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# Existence and significance of communities in the World Trade Web

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The World Trade Web (WTW), which models the international transactions among countries, is a fundamental tool for studying the economics of trade flows, their evolution over time, and their implications for a number of phenomena, including the propagation of economic shocks among countries. In this respect, the possible existence of communities is a key point, because it would imply that countries are organized in groups of preferential partners. In this paper, we use four approaches to analyze communities in the WTW between 1962 and 2008, based, respectively, on modularity optimization, cluster analysis, stability functions, and persistence probabilities. Overall, the four methods agree in finding no evidence of significant partitions. A few weak communities emerge from the analysis, but they do not represent secluded groups of countries, as intercommunity linkages are also strong, supporting the view of a truly globalized trading system.

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## I. INTRODUCTION

Among the many real-world networks studied in the literature, the World Trade Web (WTW) has recently received increasing attention because of a number of interesting features. It is quite natural to represent international transactions among countries as a network, where countries are the nodes and the connecting edges are the international trade flows between them, giving rise to an intricate system of exchanges affecting all the countries. The specific economic motivations driving international trade flows shape this network, which consequently displays characteristics that are relevant for their economic implications, as well as for the network analysis in itself.

The main topological properties observed in the WTW indicate that this network is disassortative, with a high clustering coefficient and a number of small-world properties [1–4]. Its evolution over time is slow, showing an increasing connectivity among nodes [5].

In what follows, we use the term "globalization" according to the definition given by Deardorff [6], "The increasing worldwide integration of markets for goods, services and capital," or by Robertson [7], "Globalization refers to the compression of the world and the intensification of consciousness of the world as a whole." (We note that many other definitions of globalization exist, not always in full agreement: also, in his glossary, Deardorff [6] quotes alternative definitions.) Both definitions stress the idea that globalization is affecting the world as a whole, and here we stress the economic aspects of globalization as economic integration between countries. A number of indicators show its rapid increase over time (see, e.g., Williamson [8] or Baldwin and Martin [9]), but the patterns of this integration can be quite different: for example, economic integration can increase at the regional level rather than "globally," and this has important consequences for many issues, such as the way in which economic shocks are transmitted among countries and the extent of competition between countries in the international market. This is also true using network analysis to measure the economic integration in the WTW, as the overall high degree of connectivity in the network might, nonetheless, imply quite different underlying network structures.

The aim of this paper is to study the possible existence of communities within the WTW to better understand the characteristics of economic integration. In general terms, a significant network community is a set of nodes with strong internal connections, much stronger than those with the remaining nodes of the network. Applying community analysis to the WTW should reveal groups of countries with privileged relationships, originating by geographical vicinity, trade agreements, common language or religion, traditional partnerships, etc. What defines a community, therefore, is strong commercial ties (compared to the rest of the world) rather than common individual characteristics of the nodes, such as economic size, level of development, and even the number or strength of their links.

So far, very few studies have analyzed communities, or clustering, within the WTW [10–13], possibly because of the many open issues still existing in the methodologies for community analysis [14]. It is indeed problematic to interpret the results of these studies. Reyes *et al.* [11], looking for communities in the WTW, use as a benchmark the groups of countries that signed regional trade agreements, and they find that, over time, the formation of communities follows an irregular pattern. Instead, He and Deem [13] move from a peculiar definition of distance and clusters within the network to find that clustering declined over time, making the world increasingly "global." Barigozzi *et al.* [12] examine the WTW considering sectoral trade flows, finding no clear time trend in community formation. They observe heterogeneous community structures in different sectors, even though it is impossible to compare the significance of the different communities. The above-mentioned studies define and detect

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communities in the WTW in distinct ways, but in all cases it is quite difficult to assess the significance of the partitions that emerge.

In this paper, we look for communities in the WTW in the period between 1962 and 2008, and we compare different methodologies to search for communities in networks, in order to verify the robustness of the results that we obtain. All the methods applied here base the search for a community on the identification of a group of countries sharing a disproportionate amount of trade among them compared with the trade they have with the rest of the world. From the economic point of view, the existence of communities would imply that countries trade especially with a group of preferential partners and trade tends to be "regionalized," especially if communities coincide with groups of geographically close countries [15]. Instead, in a globalized world defined as a whole, as we recalled above, we do not expect communities to be significant, as countries can be connected through trade to nearly any country in the world with similar ease.

Our analyses shed many doubts on the existence of communities in the WTW, as the results show that the network is not significantly split between different groups. Some "weak" communities emerge, but these groups of countries are not more connected with each other than with the rest of the world to the extent of forming truly privileged or exclusive relationships, supporting the view of a globalized trading system.

## II. THE WORLD TRADE WEB

Data for our analysis come from the Direction of Trade Statistics published by the International Monetary Fund (IMF) and from the NBER-UN Trade Data data set made available by the Center for International Data at the University of California, Davis, which is an elaboration from United Nations trade data by Feenstra *et al.* [18]. We use annual bilateral imports for the years 1962, 1965, 1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005, and 2008. More precisely, we use IMF data for the 1985–2008 period and NBER-UN data for the previous years, which are not covered in the IMF database. We note that the two sources are strongly consistent in the years for which they are both available (1985–2000) [19].

A number of important events affected the patterns of world trade in the period considered: the end of colonial links, changes in the exchange rate regime, the removal of many barriers to trade, and the increasing role of emerging countries in the international markets. Our observation period stops before the outbreak of the financial crisis began to affect international trade, which was still growing, by 15% in value in 2008, before the dramatic drop recorded in 2009.

We use directed flows received by an importing country from any given exporting country, measuring the value in US dollars at current prices of all merchandise imported by a country from each partner country (import data are generally more reliable and complete than export data). We prefer directed data because the direction of trade is economically important and there is no *a priori* reason to expect symmetry in bilateral trade flows. However, in order to thoroughly validate our community analysis, we also consider a symmetrized version of the trade network (see Sec. III). Finally, we note that here we are not concerned with the change in prices over time, as we do not carry out any time series analysis, but we consider the existence of communities in each year separately (for other analyses of the WTW as a directed network, see [5] and [12]).

The WTW is then modeled as a directed, weighted network composed of $N$ nodes corresponding to countries ($\mathbb{N} = \{1,2,\dots,N\}$ is the set of nodes) and $L$ edges representing the trade flows among countries. We denote by $W = [w_{ij}]$ the $N \times N$ *weight matrix*, where $w_{ij} \geqslant 0$ is the trade flow from country $i$ to country $j$. The *connectivity matrix* $A = [a_{ij}]$ is the $N \times N$ matrix, where $a_{ij} = 1$ if $w_{ij} > 0$, i.e., if there exists the edge $i \to j$, and $a_{ij} = 0$ otherwise.

The network being directed, for each node $i$ we distinguish between the in-degree $k_i^{\text{in}} = \sum_j a_{ji}$, the out-degree $k_i^{\text{out}} = \sum_j a_{ij}$, and the total degree $k_i = k_i^{\text{in}} + k_i^{\text{out}}$, and we denote the average degree $\langle k \rangle = \sum_i k_i/N$. Analogously, we define the in-, out-, and total strength of node $i$ as $s_i^{\text{in}} = \sum_j w_{ji}$, $s_i^{\text{out}} = \sum_j w_{ij}$, and $s_i = s_i^{\text{in}} + s_i^{\text{out}}$, respectively, and the total weight of the network edges as $w = \sum_{ij} w_{ij}$.

The network is *strongly connected* if, for every pair $(i,j)$ of distinct nodes, there exists an oriented path from $i$ to $j$ (e.g., [23]). If the network is not connected, the set $\mathbb{N}$ of nodes can be partitioned in components $\mathbb{K}^1,\mathbb{K}^2,\dots,\mathbb{K}^m$ having, without loss of generality, $N_1 \geqslant N_2 \geqslant \cdots \geqslant N_m > 0$ nodes, respectively ($\sum_i N_i = N$). Each component is a maximally strongly connected subnetwork (i.e., it is strongly connected and it is not part of a larger connected subnetwork). In our study, we find that the largest component $\mathbb{K}^1$ is actually a *giant component*, i.e., it has a dimension $N_1$ which has the same order of magnitude as $N$, and on the other hand, it is much larger than all the other components. Network components can be identified by means of standard algorithms of graph analysis [24].

Not all the countries in our sample are connected in every period. In fact, even if the cases in which a country does not trade at all are really exceptional, in our database a country can appear not connected in a given year for a number of reasons. For example, some countries, such as the USSR and East Germany, simply did not formally exist throughout the entire period, whereas other countries did not report their data in a given year. In 1962, the strongly connected component includes 145 countries, and it keeps slowly increasing, to reach 180–182 countries from 1995 on, including the new countries born from the dismantling of the former Soviet bloc. In the analysis in the following section we consider the giant components only.

In our sample, the total value of world imports $w = \sum_{ij} w_{ij}$ increases from about 126 billion in 1962 to 15 760 billion in 2008 (all amounts in US dollars). The value of imports in our data set represents approximately 95% of the total world imports in 2008 and slightly lower amounts in the previous years [25]. Not only the trade value but also the number of edges $L$ registers a remarkable increase, increasing from 7870 in 1962 to 21 123 in 2008. The average in-strength of each node also increases significantly, but average values in this network are not especially relevant, as nodes and edges (in our case, countries and trade flows) are very heterogeneous. For example, import flows in 2008 range from 160 million for

Tonga to more than 2000 billion for the United States. These huge differences reflect the large diversity in the economic weight of countries across the world.

## III. COMMUNITIES IN THE WTW

Consider now a directed, weighted, strongly connected network (or, if not connected, its giant component). Roughly speaking, a subset $\mathbb{C}_h \subset \mathbb{N}$ is called a *community* if the total weight of the edges internal to $\mathbb{C}_h$ is much larger than that of the edges connecting $\mathbb{C}_h$ to the rest of the network. The community analysis of a given network with nodes $\mathbb{N}$ therefore consists in finding the "best" partition $\mathbb{C}_1,\mathbb{C}_2,\ldots,\mathbb{C}_q$ (i.e., $\bigcup_h \mathbb{C}_h = \mathbb{N}$ and $\mathbb{C}_h \cap \mathbb{C}_k = \oslash$ for all $h,k$), according to some criteria (for simplicity, we do not consider possibly overlapping communities). Despite the huge number of contributions [14], there is not a consensus on formal criteria for defining communities and for testing their significance. We use four approaches to analyze communities in the WTW.

### A. Modularity optimization

Finding the partition that maximizes a quality index called *modularity* is by far the most popular method for finding communities in a given network. Originally proposed by Newman and coworkers [26,27], this approach has found plenty of applications in diverse areas and has been extended in many directions [14].

In the case of a directed and weighted network, the modularity $Q$ associated with the partition $\mathbb{C}_1,\mathbb{C}_2,\ldots,\mathbb{C}_q$ is given by

$$Q = \frac{1}{w}\sum_{h=1}^{q}\sum_{i,j\in\mathbb{C}_h}\left[w_{ij} - \frac{s_i^{\text{out}} s_j^{\text{in}}}{w}\right], \qquad (1)$$

which is the fraction of the network weight internal to communities minus the expected value of this fraction in a random network that has in common the in- and out-strengths with the original one [28].

Although the best partition (i.e., the one with $Q = Q_{\max}$) cannot be found by exhaustive search even in rather small networks, for computational reasons, many efficient algorithms are available for obtaining a presumably "close-to-optimal" solution [14]. We use the aggregative, hierarchical method devised by Blondel *et al.* [29], which is considered very effective both in terms of $Q_{\max}$ (i.e., in the capability of finding a partition with a high modularity) and in computational requirements [30,31].

Throughout this paper, the results of the application of community analysis methods to the WTW are compared with those obtained for two synthetically generated benchmark networks, purposely built with a well-defined cluster structure. We denote them Girvan-Newman (GN) and Lancichinetti-Fortunato-Radicchi (LFR) networks, respectively. The former is an undirected, unweighed network with $N = 128$ nodes, often used in the last few years for testing community analysis algorithms [14,33]. It is built by defining four blocks of 32 nodes each and by purposely (and randomly) inserting a large number of intrablock edges but only a few interblock edges. One of the possible parametrizations prescribes the average degree $\langle k\rangle$ and the average *internal* degree $\langle k\rangle_{\text{int}}$, i.e., the number of neighbors each node has in its same block. We let $\langle k\rangle = 16$ and $\langle k\rangle_{\text{int}} = 14$, which yields a strongly clusterized network. In fact, modularity optimization easily recognizes the four-community planted partition, with $Q_{\max} = 0.604$. LFR networks are instead a more complex class of benchmarks recently proposed by Lancichinetti *et al.* [34,35]. This class explicitly takes into account two properties often found in real networks, namely, the heterogeneity in the distributions of node degrees and community sizes, which are both taken as power laws. Furthermore, once the number and size of communities are defined, a tunable "mixing parameter" prescribes the fraction of edges that each node shares with the nodes of the other communities. We built a directed, weighted network with (we refer to [35] for the detailed parameter definition) $N = 171$, $\langle k\rangle = 20$, $\tau_1 = -2$, and $\tau_2 = -1$ (exponents of the power-law degree and community size distributions, respectively), $\beta = 1.5$ (coefficient of the degree-weight relationship), and mixing parameter $\mu = 0.1$. The result is a strongly clusterized network with 10 communities. Modularity optimization perfectly recognizes the planted partition, with $Q_{\max}$ as large as 0.820.

The results of modularity optimization for all the years of our WTW data set are reported in Table I [36]. In 1962 we obtain $q = 4$ communities with $Q_{\max} = 0.225$. The communities count 55, 44, and 22 countries, plus a very small community formed by only 4 countries. The largest communities essentially coincide with most of Europe and Africa, America, and Asia plus Oceania, respectively. The latter community also includes the United Kingdom and Ireland, still strongly linked to Commonwealth countries. From 1970 onward the results show $q = 3$, with a similar grouping of countries (possibly with the exception of African countries, that tend to become more scattered across communities) and with the United Kingdom and Ireland shifting to the European community, following their membership in the European Economic Community in 1973. The only exception is in 1995, when data for the new countries formed by the dismantling of the Soviet bloc start to be recorded, and indeed one of the communities is formed essentially by this group. Over time, the strong ties between these countries loosen up, as they appear no longer as a separate group, but mostly in the large Europe-based community. In 2008 the communities contain 68, 66, and 47 countries, but the largest cluster is now associated with Asia plus Oceania, confirming the rapidly increasing role of Asia in international trade. This clustering by continents is very much in line with the large body of literature showing that geographical proximity still matters for international trade (e.g., [5]). We also note that, in terms of the number $q$ of communities, our results are qualitatively consistent with [12], where a value of $q$ ranging from 2 to 4 is reported for the period 1992–2003 (no modularity value is reported, however, in that paper). Finally, note that slightly larger modularity values appear over time, reaching $Q_{\max} = 0.296$ in 2008. This increase can hardly be considered significant, however, especially because it is known that the maximum modularity (max-modularity) tends to grow with the size of the graph (e.g., [14], p. 90).

A well-known peculiarity of the WTW is the large value of the density $d = L/(N(N-1))$ (i.e., the actual number

TABLE I. (a) World Trade Web statistics in the 1962–2008 period and results of the max-modularity community analysis. (b), (c) Same as (a), but for the *filtered* (b) and *symmetrized* (c) networks. $N$, number of countries of the giant component; $\langle k_i^{\rm in}\rangle$, average number of import partner countries; $\langle s_i^{\rm in}\rangle$, average import value (million US dollars); $\langle k^{\rm sym}\rangle$, average number of partner countries; $\langle s_i^{\rm sym}\rangle$, average trade value (import + export; million US dollars); $Q_{\max}$, max modularity; No. comm., number of communities and number of countries for each community.

| | (a) Original network | | | | | (b) Filtered network | | | | | (c) Symmetrized network | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | $N$ | $\langle k_i^{\rm in}\rangle$ | $\langle s_i^{\rm in}\rangle$ | $Q_{\max}$ | No. comm. | $N$ | $\langle k_i^{\rm in}\rangle$ | $\langle s_i^{\rm in}\rangle$ | $Q_{\max}$ | No. comm. | $N$ | $\langle k^{\rm sym}\rangle$ | $\langle s_i^{\rm sym}\rangle$ | $Q_{\max}$ | No. comm. |
| 1962 | 145 | 54.2 | 870 | 0.225 | 4 [55,44,42,4] | 136 | 5.3 | 731 | 0.287 | 4 [44,40,39,13] | 146 | 52.8 | 1727 | 0.225 | 4 [55,44,43,4] |
| 1965 | 145 | 64.4 | 1197 | 0.223 | 4 [48,43,40,14] | 141 | 6.0 | 983 | 0.288 | 4 [51,41,41,8] | 149 | 72.8 | 2330 | 0.222 | 4 [49,47,38,15] |
| 1970 | 150 | 74.1 | 1949 | 0.244 | 3 [51,50,49] | 149 | 6.9 | 1618 | 0.302 | 4 [52,47,43,7] | 150 | 87.6 | 3898 | 0.244 | 3 [51,50,49] |
| 1975 | 151 | 80.8 | 5528 | 0.238 | 3 [75,40,36] | 150 | 7.8 | 4553 | 0.296 | 3 [77,71,2] | 151 | 95.1 | 11057 | 0.237 | 3 [75,40,36] |
| 1980 | 151 | 76.9 | 12322 | 0.232 | 3 [75,42,34] | 151 | 7.5 | 10009 | 0.287 | 4 [56,42,41,12] | 151 | 89.7 | 24645 | 0.232 | 3 [74,43,34] |
| 1985 | 165 | 69.2 | 11383 | 0.282 | 3 [70,64,31] | 159 | 6.6 | 9449 | 0.349 | 4 [74,61,21,3] | 167 | 86.4 | 22636 | 0.280 | 3 [71,65,31] |
| 1990 | 163 | 78.7 | 20330 | 0.260 | 3 [74,70,19] | 161 | 7.3 | 17298 | 0.312 | 4 [76,68,14,3] | 165 | 94.7 | 40353 | 0.260 | 3 [75,70,20] |
| 1995 | 182 | 92.7 | 26315 | 0.281 | 6 [77,73,18,8,4,2] | 181 | 8.4 | 22338 | 0.341 | 6 [79,75,18,5,2,2] | 183 | 113.4 | 52598 | 0.280 | 6 [74,71,18,10,8,2] |
| 2000 | 180 | 106.7 | 34432 | 0.290 | 3 [76,61,43] | 180 | 9.8 | 29545 | 0.341 | 3 [79,57,44] | 180 | 125.6 | 68865 | 0.288 | 3 [75,62,43] |
| 2005 | 181 | 113.6 | 56024 | 0.294 | 3 [70,65,46] | 181 | 10.4 | 47759 | 0.348 | 5 [68,54,49,8,2] | 181 | 130.9 | 112050 | 0.292 | 3 [69,62,50] |
| 2008 | 181 | 116.7 | 87056 | 0.296 | 3 [68,66,47] | 181 | 10.8 | 72534 | 0.360 | 3 [72,62,47] | 183 | 131.9 | 172210 | 0.295 | 3 [71,65,47] |

of edges divided by their maximum allowable number) in comparison to most real-world networks. In our data set, $d$ ranges from 0.37 in 1962 to 0.65 in 2008. Since the weights are extremely diversified, a large number of edges convey a very small import-export flow. It is reasonable to wonder whether this could be an obstacle to our analysis, in the sense that the actual communities could be concealed by the many scarcely significant intercountry connections. To assess this, we applied a filtering technique to the WTW to extract its "backbone," namely, a set of truly significant edges. Besides the trivial threshold approach (which discards all weights below a fixed level), a few filtering methods have recently been proposed which are explicitly designed to deal with multiscale weight distributions [37–39]. We apply the method proposed in [3,38], where, in deriving the filtered network, only those edges are preserved which significantly deviate from a null model which assumes that the strength of each given node is uniformly distributed among its incident edges. More precisely, once a significance level $0 < \alpha < 1$ is set, an edge is preserved if the probability that its weight complies with the null hypothesis is less than $\alpha$ (a smaller $\alpha$ value is thus more selective). Therefore the method acts locally by analyzing each single node and by discarding edges which do not carry a significant fraction of the node strength. Since the selection is done on a node-by-node basis, none of the edges (and none of the countries) is *a priori* discarded, which is instead the effect of trivially fixing a threshold.

We apply the filtering method to the WTW from 1962 to 2008, and we present in Table I the results for $\alpha = 0.01$. Consistent with [3], we find that this $\alpha$ level yields a reasonable trade-off between the simplification of the network (the number of edges is dramatically reduced, to 10% or less) and the integrity of its important features (about 80% of the total weight is preserved, and practically all nodes remain connected). If the community analysis is then performed, however, the results obtained with the original and filtered networks are not very different. As expected, the max-modularity is larger for filtered networks, but the dramatic decrease in the density does not give rise to a similar increase in $Q_{\max}$ or to a structural redesign of the communities. In fact, we note that the newly appeared communities turn out to be very small, and although geographically meaningful (e.g., Kenya, Rwanda, and Uganda in 1990), they have scarce economical importance. We conclude that, while filtering is an essential tool for unveiling important network properties, it seems not to be crucial in community analysis because different weight scales are naturally treated within the definition of modularity, (1).

The application of the max-modularity criterion to directed networks has been criticized in a few works (e.g., [40,41]) because it can yield distorted results on some specific topologies. We decided therefore to extend our analysis to the *symmetrized network*, namely, to the undirected WTW obtained by replacing the two (directed) trade flows between each pair of countries with their sum (e.g., [1,4,42]). This obviously implies a loss of information, which can be significant for some topological or economical issues. However, it is presumably not as crucial in community analysis, where one is interested in the "strength" of the relationship between two countries as a measure of their partnership. The symmetric

WTW is described by the weight matrix $W^{\text{sym}} = [w_{ij}^{\text{sym}}]$, with $w_{ij}^{\text{sym}} = w_{ji}^{\text{sym}} = w_{ij} + w_{ji}$, and the expression of modularity becomes [14]

$$Q = \frac{1}{2w^{\text{sym}}} \sum_{h=1}^{q} \sum_{i,j \in \mathbb{C}_h} \left[ w_{ij}^{\text{sym}} - \frac{s_i^{\text{sym}} s_j^{\text{sym}}}{2w^{\text{sym}}} \right], \qquad (2)$$

where $s_i^{\text{sym}} = \sum_j w_{ij}^{\text{sym}} = \sum_i w_{ji}^{\text{sym}}$ and $w^{\text{sym}} = \sum_{ij} w_{ij}^{\text{sym}}/2$. The results are reported in Table I, and they are very similar to the results obtained for the original directed network. We conclude, therefore, that the network directionality is not an obstacle to community analysis.

The problem we face now is the significance of the obtained network partitions. Maximizing the modularity obviously yields some "best" partition, but this does not imply that the network is actually structured in significant clusters. Although a large value of $Q_{\max}$ *per se* should reveal that the network has a modular organization (as it measures a kind of "dissimilarity" between the network and its randomizations), it is well known that a large value of $Q_{\max}$ can even be obtained in random (i.e., Erdös-Rényi) networks, which instead are expected to have no community structure by construction [43]. In addition, the values of $Q_{\max}$ we obtain can hardly be considered to be large (remember that the GN and LFR benchmarks have $Q_{\max} = 0.604$ and 0.820, respectively).

In other words, finding the partition that maximizes $Q$ by no means concludes the community analysis of the network [14]. For undirected, unweighted networks, some methods have been proposed for complementing the max-modularity approach with a test of statistical significance. In the simplest approach, one can consider the ensemble of networks having the same degree sequence $k_1, k_2, \ldots, k_N$ as the original one, extract a large number of random networks from this ensemble, and compute the max-modularity $Q_i$ for each one of them. Then a large value of the *z score*, $z = (Q_{\max} - \mu(Q_i))/\sigma(Q_i)$, indicates that the max-modularity obtained for the original network is likely to be significantly high. Karrer *et al.* [44], however, showed, by analyzing a pool of real networks, that the $z$ score can fail in some cases. They proposed a robustness analysis based on perturbing the original network by randomly rewiring a given fraction of edges, recomputing the best partition by max-modularity, and measuring to what extent the partitions of the original and perturbed networks overlap. The rationale is that, if communities are not significant, even a small perturbation of the network topology should induce a large reorganization of the clusters. Hu *et al.* [45] generalized this perturbational approach by defining a suitable "universal index" $R$ for measuring the significance of communities, which they proved to be fairly effective by a series of tests on both artificial and real networks.

All the above methods, however, have some features that make their use problematic in our case. First, the significance analysis is based on the modularity optimization of many instances of a random model or of a perturbed network, thus potentially it suffers from the same criticalities that affect the computation of $Q_{\max}$ (and of the associated partition) in the original network. Second, no straightforward extensions exist in the case of weighted, directed networks, for which the definition of randomized models and of suitable perturbation schemes is absolutely not trivial. The situation is especially problematic in our case, since the link weights of the WTW span many orders of magnitude and make whatever discretization scheme (such as those proposed in [46,47]) absolutely critical, as well as any technique based on fixed percentage perturbations [48] or on weight extraction from some postulated probability distribution [49].

Instead of randomizing the network, we can assess whether our results are robust by randomizing the assignment of nodes to communities. Starting from the optimal partition (i.e., the one with $Q = Q_{\max}$), imagine selecting and relabeling a fraction $0 < \alpha < 1$ of nodes, namely, assigning them to a (randomly selected) different community. As $\alpha$ increases, we obtain partitions more and more distant from the optimal one, and we can quantify this distance by the (normalized) *variation of information* $V$ [50], a measure which is 0 if and only if the two partitions are identical and 1 when they are "maximally different" (i.e., one partition has $N$ clusters and the other has only one). It is natural to expect that, as $V$ increases while departing from the optimal partition, the modularity $Q(V)$ of the associated partition decreases accordingly. And indeed this is the case, but the form of the function $Q(V)$—which *a priori* depends on which nodes are relabeled—can disclose important properties.

The function $Q(V)$ is displayed in Fig. 1 for the WTW in 3 years and for the GN and LFR benchmark networks introduced above. In all cases, two functions are depicted: one is the result of the random selection of the fraction $\alpha$ of nodes to be relabeled, and the other is obtained by relabeling the least connected nodes (i.e., the nodes with the smallest total

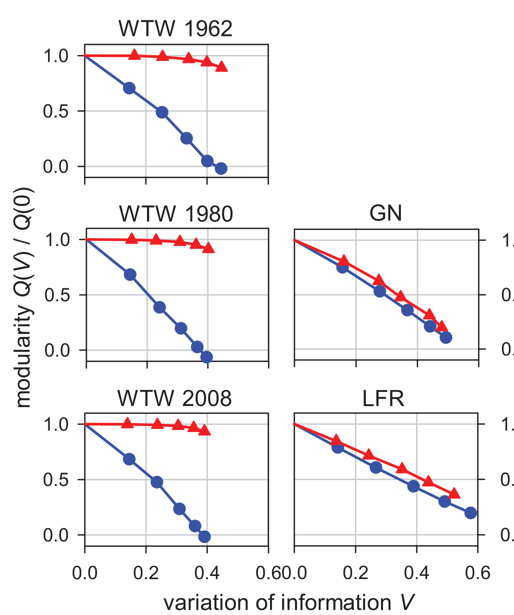


FIG. 1. (Color online) The (normalized) modularity $Q(V)$ of the perturbed partitions obtained, from the optimal one [$Q(0) = Q_{\max}$], by relabeling 10% to 50% of the nodes, i.e., by assigning them to a different community. $V$ is the variation of information of the perturbed partitions. The (blue) curve with filled circles was obtained by relabeling randomly selected nodes (each point is the average of 100 realizations); the (red) curve with filled triangles is obtained by relabeling the least connected nodes (lowest strength or degree).

strength or degree). If the communities are significant, moving even the least connected nodes should in any case produce an important effect on the modularity values, because those nodes also crucially affect the community structure. Indeed, this is what happens in the GN and LFR cases, where the two curves are qualitatively similar and quantitatively close. On the contrary, the three WTW panels strikingly put into evidence the scarce significance of the optimal partition. As a matter of fact, we can randomly relabel as many as 50% of the nodes (inducing a large variation of information), yielding just a slight decrease in the modularity $Q(V)$. This means that the positioning of a large number of countries, actually the least important ones from an economic point of view, is more or less irrelevant and that modularity is only determined by the links with the largest weights. In other words, the landscape of the modularity $Q$ is extremely insensitive, since a large number of partitions (even very distant one from the other) have very close modularity values. Overall, this suggests that the results of the modularity analysis should be taken with great caution, as they denote an extremely scarce robustness of the obtained communities: indeed, a large number of very distant partitions are qualified as nearly equally "optimal."

Given the low robustness of the partitions found via modularity optimization, in order to have a complete view of the cluster structure of the WTW, in the next sections we move to completely different approaches for testing the existence and significance of communities in the WTW.

In general terms, the weak evidence of a clusterized structure of the WTW, indicated both by the rather small modularity values and by the above sensitivity test, should not be surprising. Notice that interactions among countries are measured by their *absolute* trade values. As such, a strongly clusterized network would possibly be formed by clearly distinct groups of countries with large intra- but small intercommunity trades. But the largest edges (in terms of their weight) basically involve the largest world economies (e.g., in 2008, on at least one side of the top 20 edges, we find the United States, China, Canada, Mexico, Japan, Germany, the Netherlands, France, Korea, the United Kingdom, Belgium, and Italy). We can hardly expect these countries (or a subset of them) to form a community as defined here, because the trade flows among them are not strongly differentiated from their trade with the remaining countries. At the same time, it can also hardly be expected that the remaining countries form one or more communities. In the next sections, in order to clearly understand the possible clusterized structure of the WTW, we not only consider different community analysis techniques, but also move to analyzing *relative* trade values.

### B. Cluster analysis

Standard data clustering is aimed at organizing objects into "homogeneous groups," trying to maximize, at the same time, the intragroup similarity and the intergroup dissimilarity. This requires defining a suitable *distance* among data. When we move to *graph clustering*, i.e., grouping the nodes of a network, which distance should be used is by no means obvious.

We adopt a notion of distance among nodes which is based on *random walks*. An $N$-state Markov chain can straightforwardly be associated with the $N$-node network by row-normalizing the weight matrix $W$, i.e., by letting the transition probability from $i$ to $j$ equal

$$p_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} = \frac{w_{ij}}{s_i^{\text{out}}}. \tag{3}$$

The resulting transition matrix $P = [p_{ij}]$ is a stochastic (or Markov) matrix, i.e., $0 \leqslant p_{ij} \leqslant 1$ for all $i,j$, and $\sum_j p_{ij} = 1$ for all $i$. The study of many problems in network science benefits from some sort of Markov chain approach (e.g., epidemic spreading, navigation, etc. [23,51]). Community analysis is one of them, and several contributions in this vein have been published; we recall [40] and [52–55] among others (see, again, [14] for a comparative survey).

It is important to note that modeling the WTW by Eq. (3) corresponds to moving from *absolute* to *relative* trade values, since the flow from $i$ to $j$ is now normalized by the total export flow from country $i$. The consequence is that communities, if any, will not necessarily be composed of groups of countries related by large trading but, instead, of countries with privileged partnership, namely, whose trading is important in relative terms. This can originate from, e.g., geographical vicinity, trade agreements, common language or religion, and traditional partnerships. Since we naturally expect such communities to be composed of a mixture of large and small economies, the use of relative trade values appears to be more appropriate, as absolute measures would *a priori* obscure the position of medium-small countries.

In defining a distance among nodes, we essentially adopt the approach of [54], where a $T$-step random walk is performed, in a Monte Carlo fashion, from each of the $N$ network nodes. If the two nodes $(i,j)$ are visited along the same walk, the similarity counter $\sigma_{ij}$ is increased by 1. At the end, a *similarity matrix* $\Sigma = [\sigma_{ij}]$ is obtained which is used as a basis for agglomerative, hierarchical clustering. The rationale for the method is the following: if the number $T$ of steps is limited, the random walker starting from $i$ will more likely visit nodes strongly connected to $i$, i.e., within the same community. The choice of $T$ is thus quite critical: if $T$ is too large, the probability of visiting a given state becomes independent of the starting state (as it tends to the stationary Markov chain state probability distribution $\pi = \pi P$), whereas if $T$ is too small, the information gathered is possibly insufficient. We return to this point later.

We partially modify the above method in that we do not explicitly perform random walks. In fact, consider $M$ repetitions of a random walk started from $i$. For each repetition, the probability that the walker is in $j$ after $t$ steps is $[P^t]_{ij}$. Thus, if $M$ random walks of length $T$ are performed from $i$, the expected number of visits of $j$ in *any* time instant in $1 \leqslant t \leqslant T$ is $M\sum_{t=1}^{T}[P^t]_{ij}$. Note that this is conceptually equivalent to the above explicit random walk approach [54], but with an arbitrarily large number $M$ of repetitions from each starting node instead of only one. By averaging with respect to $M$ and $T$, and recalling that we are dealing with directed networks (thus $[P^t]_{ij} \neq [P^t]_{ji}$), we propose a similarity matrix $\Sigma = [\sigma_{ij}]$ defined by

$$\sigma_{ij} = \sigma_{ji} = \frac{1}{T}\sum_{t=1}^{T}([P^t]_{ij} + [P^t]_{ji}). \tag{4}$$

Finally, the distance $d_{ij} = d_{ji}$ between nodes $(i,j)$ is defined by complementing the similarity and normalizing the results between 0 and 1:

$$d_{ij} = d_{ji} = 1 - \frac{\sigma_{ij} - \min \sigma_{ij}}{\max \sigma_{ij} - \min \sigma_{ij}}. \quad (5)$$

At this point, a standard hierarchical, aggregative cluster analysis is used to explore the possible existence of communities [56]. More precisely, a binary cluster tree (dendrogram) is computed initially by defining $N$ groups each containing a single node and then by iteratively linking the two groups with minimal distance [57].

Cluster analysis yields a different dendrogram for each time horizon $T$, whose choice is thus nontrivial. At the two extremes, setting $T = 1$ restricts the pairs of nodes which are candidate to nonzero similarity to neighboring pairs only, whereas larger and larger values of $T$ tend to make any node equally similar to any other. We solve this indeterminacy by a sort of optimization. For each network under scrutiny, we build a dendrogram for each $T$ from 1 to a sufficiently large value $T_{\max}$ (of the order of $N$), and we take the one that maximizes the *cophenetic correlation coefficient* $C$, which is defined as the linear correlation between the distances $d_{ij}$ and the *cophenetic distances* $c_{ij}$ [56]. The latter are a product of the hierarchical cluster analysis: for any node pair $(i,j)$, the cophenetic distance $c_{ij}$ is the height of the link joining (directly or indirectly) nodes $(i,j)$ in the dendrogram. Recall that, when nodes are increasingly grouped together building the dendrogram, their distances to the other nodes (or groups) is replaced by their average. The effect is small if the nodes that are grouped together are very close each other, namely, if they form a cluster. If so, we expect similar values for the $d_{ij}$ and the $c_{ij}$ values and, thus, a large value of $C$. Although this approach can reveal criticalities in some specific applications [58], the value of $C$ is generally used to assess whether the adopted distance $d_{ij}$ induces an effective clusterization: by maximizing $C$, we thus select the best possible clusterization with respect to $T$.

The dendrograms obtained for the WTW in 1962, 1980, and 2008 (i.e., the two extremes of the time window of our data set, plus an intermediate year) are displayed in the upper three plots in Fig. 2 [36]. Each vertical line corresponds to a node (country). Horizontal lines ("links") connect two groups of nodes, and the height of the link (as read on the $y$ axis) is the distance between the two groups. In the two lower plots in Fig. 2, dendrograms of the GN and LFR benchmark networks are displayed.

A clear, visual indication of a clusterized network structure is, in the benchmark's dendrograms, the existence of long vertical segments or, equivalently, of links (i.e., horizontal segments) whose height is largely different from the heights of the links below them. In fact, this situation arises when the distance between the two groups joined by the link is much larger than the distance among the nodes forming the two groups; this exactly means that there are clusters in the network. This phenomenon is strikingly evident in the GN dendrogram: whereas there is a "continuum" of distances within a single community, the intergroup distance is large and sharply divides the four planted communities. The same happens in the LFR dendrogram.

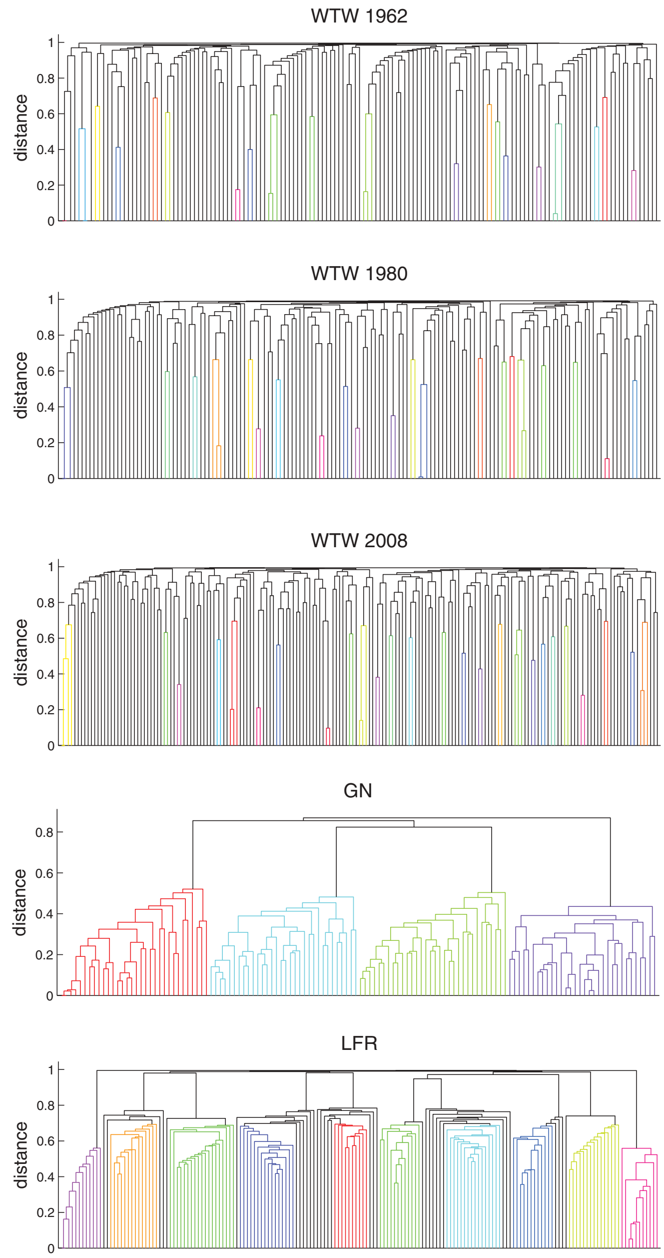


FIG. 2. (Color online) Dendrograms obtained by hierarchical cluster analysis. From top to bottom: WTW in 1962, 1980, and 2008; GN benchmark network; and LFR benchmark network. Colors (other than black) denote groups of nodes whose distances are all not larger than 0.7.

The situation appears to be markedly different for the WTW dendrograms. Only a few distinct groups appear, and they are mostly composed of few countries. Moreover, there seem to be no significant structural differences through the years, possibly with a diminishing visual distance between groups over time. As specified above, we take the dendrogram for which the cophenetic correlation coefficient $C$ is the maximum value attained by varying the time horizon $T$ of the random walker [see Eq. (4)]. Notably, such a value is attained for $T = 12$ and $T = 6$ for the GN and LFR benchmarks, respectively, while the maximum $C$ is obtained at $T = 1$ for all the WTW cases. This means that, in the latter case, the best clusterization (as measured by $C$) is obtained by assessing the node similarity only among direct neighboring countries. This is an effect of the anomalously high network density, for which direct connections carry over most of the information to the random walker.

In all years, some expected patterns can be observed. The United States and Canada form one of the strongest partnerships: their distance in the dendrogram stays constantly very small from 1970 [in seven cases of nine, their distance is actually 0, meaning that they are the closest pair consistent with (5)]. France is strongly connected to some of its former colonies, whereas Germany is close to other European countries. Some of these links are very large both in absolute and in relative terms (e.g., between the United States and Canada); others are important in relative terms (e.g., about 80% of the Guadeloupe export in 1995 is directed to France). Often very small countries are connected to much larger ones, an effect of the disassortativity already observed in the WTW [4]. These links tend to be small in absolute terms, given the small economic size of the countries, but they are very important in relative terms, as they show a strong preference for a given partner.

The scenario is not modified if we analyze the filtered and the symmetrized networks defined in Sec. III A [36]: qualitatively, they display no difference with respect to those of the original WTW.

As pointed out above, visual analysis of the dendrograms leads us to claim that the WTW, through the years, does not display a significant community structure. It is important to point out that this result is specific neither to our particular choice of node distance nor to the choice of considering relative trade values. As a matter of fact, we repeated the hierarchical cluster analysis by using the distance proposed by He and Deem [13], both on the WTW and on the benchmark networks. In [13] the node distance is defined as $d_{ij} = w_{\max} - w_{ij}$, with $w_{\max} = \max_{ij} w_{ij}$. Note that $d_{ij}$ relates country $i$ to its direct neighbors through the *absolute* trade value $w_{ij}$. Inspection of the corresponding dendrograms [36] leads to exactly the same conclusion as above: the qualitative structure of the dendrograms is markedly different passing from the benchmarks to the WTW, denoting clusterization levels very strong for the former but extremely mild for the latter.

In summary, the results of the cluster analysis, although based on visual evidence only, seem to denote the absence of the existence of a significant community structure in the WTW. This emerges both from the use of relative trade measures, a metric that appears to be more suited to a multiscale network such as the WTW (it is actually consistent with the filtering technique described in Sec. III A), and from the adoption of a node distance based on absolute trade values. Together with the small modularity level (Sec. III A), this is a further clue of a mild community structure of the WTW.

### C. Stability of partitions

A different approach for exploiting random walks in studying network communities has been devised by Delvenne *et al.* [59], who introduced the concept of *stability* of a partition. As above, the rationale is that, in a strongly clusterized network, a random walker starting in a community is likely to remain for quite a long time within that community, before leaving it to enter another community. Imagine that the walker emits a signal at each step, which has the same value as long as it remains within a community and changes upon moving to another community. Then studying the persistence of this signal provides important information on the community structure of the network.

The probability $\pi_t^i$ that the random walker is in state $i$ at time $t$ evolves according to the Markov chain equation $\pi_{t+1} = \pi_t P$, and the vector $\pi_t = (\pi_t^1 \pi_t^2 \dots \pi_t^N)$, assuming ergodicity, tends to the stationary state $\pi = \pi P$ as $t \to \infty$. Consider now a network partition $\mathbb{C}_1, \mathbb{C}_2, \dots, \mathbb{C}_q$ and assume that the walker, at each step $t$, emits a signal $s_t$ which takes value $c$ as long as it moves within $\mathbb{C}_c$. Delvenne *et al.* [59] show that the autocovariance of $s_t$ can be usefully expressed as cov $[s_t, s_{t+\tau}] = (1,2,\dots,q)' R_\tau (1,2,\dots,q)$, where $R_\tau$ is the $q \times q$ *clustered autocovariance matrix*,

$$R_t = H'(\mathrm{diag}(\pi)P^t - \pi'\pi)H, \tag{6}$$

where $H$ is a $N \times q$ binary matrix coding the partition, i.e., its entry $h_{ic}$ is 1 if and only if node $i$ belongs to community $c$. Note that $R_t$ depends on the network and on the partition only. Equation (6) provides an interpretation of each entry $[R_t]_{cd}$ as the probability of starting in community $c$ at time 0 and being in community $d$ at time $t$, minus the probability, evaluated at stationarity, that two independent walkers are in $c$ and $d$. If the partition coded by $H$ is significant, one expects a dominance of the diagonal terms $[R_t]_{cc}$ over time. The *stability* of the partition $H$ is thus defined as the following function of time:

$$r_t^H = \min_{0 \leqslant s \leqslant t} \mathrm{trace}[R_s]. \tag{7}$$

A good, significant partition will have stability $r_t^H$ which remains large over a long time span, since the random walker has a high likelihood of remaining within the same community for long time. On the contrary, a rapidly decaying $r_t^H$ denotes a scarcely significant partition, because the walker rapidly abandons the starting community [60].

We compute the stability function $r_t^H$ for all the WTW cases in our data set 1962–2008 and, for comparison, for the GN and LFR benchmark networks. In all instances, we consider the partition $H$ obtained via modularity optimization (Sec. III A). The results are depicted in the upper panel in Fig. 3 (for readability, only the WTW curves for 1962, 1980, and 2008 are plotted). These functions are, however, not easily compared, essentially for two reasons. First, the curves start from different values $r_1^H$ [61]. Second, the decay velocities are hardly comparable because of the different dimensions $N$ of the networks (recall that time $t$ is the number of steps of the random walker). For these reasons, we normalize the curves along both axes and plot, in the lower part of Fig. 3, the normalized stability $r_t^H / r_1^H$ with respect to the normalized time $t/N$. In this way the curves are directly comparable and clearly demonstrate the exponential behavior $r_t^H / r_1^H \sim \exp(-\gamma t/N)$. Visual examination of Fig. 3 is probably sufficient to grasp the much more rapid decay of the WTWs with respect to the two benchmarks, but the computation (via linear fitting) of the decay rate $\gamma$ reinforces this impression: while the GN and LFR networks have $\gamma = 23.3$ and 5.73, respectively, the

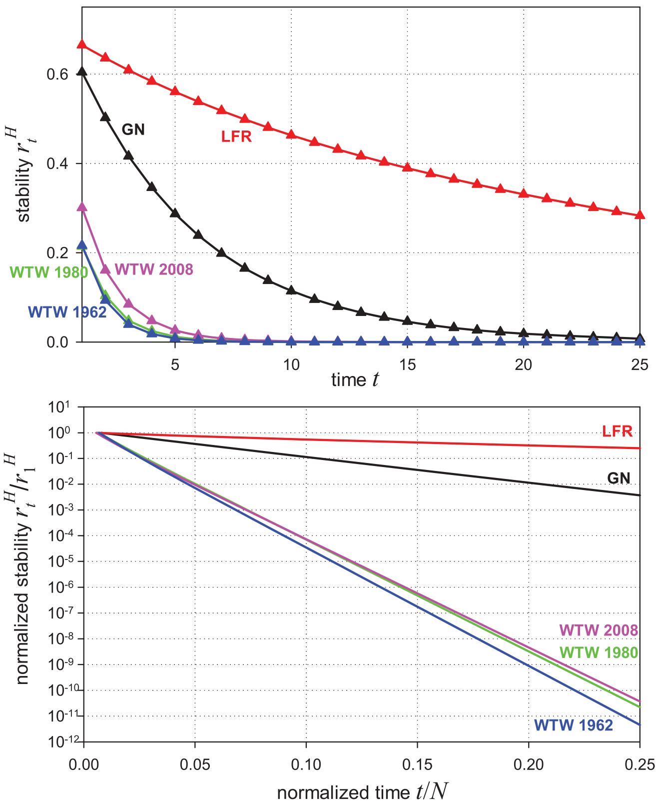


FIG. 3. (Color online) Top: Stability functions $r_t^H$ of the GN and LFR benchmark networks and those of the World Trade Web (WTW) in 1962, 1980, and 2008. For each network, we consider the partition $H$ obtained via modularity optimization. Bottom: Same as above, but the stability is normalized by the initial value $r_1^H$ and the time axis is normalized, separately for each curve, by the number $N$ of network nodes.

WTWs in 1962, 1980, and 2008 are characterized by the much higher values 106.8, 100.3, and 97.6, respectively. Similar figures ($92.4 < \gamma < 109.4$) are obtained for the other years in the data set, with no clear trend with respect to time, as well as for the symmetrized network ($90.4 < \gamma < 107.4$). A slightly slower decay, although still significantly faster than the benchmark networks, instead characterizes the filtered networks ($59.8 < \gamma < 72.5$).

### D. Persistence probabilities

We can complement the above analysis by extracting other quantitative indicators, which we call the *persistence probabilities* of the communities. Starting from the $N$-state network, a given partition $\mathbb{C}_1,\mathbb{C}_2,\ldots,\mathbb{C}_q$ induces a $q$-state metanetwork, where communities becomes metanodes. At this scale, the random walker can be described by the $q$-state *lumped* Markov chain [62] with the stochastic matrix

$$U = [\mathrm{diag}(\pi H)]^{-1} H' \mathrm{diag}(\pi) P H, \tag{8}$$

which is actually obtained by row-normalizing the term $H'\mathrm{diag}(\pi)PH$ appearing in the first term in (6). In rigorous terms, the $q$-state lumped Markov chain $\Pi_{t+1} = \Pi_t U$ provides, in general, only an approximate description of the dynamics of the random walker at the metanetwork level. Nonetheless, it becomes exact under the assumption that the Markov chain $\pi_{t+1} = \pi_t P$ is at stationarity, i.e., $\pi_t = \pi$ [63,64]. Under this assumption, the entry $u_{cd}$ of $U$ is the probability that the random walker is at time $(t+1)$ in any of the nodes of community $\mathbb{C}_d$, provided it is at time $t$ in any of the nodes of community $\mathbb{C}_c$. We define the persistence probability of the community $\mathbb{C}_c$ as the diagonal term $u_{cc}$ in $U$. Large values of $u_{cc}$ are expected for significant communities. In fact, the expected escape time from $\mathbb{C}_c$ is $\tau_c = (1-u_{cc})^{-1}$: the walker will spend a long time within the same community if the weights of the internal edges are comparatively large with respect to those pointing outside. The analysis of the persistence probabilities induced in a network by a given partition has recently been proven to be an effective tool for testing the existence and significance of communities [55].

From (8) one can derive the explicit expression for $u_{cc}$ [55],

$$u_{cc} = \sum_{i\in\mathbb{C}_c} \frac{\pi^i}{\Pi_c} \sum_{j\in\mathbb{C}_c} \frac{w_{ij}}{s_i^{\mathrm{out}}}, \tag{9}$$

where $\pi^i$ is the stationary probability of node $i$, and $\Pi_c = \sum_{i\in\mathbb{C}_c} \pi^i$. For the WTW, this means that $u_{cc}$ is a weighted average (a convex combination) of the fractions of the export flows that the countries of community $\mathbb{C}_c$ direct *within* the community itself. For example, $u_{cc} = 0.5$ denotes that, on (weighted) average, the countries of $\mathbb{C}_c$ direct half of their export flow to the countries in the same community and half to the rest of the world: a very mild requirement for a baseline level of significance. With this in mind, we compute the persistence probabilities $u_{cc}$, $c = 1,2,\ldots,q$, of the WTWs in the 1962–2008 period and of the two benchmark networks, for the partition corresponding to the max-modularity (Sec. III A). The results are shown in Fig. 4, for the original, filtered, and symmetrized WTWs. It is evident from Fig. 4 that, on average, the $u_{cc}$ values of all the WTWs under scrutiny are much smaller than those of the benchmarks (which, we recall, are purposely built with a significant cluster structure). Actually, in almost all instances the entire range of the $u_{cc}$ values of the WTWs is below the corresponding range of the benchmarks. If we then individually analyze each single community, we discover that most of them turn out to be scarcely significant, as revealed by the small persistence probability (note, for comparison, that all the $u_{cc}$ values of the 10-community partition of the LFR network are larger than 0.84, although the communities have diversified sizes, ranging from 11 to 25 nodes). Specifically, only in recent years (since 2005) are the $u_{cc}$ values of the WTW all larger than 0.5, but two of three still remain below 0.6, meaning that, on average, these countries direct more than 40% of their export outside their community. In all years in the sample, therefore, the communities dictated by the analyzed partitions are by no means secluded from the rest of the world, as their in- and out-community trade volumes have comparable magnitude.

In this respect, the results are even worse for the filtered networks. On one hand, removing several low-weight edges slightly increases the highest persistence probabilities. But on the other hand, the finer partition detected by the max-modularity approach pops up some small, scarcely significant

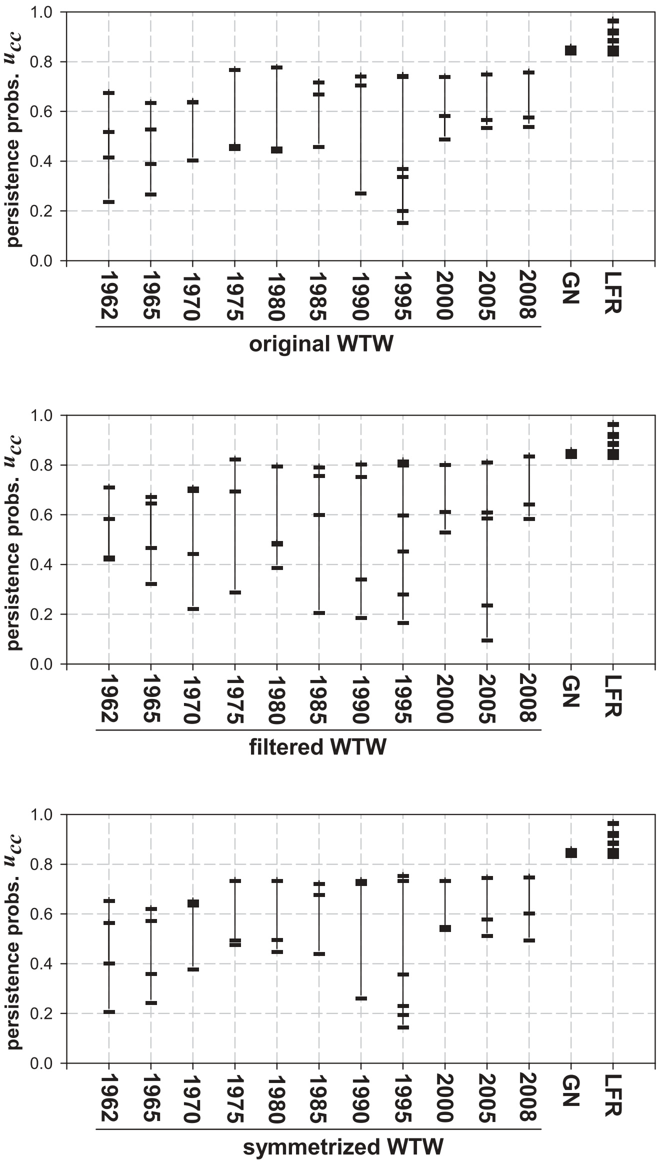


FIG. 4. Persistence probabilities of the World Trade Web (WTW; 1962–2008) and of the GN and LFR benchmark networks. The three panels refer, respectively, to the original, filtered, and symmetrized WTW, as defined in Sec. III A. For each network, we consider the $q$-community partition obtained via modularity optimization: the $q$ horizontal dashes denote the values of the diagonal terms $u_{cc}$ of the lumped Markov matrix $U$ (vertical straight lines are for visual aid only).

communities, as clearly highlighted by the larger number of small $u_{cc}$ values in the middle panel in Fig. 4.

Nonetheless, some important information is revealed by analysis of Fig. 4. Even if, in most instances, the partition of the WTW is scarcely significant as a whole, we notice that, since 1975, there is in each case (at least) one community with a rather high persistence probability, both in absolute terms and comparatively with respect to most of the other $u_{cc}$ values. It turns out that it is a large community which always includes the entire set of European countries, plus a number of minor non-European partners (partially varying from year to year), mainly from North Africa, the Near East, and the Asian republics of the former USSR. Up to 1995, there is also another large community with a high persistence probability, which includes the entire North America and most of Central and South America, plus China, Australia, and many others. Since 2000, however, the community partition dictated by the max-modularity suggests a different arrangement, with North and South America in one community and China and Australia in another. Notably, both these new communities have definitely lower persistence probabilities than before, denoting less exclusive intracommunity partnerships. The evidence emerging from this analysis is very much in line with what can be expected looking at the existence of trade agreements between countries. European countries form the oldest and deeper custom union in the world, and the persistence of their ties is confirmed by the data, which also suggest, though, that this is not a group of countries separated from to the rest of the world (in 2008, over one-third of European Union imports came from non–European Union countries). The reported evidence also captures the new active role of China, which became a major player in many areas of the world, less dependent on the US market.

Overall, we can conclude that, as well as the other methods above presented, the use of stability functions and the evaluation of the persistence probabilities seem to confirm the absence of a strong clusterized structure in the WTW, when considered as a whole. However, the capability of the persistence probabilities to assess the quality of each single community, differently from the other tools of analysis, puts forward the existence of some significant clusters of countries with privileged intracommunity partnerships.

## IV. CONCLUDING REMARKS

In this paper we have used four approaches to analyze communities in the WTW. In the literature of community analysis, these methods have been extensively tested on a variety of networks having features such as directed edges, multiscale weights, and heterogeneity in the distributions of node degrees and/or community sizes. In the case of the WTW, however, all four approaches led to similar conclusions: there is no significant evidence of the existence of a strong community structure in the WTW. The eligible communities found in the data are reasonable, but, with very few exceptions, they are not very significant according to any of the criteria adopted. Even if there is not a single robust measure to identify communities in the WTW, the convergence of results from all the approaches strengthens the robustness of this conclusion.

The configuration of the WTW therefore supports the view that the growth of international trade linkages did not occur only involving specific groups of countries. Even if the phenomenon of rapidly increasing economic integration among countries clearly appears in a number of measures computed for the WTW over time (such as the increase in density or the sharp change in the nodes' average strength), the new links that have been forming have not changed the (weak) cluster structure of the network, as they have not followed a strong or exclusive preferential pattern. Countries do select their trading partners—given that countries are on average connected to "only" approximately half of the other existing countries—but this selection is quite open. In this respect, economic integration involves the world as a whole, and in this sense it indeed appears to be a "global" phenomenon.

While a nonpreferential structure in terms of aggregate flows is quite plausible, much stronger community ties can emerge considering trade in specific sectors [12]. Future developments of this work could focus on trade flows between countries in particular commodities, using these aggregate results as a benchmark.


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[61] For undirected networks, [59] proved that $r_1^H = Q$, i.e., the modularity associated with the partition $H$. For directed networks, it can easily be checked that $r_1^H = Q^{\mathrm{lr}}$, which is the so-called *LinkRank modularity* proposed in [41].

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