Effect of external fields in Axelrod's model of social dynamics

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(Received 14 May 2012; published 21 September 2012)

The study of the effects of spatially uniform fields on the steady-state properties of Axelrod's model has yielded plenty of counterintuitive results. Here, we reexamine the impact of this type of field for a selection of parameters such that the field-free steady state of the model is heterogeneous or multicultural. Analyses of both one- and two-dimensional versions of Axelrod's model indicate that the steady state remains heterogeneous regardless of the value of the field strength. Turning on the field leads to a discontinuous decrease on the number of cultural domains, which we argue is due to the instability of zero-field heterogeneous absorbing configurations. We find, however, that spatially nonuniform fields that implement a consensus rule among the neighborhood of the agents enforce homogenization. Although the overall effects of the fields are essentially the same irrespective of the dimensionality of the model, we argue that the dimensionality has a significant impact on the stability of the field-free homogeneous steady state.

DOI: 10.1103/PhysRevE.86.031131 PACS number(s): 05.50.+q, 87.23.Ge, 89.75.Fb

I. INTRODUCTION

Axelrod's model of social dynamics was introduced to explore the mechanisms behind the persistence of cultural differences in a society [1]. The agents are represented by strings of cultural features of length F, where each feature can adopt a certain number q of distinct traits. The interaction between any two agents takes place with probability proportional to their cultural similarity, i.e., proportional to the number of traits they have in common. The analysis of this model by the statistical physics community has revealed a rich dynamic behavior with a nonequilibrium phase transition separating the culturally heterogeneous from the culturally homogeneous regime [2–4].

An interesting characteristic of Axelrod's model, which sets it apart from most lattice models that exhibit nonequilibrium phase transitions [5], is that all stationary states of the dynamics are absorbing states, i.e., the dynamics always freezes in one of these states [2]. In fact, according to the rules of Axelrod's model, two neighboring agents who do not have any cultural trait in common can not interact, and the interaction between agents who share all their cultural traits does not result in any change. Hence, at the stationary state, we can guarantee that any pair of neighbors are either identical or completely different regarding their cultural features. This allows us to easily identify the stationary regime, which is a major problem in the characterization of nonequilibrium phase transitions [5]. The problem, however, is that the dynamics can take a very large time to freeze to a homogeneous configuration for some initial conditions [2,6], which is the main reason there are so few numerical estimates of the transition lines of the phase diagram of Axelrod's model [4].

From the perspective of the statistical physics, the appealing feature of Axelrod's model in a lattice of dimension d is the existence of a threshold value $q_c = q_c(F)$ below which the stationary regime is monocultural (i.e., spatially homogeneous) and above which multicultural (i.e., spatially heterogeneous). This result holds true for both the two-dimensional [2,4] and the one-dimensional [7,8] versions of Axelrod's model. We recall that the sources of disorder in this model are the stochastic update sequence and the choice of the initial configuration: it is the competition between the disorder of the initial configuration and the ordering bias of the local interactions that is responsible for the nontrivial threshold phenomenon.

The introduction of an external global field to influence the agents' beliefs aiming at modeling the effect of the mass media [9] resulted in a surprisingly difficult problem since the external field favored the heterogeneous instead of the homogeneous regime as one would naively expect. In addition, a considerable amount of effort has been devoted to searching for a threshold on the intensity of the media influence such that above that threshold, the community is multicultural and below it, the community is monocultural (see, e.g., [10–13]). We have shown, however, that this threshold is an artifact of finite lattices, and that even a vanishingly small media influence is sufficient to produce cultural diversity in a region of the parameter space where the homogeneous regime is dominant in the absence of the media [14,15].

In this paper, we address another curious finding regarding the effect of the media (or external field) in Axelrod's model: if the control parameters are such that the stationary regime is multicultural at zero field, then turning the field on will lead to a homogeneous state in the limit of vanishingly small field intensity [11]. Here, we argue that the multicultural regime remains multicultural, though with a reduced cultural diversity, regardless of the intensity of the global external field. By global field we mean a field that is spatially uniform (i.e., it is the same for all agents), although not necessarily time independent [9].

Over and above the reexamination of previous results on the effect of external fields in Axelrod's model, in this paper we show that the effect of the media in the one-dimensional model is qualitatively identical to the two-dimensional model. Since simulations of the one-dimensional model are fast, we will use their results as clues to expose the properties of the stationary state of the two-dimensional model in the computationally prohibitive regime of large lattices.

The paper is organized as follows. In Sec. II, we briefly present the basic elements of Axelrod's model and describe the different types of external global fields (media) we study in this paper. In Sec. III, we show the results of the simulations

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for the one-dimensional model in the two cases of interest, namely, when the media-free absorbing configurations are homogeneous and when they are heterogeneous. In the first case, we show that, similarly to the two-dimensional version [14,15], even a vanishingly small field intensity is sufficient to break up the homogeneous steady state. In the second case, we show that the external field reduces the number of cultural domains, but the steady state remains heterogeneous, regardless of the field strength. This conclusion is corroborated by the simulations of the two-dimensional model described in Sec. IV. A different type of field, a spatially nonuniform field that implements the consensus or majority rule among the neighborhood of the agents, is discussed in a separate section because it can not be interpreted as a media and produces results completely distinct from the spatially uniform fields (see Sec. V). Finally, in Sec. VI, we summarize our main findings and present our concluding remarks.

II. MODEL

In the original formulation of Axelrod's model [1], which we will adhere to here, each agent is characterized by a set of F cultural features which can take on q distinct values. In the two-dimensional version, the agents are fixed in the sites of a square lattice of linear size L with free boundary conditions (i.e., agents in the corners of the lattice interact with two neighbors, agents in the sides with three, and agents in the bulk with four nearest neighbors), whereas in the one-dimensional variant, the agents are fixed in the sites of a chain of length L with the same boundary conditions. From our perspective, the advantage of using free boundary conditions is the ease of implementing the Hoshen and Kopelman algorithm for counting the number of clusters in a lattice [16], but since we are interested in the properties of the steady state for very large lattice sizes, the choice of the boundary conditions is largely irrelevant for Axelrod's model.

The initial configuration is completely random with the features of each agent given by random integers drawn uniformly between 1 and q. At each time, we pick an agent at random, i.e., the target agent, as well as one of its neighbors. These two agents interact with probability equal to their cultural similarity, defined as the fraction of common cultural features. An interaction consists of selecting at random one of the distinct features, and making the selected feature of the target agent equal to its neighbor's corresponding trait. This procedure is repeated until the system is frozen into an absorbing configuration. Axelrod's model can be viewed as F coupled voter models since it features the two main ingredients of a voter model, namely, (i) given a voter (target agent) pick a neighbor at random and (ii) the voter assumes the opinion of the neighbor [17].

The introduction of an external field or global media in the standard model follows the ingenious suggestion of adding a virtual agent which interacts with all agents in the lattice and the cultural traits of which reflect the media message [9]. In the original version, each cultural feature of the virtual agent has the trait which is the most common in the population: the consensus opinion. Henceforth, we will refer to this type of external field as the consensus field. The second type of field we consider is constant in time, i.e., the media message is fixed from the outset, so it really models some alien influence impinging on the population. We will refer to this field as the static field. Explicitly, we generate the culture vector of the virtual agent at random and keep it fixed during the dynamics [10,11]. (These two types of field were referred to as global and external media by Ref. [11], but since both fields are global and external, here we opt to use a more informative nomenclature.)

Regardless of the type of external field, the interaction of the media (virtual agent) with the real agents is governed by the control parameter $p \in [0,1]$, which may be interpreted as a measure of the strength of the external field influence. As in the original Axelrod's model, we begin by choosing a target agent at random, but now it can interact with the media with probability p or with its neighbors with probability $1 - p$. The media-free model is recovered by setting $p = 0$. Since we have defined the media as a virtual agent, the interaction follows exactly the same rules as before. We note that an alternative prescription to couple the external field with the agents, in which the target agent is affected regardless of its similarity with the media message, always leads to a homogeneous regime in the absence of noise [12].

Because of the unusually large times needed for some initial configurations to relax to a homogeneous absorbing configuration (relaxation to heterogeneous configurations is typically very fast), the simulation of the dynamics of Axelrod's model must be made as efficient as possible. In our simulations, we consider two lists of active links. The first list (list A) is composed by the active links that connect real agents, whereas the second list (list B) contains the active links that connect the virtual agent (media) with the real ones. In both cases, an active link is defined as a link that connects two agents that have at least one feature in common and at least one feature distinct from each other. Here, instead of picking the target agent at random, we first select one of the two lists, list A with probability $1 - p$ and list B with probability p, and then pick a link at random from the selected list. In case of a link from list B, the target agent is of course the real one, but if the link belongs to list A we choose the target agent at random from the two options. Regardless of the list selected, once we pick a link, the interaction occurs with probability proportional to the number of features the two agents have in common. In the case that the cultural features of the target agent are modified by the interaction with its neighbor, we need to reexamine the active/inactive status of all links associated to the target agent so as to update the lists of active links. The dynamics is frozen when the two lists of active links are emptied.

III. EXTERNAL FIELDS IN THE ONE-DIMENSIONAL MODEL

In the attempt to make Axelrod's model more "realistic," researchers have studied the model in a variety of complex networks (see, e.g., [18–20]), with the usual result that the multicultural regime is suppressed by the increase of the connectivity of the network and so the stationary regime is homogeneous regardless of the values of the parameters q and F. It is curious that the simple one-dimensional variant that preserves the phase transition [7,8] received comparatively almost no attention. In this section, we show that the effect of external fields is essentially the same in one and two dimensions as far as the favoring of the homogeneous or heterogeneous regimes is concerned. The dimensionality introduces some distinctive effects, however, as we discuss next.

A. Field-free homogeneous regime

We begin our analysis with the once puzzling situation in which the homogeneous media-free stationary regime becomes heterogeneous under the influence of the external media [9]. Since this problem was extensively studied in the two-dimensional lattice [10,11,14,15], we present here only a brief analysis of the effect of the static media, aiming at highlighting the similarities and differences between the one- and two-dimensional results.

To characterize the steady state of Axelrod's model, we focus on two basic statistical measures, namely, the normalized average number of domains $N_{\rm dom}/L$ and the average relative size of the domains that do not belong to the largest domain $1 - S_{\max}/L$. These quantities are shown in Fig. 1 together with the fraction of runs trapped into homogeneous absorbing configuration ξ_h. A domain or cluster is a bounded region in which all agents share the same culture. We note that in this figure, as well as in the next figures of this paper, the statistical error bars are smaller or at most equal to the symbol sizes. Typically, each symbol represents the result of the average over 10^3 to 10^4 independent runs of the stochastic dynamics.

The first point to note is the remarkable similarity between the results presented in the upper panel of Fig. 1 for the one-dimensional and those for the two-dimensional model [15]. For a fixed finite value of L, there seems to exist a threshold value for the media strength $p = p_c$ below which the regime is monocultural [10,11]. The quantity ξ_h shown in the middle panel illustrates this somewhat odd predominance of the homogeneous absorbing configurations, in which the agents are identical to the static media, for lattices of intermediate size and small p (the case $p = 0$ is discussed below). However, as illustrated in the figure, this "threshold" decreases with increasing L and so it is a finite size effect. Thus, our conclusion is that in the thermodynamic limit, even a vanishingly small field is sufficient to break up the monocultural regime. More pointedly, extrapolating the ratio $\langle N_{\rm dom}\rangle/L$ (upper panel of Fig. 1) for $L \to \infty$ and plotting the result against p in a log-log graph yields $\lim_{L\to\infty}\langle N_{\rm dom}\rangle/L \sim p^{2.7}$ in the limit $p \to 0$ (the quality of this fitting is similar to that found in the analysis of the two-dimensional model [15]).

The middle and lower panels of Fig. 1 reveal some remarkable differences between the one- and two-dimensional models. First, for $p = 0$, only about 35% of the samples (random initial configurations) ended up into strictly homogeneous absorbing configurations, whereas for the two-dimensional model, this happens for all samples in the homogeneous regime. The reason we keep referring to this regime as the homogeneous regime is that the order parameters $N_{\rm dom}/L$ and $S_{\max}/L$ take on values compatible with a homogeneous phase. This only happens because the heterogeneous absorbing configurations are composed of a single macroscopic domain together with a nonextensive number of microscopic domains. Hence, the probability that a randomly chosen site in such configuration belongs to the largest domain is 1 in the

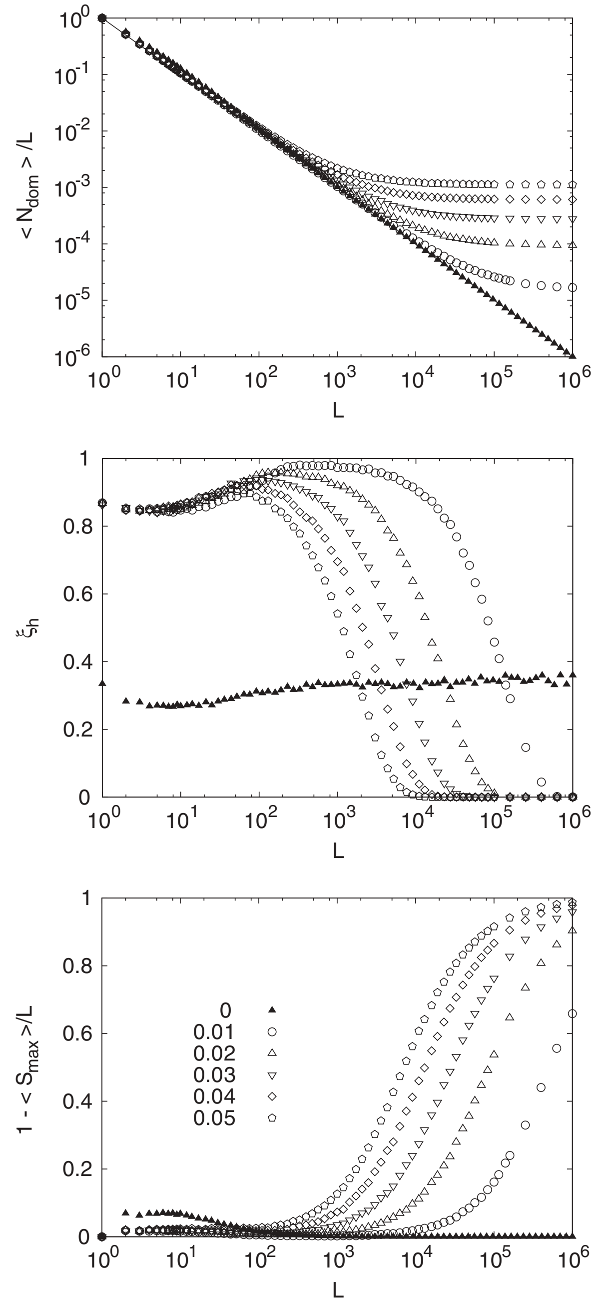

FIG. 1. Results for the static media in the one-dimensional model with $F = 5$ and $q = 3$ and for media strengths (unfilled symbols, bottom to top in the upper and lower panels and top to bottom in the middle panel) $p = 0.01$, 0.02, 0.03, 0.04, and 0.05. The filled triangles are the media-free ($p = 0$) results. Here, $\langle N_{\rm dom}\rangle$ is the average number of domains, ξ_h is the fraction of runs that ended up in homogeneous absorbing configurations, and $\langle S_{\max}\rangle$ is the average size of the largest domain. The straight line in the upper panel is $1/L$.

thermodynamic limit. Second, and more importantly, in the two-dimensional model the fraction of the sites that are not part of the largest domain (the media in that case) is finite and approaches zero for $p \to 0$ [15], whereas in the one-dimensional model that fraction tends to unity regardless of the field strength $p > 0$. This means that the effect of the field in one dimension is quite extreme: a vanishingly small field is enough to completely destroy the uniform regime, producing a fragmented configuration composed of microscopic domains. (We know that the domains are microscopic because $\langle S_{\max}\rangle/L \to 0$ in the thermodynamic limit.) This result is probably a consequence of the "weakness" of the uniform regime at $p = 0$ discussed above.

Our finding that a vanishingly small static field $p \to 0$ breaks the field-free homogeneous configuration into a highly fragmented heterogeneous configuration seems at stark contradiction with results reported in Ref. [8], which assert that in one dimension, a single single-site perturbation may only change a homogeneous configuration into another homogeneous configuration. The explanation for this conundrum has to do with the order in which we take the limits $L \to \infty$ and $p \to 0$. Since in our study we take first the thermodynamic limit $L \to \infty$ and then the zero-field limit $p \to 0$, the single-site perturbations are recurrent, i.e., they can occur many times during the dynamics. In that sense, the situation is similar to cultural drift [8,21], except that the perturbation now is not a noise but a constant field. Hence, the conclusions about the single single-site perturbation [8], which corresponds to taking the limit $p \to 0$ first and then the limit $L \to \infty$, do not apply to our problem and so there is really no contradiction between the two findings.

B. Field-free heterogeneous regime

We turn now to the problem that motivated this paper, namely, the finding that the effect of a vanishingly small external field is to turn the field-free heterogeneous steady state into a field-induced homogeneous state in the limit $L \to \infty$ [11]. This result would then be analogous to the homogenizing effect of an external noise that changes traits at random with some small probability [21]. Alas, when we consider very large lattice sizes, we find no evidence of such mind-boggling effect neither in the one- nor in the two-dimensional Axelrod's model.

In Fig. 2, we show the average number of domains for both static (upper panel) and consensus (lower panel) media. We use filled symbols to display the results for the two extreme values of the field intensity ($p = 0$ and 1), and unfilled symbols for the intermediary field strengths. This figure exhibits many noticeable results. The number of domains is maximum in the case wherein only interactions with the field are permitted ($p = 1$). Allowance of local interactions between neighboring agents results in less heterogeneous configurations, as expected. The surprise is that, even for finite L, the average number of domains jumps to a lower value as p departs infinitesimally from unity (the same phenomenon happens in the two-dimensional model [11]). Although $\langle N_{\text{dom}}\rangle$ exhibits a smooth dependence on $p \in (0,1)$, we observe another jump (now to a higher diversity value) at $p = 0$. The fact that these jumps occur at any finite value of L implies that the heterogeneous absorbing configurations are unstable to single-site changes. This conclusion is supported by the fact that the number of domains (and the diversity, as well) are reduced when the field or the local interactions are turned on. Put differently, the number of domains decreases (discontinuously) from $p = 0$ to $p > 0$ as well as from $p = 1$ to $p < 1$.

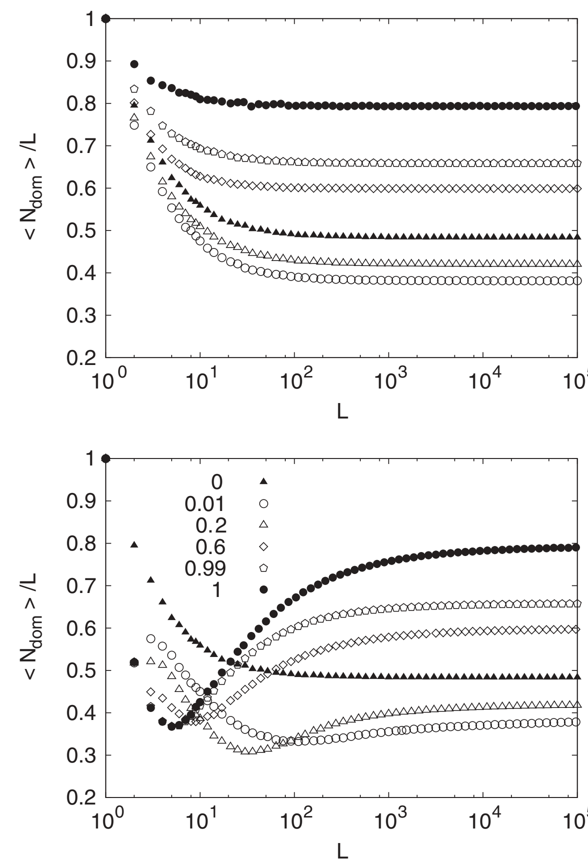

FIG. 2. Normalized number of domains in the one-dimensional model for the static (upper panel) and consensus (lower panel) media as function of the chain size L. The filled symbols are for $p = 1$ (circles) and $p = 0$ (triangles), whereas the unfilled symbols are for (top to bottom at $L = 10^5$) $p = 0.99$, 0.6, 0.2, and 0.01. The other parameters are $F = 5$ and $q = 10$.

The finding that the field-free heterogeneous adsorbing configurations are unstable even for finite L is in agreement with the results of [8], but since now the perturbation is not random (it is a spatially uniform external field) even when it is recurrent (i.e., p is nonzero) it does not lead to a homogeneous configuration, as would be the case if the perturbation were random [8].

We note that for $p = 1$, the consensus media is fixed with the traits reflecting the consensus values for the random initial configuration of the agents. However, for finite L, the results differ somewhat significantly from those of the static media (see Fig. 2). Why is that? The reason is that the traits of the static media are chosen randomly and independently of the also random initial traits of the agents, whereas in the case of the consensus media, the media traits are not independent from the agents' traits, being given by the majority rule. This correlation has a strong effect for small chains, but becomes irrelevant for large L since in this case the fraction of agents that share a given media trait is not much greater than $1/q$, which is the expected value of this fraction for the static media. In fact, the correlation between the media and the agents in the initial configuration explains the dips observed in the lower panel of Fig. 2 for $p > 0$ since it decreases the number of agents with antagonistic traits with respect to the media, resulting in less

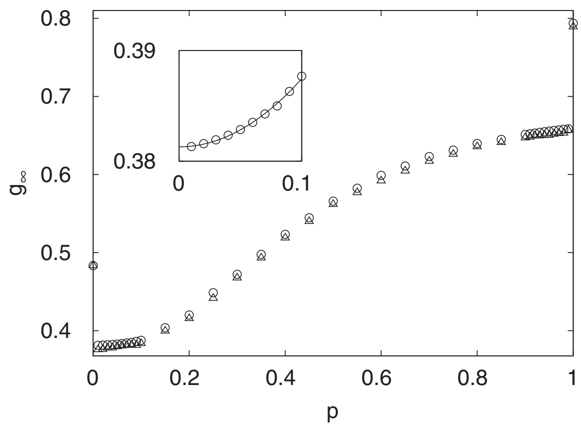

FIG. 3. Extrapolation to $L \to \infty$ of the normalized number of domains in the one-dimensional model for the static (circles) and consensus (triangle) media as function of the field strength p. At $p = 0$, we find $g_\infty = 0.483 \pm 0.001$ and at $p = 1$ we find $g_\infty = 0.793 \pm 0.001$ for both media types. The inset shows the fitting of the static media data by the function $0.38 + 0.6p^2$ (solid line) for small p. The labels of the inset axes are the same as those of the main figure. The parameters are $F = 5$ and $q = 10$.

fragmented configurations than in the case wherein the media traits are set randomly. However, as pointed out before, this effect disappears for large chains. Finally, we mention that there is an interesting mapping between the case $p = 1$, where the agents interact only with the field, and the site percolation: an occupied node corresponds to an agent that has the features of the external field and the occupation probability is given by $1 - (1 - 1/q)^F$ [20].

The discontinuities at $p = 0$ and 1 are more easily visualized in Fig. 3 where we present the extrapolation to infinite chain sizes $g_\infty = \lim_{L\to\infty} \langle N_{\text{dom}} \rangle / L$ of the results exhibited in Fig. 2. The results for both types of media are practically indistinguishable. For $p \to 0$, we find $g_\infty \to 0.38 \pm 0.01$ whereas $g_\infty \to 0.66 \pm 0.01$ for $p \to 1$, which are valid for both media types (see the legend of Fig. 3 for the values of g_∞ at $p = 0$ and 1). The important point here is that the data offer no evidence whatsoever that g_∞ would vanish in the limit $p \to 0$ (see the inset in Fig. 3). We should emphasize that the decrease on the number of domains induced by a vanishingly small field is the expected outcome of the assay since the small field destabilizes some of the field-free domains, but lacks the strength to create new field-induced domains.

In addition, we find that the average size of the largest domain $\langle S_{\max} \rangle$ grows as $\ln L$ for large L (data not shown), whereas the size of a typical domain is on the order of 1, regardless of the media type and strength. These findings are similar to those reported for the majority-vote model [22].

IV. EXTERNAL FIELDS IN THE TWO-DIMENSIONAL MODEL

We turn now to the study of the two-dimensional Axelrod's model, which is considerably more computationally demanding than the analysis of the one-dimensional version presented before. The effect of external fields on the field-free homogeneous regime is well understood by now [14,15]: the results are similar to those exhibited in Fig. 1 except for the relative size of the largest domain which in the limit $p \to 0$ tends to 1 in the two-dimensional case [15] and to 0 in the one -dimensional version (lower panel of Fig. 1). Hence, we will consider here only the effect of external fields on the field-free heterogeneous regime. Moreover, we focus most our efforts on the parameter set $F = 2$ and $q = 8$ rather than on the set $F = 5$ and $q = 30$ of Ref. [11]. In both cases, the field-free ($p = 0$) regime is heterogeneous, but simulations using our parameter selection are much faster of course.

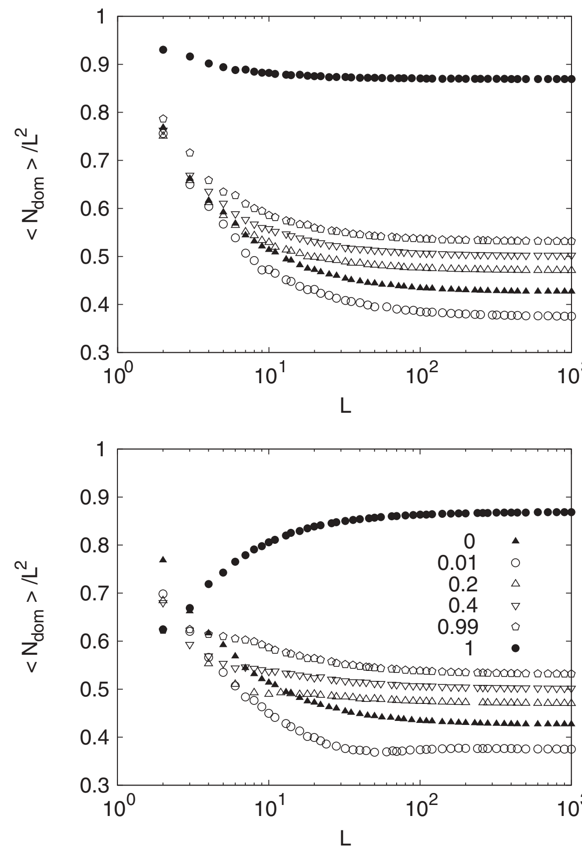

FIG. 4. Normalized number of domains in the two-dimensional model for the static (upper panel) and consensus (lower panel) media as function of the linear size L of the square lattice. The filled symbols are for $p = 1$ (circles) and $p = 0$ (triangles), whereas the unfilled symbols are for (top to bottom at $L = 10^3$) $p = 0.99$, 0.4, 0.2, and 0.01. The other parameters are $F = 2$ and $q = 8$.

Accordingly, in Fig. 4 we present the dependence of the average number of domains on the linear size of the square lattice. The results are qualitatively similar to those obtained for the one-dimensional model and summarized in Fig. 2. The correlation between the consensus media and the agents results in less heterogeneous absorbing configurations in comparison with the configurations induced by the static media, but this difference becomes negligible as the lattice size increases. This correlation effect is less dramatic than in the one-dimensional model because for the same value of L in the x axis, there are many more agents (L^2 to be precise) in the two-dimensional lattice. The result of the extrapolation to infinite lattice sizes using the definition $g_\infty = \lim_{L\to\infty} \langle S_{\text{dom}} \rangle / L^2$ is summarized in Fig. 5. For $p \to 0$, we find $g_\infty \to 0.36 \pm 0.01$, whereas

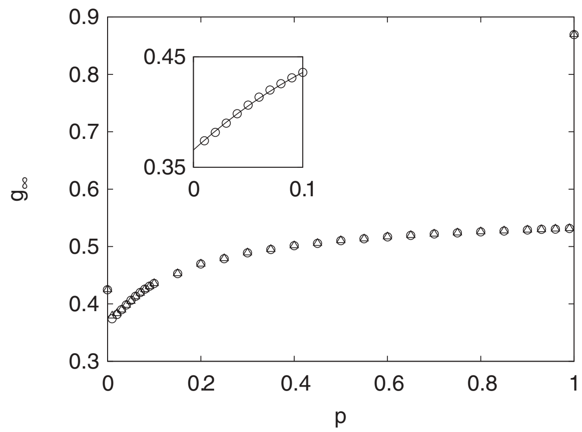

FIG. 5. Extrapolation to $L \to \infty$ of the normalized number of domains in the two-dimensional model for the static (circles) and consensus (triangle) media as a function of the field strength p. The results are indistinguishable for the two media types. At $p = 0$, we find $g_\infty = 0.425 \pm 0.001$ and at $p = 1$ we find $g_\infty = 0.869 \pm 0.001$. The inset shows the fitting of the static media data by the function $0.36 + 0.9p - 1.9p^2$ (solid line) for small p. The labels of the inset axes are the same as those of the main figure. The parameters are $F = 2$ and $q = 8$.

$g_\infty \to 0.53 \pm 0.01$ for $p \to 1$. As before, we observe discontinuities at $p = 0$ and 1 which take place even for finite L, and no evidence at all that the g_∞ would tend to 0 in the limit $p \to 0$ as claimed by Ref. [11]. We note, however, that extrapolation to $L \to \infty$ using lattices of linear size up to $L = 70$ as done in Ref. [11] may result in a significant underestimate of g_∞ since, especially for small p, one may be misled by the transient region where $g_L \equiv \langle N_{\mathrm{dom}} \rangle / L^2$ decreases abruptly with increasing L (see Fig. 4).

Our simulations for the selection of parameters $F = 5$ and $q = 30$ used in Ref. [11] led to similar conclusions. As already pointed out, the large relaxation times make an extensive analysis of this parameter set prohibitive, so we used a small number of samples (typically 100), which resulted in rather noisy data points. For small lattice sizes ($L < 70$), our results fully agree with those of [11].

V. LOCAL FIELD

Here, we consider a time and spatially nonuniform field introduced in Ref. [11], referred to as local field (or media). For a given target agent, the traits of this field reflect the consensus trait of its nearest neighbors. Hence, it can be viewed as a multistate variant of the majority-vote rule [22–27] since the interaction with the local field features the main ingredients of a majority-vote model: (i) pick a voter (target agent) at random, (ii) find the consensus opinion among its neighbors (the consensus is obtained by the majority-vote rule), and (iii) the voter assumes the consensus opinion. In that sense, we are reluctant to characterize it as a media, much less as a mass media. In fact, its local nature sets it apart from the static and consensus fields studied in the previous sections, which are spatially uniform fields, and so we see no reason to assume the system will behave similarly under the effect of such diverse fields.

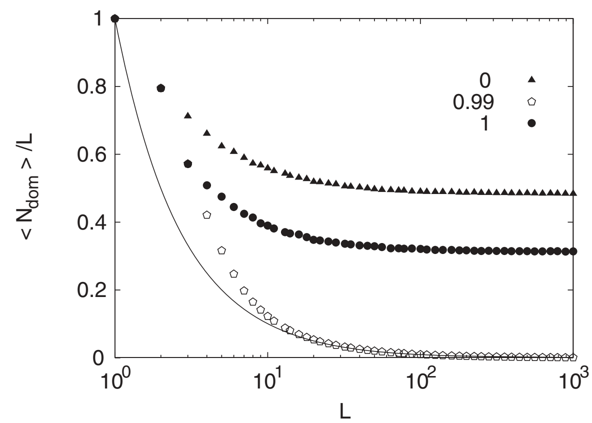

FIG. 6. Normalized number of domains in the one-dimensional model for the local field as a function of the chain size L. The filled symbols are for $p = 1$ (circles) and $p = 0$ (triangles), whereas the unfilled symbols are for $p = 0.99$. The solid curve is $1/L$ and the other parameters are $F = 5$ and $q = 10$.

The overly distinct effect of the local field on the steady state of Axelrod's model is illustrated in Fig. 6 for the one-dimensional variant. Most interestingly, in this case, the steady state is less heterogeneous when the agents interact with the field only ($p = 1$) than when the field is off ($p = 0$). This is a peculiarity of the one-dimensional model which suits well to illustrate the tendency to homogenization of the local field. The results for all values of $p \in (0,1)$ are indistinguishable from that exhibited in the figure for $p = 0.99$, which shows the dominance of homogeneous absorbing configurations already for small chains. In fact, for $L > 50$, the data points fall on the curve $1/L$ shown in the figure. So, in the thermodynamic limit, the steady state is spatially uniform except in the two extreme cases $p = 0$ and 1. A glance at the results exhibited in Fig. 6 for the local field and in Fig. 2 for the global fields reveals the great disparity of the effect of these fields on the steady-state properties of Axelrod's model.

Figure 7 shows the results of the local field in the two-dimensional Axelrod model. These simulations are incredibly time consuming for $p \in (0,1)$ due to the tendency to homogenization of the agents' cultural traits and the constant need to update the local fields. Hence, our simulations are restricted to $L < 200$ and 10^3 samples only. As in the one-dimensional case, the simulation data in this range of p is practically indistinguishable within the numerical error and so we present only the results for $p = 0.9$. However, the data do not fall on the curve $1/L^2$ (solid line in the figure), which would signal the existence of a single domain. Rather, the scaled number of domains seems to go to zero much slower than L^{-2} as the lattice size increases. This means that the average (non-normalized) number of domains grows with L^α with $\alpha < 2$, i.e., it is nonextensive in presence of the local field. Of course, the small lattice sizes as well as the reduced number of samples used in this analysis do not allow us to make quantitative claims on the scaling laws for large L. We note that it was this difficulty to make inferences on the effect of the local field (even for a more manageable parameter set than that used in Ref. [11]) that motivated our analysis of the one-dimensional model. Finally, we note that the absorbing configurations are

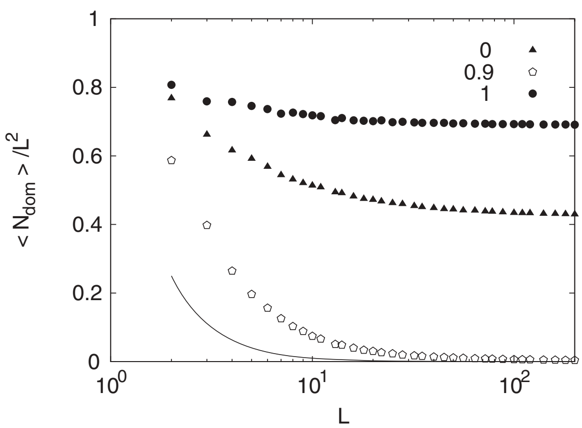

FIG. 7. Normalized number of domains in the two-dimensional model for the local field as a function of the linear size L of the square lattice. The filled symbols are for $p = 1$ (circles) and $p = 0$ (triangles), whereas the unfilled symbols are for $p = 0.9$. The solid curve is $1/L^2$ and the other parameters are $F = 2$ and $q = 8$.

more heterogeneous for $p = 1$ than for $p = 0$ in contrast to our findings for the one-dimensional model.

VI. CONCLUSION

In our effort to clarify the effects of spatially uniform fields (interpreted as mass media) as well as of nonuniform fields on Axelrod's model, we have unveiled several features of this well studied model of social influence.

First and foremost, we have revised the result that in the thermodynamic limit the field-free heterogeneous regime becomes homogeneous in the limit of vanishingly small fields, i.e., $p \to 0$ regardless of the type of the field [11]. More pointedly, we find that in the presence of a local field, the steady state does indeed become homogeneous but for all $p \in (0,1)$. This is so because the local field implements a majority-vote rule among the nearest neighbors of the target agent and so, when allied to the homogenizing interaction rule of Axelrod's model, it constitutes an insuperable force towards homogenization. However, we find that the system remains heterogeneous in the presence of the spatially uniform fields which were originally introduced in Axelrod's model to study the effect of mass media on opinion formation [9].

Second, we find that the spatially uniform but time-varying consensus media introduced in Ref. [9] and the static media yield the same results in the thermodynamic limit. It seems that the reason they produce different outcomes for small lattices is the correlation between the consensus field and the agents in the random initial configuration. This correlation becomes less pronounced for large lattices since even after application of the consensus rule, there will be a rough balance between the values of the traits of a same entry of the feature vector. From the statistical mechanics perspective, this is an important result because it allows us to make inferences about a more realistic media type using the much easier to simulate static media.

Third, we find that the heterogeneous absorbing configurations at $p = 0$ and 1 are unstable. This instability is reflected by the discontinuities that take place at those values even for finite lattices. Otherwise, the measures used to describe the steady state (i.e., average number of domains and size of the largest domain) are continuous functions of p in the range $p \in (0,1)$. The field-free homogeneous configurations for finite L are stable in both one and two dimensions. However, in the thermodynamic limit, they become unstable in one dimension (lower panel of Fig. 1) but remain stable in two dimensions [15]. We note that the instability produced by the external fields draws the system to stable (or metastable) configurations of a quite distinct nature than those deriving from noisy perturbations [8,21].

Fourth, we find that the one-dimensional version of Axelrod's model [7] yields essentially the same results as the more popular two-dimensional version. In particular, our findings about the impact of the three field types on the field-free heterogeneous regime of the two-dimensional model were corroborated by the one-dimensional model. Although this model exhibits some peculiarities, which were properly highlighted in the text, it can serve as an exceptional guide to our understanding of features that are difficult to unveil in the two-dimensional version, such as the effect of local fields.

The characterization of the absorbing configurations of Axelrod's model in the thermodynamic limit, regardless of whether or not in the presence of a field (media), remains a challenge to statistical mechanics. In fact, we do not know much about the location and the nature of the phase transition in the (q,F) space [2,4], due mainly to the huge relaxation times the dynamics needs to settle in a homogeneous configuration, which grows as N^2 where N is the number of lattice sites [7]. In that sense, the appearance of counterintuitive results about the steady-state properties of Axelrod's model in the thermodynamic limit should not be surprising. This situation is worsened by the presence of a field since lattices of intermediate sizes exhibit somewhat perversely a regime distinct from the thermodynamic one (see Fig. 1).

Nevertheless, we think that we have reached by now a good qualitative understanding of the effects of spatially uniform (mass media) as well as nonuniform (local) fields on the statistical properties of the two-dimensional Axelrod model. In the field-free homogeneous regime case, the presence of a vanishingly small field (i.e., $p \ll 1$) leads to the breaking of the single monocultural domain into a macroscopic domain (media) and a multitude of microscopic domains which occupy a finite area of the lattice [15]. The scenario is different in the one-dimensional model where the field pulverizes the giant domain into microscopic domains as shown in the lower panel of Fig. 1. Regarding the field-free heterogeneous regime case, which was our main concern in this paper, the presence of a spatially uniform field does not produce cultural homogenization, regardless of the field strength p. In contrast, a local field that implements the majority-vote rule among the neighbors of the target agent leads to a homogeneous steady state for $p < 1$.

ACKNOWLEDGMENT

This research was supported by The Southern Office of Aerospace Research and Development (SOARD), Grant No. FA9550-10-1-0006, and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

[1] R. Axelrod, J. Conflict Resolut. **41**, 203 (1997).
[2] C. Castellano, M. Marsili, and A. Vespignani, Phys. Rev. Lett. **85**, 3536 (2000).
[3] C. Castellano, S. Fortunato, and V. Loreto, Rev. Mod. Phys. **81**, 591 (2009).
[4] L. A. Barbosa and J. F. Fontanari, Theory Biosci. **128**, 205 (2009).
[5] J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge, UK, 1999).
[6] F. Vazquez and S. Redner, Europhys. Lett. **78**, 18002 (2007).
[7] D. Vilone, A. Vespignani, and C. Castellano, Eur. Phys. J. B **30**, 399 (2002).
[8] K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel, J. Econ. Dynam. Control **29**, 321 (2005).
[9] Y. Shibanai, S. Yasuno, and I. Ishiguro, J. Conflict Resolut. **45**, 80 (2001).
[10] J. C. González-Avella, M. G. Cosenza, and K. Tucci, Phys. Rev. E **72**, 065102(R) (2005).
[11] J. C. González-Avella, V. M. Eguíluz, M. G. Cosenza, K. Klemm, J. L. Herrera, and M. San Miguel, Phys. Rev. E **73**, 046119 (2006).
[12] K. I. Mazzitello, J. Candia, and V. Dossetti, Int. J. Mod. Phys. C **18**, 1475 (2007).
[13] J. Candia and K. I. Mazzitello, J. Stat. Mech. (2008) P07007.
[14] L. R. Peres and J. F. Fontanari, J. Phys. A: Math. Theor. **43**, 055003 (2010).
[15] L. R. Peres and J. F. Fontanari, Europhys. Lett. **96**, 38004 (2011).
[16] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).
[17] T. M. Liggett, *Interacting Particle Systems* (Springer, New York, 1985).
[18] J. M. Greig, J. Conflict Res. **46**, 225 (2002).
[19] K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel, Phys. Rev. E **67**, 026120 (2003).
[20] J. C. González-Avella, M. G. Cosenza, V. M. Eguíluz, and M. San Miguel, New J. Phys. **12**, 013010 (2010).
[21] K. Klemm, V. M. Eguíluz, R. Toral, and M. San Miguel, Phys. Rev. E **67**, 045101(R) (2003).
[22] L. R. Peres and J. F. Fontanari, Phys. Rev. E **82**, 046103 (2010).
[23] P. L. Krapivsky and S. Redner, Phys. Rev. Lett. **90**, 238701 (2003).
[24] D. Parisi, F. Cecconi, and F. Natale, J. Conflict Res. **47**, 163 (2003).
[25] S. Galam, Phys. Rev. E **71**, 046123 (2005).
[26] R. Lambiotte and S. Redner, Europhys. Lett. **82**, 18007 (2008).
[27] P. F. C. Tilles and J. F. Fontanari, J. Stat. Mech. (2012) P07003.