = 1$ corresponds to $p(2)$. At this time the first particle (particle 1) also can move forward deterministically with maximum velocity $V_{\max}$ since the usual maximum velocity in the solvable stochastic cellular automaton models, which corresponds to one cell per one time step, is not fast enough to treat the situation of fast walking or running. Especially, it is natural to introduce $V_{\max}$ since athletes seldom stop under our assumption of the start of a marathon. Accordingly, a particle can move forward with probability $p(h)$ for the first time after the starting wave has reached it, and then the particle can move forward deterministically with maximum velocity $V_{\max}$. After the second particle moves forward, the next particle can also start to move with probability $p(h)$ in sequence. Finally, the last (particle 5 in the case of Fig. 2) moves forward at the time $t = S - 1$. As just described, the following particles can start to move forward in our model only if the next cell is empty and the predecessor has already started, unlike the usual stochastic cellular automaton models, such as ASEP and ZRP.

The hopping probability $p(h)$, which indicates the velocity of particles as noted before, is given in analogy to the idea of the optimal velocity (OV) function, which is often introduced into mathematical models for vehicular traffic [6,11,37] and the desired velocity which is considered in pedestrian dynamics [15]. This kind of velocity of drivers and pedestrians depends on their headway distance (or density), namely, the vacant space in front of them. This function is motivated by the common expectation that drivers and pedestrians have the desired velocity and will adjust their behavior accordingly. That is, the velocity must be reduced and become small enough to prevent crashing into the preceding particle when the headway is short. Whereas when the headway is long, particles can move at their velocity in free flow, which corresponds to the legal velocity for vehicular traffic.

In order to provide the headway-dependent hopping probability, let us start the density-dependent OV function for pedestrians walking as a linear-type function for simplicity:

$$V(\rho) = V_0\left(1 - \frac{\rho}{\rho_m}\right), \tag{2}$$

which is often used in the traffic flow model built on fluid dynamics [5,37]. The constant values V_0 and ρ_m indicate the velocity of particles in free flow and the density at a complete standstill, respectively. Relation (2) gives us the headway-dependent OV function by translating the headway distance into the reciprocal of the density as $h \sim 1/\rho$. In order to translate the OV function obtained from the experimental data to the stochastic cellular automaton model, the OV function is shifted so that the function passes through the origin in consideration of the excluded-volume effect $\Delta h = 1/\rho_m$; that is, the headway h is assigned to $h + \Delta h$ in the OV function to satisfy the condition $V(h)_{|h=0} = 0$. Moreover, measurement units of length (meters) and time (seconds) are converted to model units of length (cells) and time (steps) by 1 m $= \kappa$ cells and 1 s $= \lambda$ steps. Under these transformations, we obtain the OV function for the cellular automaton model as

$$V(h) = \frac{\kappa}{\lambda} V_0\left(1 - \frac{1}{\rho_m(h + \Delta h)}\right). \tag{3}$$

In our cellular automaton model, the velocity is considered as the hopping probability $p(h)$. We assume that the free hopping probability satisfies $p(h) = 1$ if $h \geqslant \mu$, where μ is a given parameter, so that the headway is large enough to move smoothly. After the velocity is normalized so that $p(\mu) = 1$,

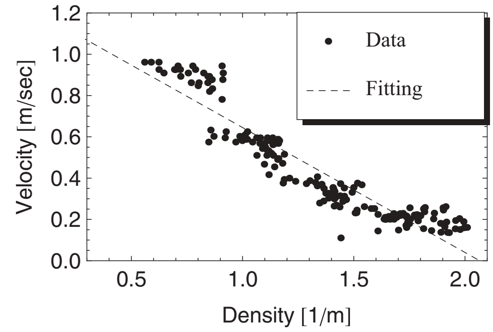


FIG. 3. The linear-type relation between walking velocity V and density ρ from the experimental data [38].

we have obtained the hopping probability as

$$p(h) = \begin{cases} \frac{\mu+\Delta h}{h+\Delta h}\,\frac{\rho_m(h+\Delta h)-1}{\rho_m(\mu+\Delta h)-1} & (h \leqslant \mu), \\ 1 & (h > \mu). \end{cases} \tag{4}$$

This hopping probability satisfies $p(0) = 0$ since $\rho_m \Delta h = 1$.

III. NUMERICAL RESULTS FOR PROPAGATION SPEED

In our numerical simulations, we have estimated the propagation speed of the starting wave under several densities, which are decided by the initial number of particles in a queue. As shown in Fig. 3, let us approximate the experimental data of pedestrians walking on a circular passage way [38] as the form (2). From this fitting, V_0 and ρ_m are 1.24994 and 2.06615, respectively. Moreover, we have obtained the conversion 1 step ~ 0.4 s and 1 cell ~ 0.5 m; that is, 1 s $\sim \lambda = 2.5$ steps and 1 m $\sim \kappa = 2$ cells, which are calculated by the value of $V_0 \sim 1.25$ m/s and $\rho_m \sim 2.0$. We have found from (4) that these values λ, κ, and V_0 do not need to obtain the form $p(h)$. However, these values are important for converting the values obtained from the model to the actual values. Here we assume $\mu = 5$ as the headway large enough to move smoothly. Then we obtain the hopping probability $p(h)$ for our mathematical model, which is given by

$$p(h) = \begin{cases} \frac{0.596798h}{0.483992+0.5h} & (h \leqslant 5), \\ 1 & (h > 5), \end{cases} \tag{5}$$

as shown in Fig. 4. Taking into account the conversions 1 step $= 0.4$ s and 1 cell $= 0.5$ m, the propagation speed a is

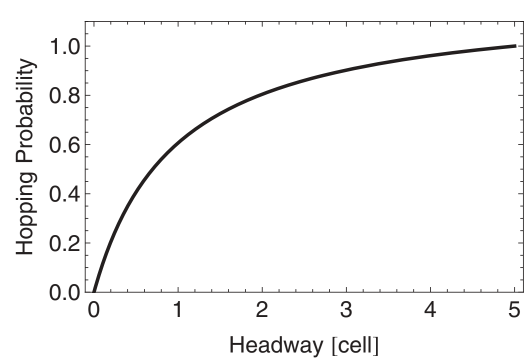


FIG. 4. Hopping probability function $p(h)$ used in our mathematical model.

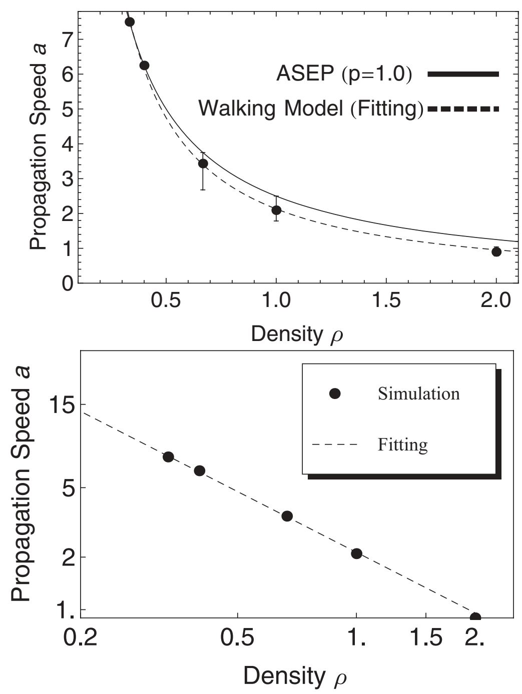


FIG. 5. The simulation results (dots) and fitting relation (dashed curve or dashed line) between the propagation speed of the starting wave and the initial density of pedestrians from our mathematical model. (top) The normal plot and (bottom) the double logarithmic plot. The case of ASEP ($p = 1.0$) is also plotted in the top panel for reference.

calculated by

$$a = \frac{0.5(L-1)}{0.4S}, \tag{6}$$

where S is the required steps for the last particle to start walking (see Fig. 2), which is obtained from the numerical simulations.

As a significant result, we have found the power law in the relation between the propagation speed of the starting wave and the initial density of pedestrians, as shown in Fig. 5. Each plot corresponds to the average velocity after 100 iterations of numerical simulations. Taking into account this power law, we have assumed the following simple relation between propagation speed and the initial density of pedestrians, in analogy with the sonic speed of gas:

$$a(\rho) = \alpha\rho^{-\beta}, \tag{7}$$

where ρ and a are the initial density and the propagation speed of the starting wave, respectively. In our mathematical model, this density ρ is defined by $\rho = N/L$. α and β indicate positive parameters. By fitting these simulation results for $V_{\max} = 6$, we have obtained the parameter values

$$(\alpha,\beta) = (2.13, 1.16). \tag{8}$$

Note that the simulation result obtained from our mathematical model in the low density region, where $\bar{h} = 4[p(h) = 1.0]$, corresponds to the case of ASEP with $p = 1.0$, as shown in Fig. 5. In the case of ASEP, we know $\beta = 1$, which corresponds to the solid curve in the top plot of Fig. 5. This indicates that the propagation speed is linearly proportional to the headway; that is, it is trivial that the required time minimizes in the lowest headway. However, the interesting fact here is that the propagation speed obtained from the mathematical model shows the nonlinearity, i.e., $\beta \neq 1$. Moreover, the value set of (α,β) is nearly independent of $V_{\max}$, as shown in Fig. 6.

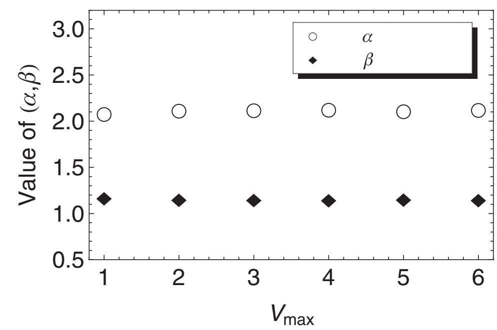


FIG. 6. The relativity plots between (α,β) and $V_{\max}$.

When the initial headway is large ($\bar{h} \geqslant 4$), the propagation speed a does not depend on the value of $V_{\max}$ in principle. However, when the initial headway is small ($\bar{h} < 4$), the hopping probability $p(h)$ does not equal to 1 because $h < 5$. From this viewpoint, the propagation speed of the starting wave depends on $V_{\max}$. However, this situation where $p(h) \neq 1$ has little effect on the propagation speed since the delay time due to $p(h) \neq 1$ is small enough compared to the total waiting time. Actually, the value set in the case $V_{\max} = 1$ is $(\alpha,\beta) = (2.08, 1.18)$.

These parameters (α,β) can be calculated when the hopping probability is described as (5) and $V_{\max} \geqslant 4$. Under these conditions, the expectation value of the required steps S for the last particle to start walking is given by

$$S = \sum_{k=0}^{N-1} \binom{N-1}{k} p(\bar{h}+1)^{N-1-k}[1-p(\bar{h}+1)]^k (N+k) \tag{9}$$

$$= N + (N-1)[1-p(\bar{h}+1)] \tag{10}$$

since particles only stop once with probability $1 - p(\bar{h}+1)$ at most. That is, if a particle stops with probability $1 - p(\bar{h}+1)$ at time t, then the predecessor moves forward with $V_{\max}$. Therefore, if $V_{\max} \geqslant 4$, the probability of the particle that did not move forward at time t changes to $p(h) = 1$ at time $t+1$ since the headway changes from $h = 1$ at time t to $h = 5$ at time $t+1$. From (6), we obtain the propagation speed for each given headway distance $\bar{h}$. This data set provides the parameters

$$(\alpha,\beta) = (2.13, 1.15) \tag{11}$$

from the fitting form (7). The parameters have a good agreement in both numerical simulations [Eq. (8)] and analytical calculations [Eq. (11)]. The results obtained from our mathematical model show that the value of β does not equal to 1. In the next section we investigate the existence of the optimal density which minimizes the required time in a queue by taking into account this nonlinearity.

IV. OPTIMIZATION OF INITIAL DISTRIBUTION

Now, let us apply this fundamental relation (7) to the optimization of initial distribution for a long queue, for example, a teeming number of athletes in a marathon. This issue is quite important for organizers to achieve smooth movement of crowds since the athletes located in the rearward position have an unavoidable delay to pass the head of the initial queue. This delay makes the time for traffic restraint longer and raises the possibility of a traffic disturbance in the surrounding area. Moreover, if there is an optimal density to minimize the delay time, controlling the density reduces the waste of time waiting in a queue. If athletes stand in line with large headway (low density), the starting wave propagates quickly, but the queue becomes long. However, if they stand in line with small headway (high density), the starting wave propagates slowly, but the queue becomes short. Which situation decreases the delay to pass the starting line? This problem, which explicitly takes into account the effect of the starting wave with a power law, has not been investigated yet. Thus, the optimal initial density which minimizes the required time in a queue is investigated here. Note that, if the hopping probability $p(h)$ is always constant and the initial headway distance corresponds to zero, our model is reduced to ASEP with the step initial condition. In this situation, the probability distribution of the required time can be obtained exactly in [39–41].

We set the problem as follows: Which density minimizes the required time for the last pedestrian to pass the head position of the initial queue? We generally set N as the total number of pedestrians. L and T indicate the length of the initial queue and the required time of the last pedestrian to pass the head of the initial queue, respectively. Note that this required time T is the sum of the waiting time until the starting wave reaches the last pedestrian in the queue and that pedestrian's travel time to the head of the initial queue. The initial equally spaced density ρ_0 is calculated as $\rho_0 = N/L$.

As shown in Fig. 7, numerical results based on our mathematical model reveal that the minimum value of the required time exists at the initial density $\rho = 1.0$ and 0.667 in the case of high maximum walking velocity $V_{\max} = 6$ and 11, respectively. If the maximum velocity is not large enough, such as in the ZRP case ($V_{\max} = 1$), the required time is minimized in the extremely high density; that is, the packed situation is optimal to reduce the delay since the propagation speed is sufficiently faster than the walking velocity. However, if the walking velocity takes a higher value, an optimal density exists in the lower-density region.

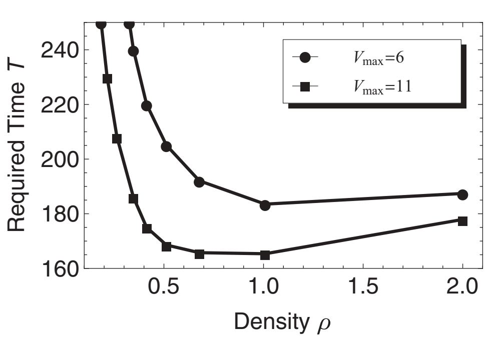


FIG. 7. Required time plots in numerical simulations for maximum velocity $V_{\max} = 6$ and 11.

The required time T for given initial density ρ_0 is calculated as

$$T(\rho_0) = \frac{L}{a(\rho_0)} + \frac{L}{V_{\max}}. \tag{12}$$

The first and second terms on the right-hand side in (12) indicate the waiting time to start walking and the travel time to reach the head position of the initial queue after starting to walk, respectively. Substituting the density $\rho_0 = N/L$, (12) translates as follows:

$$T(\rho_0) = N\left(\frac{1}{\rho_0 a(\rho_0)} + \frac{1}{\rho_0 V_{\max}}\right). \tag{13}$$

Substituting relation (7) into (13), one obtains the first derivative of $T(\rho_0)$ as

$$\frac{1}{N}\frac{dT(\rho_0)}{d\rho_0} = -\frac{1}{\rho_0^2}\left[\frac{1-\beta}{\alpha}\rho_0^{\beta} + \frac{1}{V_{\max}}\right]. \tag{14}$$

Moreover, the initial density which satisfies the extreme value $dT/d\rho_0 = 0$ is calculated by

$$\rho_0^{\beta} = \frac{\alpha}{V_{\max}(\beta - 1)}. \tag{15}$$

The condition $\beta > 1$ is necessary for the existence of ρ_0 since the parameters α, β, and $V_{\max}$ are all positive. The value of $\beta \sim 1.16$, as noted before, is larger than 1. Therefore, it is found that an optimal density does exist for given parameters α, β, and $V_{\max}$. Moreover, (15) tells us that the optimal density becomes lower as the walking velocity $V_{\max}$ becomes faster since parameters α and β are constant and are independent of the value of $V_{\max}$, as shown in Fig. 6.

V. EXPERIMENTS

The validity of the numerical simulations and analytical calculations of our mathematical model is supported by experimental measurements. In our experiments, we have measured the propagation speed of a starting wave and the required time in a queue to pass the head position of the initial queue under various densities, which are decided by the initial number of pedestrians along a line and the length of the queue as well as our mathematical model.

First, we made a long straight passage (L meters) and put marks with a distance of 0.5 m between them, as shown in Fig. 8. Detailed information of the experimental setting is shown in Table I. As an initial condition for the pedestrians, all pedestrians N stand in line with the same headway distance. For example, Fig. 9 shows the various initial densities in a queue. After that, the leader of queue starts to walk after being given a cue, and then we measure the waiting time and the required time, which are the time until the last pedestrian starts to walk and the time until the last pedestrian passes

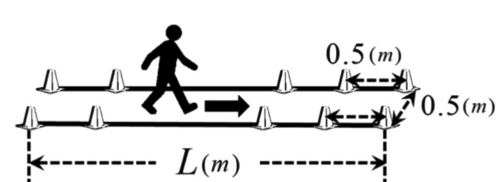


FIG. 8. Setting of our experimental passage.

TABLE I. Experimental settings: the length of the passage L, the number of pedestrians N (the number of women in a queue), the density ρ calculated by N/L, the number of trials, and information about the pedestrians.

Experiment	L	N	ρ	Number of trials	Information about the pedestrians
EX1	5	10 (1)	2.0	3	High school students.
EX2	20	20 (7)	1.0	2	Public men and women. Recruiting terms: walking speeds: over 1.0 m/s.
	20	30 (8)	1.5	3	
EX3	5	10 (1,3)	2.0	11[a]	Public men and women. Recruiting terms: walking speeds: over 1.0 m/s.
EX4	15	5 (0)	0.33	2	Public men. Recruiting terms: walking speeds: over 1.0 m/s; age restriction: 18–39.
		6 (0)	0.4	2	
		8 (0)	0.53	2	
		10 (0)	0.67	3	
		15 (0)	1.0	3	
		16 (0)	1.07	1	
		20 (0)	1.33	3	
		25 (0)	1.67	2	
		30 (0)	2.0	4	
EX5	10	10 (0)	1.0	3	Public men. Recruiting terms: walking speeds: over 1.0 m/s; age restriction: 18–39.
		20 (0)	2.0	2	
		30 (0)	3.0	2	
		40 (0)	4.0	3	
EX6	10	20 (0)	2.0	2	Public men. Recruiting terms: walking speeds: over 1.0 m/s; age restriction: 18–39.
		40 (0)	4.0	2	
EX7	20	30 (0)	1.5	2	Public men. (Recruiting terms) Walking speeds: over 1.0 meter per second Age restriction: 18–39
	12	30 (0)	2.5	2	
EX8	8.5	5 (0)	0.59	2	University students and postdoctoral researchers. Age: 20–39.
	9	7 (0)	0.78	2	
	2.5	10 (2)	4.0	14[b]	
	3.3	10 (2)	3.0	15[c]	
	5	10 (2)	2.0	10[d]	
	10	10 (2)	1.0	13[e]	
EX9	10	10 (1)	1.0	10[f]	University students and postdoctoral researchers. Age: 20–39.
	16	16 (3)	1.0	1	
	5	10 (2)	2.0	10[g]	
	8	16 (3)	2.0	1	
	3.3	10 (0)	3.0	18	
	5.5	16 (3)	3.0	1	
	2.5	10 (0)	4.0	21	
	4	16 (3)	4.0	1	

[a]One woman participated in 5 trials out of 11. Three women participated in the other 6 trials.
[b]Two women participated in 3 trials out of 14.
[c]Two women participated in 2 trials out of 15.
[d]Two women participated in 2 trials out of 10.
[e]Two women participated in 1 trial out of 13.
[f]One woman participated in 3 trials out of 10.
[g]Two women participated in 3 trials out of 10.

the head position of the initial queue, respectively. Thus, we have obtained the propagation speed of successive reactions, which is calculated from the length of the initial queue divided by the waiting time under each given density determined by a combination of L and N. Moreover, the required time corresponds to the delay time from the cue to pass the head position of the queue. Similar to the numerical simulations, we have obtained the set of parameters (α,β),

$$(\alpha,\beta) = (2.90,1.36), \tag{16}$$

by fitting our experimental data based on (7), as shown in Fig. 10. Comparing the fitting function based on (7) and the data, we have found that the fitting function is quite suitable for describing the relation between the initial density of pedestrians and the propagation speed of the starting wave in pedestrian dynamics even for results obtained from different pedestrians. If the power index β equals 1, the propagation speed is linearly proportional to the headway (reciprocal to the density), as noted before. However, the index β of our experimental results does not equal 1. What we would emphasize here is that our actual experiments have proven that the power law captures well a characteristic feature of the starting wave, that is, the relation between the propagation speed of the starting wave and the initial density, as well

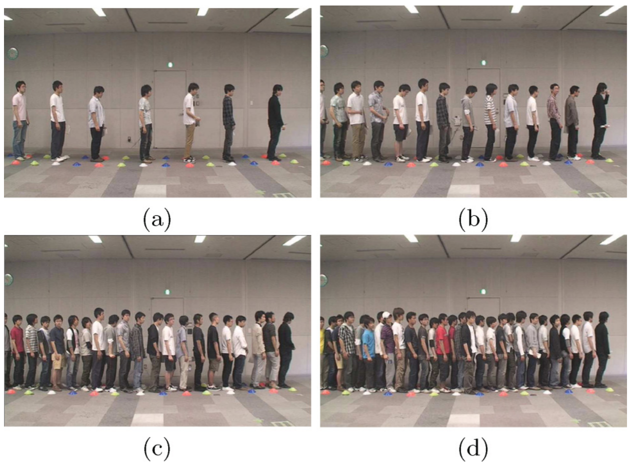


FIG. 9. (Color online) Snapshots of the various initial densities in a queue: (a) $\rho = 1.0$, (b) $\rho = 2.0$, (c) $\rho = 3.0$, and (d) $\rho = 4.0$.

as the results from our mathematical model. Moreover, its nonlinearity, i.e., $\beta \neq 1$ and $\beta > 1$, satisfies the requirements for optimizing the initial density of a queue.

On the other hand, the value of β is not in full agreement between our mathematical model and experiments (see Fig. 11). The reason for this difference is that we apply the OV function from the results of walking on a circular passage way in our model, whereas pedestrians walk in a straight passage way and they are ready to start in our experiments. That is, since they become more sensitive to the start in this situation than in the case of a circular passage way, the propagation speed becomes faster in the lower-density region. In the higher-density region, the propagation speed is almost

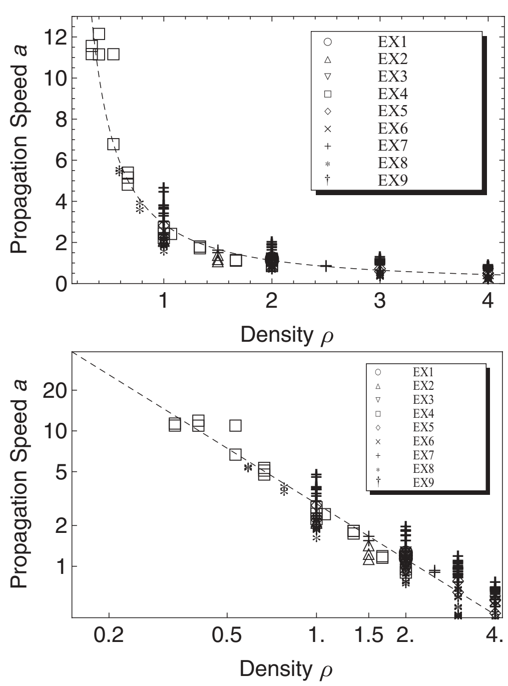


FIG. 10. The experimental results and fitting relation (dashed curve or dashed line) between the propagation speed of the starting wave and the initial density of pedestrians. (top) The normal plot and (bottom) the double logarithmic plot.

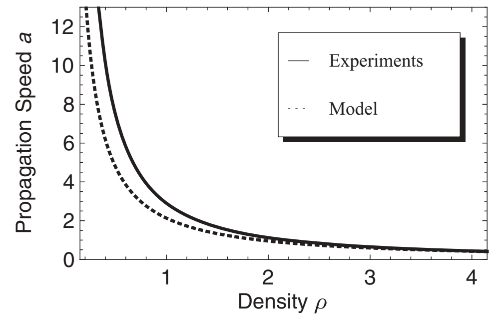


FIG. 11. The comparison between our mathematical model (8) and experiments (16). Both plots are given by fitting form (7).

the same due to physical constraint. Actually, when we neglect the data in the low-density region, the set of values $\{\alpha,\beta\}$ changes to $\{2.63,1.18\}$ in the region $1 \leqslant \rho \leqslant 4$ (the number of data is 165) and $\{2.61,1.16\}$ in the region $2 \leqslant \rho \leqslant 4$ (the number of data is 122). Our major argument in this paper is that the fundamental relation is well approximated by a power law, particularly $\beta \neq 1$ and $\beta > 1$; that is, the propagation speed of the starting wave shows nonlinearity.

In the previous section our mathematical model with a high walking velocity showed the existence of an optimal density to minimize the required time in a density region in response to the common expectation that an optimal density exists since both opposite extreme situations make the required time longer. The required time in the case of high velocity is plotted in Fig. 12. Each required time is the average value of a couple of trials. As shown in Fig. 12, we have observed that the optimal density to minimize the required time surely exists at the initial density $\rho = 3.0$ by comparing results among four densities. Thus, we have confirmed the common expectation by both a mathematical model and substantiative experiments.

VI. CONCLUDING DISCUSSION

In this paper, we have investigated the propagation speed of pedestrians' reactions in the relaxation process of a queue, the so-called starting wave, toward a smooth movement of crowds since a fast starting wave achieves a high departure rate. We first proposed our mathematical model, which is built on the stochastic cellular automaton models, in analogy with the mathematical modeling for traffic flow; then we revealed a special relation between the propagation speed of pedestrians' reactions and the initial density, which is well approximated by the power law $a = \alpha\rho^{-\beta}$ $(\beta \neq 1)$ by using numerical

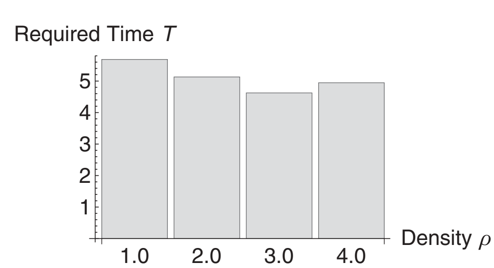


FIG. 12. The required time for each initial density from experiments.

simulations and analytical calculations. Moreover, we also found the existence of optimal density, where the required time of the last pedestrians to pass the starting line for the initial queue is minimized. The requirement for the existence of optimal density from our mathematical model is $\beta > 1$. Especially, if the walking velocity is very small, the value of the optimal density is detached from reality. However, the optimal density is suitable to apply to a real situation, such as a queue of athletes at the start in a marathon, since the optimal density becomes lower as the walking velocity increases.

The power law was verified by actual experiments, and the experimental results show a good agreement with the results from modeling and its analysis. Moreover, we have observed that the optimal density to minimize the required time surely exists at a density by comparing results among four densities in the experiments, as seen in the analysis of our mathematical model. However, experimental verification of the optimal density under various densities and velocities is an issue for the future. This optimal density inevitably plays a significant role to design not only the initial queue of pedestrians but also traffic intersections and signals.

ACKNOWLEDGMENTS

We thank Kozo Keikaku Engineering Inc. in Japan and the members of the Meiji University Global COE Program "Formation and Development of Mathematical Sciences Based on Modeling and Analysis" for assistance with the experiments, which are described in Sec. V. We acknowledge the support of the Japan Society for the Promotion of Science and the Japan Science and Technology Agency.

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