Director fluctuations in nematic liquid crystals induced by an ultrasonic wave

Caio Sátiro and Carlindo Vitoriano
Unidade Acadêmica de Garanhuns, Universidade Federal Rural de Pernambuco, Avenida Bom Pastor S/N, 55296-901, Garanhuns, Pernambuco, Brazil

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The director-density coupling theory was formulated with two parameters (u_1 and u_2) to explain the acousto-optic effect in nematic liquid crystals. The assumption that the director is not able to accompany rapid oscillations of the sound wave, so that it actually couples to the time-averaged interaction, renders it effectively a u_1-independent theory. In this paper, we investigate a route in which the time average is postponed to the end of the calculation. This approach allows us to derive measurable quantities that depend on both u_1 and u_2.

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I. INTRODUCTION

A few years ago, a theory with two parameters (u_1 and u_2) of the acousto-optic effect in nematic liquid crystals [1–4] based on direct coupling between the director and density oscillations (director-density coupling theory) has been proposed by a number of authors [5–11]. From a theoretical point of view, the theory has a clear advantage over the early model by Dion and Jacob [12] in the prediction of acoustic torque density [5]. Besides, there is good agreement between the calculated and the experimentally verified results, such as optical intensity versus (a) applied acoustic intensity [5,6], (b) the incident angle of the acoustic wave [6,7], (c) the thickness of the liquid crystal layer [7], and (d) the time after an ultrasonic wave is turned on and turned off [8]. At the same time, the model has been successfully used to explain ultrasound-induced changes of the spin-lattice relaxation time [9–11].

Quite recently, we have explored the interesting possibility of observing phase transitions driven by acoustic intensity within the framework of the director-density coupling theory [13]. In fact, we have predicted a Fréedericksz-type transition when the wave vector of the incident ultrasonic wave $\mathbf{k}$ is parallel to the z direction. In addition, we have also shown that for a small angle φ between $\mathbf{k}$ and the z direction and for acoustic intensity above a critical value (that depends on φ) the equilibrium state of the director is not obtained by solving the Euler-Lagrange equation associated with the Oseen-Frank elastic energy [14].

The director-density coupling theory has assumed that the director is not able to accompany rapid oscillations of the local density and, therefore, one shall average the interaction V_{int} at the start [5]. At first sight, this assertion seems plausible but, as pointed out in Ref. [11], it deserves further investigation. The final result is, however, that the subsequent analysis is able to derive quantities that depend only on u_2, and thus the original proposal [5] reduces to a theory with one parameter only [9]. In this context, a variant of the above assumption has been developed in which liquid crystals are treated as anisotropic Korteweg fluids at time and length scales over which changes occur in the density [15–17]. To appreciate the whole theory [5], it is desirable to obtain measurable quantities that also make clear the presence of the u_1 term in V_{int}. It is possible to accomplish this task by exploring an alternative route in which the time average is taken at the end of the calculation, letting the experiment decide about the stage of the calculation in which the time average should be carried out. In fact, the two methods of taking into account the interaction predict different results. In particular, the optical contrast ratio scales as $\mathcal{R} \propto I^2$ at low acoustic intensity I, as shown in this paper, which differs from $\mathcal{R} \propto I^4$ reported in Ref. [5] in the same regime of acoustic intensity. In principle, this discrepancy could be used to invalidate our approach. As pointed out recently [15], however, the controversy between different theories for the acousto-optic effect is far from concluded and some criticism [4,18,19] requires a response. The rest of our paper is as follows. In Sec. II, the theory is presented, together with the equation that governs the dynamics of the director and whose perturbative treatment is studied in Sec. III. In the following, in Sec. IV, we calculate some measurable quantities in terms of the parameters u_1 and u_2 at low acoustic intensity and, finally, concluding remarks are made in Sec. V.

II. DYNAMIC OF THE DIRECTOR

Consider a nematic liquid crystal of local density $\rho(\mathbf{r})$ and director $\hat{\mathbf{n}}(\mathbf{r}) = \hat{\mathbf{y}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ making an angle θ with the z direction in a cell of thickness a, as shown in Fig. 1. Let us assume that the action of a monochromatic ultrasonic plane wave of wave vector $\mathbf{k}$ and frequency ω incident into the cell causes a rapid fluctuation in the local density in the following form:

$$\rho(\mathbf{r},t) = \rho_0 + \Delta\rho \sin(\mathbf{k}\cdot\mathbf{r} - \omega t), \tag{1}$$

where ρ_0 is the average density. The acoustic intensity is given by

$$I = \frac{v^3(\Delta\rho)^2}{2\rho_0}, \tag{2}$$

where v is the sound velocity. Inhomogeneities in density couple to $\hat{\mathbf{n}}$ and give rise to an interaction energy which reads [5]

$$V_{\text{int}} = \sum_{i,j} \left[u_1 \left(\frac{\partial^2 \rho}{\partial x^i \partial x^j} \right) + u_2 \left(\frac{\partial \rho}{\partial x^i} \right) \left(\frac{\partial \rho}{\partial x^j} \right) \right] n_i n_j, \tag{3}$$

where $x^1 \equiv y$, $x^2 \equiv z$. After inserting Eq. (1) into Eq. (3), one obtains

$$V_{\text{int}} = -u_1(\Delta\rho)(\hat{\mathbf{n}}\cdot\mathbf{k})^2 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t) + u_2(\Delta\rho)^2(\hat{\mathbf{n}}\cdot\mathbf{k})^2 \cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t). \tag{4}$$

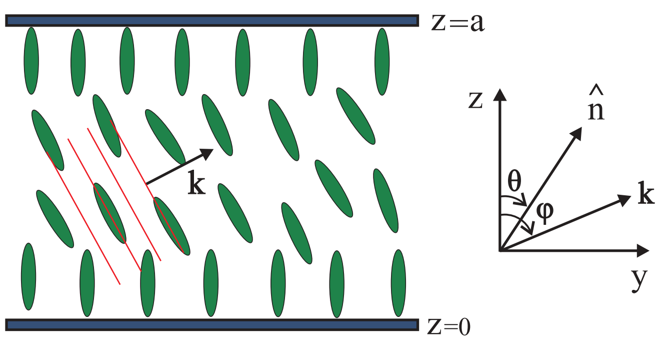

FIG. 1. (Color online) Cell of thickness a containing a nematic liquid crystal in the presence of an incident ultrasonic wave of wave vector $\mathbf{k}$. On the right, the coordinate system defines the angles θ for the director, and φ for the wave vector of the ultrasonic wave. The alignment of the molecules on the left is consistent with $\theta(z) \leqslant 0$.

Note that the u_2 term can be cast in the suggestive form $u_2(\hat{\mathbf{n}} \cdot \mathbf{E})^2$, where $\mathbf{E} \equiv \mathbf{k}(\Delta\rho)\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$ plays the role of an effective electric field [14]. However, the presence of u_1 endows the theory with a richer structure.

As pointed out in the Introduction, we shall work with V_{int} as it stands in Eq. (4), postponing the averaging procedure to the end of the calculation. Moreover, in order to avoid complications associated with phase transitions [13], we shall assume oblique incidence. To reduce $\theta = \theta(y,z,t)$ to a function of a single spatial variable z, we make the assumption that

$$|\mathbf{k}\cdot\mathbf{r}| \ll 1. \tag{5}$$

In reality, this condition can be satisfied in real experiments with $a \approx 10^{-6}$ m and $|\mathbf{k}| \approx 10^4$ m^{-1} ($\omega \approx 10^6$ Hz). As a consequence, the following approximations shall be used in Eq. (4):

$$\sin(\mathbf{k}\cdot\mathbf{r} - \omega t) = -\sin(\omega t) + O(\mathbf{k}\cdot\mathbf{r}), \tag{6}$$

$$\cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t) = \cos^2(\omega t) + O(\mathbf{k}\cdot\mathbf{r}). \tag{7}$$

The total energy per unit area is [5]

$$F = \int_0^a f(\theta,\partial\theta/\partial z,z,t)dz, \tag{8}$$

where

$$f = \tfrac{1}{2}(K_1\sin^2\theta + K_3\cos^2\theta)(\partial\theta/\partial z)^2 + V_{\text{int}}, \tag{9}$$

and K_1 and K_3 are, respectively, the Frank constants for splay and bend [14]. The dynamics of the liquid crystal director is determined by the equation [14]

$$\gamma\frac{\partial\theta}{\partial t} = -\frac{\delta f}{\delta\theta}, \tag{10}$$

where γ is the rotational viscosity coefficient and

$$\begin{aligned}\frac{\delta f}{\delta\theta} &\equiv \frac{\partial f}{\partial\theta} - \frac{\partial}{\partial z}\left[\frac{\partial f}{\partial(\partial\theta/\partial z)}\right]\\ &= -(K_1\sin^2\theta + K_3\cos^2\theta)\left(\frac{\partial^2\theta}{\partial z^2}\right)\\ &\quad -\frac{1}{2}(K_1 - K_3)(\partial\theta/\partial z)^2\sin(2\theta)\\ &\quad + u_1(\Delta\rho)k^2\sin 2(\varphi-\theta)\sin(\omega t)\\ &\quad + u_2(\Delta\rho)^2k^2\sin 2(\varphi-\theta)\cos^2(\omega t).\end{aligned} \tag{11}$$

When the ultrasonic is turned on, the initial condition reads

$$\theta(z,t=0) = f(z), \tag{12}$$

and strong-anchoring boundary conditions are imposed at the borders:

$$\theta(0,t) = \theta(a,t) = 0. \tag{13}$$

III. PERTURBATIVE SOLUTION

Let us assume that the solution of Eq. (10) can be expressed as a power series of the independent variable $\Delta\rho$:

$$\theta(z,t) = \theta^{(0)}(z,t) + (\Delta\rho)\theta^{(1)}(z,t) + (\Delta\rho)^2\theta^{(2)}(z,t) + \cdots. \tag{14}$$

If we insert (14) into (12) and (13), we get, respectively,

$$\theta^{(0)}(z,0) = f(z), \quad \theta^{(1)}(z,0) = \theta^{(2)}(z,0) = 0, \tag{15}$$

and

$$\theta^{(i)}(0,t) = \theta^{(i)}(a,t) = 0, \quad i = 0,1,2. \tag{16}$$

We now proceed by substituting Eq. (14) into Eq. (10) and, after matching coefficients in powers of $\Delta\rho$, we obtain the differential equations that the functions $\theta^{(0)}$, $\theta^{(1)}$, and $\theta^{(2)}$ must satisfy. As expected, in zero order we arrive at

$$\begin{aligned}\gamma\frac{\partial\theta^{(0)}}{\partial t} &= (K_1\sin^2\theta^{(0)} + K_3\cos^2\theta^{(0)})\left(\frac{\partial^2\theta^{(0)}}{\partial z^2}\right)\\ &\quad + \frac{1}{2}(K_1 - K_3)\left(\frac{\partial\theta^{(0)}}{\partial z}\right)^2\sin(2\theta^{(0)}).\end{aligned} \tag{17}$$

Whatever the initial condition $f(z)$ is, after a short transient period one expects that $\theta^{(0)}$ goes exponentially to zero, since this is the director configuration that minimizes the energy (8) in the absence of V_{int} and, at the same time, satisfies the constraint (13). This asymptotic behavior for $\theta^{(0)}$ is made apparent, for instance, by setting $K_1 = K_3$, given that Eq. (17) reduces to the well known diffusion equation. This exponential decay of $\theta^{(0)}$ is particularly important, because our main interest is to calculate the steady state of $\theta(z,t)$. In what follows, therefore, we shall neglect $\theta^{(0)}$, as well its derivatives, from the differential equations satisfied by $\theta^{(1)}$ and $\theta^{(2)}$ (by virtue of the fact that they are lengthy and not illuminating, we do not write them). The stationary state of $\theta^{(1)}$ is obtained by solving the simplified equation

$$K_3\frac{\partial^2\theta^{(1)}}{\partial z^2} - u_1k^2\sin(2\varphi)\sin(\omega t) = \gamma\frac{\partial\theta^{(1)}}{\partial t}. \tag{18}$$

The standard procedure to calculate the steady state of $\theta^{(1)}$ is to employ the Fourier series

$$1 = \sum_{n=1}^{\infty}\frac{2[1-\cos(n\pi)]}{n\pi}\sin\left(\frac{n\pi z}{a}\right), \quad 0 < z < a, \tag{19}$$

in Eq. (18):

$$K_3\frac{\partial^2\theta^{(1)}}{\partial z^2}-u_1k^2\sin(2\varphi)\sin(\omega t)\times\sum_{n=1}^{\infty}\frac{2[1-\cos(n\pi)]}{n\pi}\sin\left(\frac{n\pi z}{a}\right)=\gamma\frac{\partial\theta^{(1)}}{\partial t}. \quad (20)$$

The advantage is that the stationary solution we are going to obtain, i.e.,

$$\theta^{(1)}(z,t)=\sum_{n=1}^{\infty}[C_n\cos(\omega t)+D_n\sin(\omega t)]\sin\left(\frac{n\pi z}{a}\right), \quad (21)$$

automatically satisfies the boundary condition (16), although the sign of equality in Eq. (19) is not valid for $z=0$ and $z=a$.

After inserting Eq. (21) into Eq. (20), one obtains

$$C_n=\frac{2u_1\gamma\omega k^2\sin(2\varphi)[1-\cos(n\pi)]}{n\pi\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]}, \quad (22)$$

$$D_n=-\frac{2u_1K_3k^2\sin(2\varphi)[1-\cos(n\pi)](n\pi)}{a^2\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]}. \quad (23)$$

Finally, the stationary state of $\theta^{(2)}$ is obtained by solving

$$K_3\frac{\partial^2\theta^{(2)}}{\partial z^2}+2u_1k^2\theta^{(1)}\cos(2\varphi)\sin(\omega t)-u_2k^2\sin(2\varphi)\cos^2(\omega t)=\gamma\frac{\partial\theta^{(2)}}{\partial t}. \quad (24)$$

In the following, we insert Eq. (21) into Eq. (24) to obtain

$$K_3\frac{\partial^2\theta^{(2)}}{\partial z^2}+u_1k^2\cos(2\varphi)[1-\cos(2\omega t)]\sum_{n=1}^{\infty}D_n\sin\left(\frac{n\pi z}{a}\right)-u_2k^2\sin(2\varphi)[1+\cos(2\omega t)]\sum_{n=1}^{\infty}\frac{[1-\cos(n\pi)]}{n\pi}\sin\left(\frac{n\pi z}{a}\right)+u_1k^2\cos(2\varphi)\sin(2\omega t)\sum_{n=1}^{\infty}C_n\sin\left(\frac{n\pi z}{a}\right)=\gamma\frac{\partial\theta^{(2)}}{\partial t}, \quad (25)$$

with the use of Eq. (19) again. We now assume that $\theta^{(2)}$ in the stationary state is written in the form

$$\theta^{(2)}(z,t)=\sum_{n=1}^{\infty}[E_n+F_n\cos(2\omega t)+G_n\sin(2\omega t)]\sin\left(\frac{n\pi z}{a}\right), \quad (26)$$

which, after being inserted into Eq. (25), yields

$$E_n=-\frac{u_2k^2a^2\sin(2\varphi)[1-\cos(n\pi)]}{K_3(n\pi)^3}-\frac{u_1^2k^4\sin(4\varphi)[1-\cos(n\pi)]}{n\pi\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]}, \quad (27)$$

$$F_n=-\frac{u_1^2k^4\sin(4\varphi)[1-\cos(n\pi)]\left[2(\gamma\omega)^2-K_3^2\left(\frac{n\pi}{a}\right)^4\right]}{n\pi\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]\left[K_3^2\left(\frac{n\pi}{a}\right)^4+4(\gamma\omega)^2\right]}-\frac{u_2n\pi k^2K_3\sin(2\varphi)[1-\cos(n\pi)]}{a^2\left[K_3^2\left(\frac{n\pi}{a}\right)^4+4(\gamma\omega)^2\right]}, \quad (28)$$

$$G_n=\frac{3u_1^2k^4K_3\gamma\omega\sin(4\varphi)[1-\cos(n\pi)]n\pi}{a^2\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]\left[K_3^2\left(\frac{n\pi}{a}\right)^4+4(\gamma\omega)^2\right]}-\frac{2u_2\gamma\omega k^2\sin(2\varphi)[1-\cos(n\pi)]}{n\pi\left[K_3^2\left(\frac{n\pi}{a}\right)^4+4(\gamma\omega)^2\right]}. \quad (29)$$

IV. MEASURABLE QUANTITIES

We now proceed by calculating the time average of $\theta^{(1)}$ and $\theta^{(2)}$. It follows from Eq. (21) that

$$\langle\theta^{(1)}(z,t)\rangle=\lim_{T\to\infty}\frac{1}{T}\int_0^T\theta^{(1)}(z,t)dt=0, \quad (30)$$

and from Eq. (26) we obtain

$$\langle\theta^{(2)}(z,t)\rangle=\sum_{n=1}^{\infty}E_n\sin\left(\frac{n\pi z}{a}\right). \quad (31)$$

When these average values are inserted into Eq. (14), the result is

$$\langle\theta(z,t)\rangle=(I/I_0)\left\{\frac{\sin(2\varphi)}{2a^2}z(z-a)+\frac{2u_1^2k^2K_3\sin(4\varphi)}{u_2a^2}\sum_{n=1}^{\infty}\frac{[\cos(n\pi)-1]\sin\left(\frac{n\pi z}{a}\right)}{n\pi\left[K_3^2\left(\frac{n\pi}{a}\right)^4+(\gamma\omega)^2\right]}\right\}+\cdots,\quad I\to 0, \quad (32)$$

where

$$I_0\equiv\frac{v^3K_3}{u_2\rho_0k^2a^2} \quad (33)$$

defines the scale for the acoustic intensity and we have used the Fourier series

$$z(z-a) = 4a^2 \sum_{n=1}^{\infty} \frac{[\cos(n\pi)-1]\sin\left(\frac{n\pi z}{a}\right)}{(n\pi)^3}, \quad 0 \leqslant z \leqslant a. \tag{34}$$

Nicely, the first term in braces is exactly the same of Eq. (8) of Ref. [13]. In addition note that this check ensures the validity of the substitution (19) in Eqs. (18) and (24). The presence of a u_1-dependent term in $\langle\theta(z,t)\rangle$ is unexpected and it introduces additional dependence on γ and ω.

Let us now calculate the local fluctuations of the director at lower order in the parameter $\Delta\rho$:

$$\Delta\theta(z) \equiv \sqrt{\langle[\theta(z,t)]^2\rangle - \langle\theta(z,t)\rangle^2}. \tag{35}$$

From Eq. (14), one obtains

$$[\theta(z,t)]^2 = (\Delta\rho)^2[\theta^{(1)}(z,t)]^2 + \cdots = O(I/I_0), \tag{36}$$

and since $\langle\theta(z,t)\rangle^2 = O(I/I_0)^2$, it follows that the local fluctuations are dominated by the time average of $[\theta^{(1)}(z,t)]^2$:

$$[\Delta\theta(z)]^2 = \left(\frac{K_3}{u_2k^2a^2}\right)\left\{\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}(C_nC_l + D_nD_l)\sin\left(\frac{n\pi z}{a}\right)\sin\left(\frac{l\pi z}{a}\right)\right\}(I/I_0) + \cdots, \quad I \to 0. \tag{37}$$

Note that $\Delta\theta \propto \sqrt{I/I_0}$, and due to the cancellation of u_2 by the quantity I_0 previously defined in Eq. (33), it depends only on u_1 through C_n and D_n [see Eqs. (22) and (23)].

Consider now the experiment on optical transmission performed in Ref. [5]. The relevant quantity is the retardation [14]

$$\Gamma = \frac{2\pi}{\lambda}\int_0^a (\Delta n_{\mathrm{eff}})dz, \tag{38}$$

where

$$\Delta n_{\mathrm{eff}} = \frac{n_o n_e}{\sqrt{n_o^2\sin^2\theta + n_e^2\cos^2\theta}} - n_o = \frac{n_o\left(n_e^2 - n_o^2\right)}{2n_e^2}\sin^2\theta + \cdots, \quad I \to 0, \tag{39}$$

is the effective birefringence and λ is the wavelength of the light. In the steady state, one has

$$\sin^2\theta = (\Delta\rho)^2[\theta^{(1)}(z,t)]^2 + \cdots, \quad I \to 0, \tag{40}$$

so that

$$\Gamma(t) = \frac{\pi n_o\left(n_e^2 - n_o^2\right)K_3}{\lambda n_e^2 u_2 k^2 a}\left\{\cos^2(\omega t)\sum_{n=1}^{\infty}C_n^2 + \sin(2\omega t)\sum_{n=1}^{\infty}C_nD_n + \sin^2(\omega t)\sum_{n=1}^{\infty}D_n^2\right\}(I/I_0) + \cdots, \quad I \to 0. \tag{41}$$

In the limit of low I, the optical contrast ratio is

$$\begin{aligned}\mathcal{R}(t) = \sin^2(\Gamma/2) = \frac{1}{4}\Gamma^2 + \cdots = \frac{4\pi^2u_1^4n_o^2\left(n_e^2-n_o^2\right)^2(\gamma\omega)^4k^4K_3^2\sin^4(2\varphi)}{u_2^2a^2\lambda^2n_e^4A^2}\\ \times\left[S_1^2\cos^4(\omega t) + \left(\frac{K_3}{a^2\gamma\omega}\right)^2S_3^2\sin^2(2\omega t) + \left(\frac{K_3}{a^2\gamma\omega}\right)^4S_2^2\sin^4(\omega t) - 2\cos^2(\omega t)\sin(2\omega t)\left(\frac{K_3}{a^2\gamma\omega}\right)S_1S_3\right.\\ \left. + 2\cos^2(\omega t)\sin^2(\omega t)\left(\frac{K_3}{a^2\gamma\omega}\right)^2S_1S_2 - 2\sin(2\omega t)\sin^2(\omega t)\left(\frac{K_3}{a^2\gamma\omega}\right)^3S_2S_3\right](I/I_0)^2, \quad I \to 0.\end{aligned} \tag{42}$$

At this point, we take the time average to obtain

$$\langle\mathcal{R}\rangle = \frac{\pi^2u_1^4n_o^2\left(n_e^2-n_o^2\right)^2(\gamma\omega)^4k^4K_3^2\sin^4(2\varphi)}{4u_2^2a^2\lambda^2n_e^4A^2}\left[6S_1^2 + 4\left(S_1S_2 + 2S_3^2\right)\left(\frac{K_3}{a^2\gamma\omega}\right)^2 + 6S_2^2\left(\frac{K_3}{a^2\gamma\omega}\right)^4\right](I/I_0)^2 + \cdots, \quad I \to 0, \tag{43}$$

where

$$S_1 \equiv A\sum_{n=1}^{\infty}\frac{[1-\cos(n\pi)]^2}{(n\pi)^2\left[K_3^2\left(\frac{n\pi}{a}\right)^4 + (\gamma\omega)^2\right]^2}, \tag{44}$$

$$S_2 \equiv A\sum_{n=1}^{\infty}\frac{[1-\cos(n\pi)]^2(n\pi)^2}{\left[K_3^2\left(\frac{n\pi}{a}\right)^4 + (\gamma\omega)^2\right]^2}, \tag{45}$$

$$S_3 \equiv A \sum_{n=1}^{\infty} \frac{[1-\cos(n\pi)]^2}{\left[K_3^2\left(\frac{n\pi}{a}\right)^4 + (\gamma\omega)^2\right]^2}, \tag{46}$$

and $A = [(K_3/a^2)^4 + (\gamma\omega)^4]$. It should be noticed that $\langle \mathcal{R} \rangle$ depends only on u_1 [see the comment below Eq. (37)] and that the prediction $\langle \mathcal{R} \rangle \propto I^2, I \to 0$ differs from that reported in Ref. [5] (namely, $\langle \mathcal{R} \rangle \propto I^4, I \to 0$). We expect that this discrepancy can be used to decide about the correctness of each approach.

V. SUMMARY AND DISCUSSION

It has been assumed in the director-density coupling theory that the director is unable to respond rapidly to the oscillations of the sound wave and, therefore, we shall average the interaction V_{int} at the start. This assumption is not obvious [11] and the measurable quantities derived along this line of reasoning [5,13] should be compared with the corresponding ones obtained by different approaches. With this motivation, in this paper we have studied the consequences of postponing the time average to the stage of calculating the measurable quantities. Remarkably, in this case the parameter u_1 plays an important role in all quantities we have calculated. In the first place, the inclusion of u_1 gives $\langle \theta(z,t) \rangle$ an additional dependence on γ and ω, although it does not affect its linear dependence on I, as stated elsewhere [5]. We have also calculated the local fluctuations of the director at lower order in $\Delta\rho$ and have found that it depends on u_1 only. This feature dependence is also shared with $\langle \mathcal{R} \rangle$ and suggests that fluctuations are ignored when working with $\langle V_{\mathrm{int}} \rangle$. In summary, the main result of our paper is to show that high-frequency oscillations of the director can be connected with measurable quantities. In particular, we have shown that when fluctuations are properly taken into account, one has $\langle \mathcal{R} \rangle \propto I^2$.

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