Effects of nonthermal ions and polarization force on dust-acoustic waves in a density-varying dusty plasma

M. Asaduzzaman and A. A. Mamun
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
(Received 12 March 2012; published 26 July 2012)

A rigorous theoretical investigation has been made of the effects of nonthermal ions and polarization force (which arises due to the dust density inhomogeneity) on the propagation of dust-acoustic (DA) waves in a density-varying unmagnetized dusty plasma (consisting of nonthermal ions, Maxwellian electrons, and negatively charged mobile dust) by the normal mode analysis. It has been shown that the dispersion properties of the DA waves are significantly modified by the presence of nonthermal ions and polarization force. It has been also found that the phase speed of the DA waves, as well as the dust density perturbation, increases (decreases) with the increase of nonthermal ions (polarization force), and that the potential associated with the DA waves decreases with the increase of the equilibrium dust number density. The implications of our results in the specific situation of space environments (dust-ion plasma situation) are also briefly discussed.

DOI: 10.1103/PhysRevE.86.016409 PACS number(s): 52.27.Lw

I. INTRODUCTION

More than two decades ago, the dust-acoustic (DA) wave, which is one of the novel wave modes in dusty plasmas, was first theoretically predicted by Rao *et al.* [1]. The dispersion relation of the DA waves for a cold dust fluid limit is given by [1] $\omega^2/k^2 = C_d^2(1-\mu)/(1+\sigma_i\mu + k^2\lambda_{Di}^2)$, where $\lambda_{Di} = (k_BT_i/4\pi e^2 n_{i0})^{1/2}$ is the ion Debye radius, $C_d = (z_d k_B T_i/m_d)^{1/2}$ is the DA speed, $\mu = n_{e0}/n_{i0}$, and $\sigma_i = T_i/T_e$. Now, if we consider a long wavelength limit ($k^2\lambda_{Di}^2 \ll 1$), the dispersion relation for the DA waves becomes $\omega^2/k^2 = C_d^2(1-\mu)/(1+\sigma_i\mu)$. This means that in the DA waves, the dust particle mass provides the inertia, and the pressures of inertialess electrons and ions give rise to the restoring force. The theoretical prediction of Rao *et al.* [1] has been conclusively verified by a number of laboratory experiments [2,3].

A large number of investigations have been made on the linear dispersion properties of the DA waves propagating in uniform [1–7] and nonuniform [8–10] dusty plasmas. Singh and Rao [8] examined the linear propagation of the DA waves in a nonuniform dusty plasma, and their investigations [8] are limited to the consideration of Maxwellian electrons and ions. They further extended this investigation [9] by taking into account the equilibrium dust charge and density inhomogeneities. Mamun *et al.* [10] have investigated the dispersion properties of ultra-low-frequency electrostatic waves in a nonuniform dusty plasma by using kinetic theory. These works in nonuniform plasma [8–10] have not considered the effects of polarization force [11–18].

However, it is now well understood that the polarization force (which is due to the deformation of the Debye sheath in a nonuniform plasma) is very important for a nonuniform dusty plasma. Recently, Mamun *et al.* [15] and Ashrafi *et al.* [16] have theoretically studied the effects of polarization force in a uniform dusty plasma. These investigations [15,16] are not valid in general, as the polarization force arises due to the dust density inhomogeneity, and it completely disappears in a uniform dusty plasma. Very recently, Asaduzzaman *et al.* [17] have investigated the effects of polarization force on DA waves, and their assumption is valid for Maxwellian electrons and ions. Most of these studies [1–17] are concerned with the fact that electrons and ions follow the Maxwellian velocity distribution. There are a number of dusty plasma situations—namely, Earth's bow shock [19], foreshock [20], the moon's atmosphere [21], etc.—in which ions may follow non-Maxwellian distribution (now known as the Cairns distribution [22]), which represents the population of nonthermal or fast particles. The polarization force for a nonuniform plasma in the presence of nonthermal ions can be derived as $\mathbf{F}_p = -z_d e R_1 (n_{i0}/n_i)^{\frac{1}{2}} \nabla\phi$, where z_d is the number of electrons residing on the dust grain surface, e is the magnitude of an electron charge, n_{i0} is the equilibrium ion number density, ϕ is the electrostatic potential, $R_1 = (z_d e^2/4T_i\lambda_{Di0})\beta_0 \exp(-e\phi/T_i)$ is a parameter determining the modified strength of the polarization force, $\beta_0 = [1-\beta-\beta(e\phi/T_i)+\beta(e\phi/T_i)^2]$, β is a nonthermal parameter, T_i is the ion temperature in energy unit, and λ_{Di0} is the ion Debye radius. We note that for $\beta = 0$, the expression for the polarization force is exactly the same as that derived by Asaduzzaman *et al.* [17]. In our present work we take into account the effects of nonthermal ion distribution and polarization force on the dispersion properties of the DA waves in a nonuniform dusty plasma.

The manuscript is organized as follows. The basic equations of the plasma fluid model are given in Sec. II. The dispersion properties of the DA waves are examined in Sec. III. Finally, a brief summary is presented in Sec. IV.

II. BASIC EQUATIONS

We consider a three-component unmagnetized density-varying dusty plasma system containing electrons, ions, and cold negatively charged mobile dust. Thus, at equilibrium we have $n_{i0} = z_d n_{d0} + n_{e0}$, where n_{e0} is the equilibrium electron number density. We assume that electrons follow the Maxwellian distribution, and ions follow the Cairns distribution [22]. Thus, the electron and ion number densities

(n_e and n_i) are, respectively, given by

$$n_e = n_{e0}\exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

$$n_i = n_{i0}\left[1+\beta\left(\frac{e\phi}{T_i}\right)+\beta\left(\frac{e\phi}{T_i}\right)^2\right]\exp\left(-\frac{e\phi}{T_i}\right), \tag{2}$$

where $\beta = 4\alpha_i/(1+3\alpha_i)$, α_i is a parameter determining the population of the nonthermal ions, and T_e is the electron temperature in energy unit. We note that $\alpha_i = 0$ corresponds to a Maxwellian distribution. The dynamics of the DA waves in a nonthermal density-varying dusty plasma can be expressed as

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d) = 0, \tag{3}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}-\frac{z_d e}{m_d}\frac{\partial\phi}{\partial x}+\frac{z_d e}{m_d}R_1\left(\frac{n_{i0}}{n_i}\right)^{\frac{1}{2}}\frac{\partial\phi}{\partial x} + \frac{\mu_d T_d}{m_d}\frac{1}{n_d}\frac{\partial n_d}{\partial x} = 0, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2} - 4\pi e(n_e - n_i + z_d n_d) = 0, \tag{5}$$

where n_d is the dust number density, u_d is the dust fluid speed, $t(x)$ is the time (space) variable, T_d is the dust temperature, and m_d is the dust grain mass. The compressibility μ_d [23–25] can be defined as $\mu_d = (1/T_d)(\partial P_d/\partial n_d)$, where P_d is the pressure.

III. DISPERSION PROPERTIES

To study the linear propagation of the DA waves in nonuniform dusty plasma under consideration, we use normal mode analysis. We first expand the dependent variables n_d, u_d, and ϕ in terms of their equilibrium and perturbed parts as

$$n_d = n_{d0}(x) + \epsilon\tilde{n}_{d1}(x,t), \tag{6}$$

$$u_d = 0 + \epsilon\tilde{u}_{d1}(x,t), \tag{7}$$

$$\phi = 0 + \epsilon\tilde{\phi}_1(x,t), \tag{8}$$

where ϵ is a smallness parameter ($0 < \epsilon < 1$) measuring the weakness of the dispersion and subscript 1 denotes the perturbed quantities. The spatial and time dependence of all perturbed quantities can be expressed as

$$S(x,t) = S_1(x)\exp(-i\omega t), \tag{9}$$

where ω is the angular frequency, $S \equiv (\tilde{n}_{d1},\tilde{u}_{d1},\tilde{\phi})$, and $S_1 \equiv (n_{d1},u_{d1},\phi_1)$. The spatial-dependent part of the dust number density $n_{d1}(x)$ (for our purpose) satisfies the second-order differential equation [8,9,17]:

$$\frac{d^2 n_{d1}}{dx^2} + \alpha K_n\frac{dn_{d1}}{dx} + \beta_2 n_{d1} = 0, \tag{10}$$

where $\beta_2 = \omega^2/\mu_o^2$, $\mu_o^2 = \mu(1-\beta_1)C_d^2 + \mu_d u_{td}^2$, and $\alpha = \mu(1-\beta_1)C_d^2/[\mu(1-\beta_1)C_d^2 + \mu_d u_{td}^2]$, $\mu = z_d n_{do}/n_{i0}$, $\beta_1 = R(1-\beta)$, $u_{td} = (T_d/m_d)^{1/2}$, $\sigma_3(=n_{eo}/n_{i0})$, $\sigma_4(=T_i/T_e)$, $R = z_d e^2/4T_i\lambda_{Di0}$ measures the effect of the polarization force, and $C_d = [z_d T_i/m_d(1-\beta+\sigma_3\sigma_4)]^{1/2}$ is the DA speed modified by the presence of nonthermal ions. It is noted that $K_n = n_{d0}^{-1}(dn_{d0}/dx)$ is the inverse of inhomogeneity scale length $L_n = [n_d/(dn_{do}/dx)]$. We first consider the simplest dusty plasma situation, where the equilibrium dust number density is constant, i.e., $n_{d0} = \text{const}$ ($K_n = 0$ and $R = 0$). Thus, substituting (6)–(9) into (3)–(5), the dispersion relation for $K_n = 0$ and $R = 0$ is $\omega = KC_d\sqrt{1-\sigma_3}$. Applying an appropriate condition $n_{eo}/n_{i0} \ll 1$, the dispersion relation becomes $\omega = KC_d$. This special case, along with $\alpha_i = 0$, completely agrees with the work of Rao *et al.* [1]. If the equilibrium dust density is varied ($n_{d0} \neq \text{const}$), a force known as a polarization force must be introduced. We now consider a cold dust fluid limit ($T_d = 0$) and use $d/dx \to iK$. The cold dust fluid limit is valid for the DA waves since the dust thermal speed is always smaller than the phase speed of the DA waves. The dispersion relation obtained from (10) for this case reduces to

$$\omega_0 = k_0\sqrt{(1-\sigma_3)\frac{(1-R+\beta R)}{(1+\sigma_3\sigma_4-\beta)}}, \tag{11}$$

where $\omega_0/k_0(=V_p)$ is the phase speed of the DA waves, ω_0 is normalized by $\omega_{pd} = (4\pi e^2 Z_d^2 n_{d0}/m_d)^{1/2}$, k_0 is normalized by $\lambda_{Dm}^{-1} = (4\pi e^2 Z_d n_{d0}/T_{i0})^{1/2}$, and C_d is normalized by $\omega_{pd}\lambda_{Dm}$. It is clear that for $n_{eo}/n_{i0} \ll 1$ and $\beta = 0$ the dispersion relation completely agrees with that obtained by Asaduzzaman *et al.* [17] as well as with that obtained by Ashrafi *et al.* [16]. We have observed that the phase speed is significantly modified by the effects of polarization force as well as by the presence of fast ions (shown in Fig. 2). Figure 1 dictates to us that if the ratio of electron number density to ion number density increases, the phase speed is decreased. Figure 2

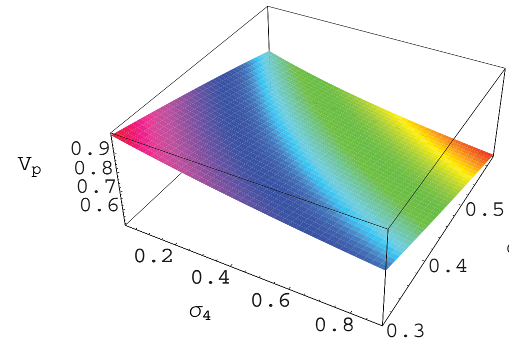

FIG. 1. (Color online) Showing the variation of V_p with σ_4 and σ_3 when $R = 0.6$ and $\alpha_i = 0.2$.

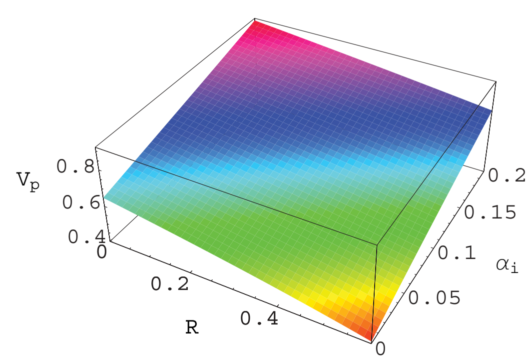

FIG. 2. (Color online) Showing the variation of V_p with R and α_i when $\sigma_3 = 0.57$ and $\sigma_4 = 0.04$.

shows that the phase speed is significantly decreased when the polarization force increases. This is physically correct because the polarization force naturally arises in a density-varying plasma system, and the dust density inhomogeneity scale length is no longer negligible, i.e., it is comparable to the wavelength of the DA waves. However, in the long wavelength limit, neglecting the effects of nonthermal ions, our present findings agree with those of Khrapak *et al.* [13].

We now revisit the polarization force [13] to imply that its effect becomes more important when the dust grain charge (or size) increases. Figure 2 also shows that the phase speed is significantly increased by the presence of nonthermal ions. These highly energetic ions may provide more energy to the motion of the wave under consideration, and therefore phase speed is increased. $\bar{n}_{d1}$ is assumed to be a slowly varying function of x, and the relation between $\bar{n}_{d1}$ and n_{d0} can be expressed as $\bar{n}_{d1} = An_{d0}^{-1/2}$, where A is an integration constant. It is obvious that the perturbed dust number density is inversely proportional to the equilibrium dust number density.

The potential associated with the DA waves corresponding to a slowly varying dust number density can be written as $\phi_1(x) = -(An_{d0}^{-1/2})[z_d T_i/e(n_{eo}\sigma_4 + n_{i0} - n_{i0}\beta)]$. The above relation concludes that the potential ϕ_1 decreases with the increase of the equilibrium dust number density and is almost directly proportional to the ion temperature (since $\sigma_3\sigma_4 \ll 1$ for highly negatively charged dust). This is physically correct because in the DA waves the restoring force is provided mainly by the ion thermal pressure $n_{i0}T_i$ (since $n_{e0}T_i \ll T_e n_{i0}$ for highly negatively charged dust), and the inertia is provided by the dust mass density $n_{d0}m_d$.

To provide some more explanation about the potential associated with the DA waves, we consider a special condition, namely, the dust-ion plasma [26] situation (i.e., $n_{i0} \approx z_d n_{d0}$). This situation is due to the fact that the electron number density is sufficiently depleted [2,26,27] during the charging of the dust grain, on account of the attachment of the background plasma electrons on the surface of the dust grains. This scenario is relevant to some space [26,28] and laboratory [2] plasma situations. The approximation $n_{i0} \approx z_d n_{d0}$ allows us to express $\phi_1(x)$ as

$$\phi_1(x) = -\frac{A}{n_{d0}^{3/2}}\left[\frac{T_i}{e(1-\beta)}\right], \tag{12}$$

which indicates that the potential is inversely proportional to $n_{d0}^{3/2}$ and directly proportional to the ion temperature. We note that for $\beta = 0$ this special situation completely agrees with the recent work of Asaduzzaman *et al.* [17]. To show the dust number density profile, we take the solution of Eq. (10) as

$$N_d = \tfrac{1}{2}L_2 \exp\left[\tfrac{1}{2}L_3(L_1 - Z_1Z_2)X\right] + \left(1 - \tfrac{1}{2}L_2\right)\exp\left[-\tfrac{1}{2}L_3(L_1 + Z_1Z_2)X\right], \tag{13}$$

where $N_d = n_{d1}/n_{d1}^{(0)}$ is the normalized perturbed dust number density, $n_{d1}^{(0)}$ represents the perturbed dust number density at $x = 0$, $L_1 = [Z_1^2Z_2^2 - 4W^2(Z_2 + \sigma_1)n^2(dn/dX)^{-2}]^{1/2}$, $L_2 = 1 + Z_1Z_2/L_1$, $L_3 = D_1^{-1}Z_1^{-1}n^{-1}D_2$, $D_1 = Z_2 + \sigma_1$, $D_2 = dn/dX$, $Z_1 = (1 + \sigma_3\sigma_4 - \beta)^{-1/2}$, $Z_2 = \mu(1 - R + \beta R)$, $\sigma_1 = \mu_d T_d \eta / z_d T_i$, $\sigma_2 = \mu_d T_d / z_d T_i$, x is normalized by $\lambda_{Dm} = (T_i/4\pi e^2 Z_d n_{d0})^{1/2}$, and n_{d0} is normalized by n_{i0}.

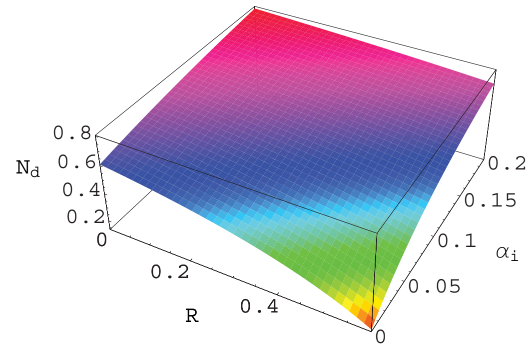

FIG. 3. (Color online) Showing the variation of N_d with R and α_i for $n = 0.43$, $W = 0.7$, $X = 1$, $\sigma_3 = 0.57$, $\sigma_4 = 0.04$, and $\sigma_2 = 0.001$.

Next we examine the basic features of the dust density N_d, which is significantly modified by different plasma parameters (viz. β, R, n, σ_2, σ_3, X, W, and σ_4). For an example, we have shown how the dust density profile changes with R (effect of polarization force) and α_i (effect of nonthermal ions). This is displayed in Fig. 3.

It is obvious that dust density (N_d) decreases with the increase of R (strength of the polarization force) but increases with the increase of α_i (number of fast ions). The typical dusty plasma parameters that we used in our numerical analysis are $\alpha_i = 0 - 0.2$, $\sigma_2 = 0.001$, $\sigma_3 = 0.3 - 0.6$, $\sigma_4 = 0.01 - 0.9$, and $R = 0 - 0.6$. This means that the ranges of the plasma parameters that we have used for our study of the DA waves are very wide and within the parametric values, which corresponds to different space [14,18] and laboratory [29] dusty plasma situations.

The magnitude of the polarization force increases with the dust charge as well as with the plasma density inhomogeneity, and that is important for most space [14,18] and laboratory [29] dusty plasma parameters. The presence of nonthermal ions drastically modifies the phase speed of the DA waves. To complement previously published results, we have used the parameter ranges corresponding to the experimental conditions of Bandyopadhyay *et al.* [29]: $n_{i0} = 7 \times 10^{13} m^{-3}$, $n_{e0} = 4 \times 10^{13} m^{-3}$, $T_i = 0.3$ eV, $T_e = 8$ eV, and $R = 0 - 0.6$. It is important to note that we have used the experimental conditions of Bandyopadhyay *et al.* [29], who used an ion temperature in their experiment that was a bit high (0.3 eV), and were consistent with similar earlier measurements reported by Thompson *et al.* [30].

IV. SUMMARY

We have considered a nonuniform dusty plasma containing nonthermal ions, Maxwellian electrons, and negatively charged mobile dust and studied the effects of nonthermal ions and polarization force on the dispersion properties of the DA waves. It is observed that the dispersion properties of the DA waves are significantly modified by the effects of nonthermal ions and polarization force. It is also shown here that the potential associated with the DA waves is inversely proportional to the square root of the equilibrium dust number density. The dispersion properties of the DA waves are found to be greatly affected by the dust density inhomogeneity. As soon as the gradient of the dust number density is increased, the polarization force can no longer be neglected in a realistic dusty plasma situation. It is seen here that the phase speed of the DA waves is significantly decreased by the effect of the polarization force, but the phase speed is significantly increased by the presence of nonthermal ions.

It may be stressed here that the results of this investigation could be useful for understanding the localized features of the electrostatic disturbances in laboratory and space dusty plasmas such as in Saturn's E ring, noctilucent clouds, interstellar molecular clouds, and inside the ionopause of Halley's comet; the present analysis can also be applied for collisionless dusty plasma systems, such as the outside ionopause of Halley's comet, Saturn's F ring, Saturn's spokes, zodiacal dust disk (1 AU), and supernovae shells, in which density-varying negatively charged dust fluid, nonthermal ions, and Boltzmann electrons are the most important plasma species. The specific dust-ion plasma situation considered here is relevant to some space [26,28] and laboratory [2] environments, and due to highly charged dust and strong nonuniformity in dust number density, the polarization force may eventually be important for such a dust-ion plasma system. In addition, the effects of the nonthermal ions are also very important to understand the localized features of the electrostatic disturbances in space and laboratory plasma systems. It may be mentioned here that our work is valid for the linear DA waves. However, the nonlinear propagation of the DA waves in the plasma system under consideration is also the a problem of great importance, but it is beyond the scope of the present work. We hope that a new experiment can be performed to identify the basic features of the localized density-varying DA waves in the presence of nonthermal ions and polarization force, which are very important for laboratory and space environments.

ACKNOWLEDGMENTS

A research grant for research equipment from the Third World Academy of Sciences (TWAS), ICTP, Trieste, Italy, is gratefully acknowledged. The constructive suggestions of an anonymous referee are gratefully acknowledged.

[1] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[2] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[3] N. D'Angelo, J. Phys. D **28**, 1009 (1995).
[4] J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
[5] R. L. Merlino, A. Barkan, C. Thompson, and N. D'Angelo, Phys. Plasmas **5**, 1607 (1998)
[6] V. E. Fortov, O. F. Petrov, V. I. Molotkov, M. Y. Poustylnik, V. M. Torchinsky, A. G. Khrapak, and A. V. Chernyshev, Phys. Rev. E **69**, 016402 (2004).
[7] R. L. Merlino and J. A. Goree, Phys. Today **57**, 32 (2004).
[8] S. V. Singh and N. N. Rao, Phys. Plasmas **5**, 94 (1998).
[9] S. V. Singh and N. N. Rao, Phys. Plasmas **6**, 3157 (1999).
[10] A. A. Mamun, M. Salahuddin, and M. Salimullah, Planet. Space Sci. **47**, 79 (1999).
[11] S. Hamaguchi and R. T. Farouki, Phys. Rev. E **49**, 4430 (1994).
[12] S. Hamaguchi and R. T. Farouki, Phys. Plasmas **1**, 2110 (1994).
[13] S. A. Khrapak, A. V. Ivlev, V. V. Yaroshenko, and G. E. Morfill, Phys. Rev. Lett. **102**, 245004 (2009).
[14] N. Shukla and P. K. Shukla, J. Plasma Phys. **76**, 677 (2010).
[15] A. A. Mamun, K. S. Ashrafi, and P. K. Shukla, Phys. Rev. E **82**, 026405 (2010).
[16] K. S. Ashrafi, A. A. Mamun, and P. K. Shukla, Europhys. Lett. **92**, 15004 (2010).
[17] M. Asaduzzaman, A. A. Mamun, and K. S. Ashrafi, Phys. Plasmas **18**, 113704 (2011).
[18] R. P. Prajapatia, Phys. Lett. A **375**, 2624 (2011).
[19] J. R. Asbridge, S. J. Bame, and I. B. Strong, J. Geophys. Res. **73**, 5777 (1968).
[20] W. C. Feldman *et al.*, J. Geophys. Res. **88**, 96 (1983).
[21] Y. Futaana *et al.*, J. Geophys. Res. **108**, 1025 (2003).
[22] R. A. Cairns *et al.*, Geophys. Res. Lett. **22**, 2709 (1995).
[23] S. Ichimaru, H. Iyetomi, and S. Tanaka, Phys. Rep. **149**, 91 (1987).
[24] M. A. Berkovsky, Phys. Lett. A **166**, 365 (1992).
[25] V. Yaroshenko *et al.*, New J. Phys. **11**, 073013 (2009).
[26] A. A. Mamun and R. A. Cairns, Phys. Plasmas **3**, 702 (1996).
[27] L. Stenflo and P. K. Shukla, IEEE Trans. Plasma Sci. **29**, 208 (2002).
[28] M. I. Abo el Maaty, E. K. El-Shewy, H. G. Abdelwahed, and M. A. Elmessary, Electronic. J. Theo. Phys. **7**, 151 (2010).
[29] P. Bandyopadhyay, G. Prasad, A. Sen, and P. K. Kaw, Phys. Rev. Lett. **101**, 065006 (2008).
[30] Thompson *et al.*, Phys. Plasmas **4**, 2331 (1997).