Collective decision dynamics in the presence of external drivers

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We develop a sequence of models describing information transmission and decision dynamics for a network of individual agents subject to multiple sources of influence. Our general framework is set in the context of an impending natural disaster, where individuals, represented by nodes on the network, must decide whether or not to evacuate. Sources of influence include a one-to-many externally driven global broadcast as well as pairwise interactions, across links in the network, in which agents transmit either continuous opinions or binary actions. We consider both uniform and variable threshold rules on the individual opinion as baseline models for decision making. Our results indicate that (1) social networks lead to clustering and cohesive action among individuals, (2) binary information introduces high temporal variability and stagnation, and (3) information transmission over the network can either facilitate or hinder action adoption, depending on the influence of the global broadcast relative to the social network. Our framework highlights the essential role of local interactions between agents in predicting collective behavior of the population as a whole.

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I. INTRODUCTION

The influences that humans have on one another's opinions and subsequent actions can shape large-scale movements [1–4] and are often facilitated by the sharing of information. Advances in information technologies are rapidly changing the way that humans exchange and share information. The widespread adoption of radio in the 1920s and television in the 1950s ushered in an era of live *broadcast media*, greatly increasing the speed and scope of mass communication from that of newspapers, which had been the dominant form of "one-to-many" information dissemination since the development of the Gutenberg printing press in the 15th century. The commercialization of the Internet in the late 1990s catalyzed the development of new digital broadcast services, consumed by a diversity of computing machines that now include laptop computers, mobile "smart" phones, and other handheld devices. These devices are now ubiquitous; by mid-2010, there were more than 5 billion mobile phone connections worldwide [5], with some regions experiencing more than 100% penetration (meaning that there is more than one mobile device per person).

A distinguishing feature of the Internet and these modern digital devices is that they enable rapid one-to-many and many-to-many communication, using services such as Facebook and Twitter, a phenomenon that has come to be known as *social media*. By the end of 2011, more than 300 million users were accessing Facebook using mobile devices [6,7]. In this modern era, anyone with a digital device and an Internet connection can publish information, and this is dramatically changing the roles that individuals and corporations play in traditional media industries such as books, music, film, and news journalism [8].

There is a general recognition that these technologies allow information to spread faster and perhaps more effectively. Such social epidemics, like biological epidemics [9,10], can be thought of as the result of single- [11] or multistage [12] complex contagion processes [13–15] that can propagate through Facebook [16], news websites [17] and other social media [18], blogs [19], and Twitter [20].

Recent events on the east coast of the United States have shown that social media can be helpful during extreme weather events and natural disasters [21]. In the case of the 5.8 magnitude earthquake that occurred in Virginia on 23 August 2011, the social network service Twitter proved to be more reliable than cellular phones, which became overloaded by increased call volume immediately after the event [22], and news of the event propagated to nearby New York on Twitter faster than the seismic waves themselves [23]. Social media also allows for rapid organization of disaster response information. For example, during the 7.1 magnitude earthquake in the Canterbury region in the South Island of New Zealand on 4 September 2010, the University of Canterbury used Facebook and other web-based technologies as a prominent source of up-to-date information and support for many months [24]. It is less clear how this faster information propagation affects the collective behavior of the individuals who consume it. Social networking technologies have been credited with facilitating the 2009 revolution in Moldova [25] and the "Arab Spring" uprisings in 2011 [26,27]. More prominently, social media has become an important component of corporate marketing and advertising, with considerable effort now directed at determining how to optimize the use of this new media [28–30].

In this paper, we consider the following question: *Do social networking technologies like Facebook and Twitter "help" to bring a group of individuals to action*? We consider the specific case of a population of individuals, each of whom must decide if and when to commit to a binary decision, and we assume that these individuals are exposed to information both from broadcast sources and over social media. The study of information diffusion on social networks has a lengthy history of illuminating the large-scale spread of rumors, social norms, opinions, fads, and beliefs [9,13,31–35]. The particular

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set of problems where individual decision making occurs in the presence of external information is known as "binary decisions with externalities" [31,36,37]. Here the individual's decision is often modeled as a threshold on their underlying opinion [31,38–41], which is modulated by the opinions of other individuals. We pose a minimal model of individual decision making and consider the effect of different forms of information exchange on the behavior of the group as a whole.

As a concrete example, we focus on a situation where a population is faced with a pending natural disaster (e.g., hurricane or wildfire). In such circumstances, it is know that each individual culls information from a wide variety of digital, sensorimotor, and social sources [42] and must decide if and when to evacuate. We assume that each individual has a simple *decision rule*: if the individual believes that the disaster is sufficiently likely, then he or she will evacuate. Individuals receive information about the disaster from a "global source" that broadcasts updates to the population as a whole [see Fig. 1(a)], and these individuals also exchange information over a social network that allows them to share opinions and observe the binary decisions of others [i.e., determine who has evacuated; see Fig. 1(b)]. By numerically exercising a series of increasingly complex models, we illustrate the tensions and trade-offs inherent in social decision dynamics.

Our results suggest that information transmission over the social network can either facilitate or hinder the action adoption depending on the influence of the global source relative to the social network. Further, we find that the sharing of binary information results in high variability, cascade-like dynamics in which the time of collective action is difficult to predict.

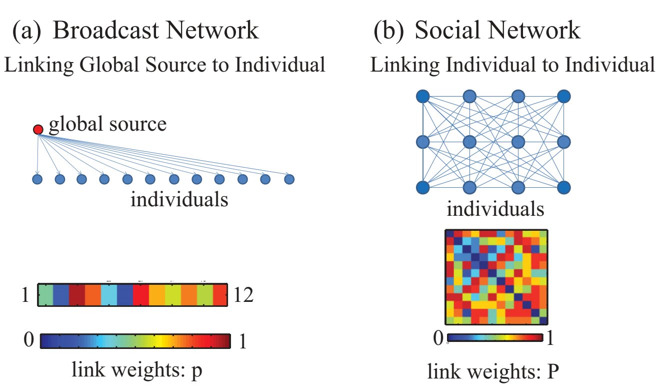

FIG. 1. (Color online) Model construct. Here we illustrate our multilayered broadcast-social network system composed of a graph allowing information diffusion between a global source and a population of 12 individuals (broadcast network) and a second graph allowing information diffusion between individuals (social network). (a) The broadcast network structure is a directed bipartite graph where the link weights are equal to the probabilities that the global source transmits information to the individual (probabilities given by the vector p, which varies over multiple numerical simulations M). (b) The social network structure is an all-to-all undirected graph where the link weights are equal to the probabilities that an individual transmits information to another individual (probabilities given by the matrix $\mathbf{P}$, which varies over multiple numerical simulations). In concrete terms, P_{ij} can be thought of as the combined rate that agent i posts and agent j reads the posting on Facebook.

II. MODEL CONSTRUCT

We build a model of decision making and social network interaction in discrete time. The social networks we study can be thought of as Facebook- or Twitter-like, in the sense that an agent posts updates or tweets to these media and subsequently other agents can check the posting at some rate. In our model framework, each agent receives a directed update from the other agents one at a time at prescribed rates.

Let $t = 0,1,2,\ldots$ index the discrete time increments. The social network consists of n individuals, in which each agent $j = 1,2,\ldots,n$ has two state variables:

(1) $S_j(t) =$ the *internal state* at time t, where $S_j(t) \in [0,1]$;

(2) $X_j(t) =$ the *externally observable state* at time t, where $X_j(t) \in \{0,1\}$.

The internal state assumes continuous values, but the externally observable state $X_j(t)$ reflects a binary *decision* on the part of agent j, derived from the *decision rule*

$$X_j(t) = \begin{cases} 1 & \text{if } S_j(t) \geqslant \tau_j \\ 0 & \text{if } S_j(t) < \tau_j, \end{cases} \tag{1}$$

and where $\tau_j \in [0,1]$ represents a *threshold* value for agent j. The use of a threshold for the binary decision $X_j(t)$ is consistent with long-standing modeling efforts of collective behavior, beginning with the work of Granovetter in the 1970s [38], and is particularly relevant for decisions that are inherently costly [31] (such as evacuations).

We think of the internal state information as "private" in the sense that, for example, it reflects an underlying belief on the part of the individual. While it is possible that an individual could share this private information with another, this exchange requires the individual to volunteer it. In contrast, the externally observable state is "public" in the sense that anyone who sees that individual would observe their binary decision. It has been argued that this separation between internal opinion and external decision is critical for an understanding of how convictions are coupled to actions [43,44].

We represent the internal state of all agents at time t using $S(t) = \{S_1(t),S_2(t),\ldots,S_n(t)\}$, and we represent the externally observable state of all agents at time t using $X(t) = \{X_1(t),X_2(t),\ldots,X_n(t)\}$.

The internal state of an agent changes over time as a result of information from three different sources: (1) a global source via a "global broadcast," (2) the internal state information of friends via "social sharing," and (3) the decision state of neighbors via "neighbor observation." Each of these sources is described in more detail below.

Global broadcast. We introduce a special external agent called the "global source" of information that influences, but is not influenced by, the other agents. Let $G(t) \in [0,1]$ represent the value that is broadcast by the global source in time period t. We assume that receipt of a broadcast message by agent j is binary (i.e., either it happens or it does not). Let $u_j(t) = 1$ represent the successful broadcast from the global source to agent j at time t, and $u_j(t) = 0$ represent no broadcast. Thus the vector $U(t) = \{u_1(t),u_2(t),\ldots,u_n(t)\}$ represents the overall broadcast from the global source at time t. This global source is the primary "external driver" of dynamics in this system.

Social sharing. We assume that the action of social sharing between agents is binary, either occurring or not occurring at any point in time. That is, we let $a_{ij}(t) = 1$ if agent i shares its internal state information with agent j in time period t, and $a_{ij}(t) = 0$ if not. Thus, the matrix $A(t) = \{a_{ij}(t)\}$ represents the adjacencies for the exchange of internal state information among the agents in time period t.

By construction, $a_{jj}(t) = 0$ for all agents j and all time periods t. In general, we use symmetric sharing [i.e., $a_{ij}(t) = a_{ji}(t)$] to indicate that any agent may address any other agent; however, the mathematics here do not require it. In our model, however, these links are directed in the sense that for a given sharing event, an agent asks for another agent's opinion or state, but does not share its own state. Whether or not there is sharing symmetry or directionality depends on the specific application we hope to model. Communication across cell phones tends to be symmetric and undirected, while that across Twitter and Facebook is not necessarily so.

Neighbor observation. Similarly, we assume that the observation of another agent's external state either happens or does not in each time period t. Let $B(t) = \{b_{ij}(t)\}$ represent an adjacency matrix for the exchange of externally observable binary state information among the agents, where $b_{ij} \in \{0,1\}$. By construction, $b_{jj}(t) = 0$ for all agents j and all time periods t. The matrix B might represent a network of interactions based on physical location, where nearby individuals literally see one another even if they do not communicate. For purposes of our discussion, we refer to agents who are adjacent in the matrix B as *neighbors*. That is, we narrowly define two agents as "friends" if they share internal state information, and we define two agents as "neighbors" if they can observe the external state of each other. Again the mathematical formulation here can be used to model both symmetric and nonsymmetric sharing.

General update rule. In the presence of all three sources of information, the internal state of agent j evolves according to the general update rule,

$$S_j(t+1) = \frac{\sum_i a_{ij}(t)S_i(t) + \sum_i b_{ij}(t)X_i(t) + u_j(t)G(t)}{\sum_i a_{ij}(t) + \sum_i b_{ij}(t) + u_j(t)}. \quad (2)$$

This update rule is a deterministic averaging of the current internal state of agent j, the internal state of agent j's friends, the external state of agent j's neighbors, and any global broadcast information. However, we assume that the coefficients in A, B, and U are stochastic in time; that is, in any given time period t, the information to agent j might or might not be received. This stochasticity is consistent with the fact that communication over these types of networks is likely to be sparse—due to geographic and energetic constraints—and dynamic—due to agent movement and constraints on communication [37].

While opinion dynamics have been studied using a variety of models [45], the averaging rule given in Eq. (2) is most consistent with the Hegselmann-Krause model of opinion dynamics [46], models of "continuous opinion dynamics" [47], and the many models of coordination and consensus of autonomous agents [37,48–51]. The discrete nature of this update rule is consistent with the fact that information is often issued at some frequency or can be obtained in discrete units from governmental, social, or technical sources [42,52].

III. COLLECTIVE DECISION DYNAMICS

In what follows, we are interested in the real-time exchange and mixing of information among the individuals, and how it affects the decision making of the collective group.

We measure the behavior of the system as a whole in several ways. Recall that the social network consists of n individuals, in which each agent $j = 1,2,\ldots,n$ has two state variables: $S_j(t)$ and $X_j(t)$. We refer to the sequence $S(t), t \geqslant 0$ which indexes over the j agents as the *information state trajectory* for the system, and the sequence $X(t), t \geqslant 0$ similarly as the *action adoption trajectory*. Let $m = 1,2,\ldots,M$ index the numerical trials associated with a particular experiment, and let $S_j^m(t)$ represent the value of $S_j(t)$ during the mth trial. We compute the average information state of the population at time t during experiment m as $\langle S_j^m(t)\rangle_n = \frac{1}{n}\sum_{j=1}^n S_j^m(t)$; accordingly, the sequence $\langle S_j^m(t)\rangle_n, t \geqslant 0$ is the *average information state trajectory* for the system during experiment m. We compute the average information state of individual j at time t as $\langle S_j^m(t)\rangle_m = \frac{1}{M}\sum_{m=1}^M S_j^m(t)$; this is the average information state of any individual in the population. Accordingly, the sequence $\langle S_j^m(t)\rangle_m, t \geqslant 0$ is the *ensemble information state trajectory* of a single individual over the ensemble of numerical trials. Finally, we can estimate the *average ensemble information state trajectory* as

$$\mathbb{E}_m\big\langle S_j^m(t)\big\rangle_n = \frac{1}{M}\sum_{m=1}^M\left[\frac{1}{n}\sum_{j=1}^n S_j^m(t)\right] = \frac{1}{n}\sum_{j=1}^n\left[\frac{1}{M}\sum_{m=1}^M S_j^m(t)\right]. \quad (3)$$

It is important to note that the variance of $S_j(t)$ over individuals could be very different from the variance expected over numerical trials; for example, one distribution might be normal and the other heavy tailed. The separation of the individual and ensemble averages allows us to independently probe these distinct sources of variability in the system.

We introduce the term $N(t) = \sum_{j=1}^n X_j(t)$, which counts the number of individuals whose internal state has exceeded their decision threshold and who therefore have taken action. With this definition, we observe that $\langle X_j(t)\rangle = N(t)/n$ measures the fraction of individuals who have chosen the binary action.

In many cases, we are also interested in the amount of time it takes for the population to collectively adopt new information or take action. We define $H_S(\alpha) = \min\{t \;|\langle S_j(t)\rangle \geqslant \alpha\}$ to be the *first hitting time* for the population to reach some *average information level* $\alpha \in [0,1]$. Similarly, we define $H_X(\alpha) = \min\{t \;|\langle X_j(t)\rangle \geqslant \alpha\}$ to be the first hitting time for the population to reach some *average adoption level* $\alpha \in [0,1]$. For example, $H_X(0.5)$ is the amount of time for half of the population to adopt the binary action.

Conceptually, we can think of two basic types of information diffusion; see Fig. 1. The first is the (one-to-many) broadcast of external information by the global source to the agents; we can think of this interaction in terms of a broadcast network. The second (many-to-many) diffusion occurs on two different social networks: one network of friends and another network of neighbors. We study the behavior of the system in the following specific cases.

Case 1: Information broadcast from the global source only. In this case, the only information exchange is the broadcast from the global source, and the update rule is given by

$$S_j(t+1) = \frac{S_j(t) + u_j(t)G(t)}{1 + u_j(t)}. \tag{4}$$

In the simple case where $S_j(0) = 0$ and where the global source broadcasts $G(t) = 1$ for all time periods t, the progression of internal state for agent j goes as $1 - 1/(k+1)$ after the kth update for $u_j(t) = 1$. That is, with each update from the global source, agent j moves "halfway" to the broadcast value of 1, reaching it only in the limit as $t \to \infty$.

As noted above, we assume that broadcast messages from the global source are received stochastically. This could occur in practice because agent j only "tunes in" to the global source sometimes or because the global source has limited success in its ability to reach agent j. For each time period t, we generate a vector $U(t)$ as an independent and identically distributed random vector, where $\text{Prob}\{u_j = 1\} = p_j$, and this probability is independent for each agent j. Thus, $\mathbb{E}[u_j(t)] = p_j$ captures the expected broadcast "rate" [52] to agent j (assumed to be stationary for now), and $\mathbb{E}[U(t)]$ represents the overall expected broadcast from the global source in time period t.

In this simple case, we can derive expected values for $S_j(t)$ and $X_j(t)$ analytically. Specifically, after t discrete time units, the probability of agent j having received $k \leqslant t$ updates is given by the binomial distribution, $\text{Binom}(k; t, p_j)$. The expected value of $S_j(t)$ is therefore

$$\mathbb{E}[S_j(t)] = \sum_{k=0}^{t} \binom{t}{k} (p_j)^k (1 - p_j)^{t-k} \left(1 - \frac{1}{k+1}\right). \tag{5}$$

In addition, $\mathbb{E}[X_j(t)]$ is simply the probability that agent j has taken action (i.e., has adopted state 1) and can be defined as

$$\begin{aligned}\mathbb{E}[X_j(t)] &= \text{Prob}\{S_j(t) \geqslant \tau_j\} \\ &= \sum_{k=0}^{t} \binom{t}{k} (p_j)^k (1 - p_j)^{t-k}\, \mathbb{I}_{\{1 - \frac{1}{k+1} \geqslant \tau_j\}},\end{aligned} \tag{6}$$

where $\mathbb{I}_{\{a\}}$ is the *indicator function*, namely, $\mathbb{I}_{\{a\}} = 1$ when condition a is true, and $\mathbb{I}_{\{a\}} = 0$ when condition a is false.

The values $\mathbb{E}[S_j(t)]$ and $\mathbb{E}[X_j(t)]$ in the simple case of the global broadcast only serve as a baseline against which we can evaluate the impact of various types of social network exchange.

Case 2: Global broadcast with social network exchange. We now add information exchange among friends alongside the global broadcast. This gives us the following update rule:

$$S_j(t+1) = \frac{\sum_i a_{ij}(t) S_i(t) + u_j(t) G(t)}{\sum_i a_{ij}(t) + u_j(t)}. \tag{7}$$

Again, we assume that exchange of information between friends is stochastic in time. Thus, we generate each matrix $A(t)$ as a weighted matrix, where $\text{Prob}\{a_{ij} = 1\} = P_{ij}$ and this probability is independent for each (i,j) pair. We let this probability be stationary in time, based on the common assumption that the information transmission occurs on a faster time scale than network changes [53]. Thus, $\mathbb{E}[a_{ij}(t)] = P_{ij}$ represents the expected rate at which agent i influences agent j, and $\mathbb{E}[A(t)]$ is the expected information exchange within the social network of individuals, as might happen using technologies such as mobile phones, Facebook, or Twitter.

This case corresponds to a particular type of consensus problem for which there are analytic results that describe the convergence of $\mathbb{E}[S_j(t)]$, specifically the conditions under which it is guaranteed and how long it will take. Jadbabaie *et al.* [49] consider the case of "leader following" in consensus problems in which one of the agents never updates its own variable, but indirectly influences all of the other agents. The role of the "leader" is equivalent to that of our global source. (Bertsekas and Tsitsiklis [54] argue that this result is essentially a special case of the more general result in [55].) Similarly, Jadbabaie [56] discusses routing in networks in which nodes iteratively update their coordinate information, but certain "boundary" nodes retain fixed locations (again, like our global source). A third related example is the model of Khan *et al.* [57], which describes consensus on random networks in which there are "anchor nodes" (again, like our global source in that their state does not change) and "sensor nodes" (like our agents who update through random mixing). Finally, Galam *et al.* [58,59] use the term "inflexible agents" to indicate nodes whose opinions do not change and find in some cases that these agents drive opinion dynamics.

Case 3: Global broadcast with binary information only among neighbors. In this final case, we consider the impact of binary exchange among neighbors alongside the global broadcast. This gives us the following update rule:

$$S_j(t+1) = \frac{S_j(t) + \sum_i b_{ij}(t) X_i(t) + u_j(t)}{1 + \sum_i b_{ij}(t) + u_j(t)}. \tag{8}$$

Again, we assume that this type of exchange is stochastic, and we generate each matrix $B(t)$ as a weighted matrix, where $\text{Prob}\{b_{ij} = 1\} = Q_{ij}$, and this probability is stationary and independent for each (i,j) pair. Thus, $\mathbb{E}[b_{ij}(t)] = Q_{ij}$, and $\mathbb{E}[B(t)]$ is the expected information exchange within local neighborhoods. In this case, we analyze the network of friends separately from the network of neighbors. In future simulations, it might be interesting to examine the special case in which the two networks are either identical or change in a dependent manner.

The behavior of this system is sufficiently complicated that analytic results do not, to our knowledge, exist for this case. We therefore turn to numerical simulations in order to analyze and compare this case to the simpler ones described above.

IV. NUMERICAL EXPERIMENTS

We conduct several numerical experiments in which we simulate the behavior of $n = 100$ agents over a maximum of $T = 1000$ time units. Each agent j starts with $S_j(0) = 0$, and the global source broadcasts $G(t) = 1$ for all time periods t. For each of our three cases, we simulate a total of $M = 100$ trials, and for each trial we select fixed broadcast rates for each agent that are uniformly distributed on the interval [0,1] (that is, $p_j \sim U[0,1]$).

Similarly, for each trial, we select social sharing rates P_{ij} or neighbor observation rates Q_{ij}, each of which remains fixed within an individual trial. We define a symmetric matrix $\mathbf{R}$ whose diagonals decrease in value:

$$R_{i,i+|k|} = (n - k + 1)/n \forall k \in \{1,2,\ldots,(n-1)\}. \quad (9)$$

The matrix $\mathbf{R}$ can be thought of as a probabilistic form of a regular lattice network. We choose $\mathbf{P}$ and $\mathbf{Q}$ as randomly scrambled versions of $\mathbf{R}$, where the scrambling maintains the symmetry of the matrix. In general, this approach provides flexibility for the examination of topological structures between random and regular graphs. In this work, we focus on random topologies to understand benchmark behavior. The distribution of probabilities across agents provides a weighted counterpart to degree heterogeneities examined in other studies of opinion dynamics [60,61], which have important consequences for collective action.

For each of our three cases, we investigate the evolution of information states and action adoption for individual agents and the collective group as a whole.

A. Average information state

Figure 2 compares the evolution of information states across the three case studies. Figure 2(a) shows the

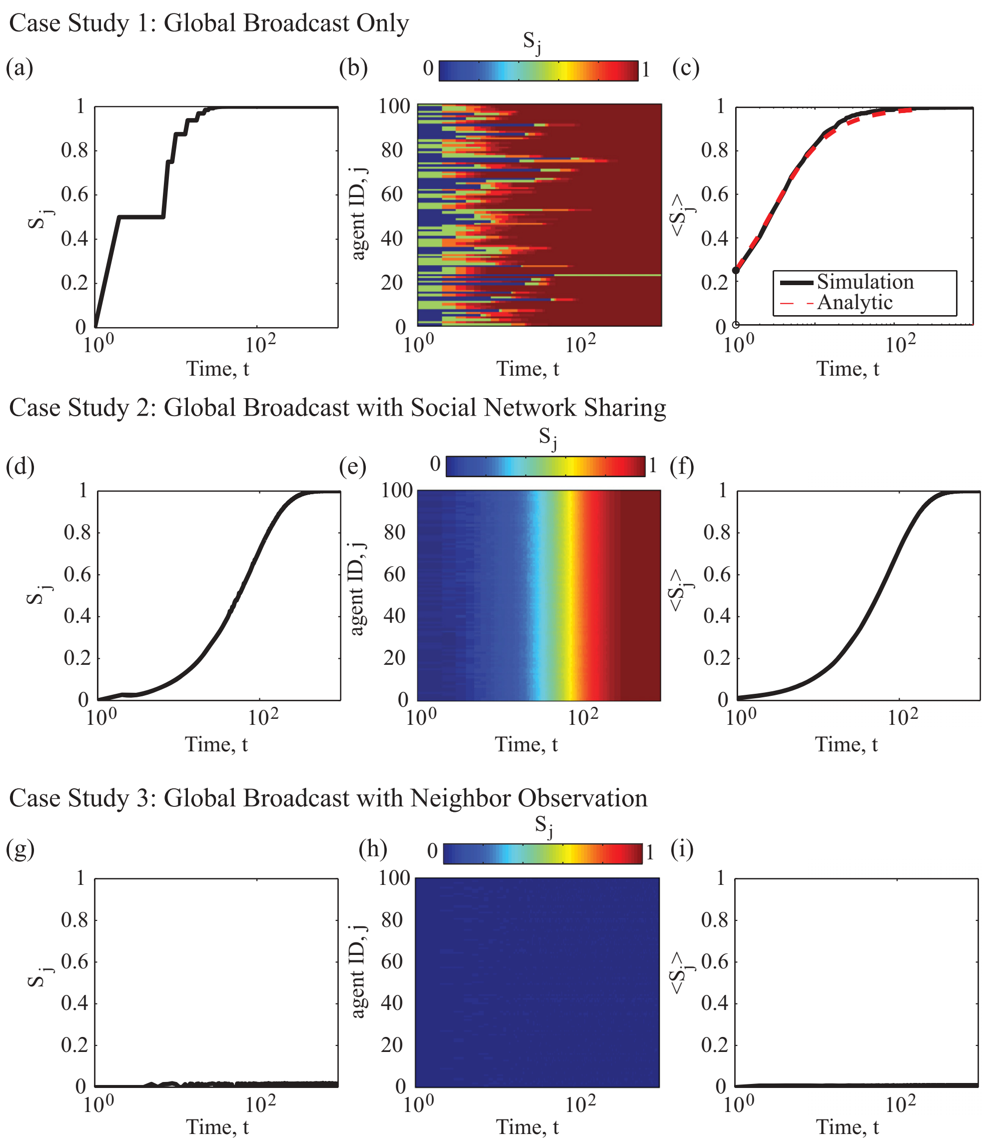

FIG. 2. (Color online) Information diffusion under three types of communication. (a), (d), (g): Information state trajectory for a single, randomly selected agent j as a function of time. (b), (e), (h): Information state trajectories for a population of $n = 100$ agents. Information states (color) for each individual (row) are shown as a function of time (x axis). (c), (f), (i): Average information state variable $\langle S_j \rangle$ averaged over $M = 100$ numerical simulations as a function of time. Note that the simulated curve (solid black line) in (c) is accompanied by a curve that was analytically calculated from Eq. (5) (dashed red line).

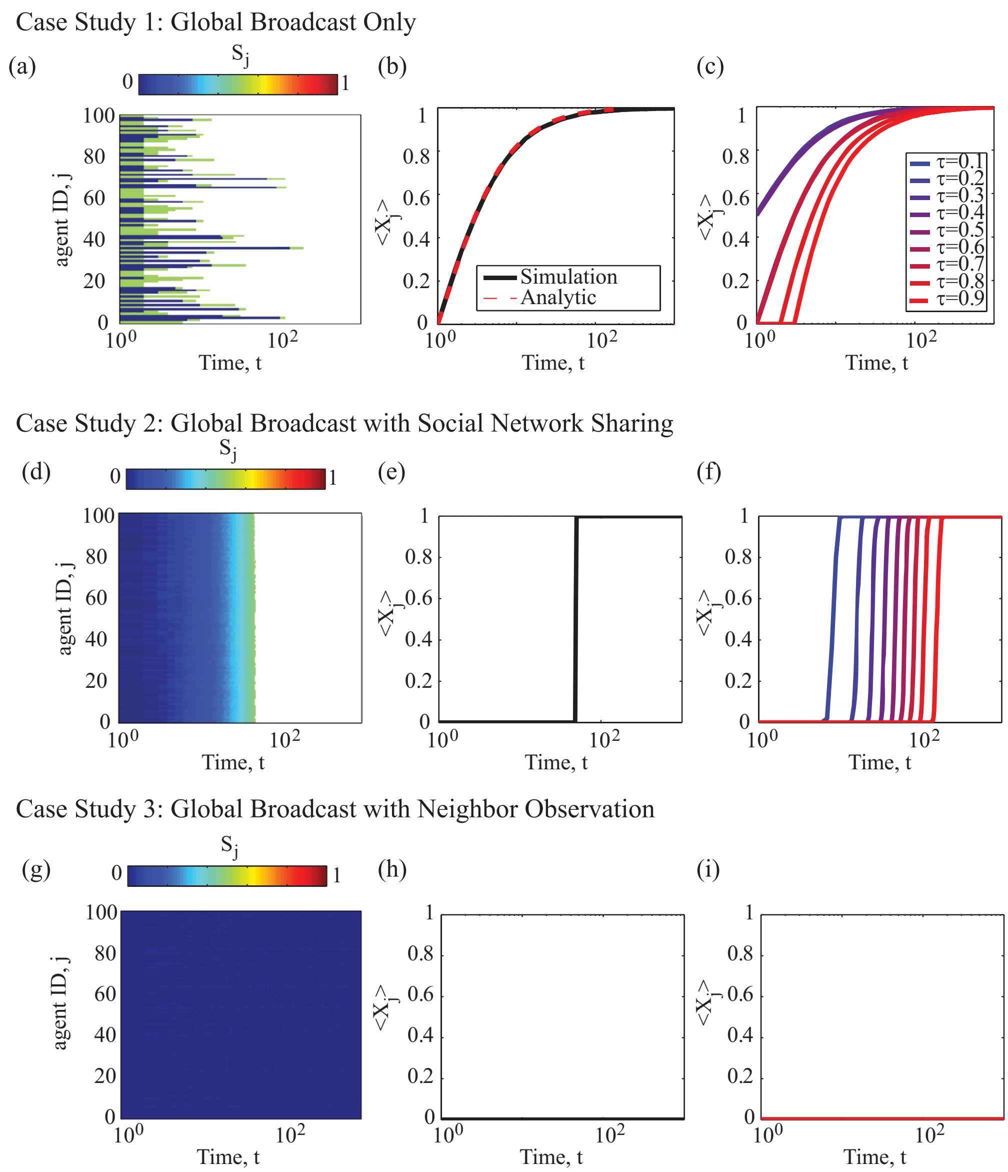

FIG. 3. (Color online) Action adoption under three types of communication. (a), (d), (g): Adoption state trajectories for a population of $n = 100$ agents. Adoption states (color) for each individual (row) are shown as a function of time (x axis). Note that the color white indicates that the individual has taken action. (b), (e), (h): Average adoption state variable $\langle X_j \rangle$ averaged over $M = 100$ numerical simulations as a function of time. (c), (f), (i): Dependence of average adoption state variable $\langle X_j \rangle$ on the threshold level τ, which we have varied in this figure from 0.1 to 0.9. Note that the simulated curve (solid black line) in (b) is accompanied by a curve that was analytically calculated from Eq. (6) (dashed red line).

stepwise trajectory $S_j(t)$ for a single agent, and Fig. 2(b) shows the overall population of agents who independently update their state information in the presence of the global broadcast only. Figure 2(c) presents the average information state trajectory for the collective group calculated both through simulation (solid black line) and through the analytic solution given in Eq. (5), which agree well.

Figures 2(d)–2(f) present the equivalent results for the case of global broadcast and social network sharing. We observe that in the presence of social networking, individual agents [Fig. 2(d)] as well as the entire group of agents evolve their state information in a collectively smooth manner [Fig. 2(e)]. However, the overall rate of increase for the average information state in the case of social network sharing [Fig. 2(f)] is *slower* than in the case of the global broadcast only [Fig. 2(c)].

Figures 2(g)–2(i) present the equivalent results for the case of neighbor observation in which the only agent-to-agent information exchange comes from observing the external state of other agents. For the given parameters ($n = 100$, $d = 1$, $M = 100$; for a description of d, see later sections), we observe that none of the individuals ever raises their information state significantly from the starting value of zero. This case represents an extreme of the previous one in which social network sharing through neighbor observation slows any increase of the information state. (Note: See later sections for other areas of the state space in which potentially more interesting behaviors like cascades and stagnation occur.)

B. Average adoption state

Figure 3 displays the adoption trajectories for the three cases, when each individual agent j has a decision threshold $\tau_j = 0.5$. In Fig. 3(a), we observe that under only global broadcast, the information state of individual agents evolves in a manner consistent with Fig. 2(b), except that we terminate each trajectory when the individual crosses the decision threshold. Figure 3(b) shows the average adoption state trajectory for a single trial calculated both through simulation and through the analytical solution given in Eq. (6), which agree well. Importantly, to remain comparable to our simulations in which p is chosen $\sim U[0,1]$, Eq. (6) was calculated for a range of $0 < p_j < 1$, and the average is plotted in Fig. 3(b). We note that the action adoption curve for a population with fixed p_j is not equivalent to that for a population with distributed p_j. Finally, Fig. 3(c) shows the effect of different threshold values τ on the average adoption state.

Figures 3(d)–3(f) present the equivalent results for the case of global broadcast and social network sharing. As before, we observe that in the presence of social networking, the entire group of agents evolves their state information in a collectively smooth manner [Fig. 3(d)]. Since this groups shares a common decision threshold, we observe that action adoption now occurs abruptly [Fig. 3(e)] and that the onset of this abrupt change depends primarily on the threshold level τ [Fig. 3(f)].

In the case of neighbor observation and global broadcast, Figures 3(g)–3(i) again show that none of the individuals ever raises their information state significantly from the starting value of zero. These results are consistent with those shown in Figs. 2(g)–2(i) and represent an extreme of social inertia keeping the information state at very low values. (Again, see later sections for other areas of the state space in which arguably more interesting behaviors like cascades and stagnation can occur.)

C. Heterogeneous decision thresholds

Up to this point, we have assumed that individual agents share a common decision threshold τ_j. In practice, this is unlikely to be the case and recent theoretic and experimental work suggests individual variability in decision thresholds might significantly alter the dynamics of the population [62,63]. We now consider the implications of a heterogeneous population of agents where the threshold for agent j is uniformly distributed, $\tau_j \sim U[0,1]$. Figure 4 displays the impact of variation in underlying τ_j on the mean adoption state of the collective group as a function of time when averaged over multiple trials, given by Eq. (3), as well as the corresponding variance of the adoption state of the collective group as a function of time.

We note that in the case of the global broadcast only, the distribution of thresholds, whether homogeneous or heterogeneous, has little effect on the mean or variance of the adoption state trajectories [Figs. 4(a) and 4(b)].

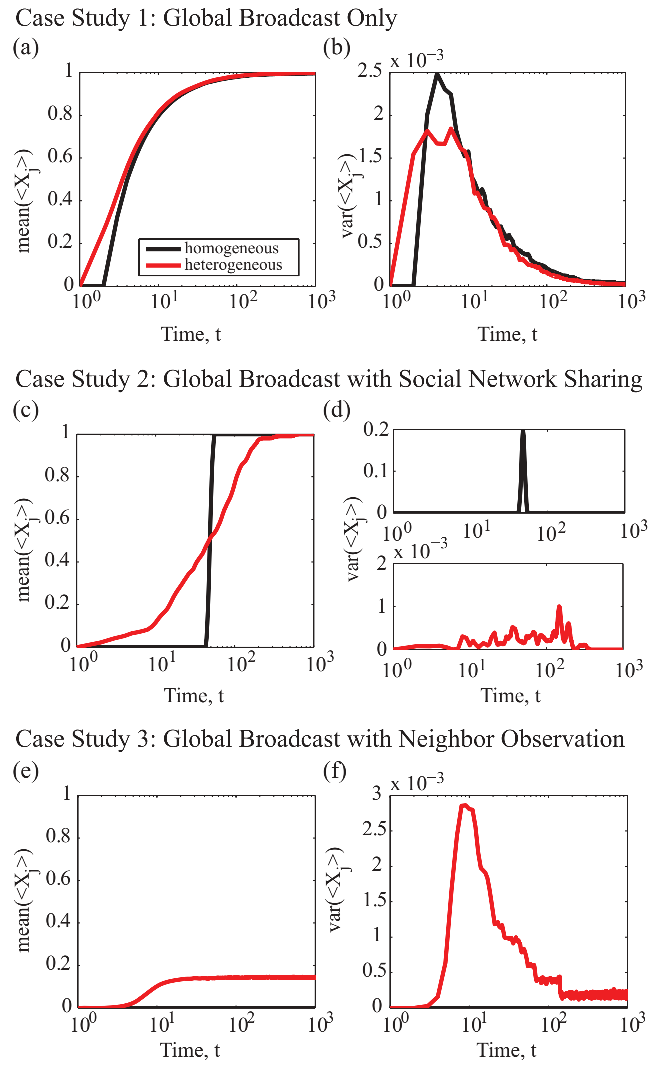

FIG. 4. (Color online) Action adoption under heterogeneous thresholds. Thresholds are either chosen to remain constant over all individuals ("homogeneous"; black lines) or to vary over individuals ["heterogeneous"; red (dark gray) lines]. (a), (c), (e): Average adoption state variable $\langle X_j \rangle$ averaged over $M = 100$ numerical simulations as a function of time. (b), (d), (f): The variance (over $M = 100$ numerical simulations) of the average adoption state variable $\langle X_j \rangle$ as a function of time. Note that the curves shown in (d) are on such different scales that we have plotted them in separate subplots to enhance visualization.

However, in the case of global broadcast and social network sharing, having a homogeneous distribution of thresholds over the population leads to a "tipping point," where at one time point, no one has taken action, while a few time steps later, everyone has taken action [Fig. 4(c), black line]. For heterogeneous thresholds, this drastic adoption is no longer evident, and instead the transition from zero adoptions to all adoptions is smooth and gradual [Fig. 4(c), red (dark gray) line]. Consistent with these results, we find that the variance in the adoption states across individuals is large precisely at the tipping point for homogeneous thresholds [Fig. 4(d), black line] and is small for all times when heterogeneous thresholds are used [Fig. 4(d), red (dark gray) line], suggestive of the formation of a "collective."

In the case of neighbor observation and global broadcast, Fig. 4(e) shows that for this set of parameters ($\tau_j = 0.5$ or $\tau_j \sim U[0,1]$), when the threshold is distributed homogeneously, the population is unable to take action (black line), but when it is distributed heterogeneously, the population can take action [red (dark gray) line]. The fact that the population cannot take action when $\tau_j = 0.5$ is true for all simulations, leading to a zero variance over simulations [Fig. 4(f), black line]. In contrast, the action adoption trajectories for the population vary over simulations when $\tau_j \sim U[0,1]$, and they do so particularly for time points close to the point of maximum adoption [Fig. 4(f), red (dark gray) line].

D. Stagnation and Cascades

The adoption state trajectories reported in the previous section were averaged over M numerical simulations. However, in order to fully understand the case of neighbor observation and global broadcast, it is important to examine individual simulation trials in addition to their average.

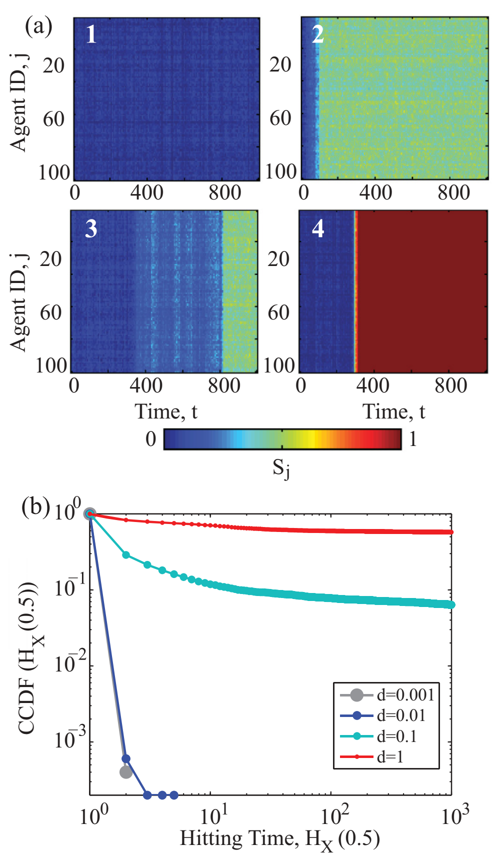

FIG. 5. (Color online) Variability in action adoption behavior. (a) Example plot of the information state (color) trajectory of agents in the population (y axis) as a function of time (x axis) showing (1) stagnation, (2) partial cascade and then stagnation, (3) late partial cascade, and (4) full cascade. (b) The complementary cumulative distribution function (CCDF) of the first hitting time for the population to reach the average adoption level of 0.5 [e.g., $H_X(0.5)$] for a set of $M = 5000$ simulations as a function of the influence parameter d.

In Fig. 5(a), we show information state trajectories for all individuals in a population in four different simulated trials. These four examples highlight several possible group behaviors, including stagnation [Fig. 5(a1)], partial cascades occurring either early [Fig. 5(a2)] or late [Fig. 5(a3)], and full cascades [Fig. 5(a4)]. This high variability is an important factor that sets Case 3 (neighbor observation and global broadcast) apart from the other two cases. Cascade behavior has previously been demonstrated in a wide variety of topological structures and update rules [31,41,64–66], and understanding when and how cascades occur is of particular importance for predicting large-scale social movements.

We can study this behavior more systematically by examining the first hitting time for the population to reach an average adoption level of 0.5 [e.g., $H_X(0.5)$]. We define a "converging simulation" as one for which $H_X(0.5)$ is identified within 1000 time steps and a nonconverging simulation as one for which it is not. In Fig. 5(b), we show the complementary cumulative distribution functions (CCDFs) of converging simulations for this model (upper red line). We note that the majority of converging simulations reach the target average adoption level in relatively short times. However, the distribution is heavy tailed such that much longer times are consistent with the statistics. In fact, out of a total of 5000 simulations, only 2127 solutions converged in 1000 time steps, indicating that stagnating periods can extend beyond the studied temporal window ($0 < t < 1000$). In the next section, we will further explore this behavior as a function of the amount of exchange between social and global information sources.

E. Impact of exchange rates

The behavior of the system under all three cases highlights the role of different types of information exchange. We observe that our stylized form of social network exchange tends to move the group as a whole.

In this section, we introduce an *influence* parameter $0 \leqslant d \leqslant 1$, which serves to tune the network effects in Case 2 (social sharing) and Case 3 (neighbor observation), respectively, as follows:

$$S_j(t+1,d) = \frac{S_j(t) + d\sum_{i\neq j} a_{ij}(t)S_i(t) + u_j(t)}{1 + d\sum_{i\neq j} a_{ij}(t) + u_j(t)} \quad (10)$$

and

$$S_j(t+1,d) = \frac{S_j(t) + d\sum_{i\neq j} b_{ij}(t)X_i(t) + u_j(t)}{1 + d\sum_{i\neq j} b_{ij}(t) + u_j(t)}. \quad (11)$$

When $d = 1$, these cases remain unchanged and the social network has full influence, but when $d = 0$, these cases each reduce to global broadcast only where the social network has no influence. This is illustrated in Figs. 6(a) and 6(f), where we see that for low values of d, the information state trajectories are similar to those found in Case 1 [global broadcast only; compare to Fig. 3(b)]. The concept of network influence is akin to the concept of information credibility, which varies over modern information transmission technologies [67,68]: a network with high credibility could be modeled as one with

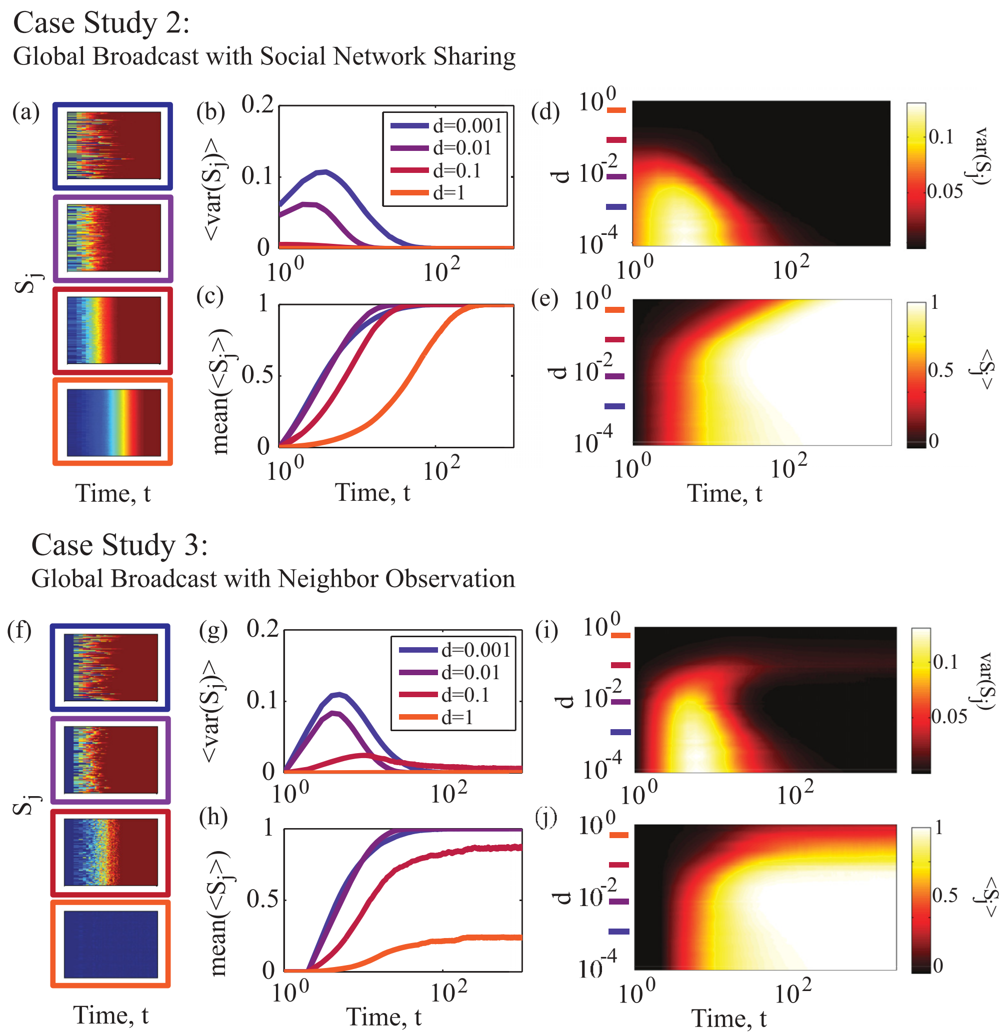

FIG. 6. (Color online) Effect of the influence parameter in (a)–(e) Case 2 (global broadcast with social network sharing) and (f)–(j) Case 3 (global broadcast with neighbor observation). (a), (f) Information states for a population of $n = 100$ individuals as a function of time for four values of the influence parameter: box outlines correspond to d values given in the legends of (b) and (g). As the influence parameter increases, the variance of information states across individuals decreases [(b), (g)], indicating the formation of a collective, and the mean of information states across individuals decreases [(c),(h)], indicating that it takes more time for individuals to reach higher values of S_j. Importantly, the four lines shown in (b), (c), and (g), (h) are average (over $M = 100$ simulations) trajectories drawn from a continuous measure of influence, d. In (d), (e), and (i), (j), we therefore show the full average contour plots for 7400 numerical simulations. Four colored bars on the left of each plot indicate the d values for which the four lines in (b), (c), and (g), (h) are taken. In these contour plots, color indicates the variance (top) or mean (bottom) of the information state trajectory as a function of time (x axis) for influence parameters in the range $0.0001 < d < 1$ (y axis).

greater influence and a network with low credibility could be modeled as one with lesser influence.

As d increases, we find that the variance in information state trajectories across agents decreases [Figs. 6(b) and 6(g)], which is consistent with the formation of a single collective state as the social network becomes stronger. As d increases, we also find that it takes longer for agents to reach any given state value, indicating that the social network is maintaining some inertia and holding agents closer and closer to their original states [Figs. 6(c) and 6(h)]. These general behaviors are also evident in a more continuous phase diagram of the state space of the system [Figs. 6(d), 6(e), 6(i), and 6(j)]. We note that the rightward tail of nontrivial behavior in Fig. 6(i) in comparison to Fig. 6(d), and the extension of low $\langle S_j \rangle$ to longer times in Fig. 6(j) in comparison to Fig. 6(e), are a result of the stagnation followed by cascading behavior present for the case of neighbor observation. These results indicate a wider range of active change or variability in model behavior relative to the case of social network sharing.

Importantly, the high variability in population behavior in Case 3 in the previous section is present over a wide range of influence parameter values [Fig. 5(b)]. Here we study the first hitting time for the population to reach an average adoption level of 0.5 [e.g., $H_X(0.5)$, when 50 out of a possible 100 agents have adopted the action]. Accumulating data over 5000 numerical simulations, we find that the first hitting time displays heavy tailed behavior: in the majority of simulations, it takes a short time for those 50 agents to take action, but in a few simulations, it takes a very long time. Furthermore, this heavy-tailed behavior is sensitive to the influence parameter

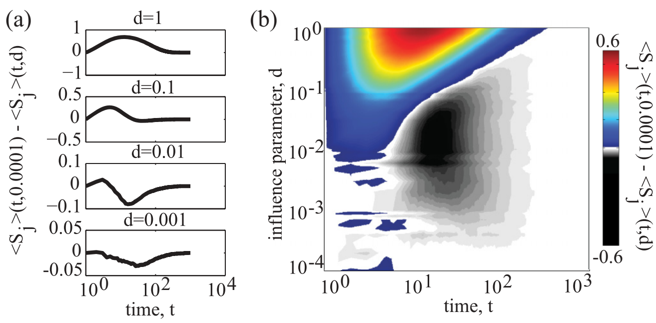

FIG. 7. (Color online) Facilitation and hinderance by the social network in Case 2. (a) The difference between the average information state at $d = 0.0001$ ($\langle S_j(t,0.0001)\rangle$) and that at four other increasing values of the influence parameter showing predominantly hinderance ($d = 1$, top; $d = 0.1$, second row), a combination of hinderance and facilitation ($d = 0.01$, third row), and predominantly facilitation ($d = 0.001$, fourth row) by the social network exchange. (b) The larger surface from which the four curves in (a) are drawn. Here we compare $\langle S_j(t,0.0001)\rangle$ to d ranging in $0.0001 < d < 1$. Color indicates $\langle S_j(t,0.0001)\rangle - \langle S_j(t,d)\rangle$.

d and therefore the strength of the social network. For low values of d, more numerical simulations have shorter first hitting times, which is consistent with what we would expect for situations that approximate Case 1 (global broadcast only). This result is consistent with the fact that the social network retards information state progress.

F. Does the social network help or hinder?

An important tradeoff is evident in Figs. 6(c) and 6(h) for low values of d. When $d = 0.0001$, the average information state trajectory curve rises the fastest initially but then slows for later values of t. In fact, for a slightly larger value of $d = 0.001$, the average information state trajectory curve lags the $d = 0.0001$ initially and then surpasses it later. This result suggests that the social network—if weakly present—can help the entire population take action sooner. However, when the social network becomes stronger (e.g., $d = 0.01$ and $d = 1$), it acts like a cage, keeping information states of all individuals from rising swiftly.

We investigate this behavior more systematically in Fig. 7, where we observe three distinct behaviors: (1) facilitation of information propagation for small rates of social exchange [e.g., small values of the influence parameter d; Fig. 7(a), bottom], (2) hinderance of information propagation for large rates of social exchange [e.g., large values of d; Fig. 7(a), top], and (3) a combination of, first, hinderance and then facilitation for intermediate rates of social exchange [e.g., intermediate values of d; Fig. 7(a), middle]. Transitions between these three distinct behaviors are smooth, as demonstrated in Fig. 7(b).

These results demonstrate that the social network can both hinder and facilitate information state changes and, by extension, action adoption. When the influence of the social network is large, the inertia of popular opinion dampens the effect of the global source attempting to inject new information into the system. On the other hand, when the influence of the social network is small, the added mixing facilitates the dissemination of the new information provided by the global source.

V. DISCUSSION

Our long-term objective is to develop a framework that enables predictive modeling of collective decision dynamics in situations involving multiple sources of information. Modern communication technologies and social networking applications provide fast, global means of information dissemination. The need to determine the impacts of these technologies on individual decisions and by extension collective action make it essential to understand the interplay between multiple information sources.

This paper lays the foundation of such a framework, by systematically exploring a sequence of models that aims to capture tradeoffs and tensions that arise when a global broadcast source competes with information transmission between individual agents. Despite our necessarily simplified scenario, we find that information transmission over the network can either facilitate or hinder action adoption, depending on the relative influence of the global and social information sources. In most situations, the social network acts overall as a damping force, homogenizing opinion states and delaying action adoption in the population.

In this work, we report results from both numerical simulations and analytical solutions. Importantly, this theoretical framework allows us to probe the effects of multiple diffusion mechanisms separately in order to disentangle their relative effects on collective behavior. However, it is also important to compare these theoretical results to recent experimental findings. Two recent studies have measured the relative influence of global and internal mechanisms of information diffusion [69,70]. In both cases, global influences—including exogenous factors and external events—appeared to be greater drivers of behavior than internal social influences, consistent with the theoretical findings reported here.

A. Insights from biology

Biological insights for the model behavior can be obtained by drawing comparisons between the multilayer information system and animal herding behavior [71–73]. The combination of the averaging update rule and the decision threshold forms the mathematical framework for *herding* or *social conformity* [74,75] in the sense that an animal can act based on inferences from the information of other animals. In humans, models of such information diffusion processes are built on a long history of empirical work in sociology known as *diffusion of innovations* [35,76,77].

For a wide range of animals including humans, group decisions to move (for food or travel) often depend on social interactions among group members [71], only a few of whom have pertinent information (e.g, food location or migration route). So-called informed individuals correspond to our global broadcast, while uninformed individuals correspond to our agents on the social network. Unlike work investigating individuals who have no decision preference [73], our update rule hard-codes the fact that agents in our system have a preference for retaining decision states. Such an inertia is consistent with the observation that individual beliefs are continually evolving variables that depend both on past beliefs [78] and newly acquired information, and in particular become less malleable as time passes [79,80].

B. Insights from statistical mechanics

Physical insights for the model behavior can be obtained by drawing an analogy with the nonequilibrium statistical mechanics of a spin system [81,82]. Our global broadcast plays the role of a uniform external field, while the information and adoption state variables of each agent can be thought of as continuous or binary spins on a directed lattice [45]. The initial state corresponds to all spins being initialized at zero, with the external field fixed at unity. The interaction between individual spins and the field is sampled stochastically, leading to a noisy dynamical transition from inaction to action (a global attractor).

Inclusion of the social network corresponds to pairwise, directed interactions between spins on a random lattice [83–85] that compete with the external field. Because the spins are initialized at zero (opposite the field), the social network initially tends to hinder action adoption, and in some cases prevents action adoption entirely. Here inclusion of the network homogenizes the collective behavior because the interaction between spins described by the update rule is intrinsically stabilizing, damping the opinions of outliers back toward the collective.

A familiar characteristic from the statistical physics of spins on a lattice that is not observed in our model is the separation of agents and spins into spatially localized domains characterized by action or inaction [86,87]. Two potential contributing factors are the damping effect of our update rule and the mixing effect of the random social network lattice, both of which inhibit local propagation of injected information. Our preliminary investigations suggest that the update rule is the larger predictor of behavior and therefore we expect that within our general framework, clustering would occur for different broadcast and opinion update rules. Exploiting parallels with well-understood systems in nonequilibrium statistical physics, operations research, and graph theory are likely to provide pathways for systematically unraveling the role of underlying network structure, communication, and influence of collective behavior of populations.

C. Future directions

(a) Update rule. Our focus here is on the collective impact of individual decision making for when to evacuate, rather than the specifics of transportation and routing. We implemented an unbiased rule for opinion updates, in which the weight assigned to state variables is independent of the state value and the time since the last update. This deliberately avoids destabilizing mechanisms promoting microscopic propagation that arise in other contexts, such as the spread of infectious diseases [88]. In the case of evacuations, it is plausible that recent information might be weighted more heavily or travel preferentially along specific paths [89,90] in opinion updates, or an agent with new information might be more likely to share information on the social network. However, inclusion of these effects requires more sophisticated assumptions about the individual agents that must be justified with cognitive and behavioral data. While extracting influence and decision rules from network databases remains a challenging problem in model identification, studies in behavioral psychology, economics, and risk [91,92] might provide useful insights for more realistic representations of how individual opinions are updated and decisions are made in the context of social networks.

(b) Network topology. In this initial investigation, our model construct is deliberately chosen to be generic, abstract, and random, setting a baseline for future work. The global broadcast is accessed at random by individuals, and the topology of the social network is random as well. In most situations, the social network acts overall as a damping force, homogenizing opinion states and delaying action adoption in the community, which is a behavior that has also been identified in experimental studies [93]. However, even in this scenario, there are situations in which the social network accelerates action adoption [94].

In each case, design and optimization could naturally play a role in policy decisions for specific scenarios. For example, the topology of the social network, both in terms of connectivity and rate of information flow, could be based on realistic measurements of network traffic as measured by cellular communication [95], Twitter [96], or Facebook [97]. Or more generally, one could employ mechanistic models to construct networks whose topologies closely match those of empirical social networks. For example, models based on collisions between particles have been shown to match the clustering, degree distribution, degree-degree correlations, and community structure in a variety of friendship and sexual contact networks [94,98,99].

Alternatively, the social network for transmission of information pertaining to action adoption during an evacuation might be chosen to correspond to the geospatial layout of neighborhoods in a community [100–103]. For a given topology, global broadcast rates and/or transmission to individuals could be tailored to the connectivity, and optimized for effectiveness as a guideline for policy. The clustering inherent in such models will likely decrease the cascade effects seen in our data [33,104].

(c) Threshold decisions. Our use of a uniform or random threshold rule for action adoption at the individual level is a traditional starting point used in the decision-making literature [38] to study collective population dynamics. Other factors that might influence decision making could incorporate a time-dependent effect—is an individual more likely to adopt an action when the rate of action adoption is increasing in the population as a whole? Moreover, the coupling between multiple networks—e.g., human decisions and transportation systems used for evacuation—can play a vital role in the collective behavior [105] (e.g., when road congestion prevents the group from evacuating effectively). A fundamental question is whether these and other feedbacks are a cause or an effect of the underlying decision process.

D. Concluding remarks

In a broader context, the systematic development of a framework for understanding the impact of social networks on collective behavior corresponds to the development of a nonequilibrium statistical mechanics for the impact of policy on decision making in populations. Historically, many fields in the social and life sciences have taken a phenomenological approach to estimating the impact of deliberate policy and other externalities on the behavior of populations. In this context, the study of social networks corresponds to an underlying statistical mechanics for the collective behavior, and must be understood systematically to obtain predictive behavior of the population as well as a characterization of the variability within the population.

It is increasingly recognized, across a broad range of fields, that understanding network phenomena is essential to characterizing behavior of the system as a whole. In the specific context of social systems, interactions between individuals can for example give rise to financial crashes [106], political revolutions [107], successful technologies [108], and cultural market sensations [109]. Constructing a statistical mechanics for these problems is only a starting point. Feedback, design, optimization, and robustness are all critical ingredients, mandating an interdisciplinary approach to developing a reliable, predictive framework that is useful for policy. Policy issues related to individual decisions, including the impact of training, identification of leaders, and signatures of stress and fatigue, will be important topics of future research.

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