Erratum: Nonperturbative renormalization group for the Kardar-Parisi-Zhang equation: General framework and first applications [Phys. Rev. E 84, 061128 (2011)]

Léonie Canet, Hugues Chaté, Bertrand Delamotte, and Nicolás Wschebor

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This Erratum corrects inconsistencies in the Fourier conventions used in the paper. The Fourier conventions that were chosen in this paper were those of Ref. [1], which are as follows:

$$\tilde{f}(\omega,\vec{p}) = \int d^d\vec{x}\, dt\, f(t,\vec{x}) e^{-i\vec{p}\cdot\vec{x}+i\omega t}, \qquad (1)$$

$$f(t,\vec{x}) = \int \frac{d^d\vec{p}}{(2\pi)^d}\frac{d\omega}{2\pi}\tilde{f}(\omega,\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega t} \qquad (2)$$

$$\equiv \int_{\mathbf{p}} \tilde{f}(\mathbf{p})e^{i\vec{p}\cdot\vec{x}-i\omega t}, \qquad (3)$$

where $\mathbf{p} = (\omega,\vec{p})$. These equations should be substituted into Eqs. (5)–(7) of the paper, respectively. With these conventions, all the equations in the paper are correct, except those of Appendix C, which should be complex conjugated. That is, the substitution $i \to -i$ should be achieved in Eqs. (C3)–(C5), (C8), and (C11).

These inconsistencies have consequences only in Sec. VI, that is, in the one-dimensional case. The numerical codes related to this section have been corrected and have been rerun. The outcome is a significant improvement of the results presented in Sec. VI. Below, we give the corrected versions of Table II and Figs. 5 and 6 of the paper, which are, respectively, Table I and Figs. 1 and 2 of this Erratum. The former Figs. 2–4 remain essentially the same.

Let us first comment on the new figures. The overall agreement between the nonperturbative renormalization group (NPRG) scaling functions (denoted $\tilde{F}$ and F) and the exact ones (denoted $\tilde{f}$ and f, respectively) of Ref. [2] is excellent. Regarding $\tilde{F}$, all the nontrivial features of the exact function $\tilde{f}$ are very accurately reproduced: the existence and the depth of the negative dip, the subsequent stretched exponential decay with superimposed oscillations, of the form

$$\tilde{f}(k) \sim \exp(-b_0 k^{3/2})\cos(a_0\, k^{3/2}).$$

The behavior of the tail is recovered not only on the correct scale $k^{3/2}$ (which is not the case for the function obtained in the mode-coupling (MC) approximation in Ref. [3]), but also with the correct coefficient b_0 for the decay and a comparable pulsation a_0 for the oscillations (see Table I). Considering the tiny magnitude over which this behavior sets in (typically, below a 10^{-6} level), the agreement is remarkable.

Regarding F, the NPRG function also precisely matches the exact one f, although it is the less accurate of our three scaling functions as it stems from two successive numerical (oscillating) integrations of the function $\mathring{F}$, which is the one directly calculated at the NPRG fixed point (see the paper). The tail of the function F is particularly sensitive to this loss of precision. The exact function $f(y)$ is found to decrease as $\exp(-cy^3)$ when $y \to \infty$ in Ref. [2], whereas, the decay of the function $F(y)$, although it starts with the correct behavior, rapidly crosses over to a simple exponential decay $\exp(-c'y)$.

This is manifest on the universal amplitude ratio g_0 defined as

$$g_0 = 4\int_0^\infty dy\, y f(y). \qquad (4)$$

The contribution of the exponential tail of the NPRG function F leads to an overestimation of the integral (4), compared

TABLE I. Characteristic parameters of the different scaling functions from the exact results of Ref. [2] and from this Erratum: (i) relative to f, universal amplitude ratio g_0; (ii) relative to $\tilde{f}$: position of the first zero k_0, coordinates of the negative dip $(k_d,\tilde{f}_d)$, coefficient of the stretched exponential b_0, and pulsation of the oscillations a_0; (iii) correction to scaling exponent ω. The error bars reflect the weak variations around plateau values when the α parameter of the cutoff function is varied between 2 and 20.

Quantity	Exact	NPRG
g_0	1.150 39	1.19(1)
k_0	4.362 36	4.60(6)
k_d	4.790 79	5.14(6)
$\tilde{f}_d$	−0.001 20	−0.0018(6)
a_0	$\frac{1}{2}$	0.28(5)
b_0	$\frac{1}{2}$	0.49(1)
ω		1.0(1)

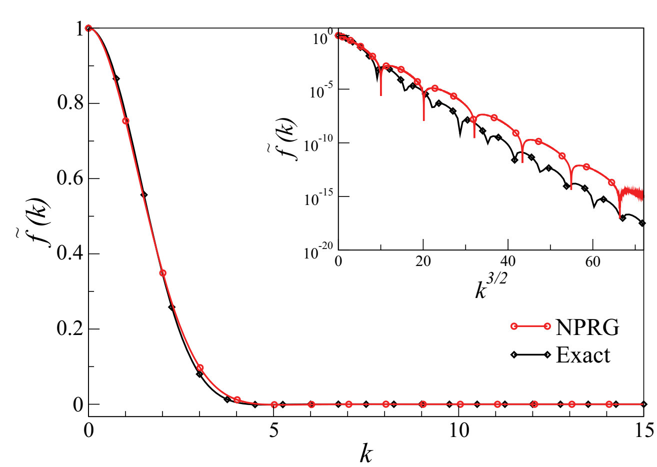

FIG. 1. (Color online) Comparison of the red curve with dots: scaling function $\tilde{F}(k)$ obtained in this Erratum with the black curve with squares: exact one $\tilde{f}(k)$ from Ref. [2]. The inset shows the stretched exponential behavior of the tail with the superimposed oscillations, developing on the same scale $k^{3/2}$. Note the vertical scale: this behavior sets in with amplitudes below typically 10^{-6}.

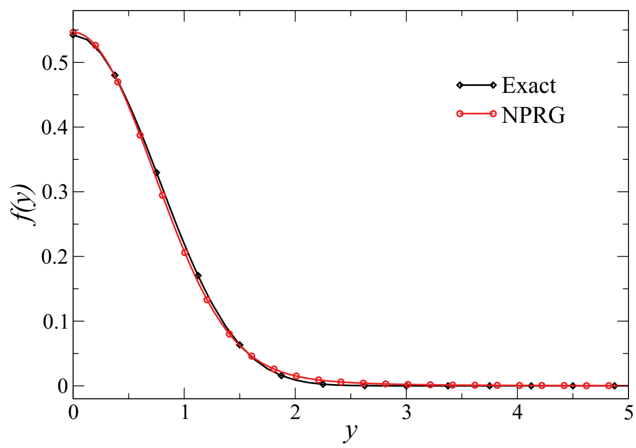

FIG. 2. (Color online) Comparison of the red curve with dots: scaling function $F(y)$ obtained in this Erratum with the black curve with squares: exact one $f(y)$ from Ref. [2].

with the exact result where this contribution is very rapidly suppressed by the $\exp(-cy^3)$ factor (see Table I). Note that our previous estimate of g_0 was closer to the exact value, but this was, in fact, a chance consequence of the greater discrepancy in the bulk of the function, which happened to compensate the contribution of the tail. We suspect that a similar mechanism occurs for the MC estimate in Ref. [3]: The discrepancies between the MC scaling function and the exact one, which are much larger than those of Fig. 2, probably cancel out to produce, *in fine*, a reasonable estimate for g_0: $g_0 \simeq 0.1137$.

Finally, we report a misprint in one of the powers of λ in Eqs. (63) and (66) that should read, respectively,

$$g(y) = \lim_{t\to\infty} \frac{C[(2\lambda^2 A t^2)^{1/3} y, t]}{\left(\frac{1}{2}\lambda A^2 t\right)^{2/3}}, \tag{5}$$

and

$$\mathring{f}(\tau) = -\frac{p^{7/2}}{2^{5/3}\lambda^{4/3}A^{5/3}t^{7/3}} C\left(\frac{p}{(2\lambda^2 A t^2)^{1/3}}, \tau\frac{p^{3/2}}{t}\right). \tag{6}$$

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