Random-field and random-anisotropy O(N) spin systems with a free surface

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We study the surface scaling behavior of a semi-infinite d-dimensional O(N) spin system in the presence of a quenched random field and random anisotropy disorders. It is known that above the lower critical dimension $d_{\rm LC} = 4$ the infinite models undergo a paramagnetic-ferromagnetic transition for $N > N_c$ ($N_c = 2.835$ for the random field and $N_c = 9.441$ for random anisotropy). For $N < N_c$ and $d < d_{\rm LC}$ there exists a quasi-long-range-order phase with a zero order parameter and a power-law decay of spin correlations. Using a functional renormalization group, we derive the surface scaling laws that describe the ordinary surface transition for $d > d_{\rm LC}$ and the long-range behavior of spin correlations near the surface in the quasi-long-range-order phase for $d < d_{\rm LC}$. The corresponding surface exponents are calculated to one-loop order. The obtained results can be applied to the surface scaling of periodic elastic systems in disordered media, amorphous magnets, and ^{3}He-A in aerogel.

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I. INTRODUCTION

The phase diagram and critical properties of spin systems with quenched disorder have attracted considerable interest for decades. One usually distinguishes two types of quenched disorder: (i) random-temperature-like (random-bond-like) disorder corresponding to randomness coupled to the local energy density as in, for example, diluted ferromagnets [1] and (ii) random-field-like disorder corresponding to the case when the order parameter couples to a random symmetry breaking field [2]. The influence of random-temperature disorder is rather well understood. There exist several powerful methods to study the phase behavior and criticality such as the perturbative renormalization group (RG) method. In particular, the Harris criterion states that uncorrelated random-temperature-like disorder affects the critical behavior if the correlation-length critical exponent ν of the corresponding pure system satisfies the inequality $\nu < 2/d$, where d is the spatial dimensionality [3]. This criterion can be generalized to the case of correlated disorder [4,5]. The effect of the random field disorder being more profound is much less studied. The prominent example is the critical behavior of the random-field Ising model (RFIM) whose complete understanding is still lacking despite significant numerical, analytical, and experimental efforts [6]. It has been found that the perturbative calculations including standard RG methods lead to incorrect results, in particular, to the so-called dimensional reduction (DR). Analysis of the Feynman diagrams giving the leading singularities [7] or using supersymmetry [8] predicts that the critical behavior of the RFIM in d dimensions is the same as that of the pure system in $d-2$ dimensions. Consequently, the lower critical dimension of the RFIM below which there is no true long-range order is expected to be $d_{\rm LC}^{\rm DR} = 3$. However, the simple Imry-Ma arguments [2] and more rigorous methods [9] show that the lower critical dimension of the RFIM is in fact $d_{\rm LC} = 2$. Deviations from the DR prediction are also confirmed by the high-temperature expansion [10] and real space RG [11]. The failure of DR can be explained by the complicated energy landscape that renders the perturbation theory useless to all orders due to unphysical averaging over multiple minima and maxima. The latter can be formulated in terms of supersymmetry or replica symmetry breaking [12,13]. To overcome this obstacle one needs nonperturbative methods or a correct resummation of the perturbation theory.

Considerable progress has been achieved in recent years for the O(N) models in which disorder couples to the N-component order parameter either linearly as in the random-field (RF) case or bilinearly as in a random-anisotropy (RA) system. These models are relevant for diverse physical systems including amorphous magnets [14], diluted antiferromagnets in a uniform external magnetic field [15], liquid crystals in porous media [16,17], nematic elastomers [18], critical fluids in aerogels [19–21], vortices in type-II superconductors [22], and stochastic inflation in cosmology [23]. Similar to the RFIM, these models suffer from DR, i.e., the perturbation theory in weak disorder wrongly suggests that the behavior of these disordered systems is the same as that of the corresponding pure systems with smaller dimensionality [7,24]. Fisher studied the effect of higher-rank anisotropies and found that they all are relevant near the lower critical dimension $d_{\rm LC} = 4$ of the RF O(N) model [25]. He showed that the higher-rank anisotropies are generated by the RG flow even if the bare model has only RF or RA disorder. Fortunately, this infinite number of relevant operators can be recast into Taylor coefficients of an auxiliary function for which one can write down a closed functional renormalization group (FRG) flow equation. However, as shown in Ref. [25], the FRG flow equation has no fixed point (FP) in the class of analytic functions. Only almost two decades later, being inspired by the progress in disordered elastic systems [26–29], it was realized that the scaling properties of the RF and RA systems are encoded in a nonanalytic FP with a cusplike behavior at the origin [30]. It was shown that the nonanalytic FPs control the paramagnetic-ferromagnetic phase transitions in the RF and RA O(N) models and allow one to compute the critical exponents within an $\varepsilon = d-4$ expansion [31]. The critical exponents obtained using the FRG differ from those predicted by DR already to one-loop order. Recently, the FRG calculations have been extended to two-loop order [32,33] and the effect of long-range-disorder correlations has been studied [21,34]. Using the truncated exact FRG developed in

Ref. [35], it has been argued that spontaneous breaking of the supersymmetry, which leads to a breakdown of DR, occurs only below a critical dimension $d_{\mathrm{DR}} \approx 5.1$ [36].

A more peculiar issue concerns the phase diagram of the RF and RA models below d_{LC}. It is known that for the RF model and models with isotropic distributions of random anisotropies true long-range order is forbidden below $d_{\mathrm{LC}} = 4$ (for anisotropic distributions, long-range order can occur even below d_{LC} [37]) for $N > 1$. Nevertheless, quasi-long-range order (QLRO) with a zero order parameter and an infinite correlation length can persist even for $d^*_{\mathrm{LC}}(N) < d < d_{\mathrm{LC}}$, where $d^*_{\mathrm{LC}}(N)$ is the lower critical dimension for the paramagnetic-QLRO transition. For example, the Gaussian variational approximation predicts that the vortex lattice in disordered type-II superconductors can form the so-called Bragg glass exhibiting slow logarithmic growth of displacements [38]. This system can be mapped onto the three-dimensional RF O(2) model (XY model), in which the Bragg glass corresponds to the QLRO phase [39]. Indeed, for $N < N_c$ and $d < d_{\mathrm{LC}}$, the FRG equations have attractive FPs that describe the QLRO phases of RF and RA models [30]. However, the question of the lower critical dimension of the paramagnetic-QLRO transition is still controversial. In order to study the transition between the QLRO phase and the disordered phase using a FRG, one has to go beyond the one-loop approximation. The truncated exact FRG [35] and the two-loop FRG [33] performed using a double expansion in $\sqrt{|\varepsilon|}$ and $N - N_{\mathrm{c}}$ give access to the additional singly unstable FP that controls the transition. Both methods give qualitatively similar pictures of the FRG flows: The critical and attractive FPs merge in some dimension $d^*_{\mathrm{LC}}(N) < d_{\mathrm{LC}}$, which is considered the lower critical dimension of the paramagnetic-QLRO transition. For the RF O(2) model, both methods give approximately the same estimation $d^*_{\mathrm{LC}} \approx 3.8(1)$ and thus suggest that there is no Bragg glass phase in $d = 3$. However, one has to take caution when extrapolating results obtained for small $\sqrt{|\varepsilon|}$ and $N - N_{\mathrm{c}}$. Moreover, in contrast to the models of Refs. [30,33], which belong to the so-called hard-spin models, the system studied in Ref. [35] corresponds to soft spins. They can belong to different universality classes since the soft-spin model allows for topological defects that destroy the QLRO.

The real systems usually are finite and have boundaries whose effect is twofold: (i) The free energy of the system in addition to the bulk contribution proportional to the volume acquires a new term proportional to the area of the surface and (ii) the presence of boundaries breaks the translational invariance. In general, this can modify the behavior in the boundary region extended in the bulk only over distances of the order of the bulk correlation length. However, at the bulk critical point or in the QLRO phase, the bulk correlation length is infinite, so one expects the effect of boundaries to be more pronounced. Indeed, the presence of the boundaries introduces a whole set of critical exponents describing the scaling behavior at and close to the boundary at criticality [40]. Several different classes of the surface transitions are known depending upon boundary conditions [41]. The *ordinary transition* corresponds to the case when the surface magnetization is suppressed due to a reduced number of close neighbors near the boundary, so that the surface ordering is completely driven by the bulk magnetization. If for some reason the coupling between spins on the surface is sufficiently enhanced with respect to the bulk coupling or there is an external surface magnetic field, the surface may order before the bulk does. The latter is called the *surface transition*. Then the system can undergo the so-called *extraordinary transition* in the presence of an ordered surface. The two lines of the extraordinary transition and the surface transition meet at the multicritical point, which is called the *special transition*. The last three transitions can take place only if the dimension of the surface $d - 1$ is above the lower critical dimension for the transition. These transitions have been studied for various systems with discrete and continuous symmetries using different methods, such as RG and numerical simulations (for reviews see Refs. [40,42,43]).

The effect of weak random-temperature-like disorder on the surface criticality was studied using RG methods in Refs. [44,45]. The modified Harris-type criteria and other exact inequalities have also been derived for the critical behavior of systems with quenched disorder restricted to the surface [46]. However, not so much is known about the surface criticality in systems with RF disorder. The phase diagram of the three-dimensional (3D) semi-infinite RFIM as a function of the ratios of the bulk and surface interaction strengths and magnetic fields has been studied using a mean field approximation in Ref. [47]. The surface criticality of the RFIM has been studied numerically in Ref. [48]. It was also shown that the RF disorder on the surface of a 3D spin system with continuous symmetry destroys long-range order in the bulk and QLRO emerges instead of it [49]. In this work we address the question of how the RF and RA disorder in the bulk affect the behavior of spin systems with continuous symmetry in the vicinity of free surfaces. In particular, we consider the ordinary surface transition of the RF and RA O(N) models for $d > 4$ and the spin correlations in the QLRO phase near a free surface for $d < 4$.

The paper is organized as follows. Section II introduces the model. In Sec. III we renormalize the theory and derive the scaling laws. In Sec. IV we calculate the surface critical exponents to one-loop order. Section V summarizes the obtained results.

II. MODEL AND SCALING LAWS

We consider a d-dimensional semi-infinite O(N) spin system whose configuration is given by the N-component classical vector field $\mathbf{s}(\mathbf{r})$ satisfying the fixed-length constraint $|\mathbf{s}(\mathbf{r})|^2 = 1$. The position vector $\mathbf{r} = (\mathbf{x}, z)$ has a $(d-1)$-dimensional component $\mathbf{x}$ parallel to the surface and a one-dimensional component $z \geqslant 0$ that is perpendicular to the surface $z = 0$. It is convenient to introduce shorthand notations for the volume integral over half space $\int_V \equiv \int_0^\infty dz \int d^{d-1}x$ and for the surface integral $\int_S \equiv \int d^{d-1}x$. The large-scale behavior of the disordered spin system can be described by the effective Hamiltonian

$$\mathcal{H}[\mathbf{s}] = \mathcal{H}_0[\mathbf{s}] + \mathcal{H}_{\mathrm{surf}}[\mathbf{s}] + \mathcal{H}_{\mathrm{dis}}[\mathbf{s}], \qquad (1)$$

consisting of the sum of three terms that result from the semi-infinite bulk, surface, and disorder in the bulk. The contribution from the semi-infinite bulk can be expressed in the form of the well-known O(N) nonlinear σ model

$$\mathcal{H}_0[\mathbf{s}] = \int_V \left[\frac{1}{2}[\nabla \mathbf{s}(\mathbf{r})]^2 - \mathbf{h}\cdot\mathbf{s}(\mathbf{r})\right], \tag{2}$$

where $\mathbf{h}$ is the magnetic field in the bulk. The surface contribution to the Hamiltonian can be written in its simplest form as [50]

$$\mathcal{H}_{\mathrm{surf}}[\mathbf{s}] = -\int_S \mathbf{h}_1\cdot\mathbf{s}(\mathbf{x}), \tag{3}$$

where for simplicity we assume that the surface magnetic field $\mathbf{h}_1$ has the same direction as the bulk field $\mathbf{h}$. We consider a quite general type of bulk disorder such that its potential can be expanded in spin variables as

$$\mathcal{H}_{\mathrm{dis}}[\mathbf{s}] = -\int_V \sum_{\mu=1}^{\infty}\sum_{i_1,\ldots,i_\mu} h^{(\mu)}_{i_1,\ldots,i_\mu}(\mathbf{r}) s_{i_1}(\mathbf{r})\cdots s_{i_\mu}(\mathbf{r}). \tag{4}$$

The coefficients $h^{(\mu)}_{i_1,\ldots,i_\mu}(\mathbf{r})$ are assumed to be Gaussian random variables with zero mean and variance given by

$$\overline{h^{(\mu)}_{i_1,\ldots,i_\mu}(\mathbf{r}) h^{(\nu)}_{i_1,\ldots,j_\nu}(\mathbf{r}')} = \delta^{\mu\nu}\delta_{i_1 j_1}\cdots\delta_{i_\mu j_\nu} r_\mu \delta(\mathbf{r}-\mathbf{r}'). \tag{5}$$

The first two coefficients have a simple physical interpretation: $h^{(1)}_i$ is a random magnetic field and $h^{(2)}_{ij}$ is a second-rank random anisotropy. The higher-order coefficients $h^{(\mu)}$ are higher-rank random anisotropies. As was shown in Ref. [25], even if the system has only a finite number of nonzero bare $h^{(\mu)}$, the RG transformations will generate an infinite set of higher-order anisotropies. However, the RG flow preserves the symmetry with respect to inversion $\mathbf{s}\to-\mathbf{s}$. For instance, starting from the bare model with only a second-rank anisotropy, only even-rank anisotropies will be generated by the RG flow. We will reserve the notation RA for the systems that possess this symmetry and the notation RF for the systems that do not.

We employ the replica trick to average over disorder. Introducing n replicas of the original system and averaging their joint partition function over disorder, we obtain the replicated Hamiltonian

$$\mathcal{H}_n = \int_V \left\{\sum_{a=1}^{n}\left[\frac{1}{2}[\nabla \mathbf{s}_a(\mathbf{r})]^2 - \mathbf{h}\cdot\mathbf{s}_a(\mathbf{r})\right] - \frac{1}{2T}\sum_{a,b=1}^{n}\mathcal{R}[\mathbf{s}_a(r)\cdot\mathbf{s}_b(\mathbf{r})]\right\} - \sum_{a=1}^{n}\int_S \mathbf{h}_1\cdot\mathbf{s}_a(\mathbf{x}), \tag{6}$$

where we have introduced the function $\mathcal{R}(z)$ defined as $\mathcal{R}(z) = \sum_\mu r_\mu z^\mu$, with r_μ given by Eq. (5). The properties of the original disordered system (1) can be extracted in the limit $n\to 0$. Both the RF and the RA models are described by the replicated Hamiltonian (6). The only difference is that the function $\mathcal{R}(z)$ is even for the RA systems and noneven for the RF systems.

Power counting shows that $d_{\mathrm{LC}} = 4$ is the lower critical dimension of the model (6) [24]. Above the lower critical dimension the RF and RA systems undergo a paramagnetic-ferromagnetic transition. The scaling behavior at criticality is controlled by a zero-temperature FP similar to the RFIM [51], reflecting the fact that disorder dominates over thermal fluctuations. However, the temperature is dangerously irrelevant. For instance, this results in violation of the usual hyperscaling relation and the appearance of an additional universal exponent θ that modifies the hyperscaling relation to Ref. [6]

$$\nu(d-\theta) = 2-\alpha, \tag{7}$$

where ν and α are the correlation-length and the specific-heat exponents. One also expects a dramatic slowing down as the transition is approached with the characteristic relaxation time $\ln\tau \sim t_1^{-\nu\theta}$, where $t_1 = |T-T_c|/T_c$ is the reduced temperature [52]. The magnetizations in the bulk and on the surface vanish at the transition according to

$$\sigma(t_1) \sim t_1^{\beta}, \quad \sigma_1(t_1) \sim t_1^{\beta_1}, \tag{8}$$

where we have introduced the bulk and the surface magnetization exponents. At the critical point $t_1 = 0$ a small magnetic field in the bulk $\mathbf{h}$ induces a magnetization in the bulk and also on the surface according to

$$\sigma(h) \sim h^{1/\delta}, \quad \sigma_1(h) \sim h^{1/\delta_1}, \tag{9}$$

where we defined the exponents δ and δ_1. The surface magnetic field $\mathbf{h}_1$ leads to the surface magnetization

$$\sigma_1(h_1) \sim h_1^{1/\delta_{11}}. \tag{10}$$

Below the lower critical dimension d_{LC} a QLRO phase with zero magnetization can emerge. At criticality or in the QLRO phase, the correlation functions of the order parameter exhibit scaling behavior. Due to dangerous irrelevance of the temperature the connected and disconnected correlation functions scale with different exponents. We define the connected and disconnected correlation functions of the two local operators A and B as

$$[A(\mathbf{r})\cdot B(\mathbf{r}')]_{\mathrm{con}} \equiv \overline{\langle A(\mathbf{r})\cdot B(\mathbf{r}')\rangle - \langle A(\mathbf{r})\rangle\cdot\langle B(\mathbf{r}')\rangle},$$

$$[A(\mathbf{r})\cdot B(\mathbf{r}')]_{\mathrm{dis}} \equiv \overline{\langle A(\mathbf{r})\rangle\cdot\langle B(\mathbf{r}')\rangle} - \overline{\langle A(\mathbf{r})\rangle}\cdot\overline{\langle B(\mathbf{r}')\rangle}.$$

Here the angular brackets denote the thermal averaging and the overbar stands for the disorder averaging. For instance, the connected and disconnected correlation functions of spins in the bulk scale independently as

$$[\mathbf{s}(\mathbf{r})\cdot\mathbf{s}(\mathbf{r}')]_{\mathrm{con}} \sim \frac{1}{|\mathbf{r}-\mathbf{r}'|^{d-2+\eta}}, \tag{11}$$

$$[\mathbf{s}(\mathbf{r})\cdot\mathbf{s}(\mathbf{r}')]_{\mathrm{dis}} \sim \frac{1}{|\mathbf{r}-\mathbf{r}'|^{d-4+\bar\eta}}. \tag{12}$$

Following the general scaling picture of the surface critical phenomena, we introduce the surface exponents $\eta_\perp$ and $\bar\eta_\perp$, which replace the bulk exponents η and $\bar\eta$ in Eqs. (11) and (12) when one of the points $\mathbf{r}$ or $\mathbf{r}'$ belongs to the surface:

$$[\mathbf{s}(\mathbf{x},z)\cdot\mathbf{s}(\mathbf{x}',0)]_{\mathrm{con}} \sim \frac{1}{[(\mathbf{x}-\mathbf{x}')^2+z^2]^{(d-2+\eta_\perp)/2}}, \tag{13}$$

$$[\mathbf{s}(\mathbf{x},z)\cdot\mathbf{s}(\mathbf{x}',0)]_{\mathrm{dis}} \sim \frac{1}{[(\mathbf{x}-\mathbf{x}')^2+z^2]^{(d-4+\bar\eta_\perp)/2}}. \tag{14}$$

We also define the surface exponents $\eta_\parallel$ and $\bar\eta_\parallel$ that describe the connected and disconnected correlation function when both points lie on the surface:

$$[\mathbf{s}(\mathbf{x})\cdot\mathbf{s}(\mathbf{x}')]_{\rm con} \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{d-2+\eta_\parallel}}, \tag{15}$$

$$[\mathbf{s}(\mathbf{x})\cdot\mathbf{s}(\mathbf{x}')]_{\rm dis} \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{d-4+\bar\eta_\parallel}}. \tag{16}$$

Schwartz and Soffer [53] showed that the bulk exponents of the RF model obey the inequality $2\eta \geqslant \bar\eta$. The same arguments can be applied to the surface correlation functions so that the surface exponents satisfy similar inequalities: $2\eta_\perp \geqslant \bar\eta_\perp$ and $2\eta_\parallel \geqslant \bar\eta_\parallel$. Note that these inequalities cannot be applied to the RA model where the coupling to disorder is bilinear.

III. FUNCTIONAL RENORMALIZATION GROUP

A. Perturbation theory

In the limit of low temperature and weak disorder the configuration of the system is fluctuating around the completely ordered state in which all replicas of all spins align along the same direction, which is parallel to $\mathbf{h}$ and $\mathbf{h}_1$. It is convenient to split the order parameter $\mathbf{s}_a = (\sigma_a, \boldsymbol{\pi}_a)$ into the $(N-1)$-component vector $\boldsymbol{\pi}_a$, which is perpendicular to this direction, and the component $\sigma_a = \sqrt{1-\boldsymbol{\pi}_a^2}$, which is parallel to this direction. Then the effective action of the system can be written as

$$\mathcal{S}[\boldsymbol{\pi}] = \frac{1}{T}\sum_{a=1}^{n}\left\{\int_V\left[\frac12(\nabla\boldsymbol{\pi}_a)^2 + \frac{(\boldsymbol{\pi}_a\cdot\nabla\boldsymbol{\pi}_a)^2}{2(1-\boldsymbol{\pi}_a^2)} - h\,\sigma_a\right] - \int_S h_1\sigma_a\right\} - \frac{1}{2T^2}\sum_{a,b=1}^{n}\int_V \mathcal{R}(\boldsymbol{\pi}_a\cdot\boldsymbol{\pi}_b + \sigma_a\sigma_b). \tag{17}$$

In general, one has to add to the action (17) the terms such as $\delta^d(0)\int_V \ln(1-\boldsymbol{\pi}_a^2)$ generated by the Jacobian of the transformation from $\mathbf{s}_a$ to $\boldsymbol{\pi}_a$. However, in what follows we will use the dimensional regularization scheme [54] in which $\delta^d(0)=0$, so we ignore these terms in action (17) from the beginning.

Let us denote averaging with the action (17) by double angular brackets and introduce the correlation functions

$$G^{(L,K)}_{\alpha,\beta}(\mathbf{r},\mathbf{x}) = \left\langle\!\!\left\langle \prod_{\nu=1}^{L}\pi_{\alpha_\nu}(\mathbf{r}_\nu)\prod_{\mu=1}^{K}\pi_{\beta_\mu}(\mathbf{x}_\mu)\right\rangle\!\!\right\rangle, \tag{18}$$

where L points $\mathbf{r}=(\mathbf{r}_1,\ldots,\mathbf{r}_L)$ are off surface and K points $\mathbf{x}=(\mathbf{x}_1,\ldots,\mathbf{x}_K)$ are siting on the surface. In Eq. (18) we have used the shorthand notation $\alpha=(\alpha_1,\ldots,\alpha_L)$, where each α_ν stands for the component number i_ν and the replica number a_ν, and similarly for β. The correlation functions (18) can be computed using the generating functional [55]

$$\mathcal{F}[\mathbf{J},\mathbf{J}_1] = \ln\int\mathcal{D}\boldsymbol{\pi}\,\exp\left(-\mathcal{S}[\boldsymbol{\pi}] + \int_V \mathbf{J}(\mathbf{r})\boldsymbol{\pi}(\mathbf{r}) + \int_S \mathbf{J}_1(\mathbf{x})\boldsymbol{\pi}(\mathbf{x})\right), \tag{19}$$

where we assume that the source $\mathbf{J}(\mathbf{r})$ vanishes at the surface. Differentiating with respect to the sources, we obtain

$$G^{(L,K)}(\mathbf{r},\mathbf{x}) = \prod_{\nu=1}^{L}\frac{\delta}{\delta J(\mathbf{r}_\nu)}\prod_{\mu=1}^{K}\frac{\delta}{\delta J_1(\mathbf{x}_\mu)}\mathcal{F}\Bigg|_{J=J_1=0}, \tag{20}$$

where for the sake of brevity we have suppressed all tensorial indices. Using correlation functions (18), one can compute the connected and disconnected functions defined in Eqs. (11) and (12). However, since we are interested only in the scaling behavior, it is more convenient to consider the similar correlation functions not for $\mathbf{s}$ but for $\boldsymbol{\pi}$ fields. For example, the correlation functions at two off-surface points read

$$[\boldsymbol{\pi}(\mathbf{r})\cdot\boldsymbol{\pi}(\mathbf{r}')]_{\rm con} = \lim_{n\to0}\sum_{i=1}^{N-1}G^{(2,0)}_{i,a;i,a}(\mathbf{r},\mathbf{r}'), \tag{21}$$

$$[\boldsymbol{\pi}(\mathbf{r})\cdot\boldsymbol{\pi}(\mathbf{r}')]_{\rm dis} = \lim_{n\to0}\sum_{i=1}^{N-1}G^{(2,0)}_{i,a;i,b}(\mathbf{r},\mathbf{r}'), \tag{22}$$

where the connected correlation function corresponds to a single replica and the disconnected one to two different replicas $a\neq b$. To compute the correlation functions at the surface such as $[\boldsymbol{\pi}(\mathbf{r})\cdot\boldsymbol{\pi}(\mathbf{x}')]_{\rm con}$ or $[\boldsymbol{\pi}(\mathbf{x})\cdot\boldsymbol{\pi}(\mathbf{x}')]_{\rm dis}$ one has to replace $G^{(2,0)}$ by $G^{(1,1)}$ and $G^{(0,2)}$, respectively.

Expanding the effective action (17) in small $\boldsymbol{\pi}$, we will treat the quadratic part as a free action and the rest of the infinite series as interaction vertices (see the Appendix). Then the correlation functions (18) can be expressed in terms of Feynman diagrams, which give the low-temperature and small disorder expansion. In practical calculations it is convenient to perform the Fourier transform with respect to $\mathbf{x}$: $\hat{\boldsymbol{\pi}}(\mathbf{q},z) = \int d^{d-1}x\,\boldsymbol{\pi}(\mathbf{x},z)e^{-i\mathbf{q}\cdot\mathbf{x}}$ and define $\int_q \equiv \int d^{d-1}q/(2\pi)^{d-1}$. The quadratic terms give the free propagator

$$\hat G^0_q(z,z') = \frac{1}{2\bar q}\left[e^{-\bar q|z-z'|} + \frac{\bar q - h_1}{\bar q + h_1}e^{-\bar q(z+z')}\right], \tag{23}$$

where we have introduced the shorthand notation $\bar q \equiv (q^2+h)^{1/2}$. The free propagator (23) satisfies the boundary conditions

$$[\partial_z - h_1]\hat G^0_q(z,z')|_{z=0} = 0. \tag{24}$$

The free surface corresponds to the limit $h_1\to0$ in which Eq. (23) becomes the Neumann propagator consisting of the bulk part and the image part. In what follows we will use the Neumann propagator as the bare one and treat the terms proportional to h_1 as soft insertions [50,56].

B. Functional renormalization group equations and critical exponents

The correlation functions (18) calculated perturbatively in small disorder and temperature suffer from UV divergences. To avoid mixture with IR singularities in the O(N)-noninvariant correlation functions it is convenient to keep $\mathbf{h}\neq0$. The UV divergences can be converted into poles in $\varepsilon = d-4$ using dimensional regularization. To renormalize the theory one has to absorb these poles into a number of Z factors. However, all the Taylor coefficients r_μ of the disorder correlator $\mathcal{R}(z)$

turn out to be relevant operators, so one has to introduce renormalization of the whole function. To simplify calculation of the disorder renormalization one can use the background field method [29], which allows one to handle functional diagrams that involve the whole function $\mathcal{R}(z)$ instead of computing one by one the counterterms to the derivatives $\mathcal{R}^{(\mu)}(0)$. Using the Legendre transform of the generating functional (19) from the sources $\mathbf{J}$ to the background fields $\mathbf{\Pi}$, one derives the effective action $\Gamma[\mathbf{\Pi}]$, which is the generating functional of the one-particle irreducible vertices. The two-replica part of the effective action gives the renormalization of the disorder. Since the scaling behavior is controlled by a zero-temperature FP, we will disregard all terms involving more than two replicas. These terms are irrelevant near $d = 4$ or their contributions are suppressed in the limit $T \to 0$ [25,32]. The renormalization of the disorder simplifies by changing variables: $\mathcal{R}(z) = R(\phi)$, where $z = \cos\phi$; for instance, $\mathcal{R}'(1) = -R''(0)$. In terms of the variable ϕ, the function $R(\phi)$ becomes periodic with period 2π in the RF case and with period π in the RA case. The relation between the renormalized and the bare correlation functions reads

$$G^{(L,K)}(\mathbf{r};T,h,h_1,R,\mu) = Z_\pi^{-(L+K)/2} Z_1^{-K/2} \mathring{G}^{(L,K)}(\mathbf{r};\mathring{T},\mathring{h},\mathring{h}_1,\mathring{R}), \quad (25)$$

where circles denote the bare quantities and μ is an arbitrary momentum scale. The UV divergences are absorbed into Z factors according to

$$\mathring{\boldsymbol{\pi}} = Z_\pi^{1/2}\boldsymbol{\pi}, \quad \mathring{\boldsymbol{\pi}}|_s = (Z_\pi Z_1)^{1/2}\boldsymbol{\pi}|_s, \quad (26)$$

$$\mathring{h} = \mu^2 Z_T Z_\pi^{-1/2} h, \quad \mathring{h}_1 = \mu Z_T (Z_\pi Z_1)^{-1/2} h_1, \quad (27)$$

$$\mathring{T} = \mu^{2-d} Z_T T, \quad \mathring{R} = \mu^{4-d} K_d^{-1} Z_R[R], \quad (28)$$

where $(2\pi)^d K_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a d-dimensional unit sphere and $\Gamma(x)$ is the Euler Gamma function. In Eq. (28) $Z_R[R]$ is a functional acting on the renormalized disorder correlator $R(\phi)$, which has the following loop expansion:

$$Z_R[R] = R + \delta^{(1)}(R,R) + \delta^{(2)}(R,R,R) + \cdots, \quad (29)$$

where $\delta^{(1)}(R,R)$ is bilinear in R and proportional to $1/\varepsilon$, while $\delta^{(2)}(R,R,R)$ is cubic in R and contains terms of order $1/\varepsilon$ and $1/\varepsilon^2$. According to Eq. (26), the surface field $\boldsymbol{\pi}|_s$ renormalizes differently from the field $\boldsymbol{\pi}$ in the bulk. The new factor Z_1 serves to cancel the additional UV divergences in Feynman diagrams arising from the image part of the Neumann propagator $\hat{G}^0_q(z,z')$ for $z' \to 0$. The renormalized theory is not unique and depends on the scale μ. Using this fact, we will derive the FRG equation.

We now consider how the scaling behavior can be extracted from the renormalized theory. Using the independence of the bare theory on the momentum scale μ, one can derive the flow equations for the renormalized correlation functions differentiating both sides of Eq. (25) with respect to μ at fixed bare quantities. One finds that the renormalized correlation functions satisfy the FRG equation

$$\Big[\mu\partial_\mu + (d-2-\zeta_T)T\partial_T - \zeta_h h\partial_h - \zeta_{h_1} h_1\partial_{h_1} + \frac{L}{2}\zeta_\pi + \frac{K}{2}(\zeta_\pi + \zeta_1) - \int d\phi\, \beta[R(\phi)]\frac{\delta}{\delta R(\phi)}\Big] G^{(L,K)} = 0, \quad (30)$$

where the integral in the last line is taken over a period, i.e., $(0,\pi)$ for RA and $(0,2\pi)$ for RF models and we have introduced the scaling functions

$$\zeta_i = \mu\partial_\mu \ln Z_i|_0 \quad (i = T,\pi,1), \quad (31)$$

$$\zeta_h = 2 + \zeta_T - \zeta_\pi/2, \quad (32)$$

$$\zeta_{h_1} = 1 + \zeta_T - (\zeta_\pi + \zeta_1)/2, \quad (33)$$

$$\beta[R] = -\mu\partial_\mu R(\phi)|_0. \quad (34)$$

Here zero indicates that the derivatives are taken at fixed bare quantities. Flow equations similar to Eq. (30) hold also for the correlation functions in which some or all the fields $\boldsymbol{\pi}_a(\mathbf{r})$ are replaced by $\sigma_a(\mathbf{r})$ and for other observables, e.g., the correlation length and the magnetization [54].

The long-distance physics can be obtained from the solution of the FRG equation (30) in the limit of $\mu \to 0$. The renormalized disorder correlator and the temperature flow according to

$$-\mu\partial_\mu R(\phi) = \beta[R], \quad (35)$$

$$-\mu\partial_\mu \ln T = 2 - d + \zeta_T. \quad (36)$$

The scaling behavior is controlled by a zero-temperature FP $\beta[R^*] = 0$, with R^* of order ε and $T^* = 0$. Indeed, according to Eq. (36), the temperature is irrelevant, i.e., it flows to 0 in the limit $\mu \to 0$ for $d > 2$ and for sufficiently small $\zeta_T = O(R)$. Although one expects that ζ_T is small in the vicinity of the FP, one has to take caution whether the zero-temperature FP survives in three dimensions where $\zeta_T \sim \varepsilon$ is negative [30]. The stability of the FP can be checked by computing the eigenvalues of the disorder flow equation (35) linearized about the FP solution: $R(\phi) = R^*(\phi) + \sum_i t_i \Psi_i(\phi)$. Since one expects that for $d > 4$ ($\varepsilon > 0$) the FP $R^*(\phi)$ describes the paramagnetic-ferromagnetic transition, it has to be unstable in a single direction $\Psi_1(\phi)$ with eigenvalue $\lambda_1 > 0$: $\beta[R^* + t_1\Psi_1] = \lambda_1 t_1 \Psi_1 + O(t_1^2)$. In the vicinity of the zero-temperature FP that controls the paramagnetic-ferromagnetic transition, the FRG equation for the correlation length ξ can be written as

$$\Big[\mu\partial_\mu - \lambda_1 t_1 \frac{\partial}{\partial t_1}\Big]\xi(\mu,t_1) = 0. \quad (37)$$

Dimensional analysis implies that $\xi(\mu,t_1) = \mu^{-1}\bar{\xi}(t_1)$. This reduces Eq. (37) to an ordinary differential equation (ODE) whose solution is given by $\xi \sim \mu^{-1} t_1^{-1/\lambda_1}$. The latter describes divergence of the correlation length on the critical line at zero temperature when the strength of disorder approaches the critical value [51]. Assuming that along the transition line at finite temperature $t_1 \sim T - T_c$, we find that the positive eigenvalue λ_1 gives the critical exponent of the correlation length $\nu = 1/\lambda_1$. For $d < 4$ ($\varepsilon < 0$) the FP becomes stable and describes a QLRO phase. The fluctuations exhibit power-law correlations in the whole QLRO phase so that the correlation length ξ is always infinite down to the lower critical dimension of the QLRO-paramagnetic transition.

Let us consider the solution of Eq. (30) for the connected two-point correlation functions. The dangerous irrelevance of the temperature manifests itself in the fact that the connected (bulk or surface) two-point functions are proportional to T in the low-temperature limit. This is explicitly shown in the Appendix for the connected correlation function $G^{(1,1)}$ to one-loop order. The linear behavior at low temperature can be checked also in higher-loop diagrams taking into account that each propagator line carries a factor of T while the one- and two-replica vertices bring factors of T^{-1} and T^{-2}, respectively. Hence, setting $h = h_1 = 0$ and $R = R^*$, we can rewrite Eq. (30) as

$$\left[\mu\partial_\mu + \frac{1}{2}(L+K)\zeta_\pi^* + \frac{K}{2}\zeta_1^* + \theta\right] G_{\rm con}^{(L,K)} = 0, \quad (38)$$

where the asterisk denotes that the function is computed at the zero-temperature FP. In Eq. (38) we have defined the exponent

$$\theta = d - 2 - \zeta_T^*, \quad (39)$$

which describes the flow of the temperature (36) in the vicinity of the FP and has been introduced *ad hoc* in the modified hyperscaling relation (7). Using the method of characteristics and dimensional analysis, one can write the solution of Eq. (38) in the form

$$G_{\rm con}^{(L,K)}(rb; R^*) = b^{-[(L+K)\zeta_\pi^*/2 + K\zeta_1^*/2 + \theta]} f_c(r; R^*). \quad (40)$$

Considering the connected two-point functions (40) with $(L = 2, K = 0)$, $(L = 1, K = 1)$, and $(L = 0, K = 2)$, we derive the critical exponents

$$\eta = \zeta_\pi^* - \zeta_T^*, \quad (41)$$

$$\eta_\perp = \zeta_\pi^* + \zeta_1^*/2 - \zeta_T^*, \quad (42)$$

$$\eta_\parallel = \zeta_\pi^* + \zeta_1^* - \zeta_T^*. \quad (43)$$

We next turn to the disconnected two-point correlation functions. At variance with the connected correlation functions, they are not proportional to the temperature. Thus, at $h = h_1 = T = 0$ they satisfy the same Eq. (38) but without the term θ in large square brackets. The solution of the latter FRG equation is given by

$$G_{\rm dis}^{(L,K)}(rb; R^*) = b^{-[(L+K)\zeta_\pi^*/2 + K\zeta_1^*/2]} f_d(r; R^*). \quad (44)$$

Repeating the analysis we did for the connected functions, we arrive at

$$\bar\eta = 4 - d + \zeta_\pi^* = 2 + \eta - \theta, \quad (45)$$

$$\bar\eta_\perp = 4 - d + \zeta_\pi^* + \zeta_1^*/2 = 2 + \eta_\perp - \theta, \quad (46)$$

$$\bar\eta_\parallel = 4 - d + \zeta_\pi^* + \zeta_1^* = 2 + \eta_\parallel - \theta. \quad (47)$$

Note that the exponents (41)–(43) and (45)–(47) are related by

$$2\eta_\perp = \eta + \eta_\parallel, \quad 2\bar\eta_\perp = \bar\eta + \bar\eta_\parallel. \quad (48)$$

Finally, we study the profile of the spontaneous magnetization below and at the paramagnetic-ferromagnetic transition for $d > d_{\rm LC}$. The magnetization as a function of the distance to the surface z, the reduced temperature t_1, and the bulk and surface magnetic fields h and h_1 satisfies the flow equation

$$\left[\mu\partial_\mu - \zeta_h^* h\partial_h - \zeta_{h_1}^* h_1\partial_{h_1} + \frac{1}{2}\zeta_\pi^* + \frac{j}{2}\zeta_1^* - \lambda_1 t_1\frac{\partial}{\partial t_1}\right] \times \sigma(z,t_1,h,h_1) = 0. \quad (49)$$

Here $j = 0$ and $z > 0$ corresponds to the bulk magnetization σ while $j = 1$ and $z = 0$ gives the surface magnetization σ_1. The solution of Eq. (49) can be written as

$$\sigma(z,t_1,h,h_1) = b^{-[\zeta_\pi^*/2 + (j/2)\zeta_1^*]}\sigma(zb^{-1}, t_1 b^{\lambda_1}, hb^{\zeta_h^*}, h_1 b^{\zeta_{h_1}^*}). \quad (50)$$

We first consider the profile for $h = h_1 = 0$. The solution (50) allows one to obtain the scaling function of the magnetization profile for $z > 0$. The magnetization approaches its bulk value $\sigma(t_1,z) \sim t_1^{\zeta_\pi^*/2\lambda_1}$ for $z \gg \xi$, but at small values of z the scaling function develops a short-distance singularity that ensures consistency with the temperature dependence of the surface magnetization given by $\sigma_1(t_1) \sim t_1^{(\zeta_\pi^* + \zeta_1^*)/2\lambda_1}$. We now reexpress the both magnetizations obtained in the limits of $z \to \infty$ and 0 in terms of ν, $\bar\eta$, and $\bar\eta_\parallel$. This yields the bulk and surface magnetization exponents defined in Eq. (8):

$$\beta = \tfrac{1}{2}\nu(d - 4 + \bar\eta), \quad \beta_1 = \tfrac{1}{2}\nu(d - 4 + \bar\eta_\parallel). \quad (51)$$

At the critical point $t_1 = 0$ and finite external fields we find that $\sigma(h) \sim h^{\zeta_\pi^*/(2\zeta_h^*)}$ in the bulk and $\sigma_1(h) \sim h^{(\zeta_\pi^* + \zeta_1^*)/2\zeta_h^*}$ or $\sigma_1(h_1) \sim h_1^{(\zeta_\pi^* + \zeta_1^*)/2\zeta_{h_1}^*}$ at the surface. Thus the exponents δ, δ_1, and δ_{11} defined in Eqs. (9) and (10) satisfy the following scaling relations:

$$\frac{\delta - 1}{2 - \eta} = \frac{\nu}{\beta}, \quad \frac{\delta_1 - \beta/\beta_1}{2 - \eta} = \frac{\nu}{\beta_1}, \quad \frac{\delta_{11} - 1}{1 - \eta_\parallel} = \frac{\nu}{\beta_1}. \quad (52)$$

These scaling relations are expected to be general for the RF systems and thus can be applied to not only the O(N) models but also to the semi-infinite RFIM.

IV. SURFACE EXPONENTS TO ONE-LOOP ORDER

We now renormalize the both semi-infinite RF and RA models to one-loop order and explicitly calculate the surface critical exponents to first order in $\varepsilon = d - 4$. The factors Z_π, Z_T. and $Z_R[R]$ defined in Eqs. (26)–(29) are the same that appear in the case of the infinite systems. They have been calculated in several works up to two-loop order [25,30–33]. To one-loop order they read

$$Z_\pi = 1 - (N-1)\frac{R''(0)}{\varepsilon} + O(R^2), \quad (53)$$

$$Z_T = 1 - (N-2)\frac{R''(0)}{\varepsilon} + O(R^2), \quad (54)$$

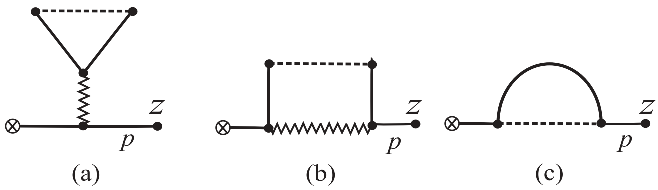

FIG. 1. One-loop diagrams contributing to the connected two-point function $\mathring{G}^{(1,1)}_{1,a;1,a}(z,p)$. The solid lines stand for the Neumann propagator (23). The wavy and dashed lines are vertices defined in Eqs. (A1)–(A3). The crossed circles denote the points on the surface.

$$\varepsilon\delta^{(1)}(R,R) = \frac{1}{2}R''(\phi)^2 - R''(0)R''(\phi) - (N-2)\left\{R''(0)[2R(\phi)+R'(\phi)\cot\phi] - \frac{1}{2\sin^2\phi}\left[R'(\phi)\right]^2\right\}. \tag{55}$$

The new factor Z_1 that eliminates the poles resulting from the presence of the surface can be determined from the renormalization of the two-point function $\mathring{G}^{(1,1)}(p,z;\mathring{h},\mathring{T},\mathring{R})$. The one-loop diagrams contributing to this function are shown in Fig. 1. The corresponding integrals are computed in the Appendix and give

$$\mathring{G}^{(1,1)}(p,z;\mathring{h},\mathring{T},\mathring{R}) = \mathring{T}\frac{e^{-\bar{p}z}}{\bar{p}}\left\{1 - \frac{K_d}{4\varepsilon}\mathring{R}''(0)\left[(N-3)\left(\frac{\mathring{h}}{\bar{p}^2}+\frac{z\mathring{h}}{\bar{p}}\right) + 2(N+1)\right] + O(\mathring{R}^2)\right\}, \tag{56}$$

where $\bar{p} = (p^2+\mathring{h}^2)^{1/2}$. The factor Z_1 can be found from the renormalization condition

$$Z_\pi^{-1}Z_1^{-1/2}\mathring{G}^{(1,1)}(p,z;\mathring{h},\mathring{T},\mathring{R}) = \text{finite for} \quad \varepsilon \to 0, \tag{57}$$

where the bare $\mathring{h}$, $\mathring{T}$, and $\mathring{R}$ are replaced by the renormalized h, T, and R according to Eqs. (26)–(28). We obtain

$$Z_1 = 1-(N-1)\frac{R''(0)}{\varepsilon} + O(R^2). \tag{58}$$

Thus, to one-loop order we have $Z_1 = Z_\pi + O(R^2)$. Using Eqs. (31) and (34) we calculate the scaling functions

$$\zeta_T = -(N-2)R''(0) + O(R^2), \tag{59}$$

$$\zeta_\pi = \zeta_1 = -(N-1)R''(0) + O(R^2) \tag{60}$$

and the beta function

$$\beta[R] = -\varepsilon R(\phi) + \frac{1}{2}R''(\phi)^2 - R''(0)R''(\phi) - (N-2)\left\{R''(0)[2R(\phi)+R'(\phi)\cot\phi] - \frac{1}{2\sin^2\phi}[R'(\phi)]^2\right\} + O(R^2) \tag{61}$$

to one-loop order. The solution of the FP equation

$$\beta[R^*] = 0 \tag{62}$$

with the β function (61) has been analyzed for different values of N and different sign of ε in Refs. [30–33]. We first assume that the flow has a FP $R^*(\phi)$ that is a π-periodic function for the RA model and a 2π-periodic function for the RF model. Then the surface critical exponents can be computed to one-loop order using Eqs. (41)–(43) and (45)–(47), which yields

$$\eta = -R^{*\prime\prime}(0), \quad \bar{\eta} = -\varepsilon - (N-1)R^{*\prime\prime}(0), \tag{63}$$

$$\eta_\perp = -\frac{N+1}{2}R^{*\prime\prime}(0), \quad \bar{\eta}_\perp = -\varepsilon - \frac{3}{2}(N-1)R^{*\prime\prime}(0), \tag{64}$$

$$\eta_\parallel = -NR^{*\prime\prime}(0), \quad \bar{\eta}_\parallel = -\varepsilon - 2(N-1)R^{*\prime\prime}(0). \tag{65}$$

The other surface exponents are related to Eqs. (63)–(65) by the scaling relations (51) and (52).

Before we explicitly calculate the surface exponents for the semi-infinite RF and RA models let us recall how the FRG allows one to overcome the DR problem. The incorrect DR prediction results from the assumption that the flow equation (35) with the β function (61) has a FP that is an analytic function. Indeed, in this case one can obtain a closed flow equation for the $R''(0)$:

$$-\mu\partial_\mu R''(0) = -\varepsilon R''(0) - (N-2)R''(0)^2. \tag{66}$$

Equation (66) has a nontrivial FP solution $R^{*\prime\prime}(0) = -\varepsilon/(N-2)$ with the eigenvalue $\lambda_1 = \varepsilon$. This FP is unstable for $\varepsilon > 0$, as one expects for a FP corresponding to the transition, and gives the DR exponents $\nu^{(\mathrm{DR})} = 1/\varepsilon$ and

$$\eta^{(\mathrm{DR})} = \bar{\eta}^{(\mathrm{DR})} = \frac{\varepsilon}{N-2}, \tag{67}$$

$$\eta_\perp^{(\mathrm{DR})} = \bar{\eta}_\perp^{(\mathrm{DR})} = \frac{N+1}{2(N-2)}\varepsilon, \tag{68}$$

$$\eta_\parallel^{(\mathrm{DR})} = \bar{\eta}_\parallel^{(\mathrm{DR})} = \frac{N}{N-2}\varepsilon. \tag{69}$$

The one-loop DR exponents for the magnetization read

$$\beta^{(\mathrm{DR})} = \frac{N-1}{2(N-2)}, \quad \beta_1^{(\mathrm{DR})} = \frac{N-1}{N-2}. \tag{70}$$

For $\varepsilon < 0$ the FP is stable but the η critical exponents become negative and hence unphysical.

A more accurate analysis of the RG flow shows that $R'''(0)$ diverges at a finite scale μ. Thus no analytic FP can exist and one has to look for a nonanalytic FP with $R^{*\prime\prime\prime}(0^+) \neq 0$ that would violate the DR predictions. This requires solution of the boundary-value problem for the nonlinear ODE with periodic boundary conditions, which depend on the universality class. We assume that the small ϕ expansion of the FP solution $R^*(\phi)$ has the form

$$R^*(\phi) = a_0 + a_2\phi^2 + a_3|\phi|^3 + a_4\phi^4 + a_5|\phi|^5 + \cdots, \tag{71}$$

meaning that $R^{*\prime\prime}(\phi)$ has a cusp at the origin with $R^{*\prime\prime\prime}(0^+) \neq 0$. Substituting the ansatz (71) into the FP equation, we find that

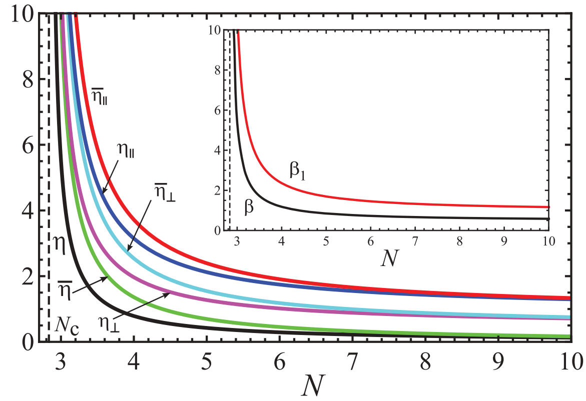

FIG. 2. (Color online) Critical exponents η_i and $\bar{\eta}_i$ (divided by ε), which describe the paramagnetic-ferromagnetic transition of the RF model above the lower critical dimension, as functions of N for $N > N_c$. The inset shows the corresponding bulk magnetization exponent β and the surface magnetization exponent β_1 as functions of N.

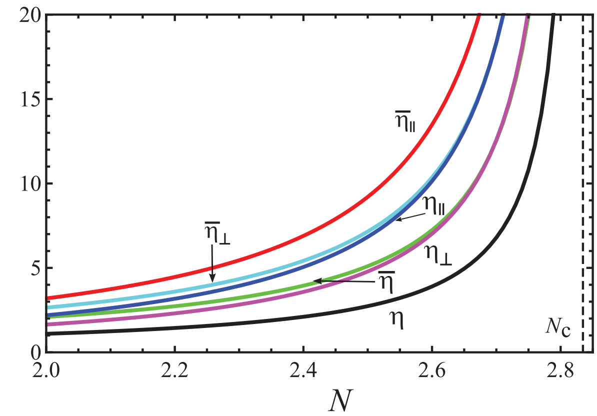

FIG. 3. (Color online) Critical exponents η_i and $\bar{\eta}_i$ (divided by $|\varepsilon|$), which describe the power-law decay of correlations in the QLRO phase of the RF model below the lower critical dimension, as functions of N for $N < N_c$.

the first coefficients are given by

$$a_0 = -\frac{2a_2^2(N-1)}{4(N-2)a_2+\varepsilon}, \quad a_2 = \frac{R^{*\prime\prime}(0)}{2}, \tag{72}$$

$$a_3 = -\text{sgn}(\varepsilon)\sqrt{\frac{2\varepsilon a_2 + 4a_2^2(N-2)}{9(N+2)}}. \tag{73}$$

The value of $R^{*\prime\prime}(0)$ and the sign of a_3 are constrained by the boundary conditions; $R^{*\prime\prime}(0)$ can be determined using the shooting method to fulfill the appropriate periodicity requirement.

A. Random-field O(N) model

1. Paramagnetic-ferromagnetic transition for d > 4 (ε > 0)

The RF model is described by $R(\phi)$, which is a 2π-periodic function. Numerical solution of the FP equation (62) shows that for $d > 4$ a 2π-periodic solution of the form (71)–(73) exists only for $N > N_c = 2.834\,74$. It has $R^{*\prime\prime}(0) < 0$ and it disappears when $N \to N_c^+$. This cuspy FP is once unstable with the positive eigenvalue $\lambda_1 = \varepsilon$. Thus the correlation-length exponent $\nu = 1/\varepsilon + O(\varepsilon^0)$ coincides with the DR prediction to one-loop order. Remarkably, the nonzero $R^{*\prime\prime\prime}(0^+)$ vanishes for $N > N^* = 18 + O(\varepsilon)$. The nonanalyticity becomes weaker as N increases and starts with $R^{*[2p(N)+1]}(0^+) \neq 0$, where $p \sim N$ [32,33,57]. Weaker nonanalyticity results in restoring the DR critical exponents for $N > N^*$. The critical exponents η_i and $\bar{\eta}_i$ computed using Eqs. (63)–(65) as functions of N are shown in Fig. 2. With increasing N they monotonically decay approaching the DR values at $N = N^*$ and satisfying the inequalities $\eta < \bar{\eta} < \eta_\perp < \bar{\eta}_\perp < \eta_\parallel < \bar{\eta}_\parallel$. The bulk and surface magnetization exponents β and β_1 calculated for different N are shown in inset of Fig. 2. To one-loop order they obey the relation $\beta_1 = 2\beta$. Up to now both magnetization exponents have been studied only for the 3D RFIM, where numerical simulations give $\beta = 0.0017 \pm 0.005$ [58] and $\beta_1 = 0.23 \pm 0.03$ [48]. Thus the ratio β_1/β for the RF O(N) systems in $d > 4$ is much smaller than for the 3D RFIM.

2. Quasi-long-range order for d < 4 (ε < 0)

Below the lower critical dimension the flow equation for the disorder correlator (35) has an attractive 2π-periodic FP solution of the form (71)–(73). This cuspy FP appears only for $2 \leqslant N < N_c$, where it controls the scaling behavior of spin fluctuations in the QLRO phase. The corresponding exponents η_i and $\bar{\eta}_i$ as functions of N are shown in Fig. 3. In the case $N = 2$ the FP equation admits for an explicit nonanalytic ϕ_0-periodic solution given by

$$R^*(\phi) = \frac{|\varepsilon|\phi_0^4}{72}\left[\frac{1}{36} - \left(\frac{\phi}{\phi_0}\right)^2\left(1-\frac{\phi}{\phi_0}\right)^2\right]. \tag{74}$$

Using Eqs. (63)–(65) one obtains

$$\eta = \frac{\phi_0^2}{36}|\varepsilon|, \quad \bar{\eta} = \left(1+\frac{\phi_0^2}{36}\right)|\varepsilon|, \tag{75}$$

$$\eta_\perp = \frac{\phi_0^2}{24}|\varepsilon|, \quad \bar{\eta}_\perp = \left(1+\frac{\phi_0^2}{24}\right)|\varepsilon|, \tag{76}$$

$$\eta_\parallel = \frac{\phi_0^2}{18}|\varepsilon|, \quad \bar{\eta}_\parallel = \left(1+\frac{\phi_0^2}{18}\right)|\varepsilon|, \tag{77}$$

with $\phi_0 = 2\pi$ for the RF system.

The semi-infinite RF O(2) model can be mapped onto a semi-infinite periodic disordered elastic system with a free surface. There is one to one correspondence between the Bragg glass phase of the elastic system and the QLRO phase of the studied spin model [38]. The power-law decay of the spin correlations in the QLRO phase corresponds to the logarithmic growth of the displacements in the disordered elastic system. Moreover, the exponents η, $\eta_\perp$, and $\eta_\parallel$ provide the universal amplitudes of the logarithmic growth of the displacements in the bulk, at the surface, and along the surface, respectively. For a ϕ_0-periodic elastic system with a free surface these amplitudes are given by Eqs. (75)–(77). In particular, we find that the logarithmic growth of the displacements along the

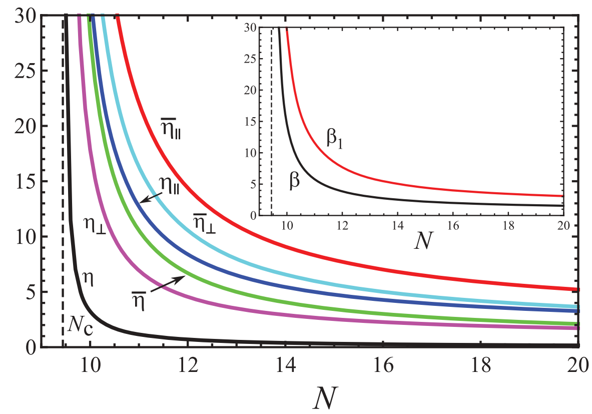

FIG. 4. (Color online) Critical exponents η_i and $\bar{\eta}_i$ (divided by ε), which describe the paramagnetic-ferromagnetic transition of the RA model above the lower critical dimension, as functions of N for $N > N_c$. The inset shows the corresponding bulk magnetization exponent β and the surface magnetization exponent β_1 as functions of N.

surface is twice as large as the logarithmic growth in the bulk. In the case when only one point is on the surface the growth is enhanced by 50%. The presence of a free surface can be considered as an extended defect of a special kind. The influence of potential-like extended defects on the Bragg glass has been recently studied in Refs. [34,59].

B. Random anisotropy O(N) model

1. Paramagnetic-ferromagnetic transition for $d > 4$ ($\varepsilon > 0$)

The FP equation (62) has a cuspy π-periodic solution of the form (71)–(73) that is singly unstable giving the correlation length exponent $\nu = 1/\varepsilon + O(\varepsilon^0)$. It exists for any $N > N_c = 9.4412$ with a nonzero $R^{*'''}(0^+)$. Therefore, at variance with the RF case in the RA model, the DR breaks down for all values $N > N_c$, i.e., $N^* = \infty$ [33]. The N dependence of the critical exponents η_i and $\bar{\eta}_i$ is shown in Fig. 4. For large N one can find the asymptotic behavior of the FP solution [32,33,57,60]. Following Ref. [32], we look for the π-periodic solution of the FP equation $\beta[R] = 0$ with the β function (61) of the form

$$R^{*\prime}(\phi) = -\frac{3}{2}\delta\varepsilon \sin\left(\frac{\pi - 2\phi}{3}\right)[2x(\phi) - 1]G(x). \quad (78)$$

Here we have introduced a small parameter $\delta = 1/(N-2)$ and defined variable $x(\phi) = \cos(\frac{\pi-2\phi}{3})$. Substituting the ansatz (78) into the FP equation and expanding the function $G(x)$ in small δ, one finds that the coefficients are polynomials in x:

$$G(x) = 1 + \frac{2}{9}(95 - 44x - 16x^2)\delta - \frac{4}{81}(11\,737 - 5040x - 3624x^2 - 3104x^3 - 768x^4)\delta^2 + \frac{8}{10\,935} \times (103\,378\,933 - 45\,854\,072x - 23\,128\,624x^2 - 16\,172\,328x^3 - 9\,791\,216x^4 - 4\,642\,048x^5 - 901\,120x^6)\delta^3 + O(\delta^4). \quad (79)$$

This implies that [60]

$$\frac{R^{*\prime\prime}(0)}{\varepsilon\delta} = -\frac{3}{2} - 23\delta + \frac{1750}{3}\delta^2 - \frac{2\,129\,692}{27}\delta^3 + O(\delta^4). \quad (80)$$

Substituting the solution (80) into Eqs. (63)–(65), we find the correlation function exponents to leading order in $1/N$ as

$$\eta = \frac{3\varepsilon}{2N}\left(1 + \frac{52}{3N} + \cdots\right), \quad \bar{\eta} = \frac{\varepsilon}{2}\left(1 + \frac{49}{N} + \cdots\right), \quad (81)$$

$$\eta_\perp = \frac{3\varepsilon}{4}\left(1 + \frac{55}{3N} + \cdots\right), \quad \bar{\eta}_\perp = \frac{5\varepsilon}{4}\left(1 + \frac{147}{5N} + \cdots\right), \quad (82)$$

$$\eta_\parallel = \frac{3\varepsilon}{2}\left(1 + \frac{52}{3N} + \cdots\right), \quad \bar{\eta}_\parallel = 2\varepsilon\left(1 + \frac{49}{2N} + \cdots\right), \quad (83)$$

$$\beta = \frac{3}{4}\left(1 + \frac{49}{3N} + \cdots\right), \quad \beta_1 = \frac{3}{2}\left(1 + \frac{49}{3N} + \cdots\right), \quad (84)$$

where in the last line are the bulk and the surface magnetization exponents.

2. Quasi-long-range order for $d < 4$ ($\varepsilon < 0$)

For $2 \leqslant N < N_c$ the flow equation (35) has a stable π-periodic FP solution of the form (71)–(73) that controls the scaling behavior of spin fluctuations in the QLRO phase of the RA model for $d < 4$. The correlation function exponents η_i and $\bar{\eta}_i$ computed for different N are shown in Fig. 5. For $N = 2$ the FP equation has an explicit nonanalytic π-periodic solution given by Eq. (74) with $\phi_0 = \pi$. The critical exponents of the RA $O(2)$ model are given by Eqs. (75)–(77) with $\phi_0 = \pi$.

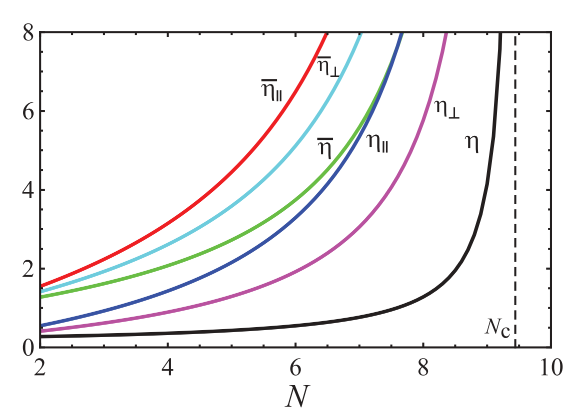

FIG. 5. (Color online) Critical exponents η_i and $\bar{\eta}_i$ (divided by $|\varepsilon|$), which describe the power-law decay of correlations in the QLRO phase of the RA model below the lower critical dimension, as functions of N for $N < N_c$.

V. CONCLUSION

In this paper we have studied the RF and RA semi-infinite O(N) models with a free surface. The both models have the lower critical dimension $d_{\rm LC} = 4$. Above $d_{\rm LC}$ they undergo a paramagnetic-ferromagnetic transition for $N > N_c$, while below $d_{\rm LC}$ and for $N < N_c$ they exhibit a QLRO phase with zero magnetization and a power-law decay of spin correlations. The critical value N_c is $N_c = 2.835$ for the RF and $N_c = 9.441$ for the RA models. Using a FRG, we have studied the surface scaling behavior of these models at criticality as well as in the QLRO phase. We have shown that the connected and disconnected correlation functions have different scaling behavior in the vicinity of the surface due to dangerous irrelevance of the temperature. The DR prediction fails to describe a surface scaling behavior similar to what happens in the bulk. We have derived the exact scaling relations between different surface exponents and computed the critical exponents to first order in $\varepsilon = d - 4$.

Whereas the critical behavior of the systems in $d > 4$ is of pure theoretical interest, the surface scaling behavior of the QLRO phase in $d < 4$ is of relevance for several experimental systems. In particular, the surface scaling behavior found for the 3D RF O(2) model describes the growth of displacements in disordered periodic elastic systems near a free surface, e.g., in the Bragg glass phase of disordered superconductors [39]. The results obtained for the Heisenberg ($N = 3$) RA model are relevant for the surface scaling behavior of amorphous magnets [14,61]. Another interesting example of a system with continuous symmetry in the presence of RA disorder is the anisotropic superfluid ^{3}He-A in aerogel. The silicon strands of aerogel play the role of quenched RA disorder acting on the orbital anisotropy vector $\hat{\mathbf{l}}$ characterizing the superfluid properties of ^{3}He-A [20,21].

Finally, let us mention several issues that have been left beyond the scope of this paper. A natural extension of this work would be to study the special and extraordinary surface transitions. Another interesting problem is the presence of surface disorder. According to Ref. [49], the RF disorder restricted to the surface of a system with continuous symmetry destroys true long-range order in the bulk for $d \leqslant 3$. Moreover, it generates a QLRO phase that is different from the QLRO phase studied here. It would be interesting to study competitions between the two different QLRO phases in the case when both the bulk and surface RF disorder are present in the system. In this paper we assumed that the bulk O(N) symmetry persists on the surface. However, in some physical systems one could expect surface spin anisotropies, e.g., the formation of an easy axis on the surface. In pure systems the surface anisotropies are irrelevant at the ordinary phase transition considered here but allow for different universality classes of the anisotropic special transition [62]. In the presence of RF disorder the surface anisotropies may be relevant even at the ordinary transition and also in the QLRO phase.

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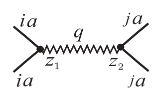
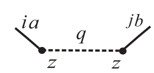
APPENDIX: ONE-LOOP DIAGRAMS CONTRIBUTING TO $\mathring{G}^{(1,1)}$

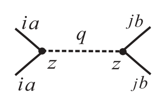
In this Appendix we calculate the correlation function $\mathring{G}^{(1,1)}_{1,a;1,a}$ to one-loop order. Expanding the action (17) in small $\boldsymbol{\pi}_a$, we find that the only vertices we need are



$$= -\frac{1}{8\mathring{T}}\delta(z_1 - z_2)[q^2 + \partial_{z_1}\partial_{z_2} + \mathring{h}], \qquad \text{(A1)}$$



$$= -\frac{1}{2\mathring{T}^2}\mathring{R}''(0), \qquad \text{(A2)}$$



$$= -\frac{1}{8\mathring{T}^2}\mathring{R}''(0). \qquad \text{(A3)}$$

The one-loop diagrams contributing to the correlation function $\mathring{G}^{(1,1)}_{1,a;1,a}$ are shown in Fig. 1. The solid line corresponds to the Neumann propagator (23) with $\mathbf{h}_1 = 0$ and the wavy and dashed lines correspond to vertices (A1)–(A3). The first diagram (a) in Fig. 1 gives

$$\begin{aligned}(a) &= \frac{N-1}{2\mathring{T}^3}\mathring{R}''(0)\int_q\int_0^\infty dz_1\int_0^\infty dz_2\int_0^\infty dz_3\,\delta(z_2 - z_1)\\ &\quad\times[\partial_{z_1}\partial_{z_2} + \mathring{h}]\hat{G}^0_p(0,z_1)\hat{G}^0_p(z_1,z)\big[\hat{G}^0_q(z_2,z_3)\big]^2\\ &= \frac{N-1}{16\bar{p}}\mathring{T}\mathring{R}''(0)e^{-\bar{p}z}\left(\frac{z\mathring{h}}{\bar{p}} + \frac{\mathring{h}}{\bar{p}^2} + 2\right)I_2 + \text{finite}, \qquad \text{(A4)}\end{aligned}$$

where we have used $\bar{p} = (p^2 + \mathring{h}^2)^{1/2}$ and omitted the terms finite in the limit $\varepsilon \to 0$. The logarithmically divergent one-loop integral reads

$$\begin{aligned}I_2 &= \int_q \frac{1}{\bar{q}(\bar{p}+\bar{q})^2} = K_{d-1}\int_0^\infty \frac{q^{d-2}dq}{(q^2+\mathring{h}^2)^{3/2}} + O(\varepsilon^0)\\ &= -\frac{4K_d}{\varepsilon} + O(\varepsilon^0). \qquad \text{(A5)}\end{aligned}$$

The second (b) and third (c) diagrams in Fig. 1 yield

$$\begin{aligned}(b) &= \frac{1}{\mathring{T}^3}\mathring{R}''(0)\int_q\int_0^\infty dz_1\int_0^\infty dz_2\int_0^\infty dz_3\delta(z_2 - z_1)[(\mathbf{p}+\mathbf{q})^2 + \partial_{z_1}\partial_{z_2} + \mathring{h}]\hat{G}^0_p(0,z_1)\hat{G}^0_p(z_2,z)\hat{G}^0_q(z_1,z_3)\hat{G}^0_q(z_3,z_2)\\ &= -\frac{1}{8\bar{p}}\mathring{T}\mathring{R}''(0)e^{-\bar{p}z}\left[\left(\frac{z\mathring{h}}{\bar{p}} + \frac{\mathring{h}}{\bar{p}^2} - 2\bar{p}z - 7\right)I_2 - 2I_3\right] + \text{finite}, \qquad \text{(A6)}\end{aligned}$$

$$(c) = -\frac{1}{\mathring{T}^2}\mathring{R}''(0)\int_q\int_0^\infty dz_1\hat{G}^0_p(0,z_1)\hat{G}^0_q(z_1,z_1)\hat{G}^0_p(z_1,z) = -\frac{1}{8\bar{p}}\mathring{T}\mathring{R}''(0)e^{-\bar{p}z}[(2\bar{p}z+5)I_2 + 2I_3] + \text{finite}, \qquad \text{(A7)}$$

respectively, where we have defined the algebraically divergent integral

$$I_3(\bar{p},z) = \int_q \frac{3\bar{p} + \bar{q} + (2\bar{p} + \bar{q})\bar{p}z}{\bar{p}^2(\bar{p} + \bar{q})^2}. \tag{A8}$$

Summing up the three diagrams, we find that the algebraically divergent integral (A8) cancels and we obtain Eq. (56).

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