Referring to the social performance promotes cooperation in spatial prisoner's dilemma games

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We propose a new pairwise Fermi updating rule by considering a social average payoff when an agent copies a neighbor's strategy. In the update rule, a focal agent compares her payoff with the social average payoff of the same strategy that her pairwise opponent has. This concept might be justified by the fact that people reference global and, somehow, statistical information, not local information when imitating social behaviors. We presume several possible ways for the social average. Simulation results prove that the social average of some limited agents realizes more significant cooperation than that of the entire population.

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I. INTRODUCTION

The emergence and maintenance of cooperation have attracted considerable attention from natural and social disciplines [1,2]. In order to understand the survival of cooperation, a theoretical framework that has shed light onto this long-standing issue is the evolutionary game theory [3,4]. In particular, one of the most fascinating models is the prisoner's dilemma game where two players simultaneously decide to adopt one of two strategies: cooperation (C) and defection (D). When a population of players has interaction in the well-mixed case, cooperation soon disappears. Over the past few decades, a great number of scenarios has been identified that can offset an unfavorable outcome of social dilemmas and can lead to the evolution of cooperation [5–9]. Whereas, Nowak attributed all these to five mechanisms: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [10], these mechanisms can be somewhat related to the reduction of an opposing player's anonymity relative to the so-called well-mixed situation.

Among the five mechanisms, network reciprocity, where players are arranged on the spatially structured topology and interact only with their direct neighbors, has attracted the greatest interest (for comprehensive reviews refer to Refs. [11–16]) because cooperators can survive by means of forming compact clusters, which minimize the exploitation by defectors and protect those cooperators that are located in the interior of such clusters. In addition, the strategy updating rule and dynamics on spatial topology also play a key role in the evolution of cooperation. Typically, players can alter their strategies according to pairwise Fermi, pairwise linear, imitation max, roulette selection, or others [17–21]. To understand what creates the influence of network reciprocity, Yamauchi *et al.* [17,18] and Wen *et al.* [22] performed a full factorial design of experiments for a comprehensive factorial analysis focusing on network reciprocity and qualitatively found that strategy update rules as well as update dynamics (synchronous or asynchronous) possessed more influence on network reciprocity than that of network topology alone. Following these seminal papers, Tanimoto attempted to answer the question of what is substantially important when we say network reciprocity [23]. He implied that allowing strategy adaptation speed slower than gaming speed can be considered as the substance of network reciprocity. For example, Tanimoto showed that, when the prisoner's dilemma game on the scale-free (SF) network with a spatial distribution of the strategy updating the time scale was assumed, a negative correlation between degree and strategy updating speed caused an extremely large cooperation-enhancing effect. This occurs because a cooperative hub agent, who is insensitive to strategy adaptation, can protect herself from copying neighboring defection for several initial time steps of each simulation episode, leading to survival and growth of the cooperators' cluster (C cluster).

Concerning the most studied pairwise-Fermi (PW-Fermi) process where a focal player adopts the strategy of a randomly chosen neighbor based on the difference of accumulated payoffs, Wang and Perc [24], Perc and Wang [25], and Tanimoto *et al.* [26] found a significant enhancement effect for cooperation when assuming the simple rule in which a higher payoff neighbor was chosen as a pairwise opponent instead of random selection. This framework makes a cooperator (who is on the border of cooperation and defection clusters) insensitive to copy defection because the richer pairwise opponent (perhaps one of neighboring cooperators who collects a high payoff through mutual cooperation with her cooperative neighbors) prevents him from changing from C to D. Meanwhile, other scholars [27–29] reported that when the copying probability directly attenuated through the so-called "letting learning activity level decrease," the evolution of cooperation could be guaranteed. Evidently, the factor that improves cooperation in those models is that the slower strategy adaptation speed rather than gaming speed causes less frequent copies of defection.

Observing the social learning attitude in human society, we know that the individual becomes reluctant to learn other attitudes when he is satisfied with his current situation. With respect to this point, we previously proposed a model for the PW-Fermi adaptation process considering copy resistance [30] and reported that it enhanced cooperation more significantly than the usual PW-Fermi update rule. The model took account

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of both payoff differences between a focal player and the randomly selected opponent and between the focal player and the social average. The central idea of that model is that a focal player obtaining a relatively richer payoff than the social average has less incentive to copy her opponent's strategy.

Incidentally, in the usual PW Fermi, a focal agent compares her payoff with that of one of her neighbors, which has been widely accepted and unquestioned. Observing realistic human social networks, we can find many proofs that people refer to not local information but the global and, perhaps, statistical information, provided by media, for example. Here, we demonstrate a plausible model to reproduce this concept. Simulation results reveal that our model, based on the revised PW-Fermi principle, significantly enhances cooperation in the prisoner's dilemma game.

II. MODEL

We evaluate network reciprocity in the prisoner's dilemma game played on a static network with the assumption of pairwise strategy adaptation, described as follows.

A. The 2 × 2 game

We consider a 2×2 game as an archetype. Each player can adopt one of the two strategies cooperation (C) or defection (D). Players are rewarded (R) for mutual cooperation and are punished (P) for mutual defection. If one player chooses C and the other D, the latter receives a temptation payoff (T), and the former is labeled as a saint (S). According to the seminal idea [31], we define the stag-hunt-type dilemma as $D_r = P - S$ and the chicken-type dilemma as $D_g = T - R$, and the payoffs can be rescaled such that $R = 1$ and $P = 0$. In this situation, the payoff matrix can be given as

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}, \tag{1}$$

where the elements are assumed to satisfy $T > R > P > S$ and $2R > T + S$. We limit the prisoner's dilemma game class by assuming $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$.

B. Network

As the interaction network, we use either a regular lattice with a periodic boundary condition and four nearest neighbors or the SF network with an average degree of 4 (i.e.,$\langle k \rangle = 4$) generated via the Barabási-Albert algorithm [32]. The total number of agents is $N = 4900$.

C. Strategy updating

The game is iterated forward in accordance with the sequential simulation procedure comprising the following elementary steps. First, player i acquires its payoff π_i by playing the game with all its neighbors. Then, in the same way, we evaluate the payoffs of all the neighbors of player i. Last, player i randomly selects a neighbor j and adopts the strategy s_j of neighbor j with the probability (dependent on the difference of payoffs),

$$P_{s_i \to s_j} = \frac{1}{1 + \exp\left[\frac{\pi_i - \langle \pi \rangle}{\kappa}\right]}, \tag{2}$$

$$\langle \pi \rangle = \begin{cases} \langle \pi_C \rangle, & \text{if } s_j = C, \\ \langle \pi_D \rangle, & \text{if } s_j = D, \end{cases} \tag{3}$$

where $\langle \pi_C \rangle$ and $\langle \pi_D \rangle$ indicate average payoffs of all the cooperators and defectors in a defined sample set, respectively, and κ denotesthe amplitude of noise. $\kappa = 0$ and $\kappa \to \infty$ denote the completely deterministic and completely random selections of the neighbor's strategy, whereas, for any finite positive values, κ incorporates the uncertainties in the strategy adoption. As a previous setting [28], we simply fix the value of κ to be $\kappa = 0.1$ in the present paper. Moreover, it is evident that the focal player i refers to the average payoff of the strategy of his opponent rather than his accumulated payoff. This seems acceptable that the average payoff can show a better social performance at the current moment. With respect to the definition of the sample set, one may think that the entire population is the simplest idea, which implies $\langle \pi_C \rangle$ and $\langle \pi_D \rangle$ represent the total average of all the cooperators and defectors, respectively. In practice, however, it is difficult to obtain accurate statistical data covering the entire population in the real world. Thus, we presume two types of ways: One is the "neighborhood case," and the other is the "limited sample case." The first idea is to take averaged $\langle \pi_C \rangle$ and $\langle \pi_D \rangle$ among the immediate neighborhood, the second neighborhood, or the third neighborhood. Especially, when determining the social average payoffs of cooperators and defectors with the second and third neighborhoods, all individuals, until the second (or third) neighbors, are involved in the simulations. This idea comes from actual situations where we are able to obtain statistical information about people who live in our neighborhood. As the second idea, we take randomly sampled agents for drawing $\langle \pi_C \rangle$ and $\langle \pi_D \rangle$. For example, we can take 10%, 1%, or an arbitrary sample ratio of the entire population. This might be justified because we can know some statistical information for "typical people," who are randomly selected from a mother population.

D. Simulation setting

Initially, an equal percentage of strategies cooperation (C) and defection (D) are randomly distributed among players allocated on different vertices of the network. Then, several generations are run until the frequency of cooperation arrives at quasiequilibrium. If the frequency of cooperation continues to fluctuate, we obtain it for the final 1000 generations over the total 10 000 generations. From the viewpoint of statistical robustness, we take 100 ensemble averages where 100 simulation realizations give the average value for each parameter setting. In the simulation, the agents update their strategies synchronously. We focus primarily on the cooperation fraction in the following discussion.

III. RESULTS AND DISCUSSION

A. Enduring and expanding periods

For the sake of the following discussion, we define the terminology as the enduring (END) period and the expanding

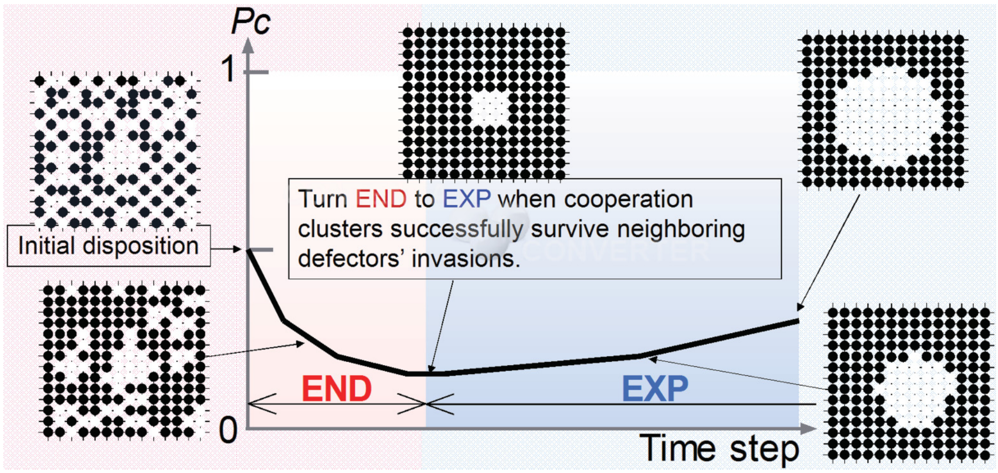

FIG. 1. (Color online) Schematic for the evolution of the spatial prisoner's dilemma game with the concept of END and EXP of the fraction of cooperators P_C. END period: initial cooperators are rapidly plundered by defectors, which cause only a few cooperators to be left through forming compact C clusters. EXP period: C clusters start to expand since a cooperator on the clusters' border can attract a neighboring defector into the cluster.

(EXP) period as shown in Fig. 1. In a typical evolution process where the initial value of the cooperation fraction is 0.5, there usually are two periods: The first one features the rapid decrease in cooperation, whereas, the following period is along the increasing cooperation level as long as the evolutionary trail is not absorbed by the all-defectors state during the initial period. In our paper, the first is the so-called END period because cooperators try to endure defectors' invasion (or cooperators avoid copying defection from neighbors). Correspondingly, we call the other the EXP period since cooperators who successfully survive in the END period by forming cooperative clusters (C clusters) expand their area by converting defectors into cooperators.

B. Result of the fundamental case

Figure 2 shows the average cooperation fraction in the entire region of the prisoner's dilemma game on the lattice network (A,B) and the scale-free network (C,D) where the traditional pairwise-Fermi process (B,D) and the proposed model (A,C) are taken into account. In this section, we assume that $\langle \pi_C \rangle$ and $\langle \pi_D \rangle$ are social averages for all the cooperators and defectors over the whole population. Figure 3 illustrates the time evolution of the cooperation fraction. It is clearly shown, compared to the case of the traditional version, that the frequency of cooperators in the proposed model is higher on the lattice network.

We can explain the cause of this enhancement mechanism on the lattice network as follows. In the proposed model, the agent compares his own payoff with the social average payoff of cooperators or defectors, which implies that the strategy updating speed may largely differ by the strategy of a selected pairwise opponent. When the pairwise opponent is a cooperator and lies on the C clusters, he obtains R through each interaction with other cooperators. As a result, the cooperator agents' social average payoff increases, which makes the defectors among the cooperator neighborhood tend to change the current strategy. On the contrary, when the pairwise opponent is a defector and lies on the defective clusters, he can only get P from mutual defection. This makes the defectors' social average payoff decrease. Thus, cooperators within the neighborhood of defectors find it impossible to copy defection.

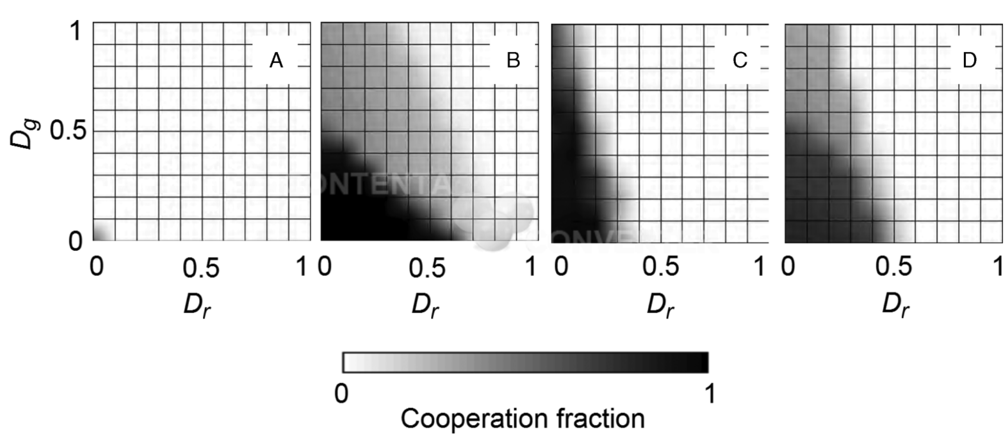

FIG. 2. Average cooperation fraction for the traditional model (A,C) and proposed model (B,D) within the limit of $D_g \in [0,1]$, $D_r \in [0,1]$. Games are played on a lattice network (A,B) and a scale-free network (C,D) with $\langle k \rangle = 4$.

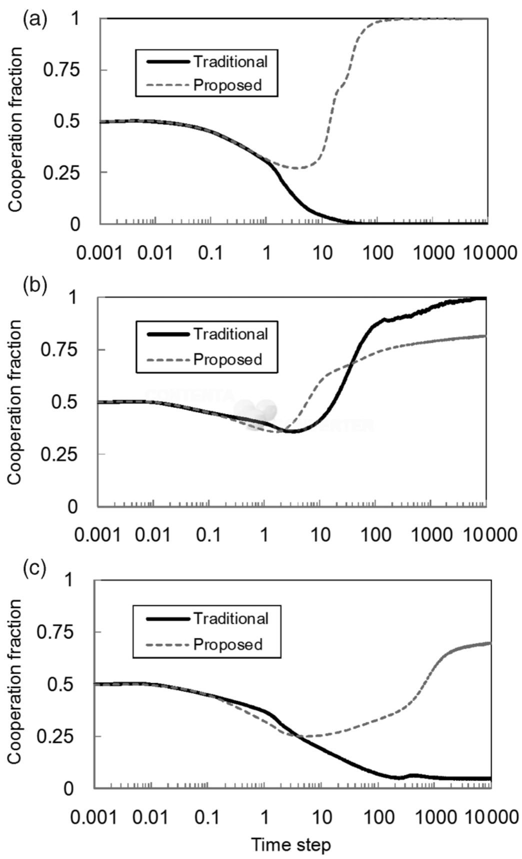

FIG. 3. Time evolution of the cooperation fraction when assuming (a) the lattice network $D_g = D_r = 0.1$, (b) the scale-free network, weaker dilemma $D_g = D_r = 0.1$, and (c) the scale-free network, stronger dilemma $D_g = D_r = 0.3$. Black solid lines show the traditional model, and gray dashed lines show the proposed model case, respectively. Each of the time evolution curves indicates an ensemble average of 100 realizations. We confirmed small standard deviations for each of those ensemble averages. This means the proposed case in (c), for example, has a polymorphic equilibrium where the cooperators and defectors coexist, not bistable equilibrium.

Those two situations jointly lead to the fact that constructing clusters brings more benefits to cooperators than defectors, namely, C clusters can easily survive and expand. Observing the time evolution of the cooperation fraction in Fig. 3(a), we note that cooperators in the traditional model cannot escape from the defectors' invasion during the END period. On the other hand, cooperators, who successfully construct compact C clusters in the proposed model, can survive in the END period and can show a rapid increase in the cooperation level during the EXP period. By the way, when the asynchronous update rule is assumed, the cooperation level is lower than that of the synchronous update rule. We can explain the cause of this deterioration mechanism on the lattice network with an asynchronous update as follows. In a typical evolution process, it is important to expand the C clusters after the END period. In the EXP period with stochastic update rules, so to speak, the asynchronous update boundaries between C clusters and D clusters are disturbed. These irregular boundaries help D agents to get T. Hence, when the synchronous update rule is assumed, C clusters are able to expand in unison, on the other hand, the asynchronous update rule is assumed, and C clusters cannot expand in unison.

Next, let us examine the results on the scale-free network. The proposed strategy adaptation process shows less cooperation than the traditional model does in the weaker dilemma region, although there exists more cooperation in the relatively stronger dilemma area (that indicates the proposed model brings a larger critical dilemma strength than the traditional model). This seems slightly different from the tendency of the homogeneous network (lattice) case [see Fig. 3(a)]. By means of our proposed PW-Fermi process, the adaptation speed of the agents who obtain more payoff than the social average of the opponent's strategy becomes slow, which makes the change in the strategy more difficult. When the game is implemented on a heterogeneous network (such as, a scale-free network), highly linked agents (hub agents) earn more than the social average because they engage in more games. Therefore, hub cooperators hardly copy defection, which means a reasonable number of cooperators around those hub cooperators can survive even in a stronger dilemma environment [Fig. 3(c)]. Similarly, hub defectors hardly copy cooperation, which means that large numbers of defectors do not convert to cooperators even in a weaker dilemma situation where the traditional PW-Fermi model can convert all the defectors into cooperators [Fig. 3(b)].

C. Result of neighborhood cases

Figure 4 shows the average cooperation fraction in the entire region of the prisoner's dilemma for the neighborhood cases. On the lattice network, it is noteworthy that the increase in neighborhood cases exhibits a significant enhancement for cooperation, despite less sensitivity on the scale-free network.

When presuming the immediate neighborhood on the lattice network, a cooperator with defective neighbors tends to change

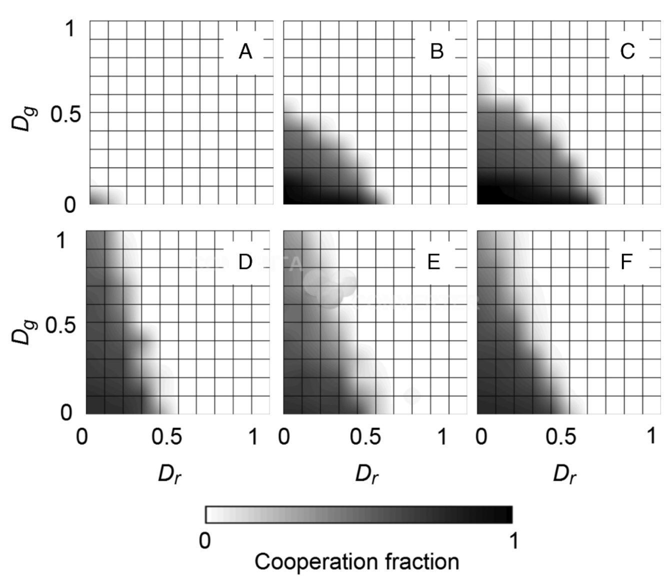

FIG. 4. Average cooperation fraction for the neighborhood case. From A to C (and from D to F), the neighborhoods are varying from first, second, and third within the limit of $D_g \in [0,1]$, $D_r \in [0,1]$. Games are played on a lattice network (A,B,C) and a scale-free network (D,E,F) with $\langle k \rangle = 4$.

his strategy because the defectors around this cooperator take a relatively higher payoff by obtaining T and imposing S on him. This inevitably comes to an end with a meager cooperation level. In more than the second neighborhood cases, the neighborhood area on a cooperator-defector border contains several cooperators who construct a C cluster with obtaining R as well as several defectors who gain P. This particular situation is beneficial for emerging cooperation because cooperators hardly copy defection and defectors easily copy cooperation. In the scale-free network, the irregularity of the topology usually cannot produce regular clusters, such as the lattice network case. Therefore, although C clusters are made, these members are not only C agents, but also D agents, which makes the boundaries of C clusters and D clusters not legible enough. In such a case, C agents existing in the C cluster cannot get only R, and D agents existing in the D cluster get both T and P. Naturally, there is not a big difference between the first neighbors' social average payoff and the second (third) neighbors' case.

Moreover, we think that clustering coefficients are related to the cooperation fraction when determining the neighborhoods. In general, low connectivity cases are an advantage to cooperation enhancement, although in this case, this statement does not always apply to the range of strategy updating. Hence, spreading the scope of updating, that is, increasing the clustering coefficient, plays an important role in cooperation enhancing, especially in the homogeneous network case.

D. Result of limited sample cases

Figure 5 shows the average cooperation fraction in the entire region of the prisoner's dilemma game for the limited sample cases. Compared with the cases of the lattice network, we note that lower sampling ratios (such as, 0.05 and 0.01) exhibit significant enhancement for cooperation, although a too small ratio looks slightly counterproductive. Moreover, some stronger dilemma regions (highlighted by the circle) singularly exhibit higher cooperation than the region with a relatively weaker dilemma.

Let us first discuss why the relatively lower sampling ratio enhances cooperation. Figure 6(a) shows the time evolution of the cooperation fraction, average payoffs of cooperators and defectors for the sampling ratio 0.05 ($\langle\pi\rangle_{C\_0.05}$ and $\langle\pi\rangle_{D\_0.05}$) as well as the entire population ($\langle\pi\rangle_{C\_\mathrm{all}}$ and $\langle\pi\rangle_{D\_\mathrm{all}}$) when assuming $D_g = 0.8$, $D_r = 0.2$. We see that, after the END period, $\langle\pi\rangle_{C\_0.05}$ significantly oscillates. When this oscillation makes $\langle\pi\rangle_{C\_0.05}$ exceed $\langle\pi\rangle_{C\_\mathrm{all}}$, the defector possessing cooperative neighbor's C agent tends to change his strategy. At the same time, the oscillation does not produce any influence on the cooperator with defective neighbors because what the cooperator refers to is $\langle\pi\rangle_{D\_0.05}$ rather than $\langle\pi\rangle_{C\_0.05}$. That is, cooperator agents are not affected under smaller $\langle\pi\rangle_{C\_0.05}$. Thus, this asymmetric effect gives chances of converting defection to cooperation.

Next, we discuss why more cooperation can be observed in the stronger dilemma than in the weaker dilemma environment. Figure 7 presents some typical snapshots for sampling ratio 0.05 when assuming $D_g = 0.2$, $D_r = 0.7$ (upper panel) and $D_g = 0.2$, $D_r = 0.8$ (bottom panel). During the END period, obviously, there is more C clusters' survival in the weaker dilemma case ($D_r = 0.7$) than in the stronger dilemma case ($D_r = 0.8$). But concerned with the EXP period, we see that, in the case of the weaker dilemma, the surviving C clusters stop growing and finally reach a relatively meager cooperation level [Fig. 7(d)], whereas, in the stronger dilemma case, the surviving C clusters (despite the lower number) can expand and can even reach a higher cooperation level at equilibrium

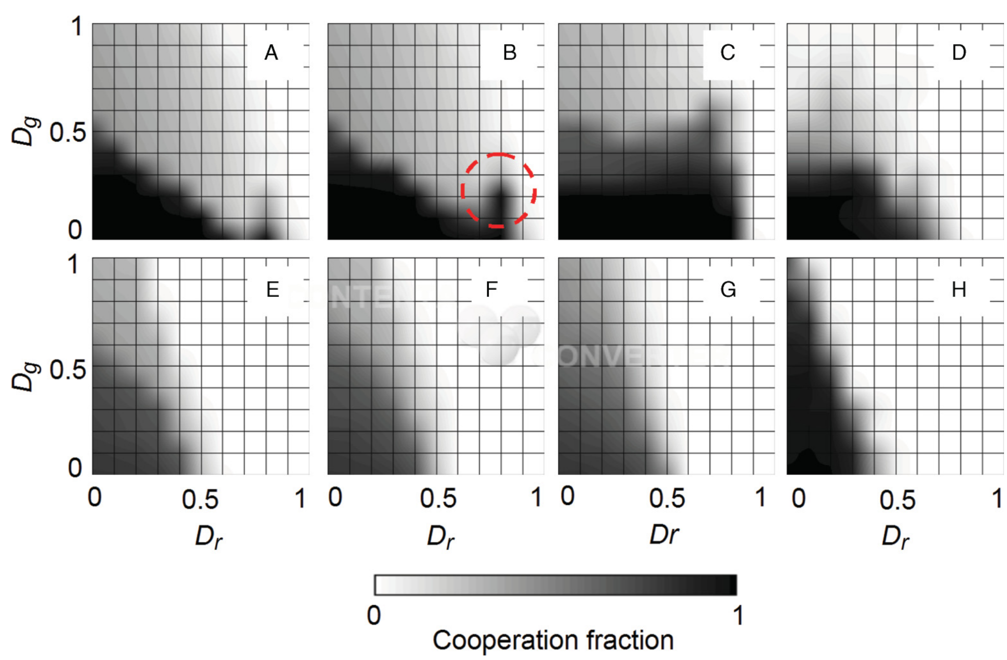

FIG. 5. (Color online) Average cooperation fraction for the limited sample case. From A to D (and from E to H), the sampling ratios are 0.1, 0.05, 0.01, and 0.001 within the limit of $D_g \in [0,1]$, $D_r \in [0,1]$. Games are played on a lattice network (A, B, C, and D) and a scale-free network (E, F, G, and H) with $\langle k\rangle = 4$.

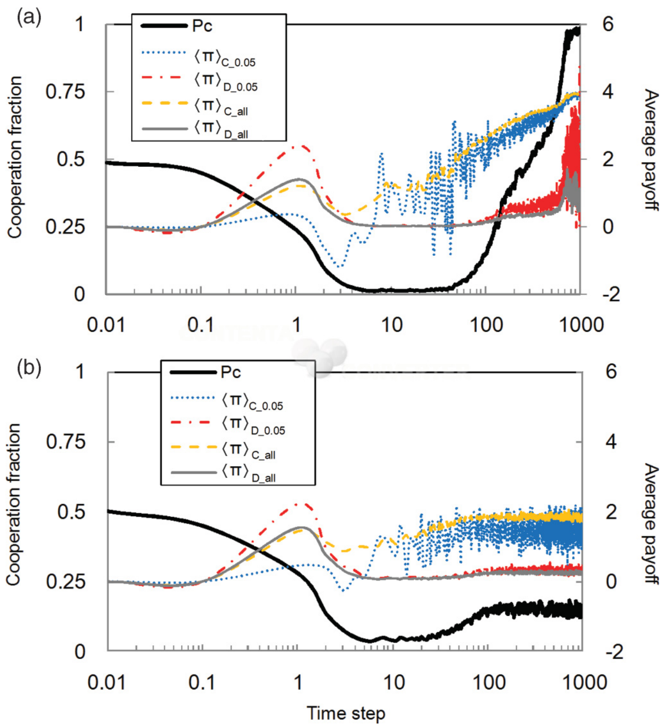

FIG. 6. (Color online) The time evolution of the black solid line: cooperation fraction; blue dotted line: sampled cooperators' average payoff; red dashed-dotted line: sampled defectors; orange dashed line: average payoff of all the cooperators; and gray solid line: all the defectors when presuming (a) the stronger dilemma $D_r = 0.8$, $D_g = 0.2$ and (b) the weaker dilemma $D_r = 0.7$, $D_g = 0.2$, respectively. We assume the sampling ratio of the total population to be 0.05.

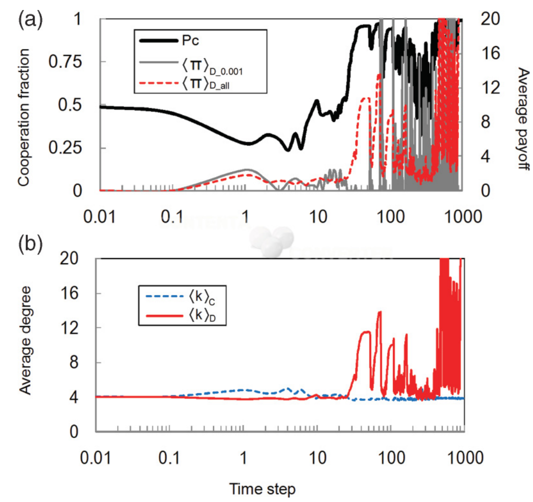

FIG. 8. (Color online) (a) Black solid line P_C: the time evolution of the cooperation fraction; gray solid line: sampled D agents' average payoff; and red dashed line: all defectors. (b) Blue dashed line: the average degrees of both sampled cooperators and red solid line: defectors when presuming $D_g = D_r = 0.2$.

[Fig. 7(h)]. Returning to Figs. 6(a) and 6(b), we also notice that the oscillation of $\langle \pi \rangle_{C\_0.05}$ of the stronger dilemma case (A) instantaneously exhibits higher values around the ten steps than that of the weaker dilemma case (B). This is because a relatively smaller number of C clusters, with a larger size in the case of the stronger dilemma case, can make more cooperators belong to the C clusters where they have less opportunities to be exploited by defectors.

Lastly, it is interesting to explore why we see a higher cooperation fraction with the smallest sampling ratio on the scale-free network [Fig. 5(h)]. In the case of the sampling ratio being 0.001, the number of selected agents is less than 5 ($4900 \times 0.001 = 4.9$), implying the statistical sampling size is very small. Meanwhile, the scale-free network has a power-law distribution of degree, which makes the majority of agents have smaller degrees. Therefore, sampled agents can only gain a limited payoff with a high possibility. Figure 8(a) illustrates the time evolution of the cooperation fraction and average payoffs of defectors ($\langle \pi \rangle_{D\_0.001}$ and $\langle \pi \rangle_{D\_\mathrm{all}}$), and Fig. 8(b) shows

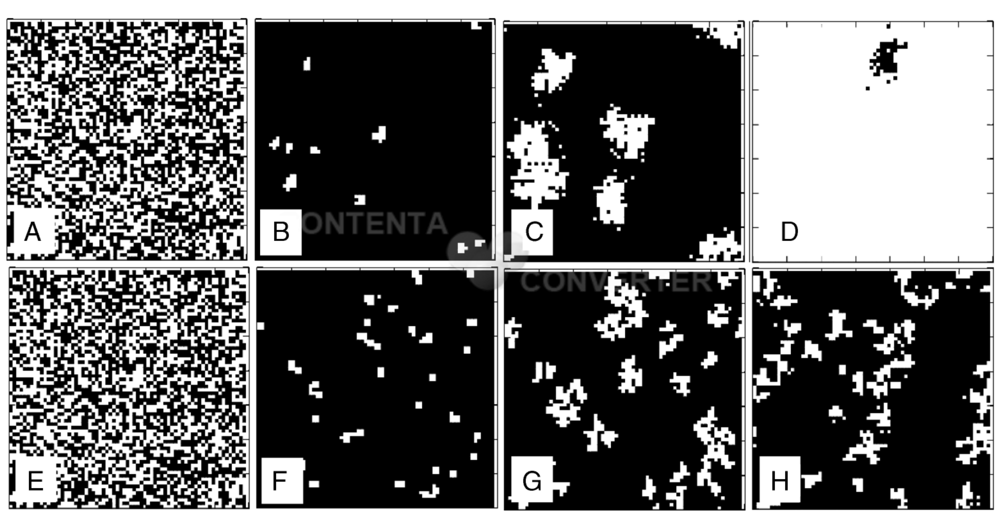

FIG. 7. White: snapshots of cooperators and black: defectors for two different dilemma strengths. From A to D, the corresponding time steps are 0, 10, 100, and 1000 steps ($D_r = 0.8$ and $D_g = 0.2$), respectively. From E to H, we show consistent time steps with the upper panel ($D_r = 0.7$, $D_g = 0.2$). The sampling ratio is 0.05 on a 70×70 lattice.

the time evolution of the average degrees of both sampled cooperators and defectors. In the END period, the cooperation fraction shows the bottom, which indicates defectors play games with defectors and obtain P. This situation makes $\langle \pi \rangle_{D\_0.001}$ quite small (nearly equaling zero). As a result, the few cooperators who successfully construct the C cluster can obtain R. However, the trend will alter in the subsequent EXP period. The defectors have smaller degrees, and cooperator neighbors gradually change their strategies, which makes the C clusters expand. This is why we can see both average degrees of sampled defectors and $\langle \pi \rangle_{D\_0.001}$ coherently increase with an increase in cooperation fraction. After hundreds of time steps, the remaining defectors are relatively large degree agents. In this manner, a smaller sampling ratio with a scale-free network realizes a high cooperation level.

IV. CONCLUSION

To emulate a more plausible strategy adaptation mechanism observed in human society, we established a new concept for the PW-Fermi process where a focal agent refers to the social average payoff of the strategy that her opponent has instead of her opponent's payoff. A series of simulations reveals that the proposed model creates an evident effect on the evolution of cooperation, especially when the lattice is assumed as an underlying topology.

In a heterogeneous network, however, only limited enhancement can be observed because the proposed model works counterproductively under a weaker dilemma by producing many stubborn defectors who reduce the cooperation level at a coexisting equilibrium.

Furthermore, we studied two possible ways to consider the social average: taking the average of relatively close agents in her neighborhood (neighborhood case) and taking the average of a limited number of randomly sampling agents from the whole population (limited sample case). For the neighborhood case on the homogeneous network, it is noteworthy that more than the second neighborhood settings exhibit significant enhancement for cooperation, although the heterogeneous network shows less sensitivity to the range of neighborhoods. As for the limited sample case, it is notable that the lower sampling ratio exhibits significant enhancement for cooperation.

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[1] S. J. Maynard and E. Szathmary, Nature (London) **374**, 227 (1995).
[2] J. W. Weibull, *Evolutionary Game Theory* (MIT Press, Cambridge, MA, 1995).
[3] M. A. Nowak, *Evolutionary Dynamics* (Harvard University Press, Cambridge, MA, 2006).
[4] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).
[5] Z. H. Rong, X. Li, and X. Wang, Phys. Rev. E **76**, 027101 (2007).
[6] G. Szabó and C. Tőke, Phys. Rev. E **58**, 69 (1998).
[7] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992).
[8] M. H. Vainstein and J. J. Arenzon, Phys. Rev. E **64**, 051905 (2001).
[9] B. Wen, B. Xian, and B. Mao, Physica A **388**, 5005 (2009).
[10] M. A. Nowak, Science **314**, 1560 (2006).
[11] G. Szabo and G. Fath, Phys. Rep. **446**, 97 (2007).
[12] M. Perc and A. Szolnoki, BioSystems **99**, 109 (2010).
[13] J. Gómez-Gardeñes, M. Campillo, L. M. Floría, and Y. Moreno, Phys. Rev. Lett. **98**, 108103 (2007).
[14] Z. Wang, A. Szolnoki, and M. Perc, Sci. Rep. **2**, 369 (2012).
[15] S. Meloni, A. Buscarino, L. Fortuna, M. Frasca, J. Gomez-Gardeñes, V. Latora, and Y. Moreno, Phys. Rev. E **79**, 067101 (2009).
[16] W. Shijun, S. S. Mate, Z. Changshui, and S. Peter, PLoS ONE **3**, e1917 (2008).
[17] A. Yamauchi, J. Tanimoto, and A. Hagishima, BioSystems **102**, 82 (2010).
[18] A. Yamauchi, J. Tanimoto, and A. Hagishima, BioSystems **103**, 85 (2011).
[19] Z. Wang, A. Szolnoki, and M. Perc, Phys. Rev. E **85**, 037101 (2012).
[20] P. R. Carlos, A. C. Jose, and S. Angel, Phys. Life Rev. **6**, 208 (2009).
[21] Y. X. Cheng, Q. M. Zhi, L. W. Yi, S. W. Jin, and Q. C. Zhen, Physica A **390**, 4602 (2011).
[22] B. D. Wen, B. C. Xian, Z. Lin, and B. H. Mao, Physica A **388**, 4509 (2009).
[23] J. Tanimoto, Sociobiology **58**, 315 (2011).
[24] Z. Wang and M. Perc, Phys. Rev. E **82**, 021115 (2010).
[25] M. Perc and Z. Wang, PLoS ONE **5**, e15117 (2010).
[26] J. Tanimoto, M. Nakata, A. Hagishima, and N. Ikegaya, Physica A **391**, 680 (2011).
[27] X. Chen, F. Fu, and L. Wang, Int. J. Mod. Phys. C **19**, 1377 (2008).
[28] A. Szolnoki and G. Szabo, Europhys. Lett. **77**, 30004 (2007).
[29] H. Zhang, M. Small, H. Yang, and B. Wang, Physica A **389**, 4734 (2010).
[30] K. Shigaki, S. Kokubo, J. Tanimoto, A. Hagishima, and N. Ikegaya, Europhys. Lett. **98**, 40008 (2012).
[31] J. Tanimoto and H. Sagara, BioSystems **90**, 105 (2007).
[32] A. L. Barabasi and R. Albert, Science **286**, 509 (1999).