# Correlation and network analysis of global financial indices

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Random matrix theory (RMT) and network methods are applied to investigate the correlation and network properties of 20 financial indices. The results are compared before and during the financial crisis of 2008. In the RMT method, the components of eigenvectors corresponding to the second largest eigenvalue form two clusters of indices in the positive and negative directions. The components of these two clusters switch in opposite directions during the crisis. The network analysis uses the Fruchterman-Reingold layout to find clusters in the network of indices at different thresholds. At a threshold of 0.6, before the crisis, financial indices corresponding to the Americas, Europe, and Asia-Pacific form separate clusters. On the other hand, during the crisis at the same threshold, the American and European indices combine together to form a strongly linked cluster while the Asia-Pacific indices form a separate weakly linked cluster. If the value of the threshold is further increased to 0.9 then the European indices (France, Germany, and the United Kingdom) are found to be the most tightly linked indices. The structure of the minimum spanning tree of financial indices is more starlike before the crisis and it changes to become more chainlike during the crisis. The average linkage hierarchical clustering algorithm is used to find a clearer cluster structure in the network of financial indices. The cophenetic correlation coefficients are calculated and found to increase significantly, which indicates that the hierarchy increases during the financial crisis. These results show that there is substantial change in the structure of the organization of financial indices during a financial crisis.

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## I. INTRODUCTION

Over the last few years, there has been a growing interest of physicists in economic systems [1,2]. The present study is to understand the origin of a financial crisis and its effect on the structure of organization of financial indices. This paper is focused on the global financial crisis of 2008 and uses correlation and network methods to investigate its effect on the structure of organization of 20 financial indices. Random matrix theory (RMT) was developed [3–6] to deal with the statistics of eigenvalues and eigenvectors of complex many-body systems. It has been successfully used to investigate phenomena from different areas such as quantum field theory, quantum chaos, disordered systems, and has recently been applied to a large number of financial markets [7–12] to investigate the structure of cross-correlations in financial markets. In the random matrix theory approach the first few largest eigenvalues deviate significantly from the RMT prediction and the deviation changes during the crisis. The largest eigenvalue represents the collective information about the correlation between different stocks and its trend depends on the market conditions. The components of eigenvectors corresponding to the remaining large eigenvalues are associated with the formation of clusters (organization) in the financial market, which respond and reorganize during a crisis.

Another powerful technique is the complex network technique which has become an important method for studying properties of complex systems in the real world (physical systems, social sciences, biological sciences, and financial markets) [13–20]. The study of complex networks was initiated by a desire to understand various real systems from the empirical data [13]. Complex networks display the spatial topological structure of a system. In this paper the threshold and hierarchical methods are used to construct the correlation network of financial indices. Networks generated by the threshold method [20] using the Fruchterman-Reingold layout display the network structure in a simple and clear way. If a system presents a cluster organization then the threshold method is typically able to detect it. Hence, here the threshold method is used to study the network organization before and during the crisis. To detect a possible hierarchical structure hidden in the global financial data we apply the minimum spanning tree (MST) [21] method (the MST has been applied to stock market indices in [22–25]). This method selects the indices with the closest interactions among all indices and generates a visual presentation of the linkage relationship among selected interactions of the financial indices [26–28]. This idea is used here for the global financial indices before and during the crisis of 2008. The hierarchical clustering method which organizes the financial indices in terms of dendrograms is also used to strengthen the hierarchical results.

The paper is organized as follows: The financial data are discussed in Sec. II. In Sec. III, the RMT approach is applied to the global financial indices. The techniques of network analysis and results are discussed in Sec. IV. Finally, we end with a conclusion in Sec. V.

## II. DATA ANALYZED

We analyze the daily closing prices of 20 financial markets around the world traded from the period July 2, 1997 to June 1, 2009. These indices are as follows: Argentina, MERV; Brazil, BVSP; Egypt, CCSI; India, BSESN; Indonesia, JKSE; Malaysia, KLSE; Mexico, MXX; South Korea, KS11; Taiwan, TWII; Australia, AORD; Austria, ATX; France,

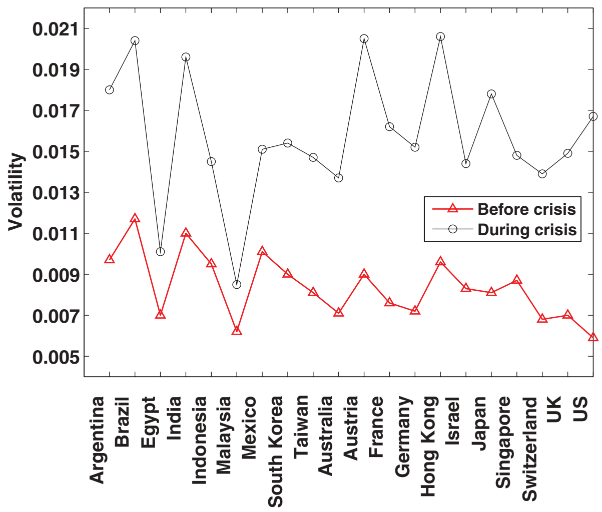
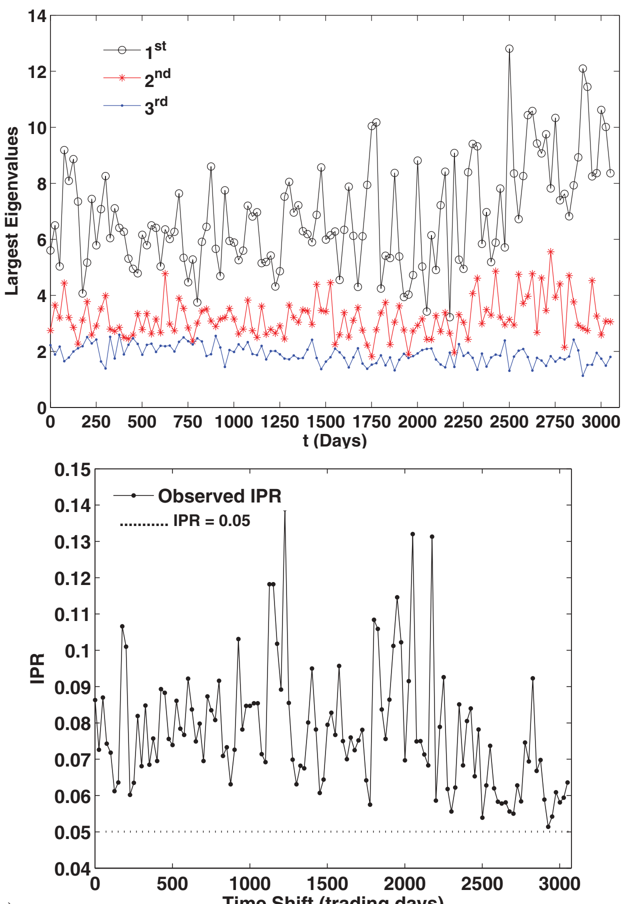



FIG. 1. (Color online) Volatility as a measure of fluctuation in financial indices before (△) and during (○) the crisis.

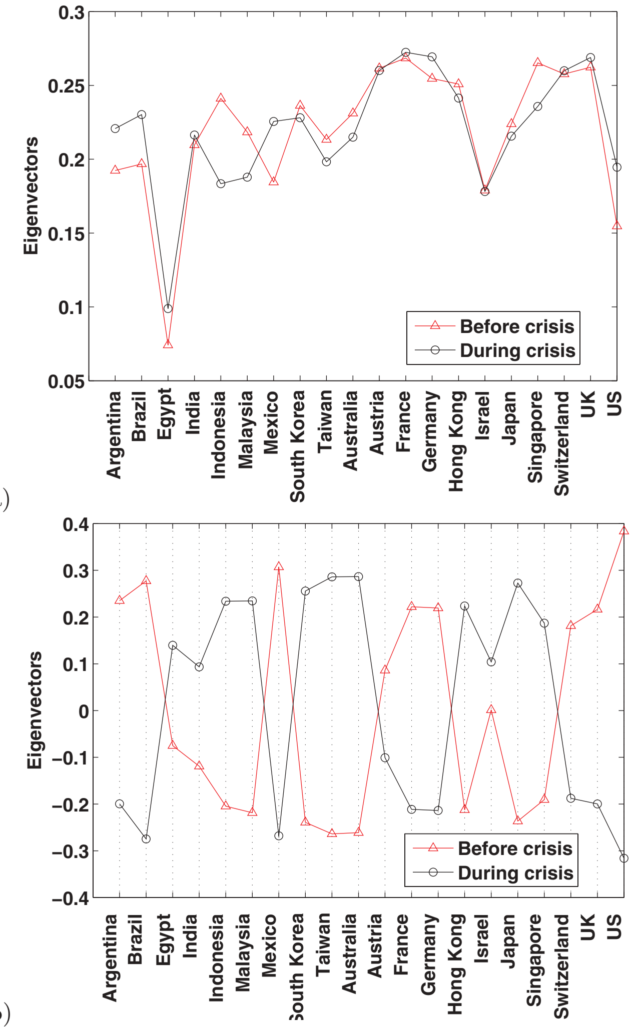
FCHI; Germany, GDAXI; Hong Kong, HSI; Israel, TA100; Japan, N225; Singapore, STI; Switzerland, SSMI; the United

(a)




(b)

FIG. 2. (Color online) (a) First three largest eigenvalues of the correlation matrices of financial indices using sliding time windows of 25 days. (b) IPR for the eigenvector $U^{20}$ as a function of time, obtained from correlation matrices of financial indices using windows of 25 days. The dashed line marks the value 0.05 of the IPR when all components contribute equally.




(a)

(b)

FIG. 3. (Color online) (a) Components of eigenvectors corresponding to first largest eigenvalue. The fact that all components are positive reflects a common global financial market mode. (b) Components of eigenvectors corresponding to second largest eigenvalue. The financial indices form two clusters in the positive and negative directions, respectively. The positive significant contributions of the components are associated with the cluster of American (Argentina, Brazil, Mexico, United States) and European (Austria, France, Germany, Switzerland) indices. The negative significant contributions of the components are associated with the cluster of indices corresponding to Asia-Pacific (Egypt, India, Indonesia, Malaysia, South Korea, Taiwan, Australia, Hong Kong, Japan, Singapore) indices (except Israel). The components of the two clusters switch in opposite directions during the crisis.

Kingdom, FTSE; and the United States, GSPC. The data were obtained from [29]. There are differences in public holidays or weekends among countries so the data are shifted according to the rule that when more than 30% of markets did not open on a certain day, we remove that day from the data, and when it was fewer than 30%, we kept existing indices and inserted the last closing price for each of the remaining indices. Also these markets do not operate in the same time zones. It has been reported [11] that correlations of Asian with US indices increases when one considers the correlation of the US indices with the next day indices of the Asian

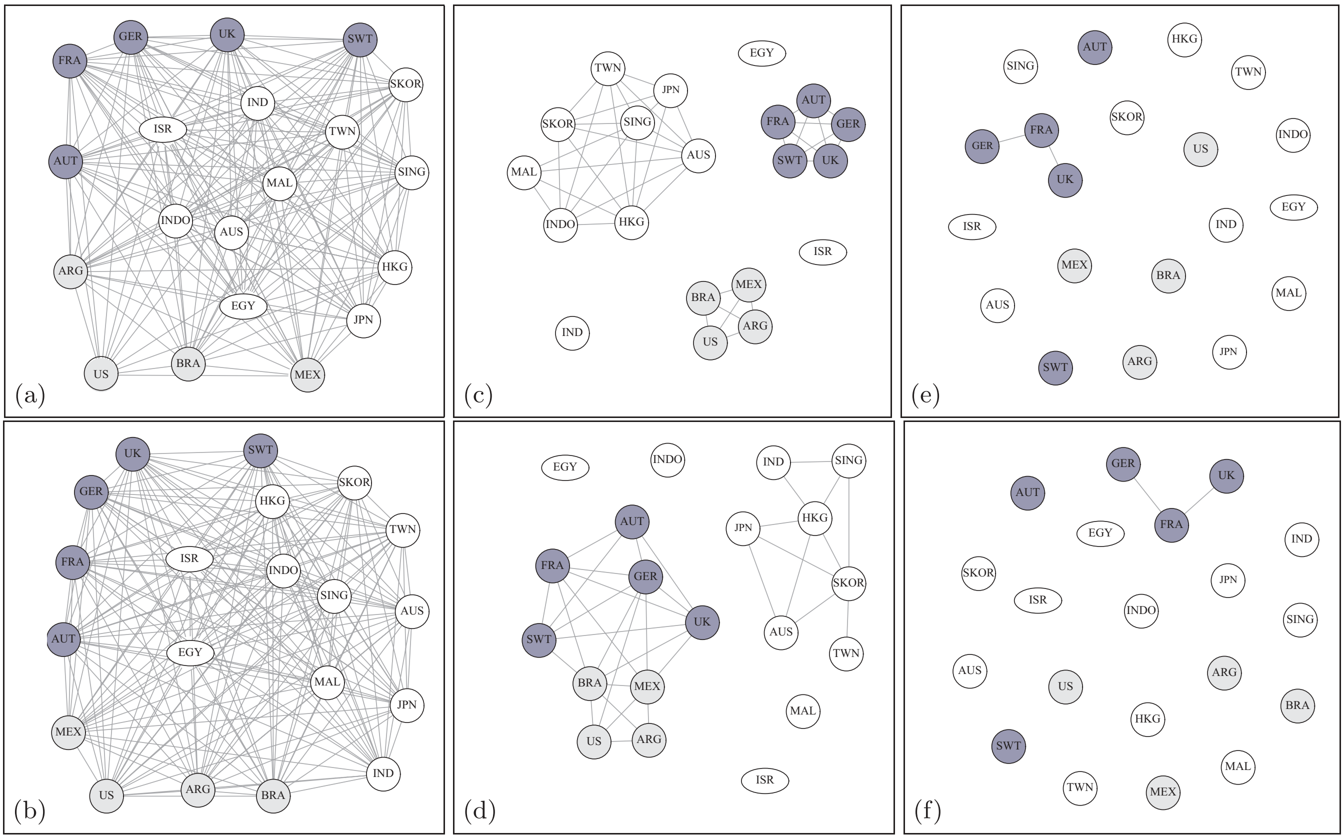



FIG. 4. (Color online) The network at $\theta = 0.1$, (a) before and (b) during the crisis. All financial indices are well connected. (c) At $\theta = 0.6$ (before the crisis), there is a formation of three separate clusters: *American* (gray vertices), *European* (dark gray vertices), and *Asia-Pacific* (colorless vertices) with disconnected node of India. The Egypt and Israel (elliptical vertices) nodes of the *Africa–Middle East* cluster disconnect from the global financial network at $\theta = 0.5$. (d) At $\theta = 0.6$ (during the crisis), the American and European clusters combine together to form a strongly linked cluster, and the Asia-Pacific cluster connects India and disconnects Indonesia and Malaysia. (e),(f) If the threshold is further increased to 0.9 then the European indices (France, Germany, and the United Kingdom) consistently constitute the most tightly linked markets.

market. We did not consider weekly data to avoid the problem of different operating hours between international markets so that we do not miss major changes in markets which tend to occur during a small interval of days. Thus, we considered all indices taken on the same date and filtered the data as in [11]. The global financial crisis of 2007–2009 is known to be the worst financial crisis since the Great Depression of the 1930s and had its origins in the United States and then spread to the world [19]. To investigate the effect of the financial crisis on the structure of organization of financial indices, we considered the period before the crisis (June 7, 2006 to November 30, 2007) and during the crisis (December, 2007 to June, 2009). The periods before (calm) and during the crisis are chosen by observing the volatility, which gives a measure of the market fluctuations. The calm and crisis periods, corresponding to the red line with triangles and the black line with circles in Fig. 1, show an increase in the value of the volatility of each index during the crisis period.

## III. RANDOM MATRIX THEORY APPROACH

Let $S_i(t)$ and $R_i(t)$ denote the daily closing prices and returns of indices $i$ at time $t$ ($i = 1,2,\ldots,N;\ \ t = 1,2,\ldots,T$), respectively. The logarithmic returns is defined as $R_i(t) \equiv \ln[S_i(t+\Delta t)] - \ln[S_i(t)]$, where $\Delta t = 1$ day is the time lag. The normalized return for index $i$ is defined as $r_i(t) \equiv (R_i(t) - \langle R_i\rangle)/\sigma_i$, where $\sigma_i \equiv \sqrt{\langle R_i^2\rangle - \langle R_i\rangle^2}$ is the standard deviation of $R_i$, and $\langle\cdots\rangle$ denotes the time average over the period studied. The equal-time cross-correlation matrix is computed with elements $C_{ij} \equiv \langle r_i(t) r_j(t)\rangle$ which are limited to the domain $[-1,1]$. For the global financial indices $C_{ij} = 1\ (-1)$ corresponds to perfect correlation (anticorrelation) in indices and $C_{ij} = 0$ corresponds to no correlation. The financial data of $N = 20$ indices for $T = 387$ days have been used to analyze the crisis. The value of the average correlation coefficient $\langle C_{ij}\rangle$ increases from 0.4353 (before the crisis) to 0.4634 (during the crisis) in response to the financial crisis. The statistical properties of a Wishart matrix (a correlation matrix of uncorrelated time series with finite length) are known [6]. In the limit $N \to \infty, T \to \infty$ with $Q \equiv T/N\ (\geqslant 1)$, the probability distribution of the eigenvalue $\lambda$ is given by $P_{\rm RM}(\lambda) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda_{\max}-\lambda)(\lambda-\lambda_{\min})}}{\lambda}$ within the bounds $\lambda_{\min} \leqslant \lambda_i \leqslant \lambda_{\max}$ and 0 otherwise [8]. The smallest (largest) eigenvalue of the random matrix is given by $\lambda_{\min(\max)} = [1 \mp (1/\sqrt{Q})]^2$. If there is no correlation between financial indices then the eigenvalues should be bounded between the RMT predictions, i.e., $\lambda_{\min(\max)} = 0.597\ (1.5063)$. But for the global financial indices we find that the smallest (largest) eigenvalue

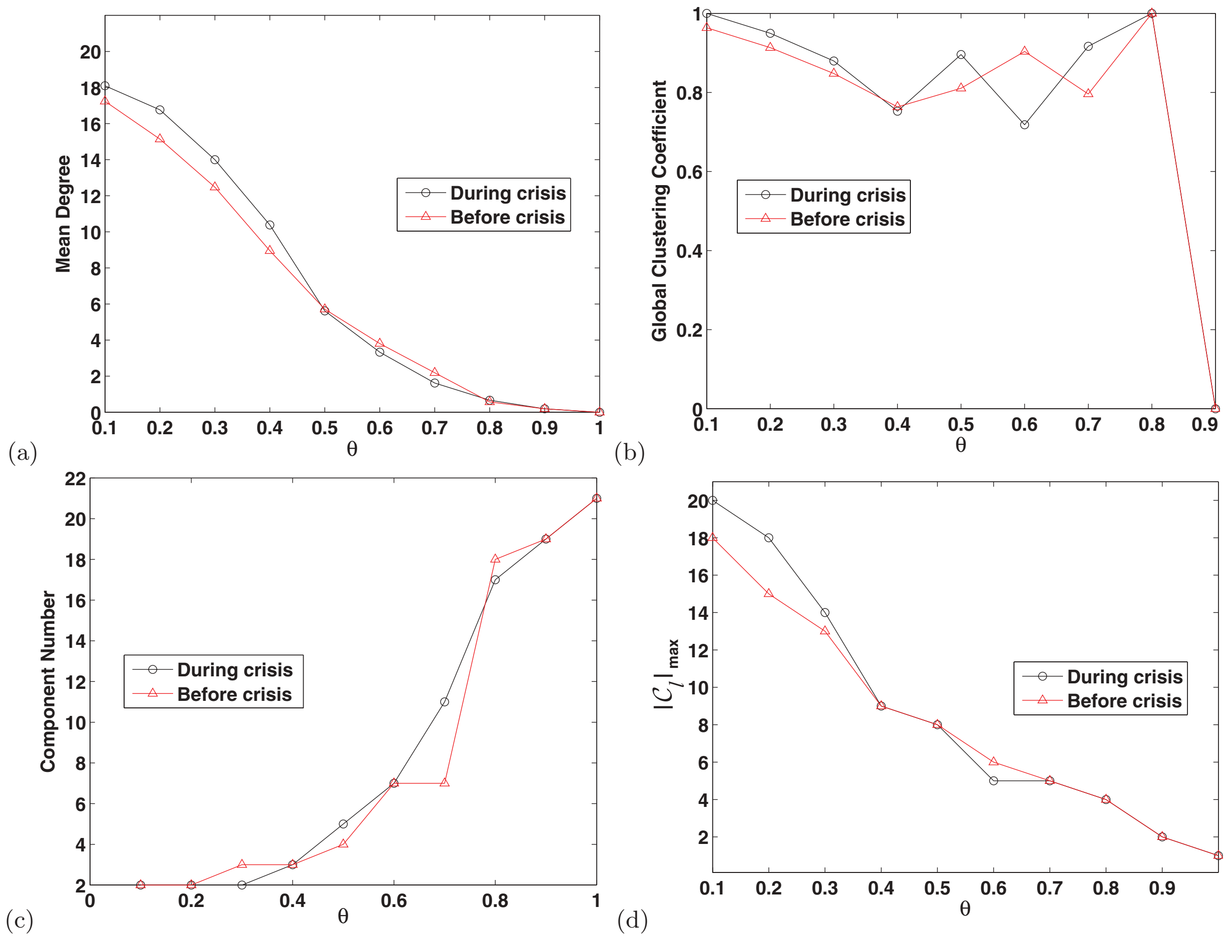



FIG. 5. (Color online) (a) Mean degree for various thresholds before and during the crisis. (b) Global clustering coefficients for various thresholds before and during the crisis. At $\theta = 0.9$ there is no triangle formation in the correlation network and there is only one triplet so its clustering coefficient is zero. (c) Component number in the index correlation network under different correlation thresholds. When $\theta > 0.9$, the vertices are nearly all isolated and the component number is approximately the vertex number. When $\theta \leqslant 0.2$, the networks are fully connected and the component number is just 2. (d) For $\theta \geqslant 0$, all indices in a clique are positively correlated with each other. The maximum clique size of the correlation networks of global indices decreases with increase in the threshold.

is $\lambda^{\text{real}}_{\text{min(max)}} = 0.0527$ (9.0454) before the financial crisis and $\lambda^{\text{real}}_{\text{min(max)}} = 0.0388$ (9.5282) during the financial crisis. Thus, the largest eigenvalues deviate significantly from the RMT prediction in both periods, which shows that there is a strong correlation in the financial indices. The largest eigenvalue represents the collective information about the correlation between different indices therefore its trend is expected to be dependent on the market conditions. Figure 2(a) shows the trend of first, second, and third largest eigenvalues over sliding windows of 25 days. There is an increase in the first and second largest eigenvalues during the financial crisis of 2008 while the third largest eigenvalues do not show significant variation. Next we compute the inverse participation ratio (IPR) which allows the quantification of the number of components that participate significantly in each eigenvector and tells us more about the level and nature of the deviation from the RMT. The IPR of the eigenvector $u^k$ is defined by $I^k \equiv \sum_{l=1}^{N} [u_l^k]^4$, where $u_l^k, l = 1, \ldots, N$, are the components of the eigenvector $u^k$. Figure 2(b) shows the IPR of 20 financial indices using time windows of 25 days for the eigenvector $U^{20}$ corresponding to the largest eigenvalue.

The eigenvectors corresponding to the first largest eigenvalue are shown in Fig. 3(a). The fact that all the components are positive reflects a common global financial market mode and this does not show appreciable change due to the crisis. Figure 3(b) shows the components of the eigenvectors corresponding to the second largest value. We find that the positive components of the second eigenvector (Argentina, Brazil, Mexico, Austria, France, Germany, Switzerland, the UK, and the US) switch to negative values during the crisis while the negative components (Egypt, India, Indonesia, Malaysia, South Korea, Taiwan, Australia, Hong Kong, Japan, and Singapore) switch to large positive values during the crisis. However, the eigenvectors corresponding to the third largest eigenvalue does not carry much information as the third largest eigenvalue is near the random matrix bound.

## IV. CONSTRUCTION AND ANALYSIS OF NETWORK OF FINANCIAL INDICES

The network of global financial indices using the threshold method is constructed as follows: Let the set of financial indices define the set of vertices ($V$) of the network. We specify a certain threshold $\theta$ ($-1 \leqslant \theta \leqslant 1$) and add an undirected edge connecting the vertices $i$ and $j$ if $C_{ij}$ is greater than or equal to $\theta$. Therefore different values of threshold generate networks with the same set of vertices but different sets of edges [20]. The edges ($E$) in the graph $G = (V,E)$ which represents the network of global financial indices are defined by

$$E = \begin{cases} e_{ij} = 1, & i \neq j \quad \text{and} \quad C_{ij} \geqslant \theta, \\ e_{ij} = 0, & i = j. \end{cases}$$

We construct financial correlation networks of indices at different thresholds (in the range 0 to 0.9). The Fruchterman-Reingold layout is used to find clusters in all these networks. The Fruchterman-Reingold algorithm is a force-directed layout [30]. In the force-directed layout, vertices are replaced by steel rings and each edge with a spring to form a mechanical system. The attractive force is analogous to the spring force and the repulsive force is analogous to the electrical force. The basic idea is to minimize the energy of the system by moving the nodes and changing the forces between them. At thresholds $\theta \leqslant 0.2$ the network of global financial indices is fully connected as shown in Figs. 4(a) and 4(b). On further increasing the value of the threshold, first of all, the nodes of Egypt and Israel corresponding to the Africa–Middle East cluster disconnect from the global network at threshold 0.5. The interesting feature of reorganization of financial indices is found in this financial network at the threshold 0.6. Before the crisis, Fig. 4(c) shows three separate clusters of indices namely, the American, European, and Asia-Pacific (except India). In Fig. 4(d) during the crisis, we find that the American and European clusters combine together to form a strongly linked cluster while the Asia-Pacific cluster forms a weakly linked cluster which connects India and disconnects Indonesia and Malaysia. These results show that the organization of financial indices changes when a crisis occurs. The threshold method using the force-directed layout shows this change in the organization of financial indices. If the threshold is further increased to $\theta = 0.9$ as shown in Figs. 4(e) and 4(f), we find that the European indices (France, Germany, and the United Kingdom) consistently constitute the most tightly linked financial indices.

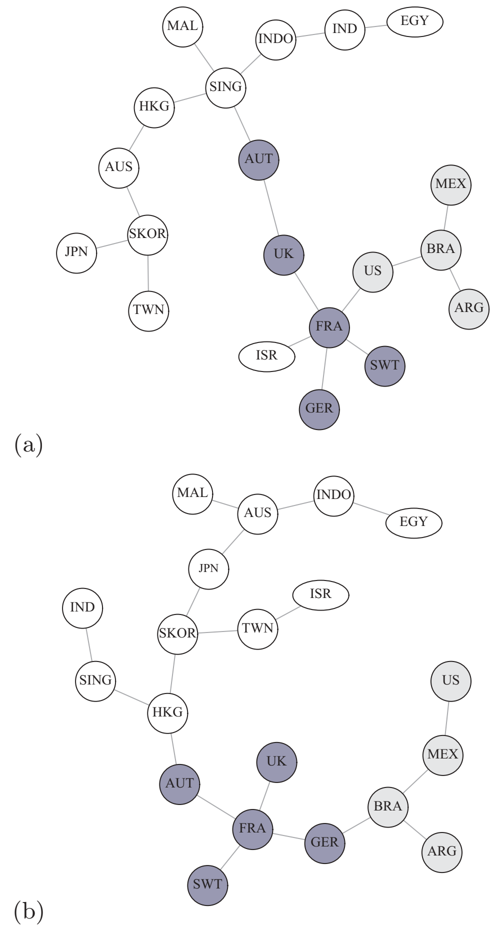



FIG. 6. (Color online) The MST is composed of the American (gray nodes), European (dark gray nodes), and Asia-Pacific (colorless nodes) clusters of indices. Egypt and Israel from the Africa–Middle East cluster are shown by elliptical nodes. (a) Before the crisis, the American and European clusters are connected via the United States and France. The European cluster connects to the Asia-Pacific via Austria and Singapore. Israel is connected with Europe while Egypt is linked with Asia-Pacific. (b) During the crisis, the American and European clusters link together via Brazil and Germany. Financial indices are organized by their geographical location.

### A. Topological structure of financial networks

*Mean degree*. The degree of vertex $i$ can be defined as $k_i = \sum_{j \neq i} e_{ij}$ [20]. The mean degree is based upon the degree and shows how many neighbors a node in the network has on average. This measure can be calculated only when the network has at least one edge connecting the nodes. Figure 5(a) shows that the mean degree decreases with increase in the threshold as the number of connected vertices decreases with increase in the threshold. In the correlation network of financial indices, a large value of the degree indicates that it is correlated with many other indices.

*Clustering coefficient*. If $k_i$ nearest neighbors of vertex $i$ have $m_i$ edges among them, the ratio of $m_i$ to $k_i(k_i - 1)/2$ is the clustering coefficient of vertex $i$. The global clustering coefficient is simply the ratio of triangles and connected triples in the correlation network of financial indices. Figure 5(b) shows that the clustering coefficients become smaller with increase in threshold up to 0.4. At $\theta = 0.9$ there is no triangle formation in the correlation network; there is only one triplet so its clustering coefficient is zero.

*Connected components*. If the graph $G = (V,E)$ is disconnected, it can be decomposed into several subgraphs which are known as connected components of $G$ [18,20]. The component number in the financial correlation network represents the financial indices that are correlated with each other. Figure 5(c) shows that the component number depends on the value of the correlation threshold and increases with increase in the value of the threshold.

*Clique*. A clique in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge. In the network of financial indices, the clique finds the cluster of indices that interact closely with each other. A maximum clique ($|C_l|_{\max}$) is a clique of the largest possible size in a given graph. We have used the IGRAPH and R software to find the maximum clique in the correlation network of global

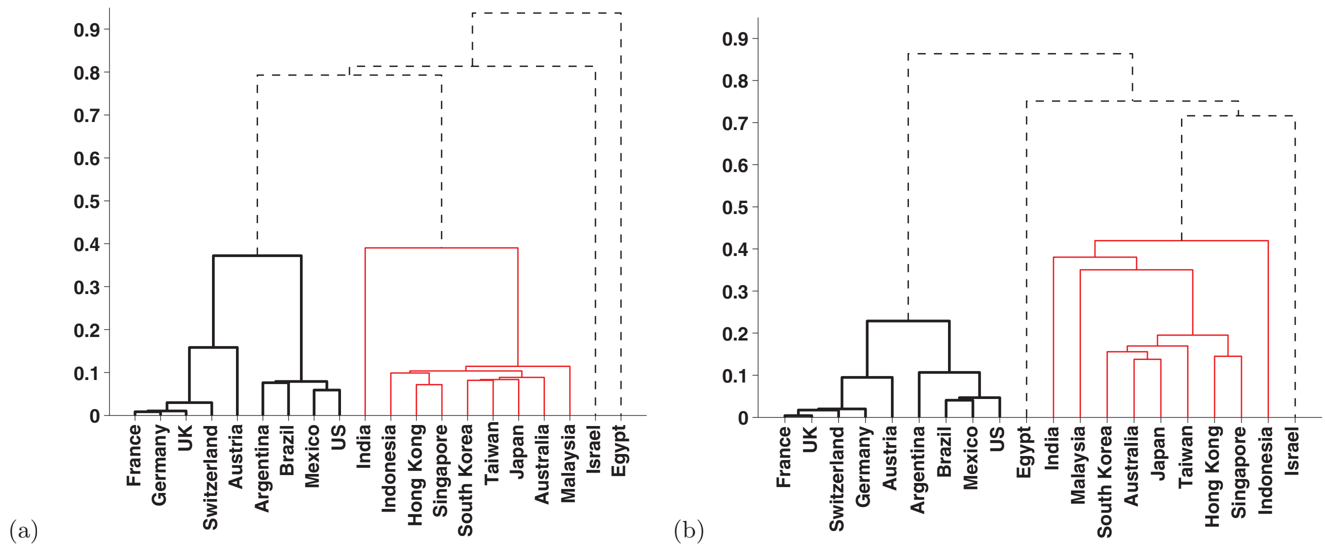



FIG. 7. (Color online) The dendrogram representation of financial indices (a) before the crisis and (b) during the crisis, showing the European-American (thick black lines) and Asia-Pacific (thin red lines) clusters including Egypt and Israel (dashed lines). France, United Kingdom, Switzerland, and Germany are correlated strongly as compared with other indices. The height of dendrograms of the European-American cluster decreases while the height of dendrograms of Asia-Pacific increases during the crisis.

financial indices. Figure 5(d) shows that the maximum clique size in the index correlation network decreases with increase in the threshold. We observe that up to $\theta = 0.4$ the maximum clique size is larger during the crisis as compared with the period before the crisis. For $\theta \geqslant 0.4$, there is no change in its value except at $\theta = 0.6$ where the indices form three clusters, namely, the American, European, and Asia-Pacific before the crisis, and two clusters, namely, the American-European and the Asia-Pacific during the crisis.

### B. Minimum spanning tree

We construct the network of financial indices by using the metric distances $d_{ij} = \sqrt{2(1 - C_{ij})}$ forming an $N \times N$ distance matrix whose elements vary between 0 and 2 [21]. The number of possible nodal connections of financial indices is large, i.e., $N(N-1)/2$. The MST reduces this complexity by showing only the $N - 1$ most important nonredundant connections in a graphical manner. We use the Prim algorithm [31] for drawing the MST. In Fig. 6(a), before the crisis, the MST shows that the American cluster is linked with the European cluster through nodes of the United States and France. The American and European clusters have Brazil and France as their hub vertices. Singapore and South Korea are the hub vertices in the Asia-Pacific cluster. Austria from the European cluster is linked with Singapore of the Asia-Pacific cluster. Egypt is connected with India of the Asia-Pacific cluster while Israel is connected with France from the European cluster. With Brazil, France, Singapore, and South Korea at the center, the structure of the MST is more starlike before the crisis. In Fig. 6(b), during the crisis, Brazil and France still remain at the centers of the American and European clusters, respectively, and Brazil is linked with Germany, connecting the American and European clusters. The node of Austria is linked with Hong Kong, connecting the European and Asia-Pacific clusters. Hong Kong, South Korea, and Australia are the major hub vertices in the Asia-Pacific cluster. With Brazil, France, Hong Kong, South Korea, and Australia connected in a chain the structure of the MST is more chainlike during the crisis [28]. We observe that there is a strong tendency for financial indices to organize by geographical location.

### C. Hierarchical clustering

The average linkage hierarchical clustering algorithm is applied to the distance matrix to produce the best treelike dendrogram [32]. The dendrogram is a mathematical and pictorial representation of the complete clustering procedure and the arrangement of nodes and stems is the topology of the tree. The dendrogram itself describes the process by which the hierarchy has been obtained [32]. The structures of the European-American (thick black lines) and Asia-Pacific (thin red lines) clusters are shown in Figs. 7(a) and 7(b); Egypt and Israel are shown by dashed lines. We find that during the crisis, the height of the dendrograms of the European-American cluster decreases while the height of the dendrograms of the Asia-Pacific cluster increases. This shows that the European-American indices interact (correlate) strongly while the Asia-Pacific indices (including Egypt and Israel) correlate weakly during the crisis. France is the tightly linked index in the European cluster in both periods. This further distinguishes the behavior of the European-American cluster from the Asia-Pacific cluster and indicates that the hierarchy of European and American indices increases while the hierarchy of Asia-Pacific indices decreases during the crisis.

We compute the correlation between the distances implied by the tree construction and distances defined by the original data, quantified by the cophenetic correlation coefficient (CCC) that measure the amount of hierarchy [32,33]. The CCC value is found to be 0.903 (before the crisis) and 0.933 (during the crisis) which is a significant change in the case of financial indices. This indicates that hierarchy increases during the financial crisis.

## V. CONCLUSION

The RMT analysis of correlation matrices provides information about the formation of clusters of indices. We show that the components of the eigenvectors corresponding to the second largest eigenvalue form two clusters of indices in the positive and negative directions. The components of these two clusters switch in opposite directions during the financial crisis. We constructed networks at different thresholds. The Fruchterman-Reingold layout is used in order to find the clusters in the networks. At threshold 0.6, we show that clusters of the European and American indices combine together to form a strongly linked cluster during the financial crisis. However, the behavior of the cluster of indices corresponding to the Asia-Pacific cluster is found to be different from that of the American and European cluster, and forms a weakly linked cluster during the crisis. The topological properties of financial correlation networks at different thresholds are studied before and during the crisis. The MST, dendrogram, and CCC analyses reinforce these results and show that the hierarchy increases during the crisis period. In conclusion, our findings show that there are major changes in the structure of the organization of financial indices during the financial crisis.

In a financial market, crisis and booms are some of the most important phenomena. Studying the crisis and finding the organizational changes of clusters during a crisis period is useful and interesting as similar changes may occur during other crises, leading to innovative ways for prevention and control.

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