Scaling of seismic memory with earthquake size

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It has been observed that discrete earthquake events possess memory, i.e., that events occurring in a particular location are dependent on the history of that location. We conduct an analysis to see whether continuous real-time data also display a similar memory and, if so, whether such autocorrelations depend on the size of earthquakes within close spatiotemporal proximity. We analyze the seismic wave form database recorded by 64 stations in Japan, including the 2011 "Great East Japan Earthquake," one of the five most powerful earthquakes ever recorded, which resulted in a tsunami and devastating nuclear accidents. We explore the question of seismic memory through use of mean conditional intervals and detrended fluctuation analysis (DFA). We find that the wave form sign series show power-law anticorrelations while the interval series show power-law correlations. We find size dependence in earthquake autocorrelations: as the earthquake size increases, both of these correlation behaviors strengthen. We also find that the DFA scaling exponent α has no dependence on the earthquake hypocenter depth or epicentral distance.

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I. INTRODUCTION

Many complex physical systems exhibit complex dynamics in which subunits of the system interact at widely varying scales of time and space $[1,2]$. These complex interactions often generate very noisy output signals which still exhibit scale-invariant structure. Such complex systems span areas studied in physiology $[3]$, finance $[4]$, and seismology $[5-14]$.

In seismology, temporal and spatial clustering are considered important properties of seismic occurrences and, together with the Omori law (dictating aftershock timing) and the Gutenberg-Richter law (specifying the distribution of earthquake size), comprise the main starting requirements to be fulfilled in any reasonable seismic probabilistic model. Analyzing the timing of individual earthquakes, the authors of Ref. [\[5\]](#page-6-0) introduced the scaling concept to statistical seismology. The recurrence times are defined as the time intervals between consecutive events, $\tau_i \equiv t_i - t_{i-1}$. In the case of stationary seismicity, the probability density $P(\tau)$ of the occurrence times was found to follow a universal scaling law

$$
P(\tau) = Rf(R\tau),\tag{1}
$$

where f is a scaling function and R is the rate of seismic occurrence, defined as the mean number of events with $M \geq M_c$ [\[6\]](#page-6-0). References [\[7,8\]](#page-6-0) has demonstrated how the structure of seismic occurrence in time and magnitude can be treated within the framework of critical phenomena.

The study of seismic waves is both scientifically interesting and of practical concern, particularly in such applied areas as engineering. A better understanding of seismic waves is immediately applicable in the design of structures for earthquakeprone areas [\[15–17\]](#page-6-0). It also allows scientists to better understand the underlying mechanisms that drive earthquakes [\[18–23\]](#page-7-0). The factors that affect the seismic waves include source, medium, and instrument response, and Refs. [\[24,25\]](#page-7-0) show the importance of studying wave forms in the frequency domain and the connection between frequency content and seismic source. Reference [\[26\]](#page-7-0) indicates the fractal spectrum structure of random-pulse time series. In Refs. [\[27,28\]](#page-7-0) the correlation between the fractal nature of inhomogeneities and seismic wave propagation is reported. These results are in agreement with the intriguing hypothesis of the existence of long-range autocorrelations in seismic waves. Recently, with development in the measurement technology of seismic waves, the seismometer has become capable of recording seismic motion over a very wide frequency range.

Recently, a few papers have analyzed the existence of correlations between magnitudes of subsequent earthquakes [\[7,8\]](#page-6-0). Analyzing earthquakes with τ greater than 30 min, Ref. [\[7\]](#page-6-0) reported possible magnitude correlations in the Southern California catalog. Magnitude correlations have often been interpreted as a spurious effect due to so-called short-term aftershock incompleteness [\[9\]](#page-6-0). This hypothesis assumes that some aftershocks, especially small events, are not reported in the experimental catalogs, which is in agreement with the standard approach that assumes interdependence of earthquake magnitudes, implying no memory in earthquakes.

However, recent work has also challenged this interpretation. The authors of Ref. [\[10\]](#page-6-0) report the existence of magnitude clustering in which earthquakes of a given magnitude are more likely to occur close in time and space to other events of similar magnitude. They find that a subsequent earthquake tends to have a magnitude similar to but smaller than the previous earthquake. Reference [\[8\]](#page-6-0) also reports the existence of magnitude correlations and additionally demonstrates the structure of these correlations and their relationship to Δt and Δr , where the latter represents the distance between subsequent epicenters. In Ref. [\[12\]](#page-6-0) a model to explain these magnitude correlations is created. Additionally, Refs. [\[13\]](#page-6-0) and [\[14\]](#page-6-0) find that the distribution of recurrence times strongly depends on the previous recurrence time, such that small and large recurrence

times tend to cluster in time. This dependence on the past is reflected in both the conditional mean recurrence time and the conditional mean residual time until the next earthquake.

In addition to analyzing the raw wave form, it is also common to analyze related time series, such as the time series generated by taking the sign or magnitude of the wave form [\[3\]](#page-6-0). Reference [\[3\]](#page-6-0) reports an empirical approximate relation at small time scales for the scaling exponents calculated for the sign, the magnitude, and the original time series, $\alpha_{sign} = (1/2)(\alpha_{magnitude} + \alpha_{original})$, in physiology. The study of magnitude and sign time series is important in physiology because the magnitude time series exhibits weaker autocorrelations and a scaling exponent closer to the exponent of an uncorrelated series when the subject is unhealthy [\[3\]](#page-6-0). Diagnostic power in physiology has been confirmed for sign time series as well—the sign time series of heart failure subjects exhibit scaling behavior similar to that observed in the original time series, but significantly different from that of healthy subjects [\[3\]](#page-6-0). Understanding the correlation properties of these three time series allows us to also understand the underlying processes generating them.

As stated in Ref. [\[25\]](#page-7-0), "As in all other branches of geophysics, the effects of source and medium are strongly coupled in seismology*...* The motions recorded in a seismogram are a result both of propagation effects and of source effects." In practice, when we apply a single time-series correlation method to a given physical output, clearly, we do not take into account the potential influence of other time series that may influence the given time series. For instance, many physical outputs exhibit long-range autocorrelations and they also cross-correlate with other physical outputs. So the fact that many physical outputs mutually affect each other does not prevent us from studying single statistical properties. Here we test a challenging hypothesis that long-range autocorrelations exist in seismic waves. We first note that long-range power-law autocorrelations are quite common in a large number of natural phenomena, ranging from weather [\[29–31\]](#page-7-0) and physiological systems [\[3](#page-6-0)[,32–35\]](#page-7-0) to financial markets [\[36–44\]](#page-7-0). Specifically, in the latter area, the authors of $[45]$ have investigated the relationship between financial aftershock memory and the size of the nearest major event, in close analogy to our work here.

Our investigation and discussion are organized as follows. First, we study the autocorrelations of interval series by using the mean conditional technique. Second, we employ detrended fluctuation analysis (DFA) [\[46–48\]](#page-7-0) to investigate the possible existence of long-range autocorrelations in the sign and interval time series. For the interval time series we find a positive regression between the DFA scaling exponent *α* and earthquake size (measured by the Richter magnitude scale *M* or seismic moment M_0), while for the sign time series we find an inverted regression between α and earthquake magnitude. Thus we report that the observed autocorrelation depends on earthquake size, in both the sign and interval time series. We also find that the scaling exponent α has no dependence on hypocenter depth or epicentral distance.

II. DATA

Seismic waves are unique in that they have nonstationarities of a much larger order than those of any other known natural

FIG. 1. (Color online) (a) Location map for the 46 broadband stations of the Full Range Seismograph Network of Japan (F-net) (red asterisks). Inset: An example of a record of a seismic wave (up-down component). (b) A part of the coda wave series indicated in the inset of (a), as an example. (c) An example of a sign time series, where the positive sign $(+1)$ represents a positive wave form and the negative sign (−1) represents a negative wave form in coda wave series of seismic waves. (d) Interval time series (τ) of the coda wave series for a subset of the record shown in (b).

signal. Large earthquakes are characterized by a maximum amplitude that is often *>*100 times larger than the mean amplitude [see Fig. 1]. This is a limitation that makes seismic waves difficult to analyze using traditional analysis. Although we might want to use detrended fluctuation analysis [\[46–49\]](#page-7-0), originally proposed to study the correlations in a time series in the presence of nonstationarities commonly observed in natural phenomena, the level of nonstationarity in earthquakes is so large that DFA is inappropriate regardless of the order of the polynomial fit applied [\[48\]](#page-7-0). Thus, due to lack of methods for highly nonstationary signals, we do not analyze correlations in the series of magnitudes, but instead analyze the correlations in the sign series [Fig. $1(c)$] and interval series [Fig. $1(d)$]. For our data, we use the seismic wave form database from the National Research Institute for Earth Science and Disaster Prevention (NIED) F-net (Full Range Seismograph Network of Japan), which records continuous seismic wave form data w_t by using broadband sensors in 64 stations in Japan [see Fig. $1(a)$]. In our study we select 46 stations (ADM, AOG, ASI, HID, HJO, HRO, IGK, IMG, INN, IYG, IZH, KGM, KMU, KNM, KNP, KNY, KSK, KSN, KSR, KYK, MMA, NKG, NOK, NOP, NRW, NSK, OSW, SAG, SHR, SIB, TANS, TGA, TGW, TKO, TMC, TSA, TYM, TYS, UMJ, WTR, YAS, YNG, YSI, YTY, YZK, and ZMM), based on the locations and integrity of the data series. Seismic signals are recorded in three directions: U (up-down with up positive), N (north-south with north positive), and E (east-west with east positive) $[20]$. We find that the data recorded in those three directions show very similar memory and scaling rules. In this paper, we present the quantitative results for each of those three directions, and the qualitative results only for the U direction as a representative. Sampling intervals have five recording frequencies: 80, 20, 1, 0.1, and 0.01 Hz. We study earthquake coda wave data with a 1 Hz sampling interval for the year 2003, together with selected earthquake coda wave data from 11 March 2011. We note that, because of the interaction between earthquakes, not all earthquakes can be employed in our analysis (see the Appendix). The data from 11 March 2011 are selected because they contain the notable 2011 Tōhoku earthquake ("Great East Japan Earthquake"), which resulted in the tsunami that caused a number of nuclear accidents. We also add two large earthquakes ($M = 7.3$ and $M = 7.6$) to our study, which also occurred the same day as aftershocks.

We employ the following procedure to create our time series:

(i) For each selected earthquake (see the Appendix) we create a new time series, the normalized wave form, denoted by w_t , out of the raw seismic acceleration wave form data,

$$
w_{\text{norm}} = (w_t - \overline{w})/\sqrt{\overline{w_t^2} - \overline{w}^2}.
$$
 (2)

(ii) From the normalized time series w_{norm} we define a new subseries w'_t , starting at the time coordinate where maximum w_t occurs and terminating at the end of the normalized wave form w'_t [see the inset in Fig. [1\(a\)\]](#page-1-0).

(iii) Let the time series t_i denote the points in time when w_t changes sign, with $t_i < t_{i+1}$. We define the interval series by $\tau_i \equiv t_i - t_{i-1}$ [Fig. [1\(c\)\]](#page-1-0).

(iv) The sign series is defined by $s_t \equiv \text{sgn}(w_t')$ [Fig. [1\(d\)\]](#page-1-0).

Note that our definition of interval is different from that recently defined in several papers, where the return intervals *τ* between consecutive fluctuations above a volatility threshold *q* in different complex systems have been studied. The probability density function of return intervals $P_q(\tau)$ scales with the mean return interval as

$$
P_q(\tau) = \overline{\tau}^{-1} f(\tau/\overline{\tau}), \tag{3}
$$

where $f()$ is a stretched exponential $[37-40]$. Since, on average, there is one volatility above the threshold *q* for every $\overline{\tau}_q$ volatilities, then it holds that [\[43\]](#page-7-0)

$$
1/\overline{\tau}_q \approx \int_q^{\infty} P(|R|)d|R| = P(|R| > q) \sim q^{-\alpha}.
$$
 (4)

For the time intervals τ_q between events given by fluctuations *R* where $R > q$, Ref. [\[43\]](#page-7-0) derived that $\overline{\tau_q}$, the average of τ_q ,

FIG. 2. (Color online) Scaled mean conditional interval $\langle \tau | \tau_0 \rangle / \overline{\tau}$ vs $\tau_0/\overline{\tau}$. Five groups, one with no proximal earthquake, and earthquakes with Richter magnitude scale $M < 4.5$, $M = 4.5-5.5$, $M = 6.5 - 6.5$, $M > 6.5$. An increasing trend implies a short-range correlation in the interval series.

obeys a scaling law,

$$
\overline{\tau_q} = q^{\alpha},\tag{5}
$$

where α denotes our estimate of the tail exponent probability density function, $P(|R|^{1+\alpha})$. Similarly, if $P(|R|)$ follows an exponential function $P(|R|) \propto \exp(-\beta |R|)$, then employing Eq. (4) we easily derive

$$
\overline{\tau}_q \propto \exp(\beta q). \tag{6}
$$

Equation (6) can be used for estimation of the exponential parameter *β*.

III. MEMORY OF INTERVAL TIME SERIES

Returning to wave form data, we begin analyzing the series by studying the conditional mean

$$
\langle \tau | \tau_0 \rangle / \overline{\tau}, \tag{7}
$$

which gives the mean value of τ immediately following a given term τ_0 , normalized in units of $\overline{\tau}$. The conditional mean gives evidence of whether seismic memory exists in the intervals in the form of correlations or anticorrelations. For example, should correlations exist, one would expect the mean interval to be shorter in the window immediately following a small interval.

Indeed, Fig. 2 shows that the large intervals τ tend to follow large initial τ_0 and small τ follow small τ_0 , indicating the existence of (positive) correlations in the interval time series. We also note that the autocorrelations tend to be stronger for the subsets associated with larger earthquakes than for those associated with smaller earthquakes.

To expand on this we also extend our investigation to longerrange effects. We investigate the mean interval after a cluster of *n* consecutive intervals that are either entirely above the series mean or entirely below it. We denote clusters that are entirely above the series mean with a "+" and clusters below the series mean with a "−". Figure [3](#page-3-0) shows the mean interval *τ* that follows a $\tau_0(n)$ defined as a cluster size of *n*. We find that

FIG. 3. (Color online) Long-range memory in interval clusters. *τ*⁰ signifies a cluster of intervals, consisting of *n* consecutive values that all are above (denoted as "+") or below (denoted as "−") the median of the entire interval records. Plots display the scaled mean interval conditioned on a cluster, $\langle \tau | \tau_0 \rangle / \overline{\tau}$, vs the size *n* of the cluster for five group intervals. The upper curves are for $+$ clusters while the lower part is for $-$ clusters. The plots show that $+$ clusters are likely to be followed by large intervals and − clusters by small intervals, consistent with long-term correlations in interval records. As in Fig. [2,](#page-2-0) the long-term correlation increases with earthquake size, with exceptions for very large earthquakes.

for + clusters—shown by open symbols—the mean interval increases with the size of the cluster *n*. This is the opposite of what we find for − clusters—shown as closed symbols. The results indicate the existence of at least short-term memory in the interval time series. Furthermore, we find that the mean interval increases with the seismic magnitude. However, this relationship breaks at the high end of the Richter magnitude scale, $M > 6.5$.

IV. DETRENDED FLUCTUATION ANALYSIS

Many physical, physiological, biological, and social systems are characterized by complex interactions between a large number of individual components, which manifest in scale-invariant correlations $[1,2,50,51]$ $[1,2,50,51]$. Since the resulting observable at each moment is the product of a magnitude and a sign, many recent investigations have focused on the study of correlations in magnitude and sign time series $[3,4,33,47,52-54]$ $[3,4,33,47,52-54]$. For example, the time series of changes $\delta \tau_i$ of heartbeat intervals [\[3](#page-6-0)[,33,54\]](#page-7-0), physical activity levels [\[47\]](#page-7-0), intratrading times in the stock market [\[52\]](#page-7-0), and river flux values [\[53\]](#page-7-0) all exhibit power-law anticorrelations, while their magnitudes $|\delta \tau_i|$ are positively correlated. A common means of finding autocorrelations hidden within a noisy nonstationary time series is detrended fluctuation analysis [\[46–48\]](#page-7-0). In the DFA method, the time series is partitioned into pieces of equal size *n*. For each piece, the local trend is subtracted and the resulting standard deviation over the entire series is obtained. In general, the standard deviation $F(n)$ of the detrended fluctuations depends on *n*, with smaller *n* resulting in trends that more closely match the data. The dependence of *F* on *n* can generally be represented as a power law such that

$$
F(n) \propto n^{\alpha},\tag{8}
$$

where α is the scaling exponent—sometimes referred to as the Hurst exponent—to be obtained empirically. DFA therefore can conceptually be understood as characterizing the motion of a random walker whose steps are given by the time series. $F(n)$ gives the walker's deviation from the local trend as a function of the trend window. Because the root mean square displacement of a walker with no correlations between his steps scales as \sqrt{n} , we can expect a time series with no autocorrelations to yield an α of 0.5. Similarly, long-range power-law correlations in the signal (i.e., large terms follow large terms and small terms follow small terms) manifest as $\alpha > 0.5$. Power-law anticorrelations within a signal will result in α < 0.5. Additionally, DFA can be related to the autocorrelation as follows: if the autocorrelation function*C*(*L*) can be approximated by a power law with exponent *γ* such that

$$
C(L) \propto L^{-\gamma},\tag{9}
$$

then γ is related to α by [\[46\]](#page-7-0)

$$
\alpha \approx 1 - \gamma/2. \tag{10}
$$

Another reason we employ the DFA method is that it is appropriate for sign time series [\[33\]](#page-7-0). Other techniques for the detection of correlations in nonstationary time series are not appropriate for sign time series. Also, because the sign and interval time series have affine relations, the analysis of sign will be helpful in understanding the intervals. However, the DFA gives biased estimates for the power-law exponent in analysis of anticorrelated series [\[47\]](#page-7-0), and so in order to improve the accuracy of analysis, we integrate the time series before we employ the standard DFA procedure.

For the 2011 Tōhoku earthquake, also known as the "Great" East Japan Earthquake," we present the fluctuation functions $F(n)$ of the coda wave, measured at the KSN station, as typical examples of sign and intervals time series (Fig. [4\)](#page-4-0). By using DFA, we find, for most coda waves after earthquakes, that the time series of the intervals are consistent with a power-law correlated behavior $\alpha = 0.69$, while the sign time series are consistent with a power-law anticorrelated behavior $(\alpha = 0.32)$. The results therefore indicate that for the interval series large increments are more likely to be followed by large increments and small increments by small increments. These results are in agreement with the results of the correlation analysis reported in Sec. [III.](#page-2-0) In contrast, anticorrelations in the sign time series indicate that positive increments are more likely to be followed by negative increments and vice versa.

For the entire set of sign time series comprising our sample we calculate the average DFA scaling exponent $\bar{\alpha}$ = 0.34 ± 0.09 , indicating anticorrelations, and for the interval time series we calculate the average DFA scaling exponent $\bar{\alpha} = 0.58 \pm 0.08$, indicating correlations. For the different stations measuring the 2011 Tōhoku earthquake we find that for the sign time series, $\bar{\alpha} = 0.29 \pm 0.05$ and for the interval time series, $\overline{\alpha} = 0.66 \pm 0.07$.

FIG. 4. (Color online) DFA fluctuation function *F*(*n*) of 2011 Tōhoku earthquake as a function of time scale $n \lfloor F(n) \propto n^{\alpha} \rfloor$ for (a) sign time series $[\alpha + 1 = 1.32 \; (\alpha < 0.5)$ indicates anticorrelations] and (b) interval time series $\alpha + 1 = 1.69$ ($\alpha > 0.5$) indicates correlations].

V. RELATION BETWEEN EARTHQUAKE MOMENTS AND SCALING EXPONENTS OF SIGN AND INTERVAL SERIES

Because large earthquake events release such extraordinary amounts of energy, it is reasonable to ask whether their occurrence influences local wave dynamics. To this end, we study interval time series of coda waves from earthquakes occurring in 2003, also including the particularly large events of 11 March 2011, when three events with *M >* 7 occurred in the same day. Figure $5(a)$ shows the DFA scaling exponent of the sign series versus the seismic moment, where the seismic moment is a quantity used to measure the size of an earthquake. We find a decreasing functional dependence between the DFA exponent of the sign series and the seismic moment of the proximal earthquake, with slope -0.028 ± 0.002 (slope −0*.*042 ± 0*.*003 for the N direction and slope −0*.*033 ± 0*.*003 for the E direction) indicating that the DFA exponent decreases with increasing seismic moment. Note that because most of the exponents are below 0*.*5, this indicates the presence of ever stronger anticorrelations in the time series as earthquake magnitude increases. Note, however, that the data break from this trend for very large earthquakes (Richter magnitude scale *>*6*.*6 or seismic moment *>*1019).

We also find similar results in the interval series, the difference being that the anticorrelations become correlations. Figure $5(b)$ shows that the DFA interval exponent and seismic moment exhibit a *positive* functional dependence with slope 0.025 ± 0.002 (slope 0.022 ± 0.002 for the N direction and slope -0.020 ± 0.002 for the E direction) so that the DFA exponent increases with increasing seismic moment. Because most of the exponents for the interval series are greater than 0*.*5, this indicates that the series show stronger correlations for

FIG. 5. (Color online) Scaling exponent *α* vs seismic moment (Richter magnitude scale) for (a) sign time series (correlation coefficient $\rho = -0.3604$), and (b) interval time series (correlation coefficient $\rho = 0.3602$). Inset: Scaling exponent α vs seismic moment of data in N (north-south with north positive) and E (east-west with east positive) directions. The values of α show negative slope in the regression α vs seismic moment of the sign series, and positive slope in the regression of the interval series. Triangular symbols show the mean of the exponent within each bin (bins: $\langle 1 \times 10^{15}, 1 \times 10^{15} - 1 \times 10^{16}, 1 \times 10^{16} - 1 \times 10^{17}, 1 \times 10^{17} \rangle$ $10^{17} - 1 \times 10^{18}$, $1 \times 10^{18} - 1 \times 10^{19}$, $1 \times 10^{19} - 1 \times 10^{20}$, $1 \times 10^{20} 1 \times 10^{21}$, $>1 \times 10^{21}$); the error bar shows the \pm standard deviation. The plots show a linear relationship between the logarithmic earthquake moment and the scaling exponent α in the sign and interval series, with exceptions for very large earthquakes.

increasing seismic moment. Again, as with the sign series, we find a deviation from this trend for very large earthquakes.

Having observed the influence of seismic moment on autocorrelations, we now investigate whether other readily observable factors such as hypocenter depth and epicentral distance (the distance from the event to the recording station) also contribute. Specifically, we would like to explore whether there is evidence that such long-term memory is affected by the

FIG. 6. (Color online) Scaling exponent *α* vs hypocenter depth for events where the Richter magnitude scale *M <* 5 for (a) sign time series and (b) interval time series. Inset: scaling exponent α vs hypocenter depth requiring that the Richter magnitude scale $M > 5$. (c),(d) Scaling exponent α vs epicentral distance for events where the Richter magnitude scale $M < 5$ for (c) sign time series and (d) interval time series. Inset: scaling exponent α vs epicentral distance requiring that the Richter magnitude scale $M > 5$. All absolute values of the correlation coefficient $ρ$ are smaller than 0.1, showing that $α$ is uncorrelated with both hypocenter depth and epicentral distance.

spreading process as seismic waves disseminate outward from their epicenter to a recording station, or whether the memory observed is strictly due to the seismic activity. Figure 6 shows that the DFA exponents for both interval and sign series are independent of both hypocenter depth and epicentral distance. From these results we speculate that the DFA exponent is mainly a result of the characteristics of the hypocenter rather than of the process by which the seismic waves are spread.

For moderately large earthquakes ($M_0 = 10^{14} \sim 10^{19}$), we approximate the relation between the DFA scaling exponent and the seismic moment through the empirical formula

$$
\alpha \approx a \log_{10}(M_0) + c,\tag{11}
$$

where $a = -0.028$, $c = 0.797$ (N direction, $a = -0.041$, $c = 1.096$; E direction, $a = -0.033$, $c = 0.946$) for the sign time series and where $a = 0.025$, $c = 0.174$ (N direction, $a = 0.022$, $c = 0.221$; E direction, $a = 0.020$, $c = 0.25$) for the interval time series. Since

$$
M = [\log_{10}(M_0) - 9.1]/1.5, \tag{12}
$$

we can also write

$$
\alpha \approx a(1.5M + 9.1) + c = a'M + c'. \tag{13}
$$

For example, in the U direction we obtain $a' = -0.042$, $c' =$ 0.542 for the sign series, and $a' = 0.037$, $c' = 0.398$ for the interval series.

We note that similar size dependence in the Hurst exponent was found in Ref. [\[55\]](#page-7-0), where the Hurst exponents of financial time series increase logarithmically with company size.

VI. SUMMARY

We analyze seismic coda waves during earthquakes, finding long-range autocorrelations in both the interval and sign time series. The sign series generally display power-law anticorrelated behavior, with anticorrelations becoming stronger with larger earthquake events, while the interval series generally display power-law correlated behavior, with correlations also becoming stronger with larger earthquake events. We also show that while the DFA autocorrelation exponent is influenced by the size of the earthquake seismic moment, it is unaffected by earthquake depth or epicentral distance.

The factors that affect the seismic waves include source, medium, and instrument response. In this study, we use the database from a wide area (the global area of Japan); moreover, the broadband seismometer sensors have equivalent performance and response characteristics [\[20\]](#page-7-0). So we may ignore the influence of instrumental response and assume that the wave forms are affected only by the medium and the source. In the empirical analysis of seismic waves, the impacts from media that work on the wave form are complex and difficult to measure. Fortunately, from our statistical analysis results, we can conclude that the power-law correlation or anticorrelation of the autocorrelation function, which is described as a DFA scaling exponent empirically, has no dependence on earthquake hypocenter depth or epicentral distance. These results indicate that the impact of the medium on the wave form may be considered as negligible, at least when the records are taken from a large area and over a long period of time. In other words, in our quantitative analysis of the power-law correlation or anticorrelation of autocorrelation functions, we can assume that the significant influences are all from the source, in the form of the earthquake size.

Seismologists usually consider the source spectrum, the attenuation laws, and the instrument response to be the primary three factors which affect the wave form of seismic waves. Additionally, in theory, the power spectral density function and power-law autocorrelation of a real, stationary signal can be calculated from each other $[36]$. For these reasons we can assume that the long-term memory of sign and interval time series of seismic waves will be strongly related to the source spectrum. From the seismic wave models that refer to the source spectrum, we can infer some fractal characteristics [\[24,25\]](#page-7-0). Because there is a lack of methods for the empirical evaluation of the spectral frequency function of seismic waves—waves which are highly nonstationary and may be affected by many factors—researchers usually make theoretical models and hypotheses to match the spectral frequency function. On the other hand, in physics and economics, researchers are more likely to analyze the time domain characteristics of the signals, using analyses such as the autocorrelation function and power-law long-term memory, described by the Hurst exponent. However, in the latter fields, some physical models are also applied to analyze seismic wave forms, such as the locally stationary autoregressive and varying coefficient autoregressive models [\[56–58\]](#page-7-0).

In this paper, we employed statistical physical methods to investigate the long-term memory in sign and interval time series. Our findings are in contrast with the standard approach, which assumes independence in earthquake signals, and thus have strong implications for the ongoing debate about earthquake predictability [\[59\]](#page-7-0).

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APPENDIX: THE SELECTION OF EARTHQUAKES

In some regions it is common for multiple earthquakes to occur in short succession. In many cases, because the interoccurrence times are so short, the coda waves can be derived from more than one earthquake. This is especially true for large earthquakes with many aftershocks [\[24\]](#page-7-0). In order to make sure that the coda waves we study are the effects of only one earthquake, we need a way of determining which earthquakes are independent. We use the following two functions to determine the sphere of influence and duration of each earthquake by using the Richter magnitude scale *M* [\[24\]](#page-7-0). We select only those earthquakes that have no larger earthquake in their spatiotemporal sphere of influence, such that

$$
t \approx 10^{(M-4.71)/1.67} \tag{A1}
$$

and

$$
R \approx 2 \times 10^{(M+1.0)/2.7}, \tag{A2}
$$

where t is the duration and R is the sphere radius of influence. The two functions are empirical formulas based on an analysis of earthquakes in Japan $[24]$. $10^{M+1.0}/2.7$ is an empirical formula that indicates the maximum radius over which a human can feel an earthquake, especially for the earthquakes in Japan.

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