Dust ion-acoustic solitary waves in a dusty plasma with nonextensive electrons

Mustapha Bacha,[1] Mouloud Tribeche,[1,2] and Padma Kant Shukla[2]
[1]*Plasma Physics Group, Theoretical Physics Laboratory, Faculty of Sciences-Physics, University of Bab-Ezzouar, USTHB, B.P. 32, El-Alia, Algiers 16111, Algeria*
[2]*International Centre for Advanced Studies in Physical Sciences, Faculty of Physics and Astronomy, Ruhr-University Bochum, D-44780 Bochum, Germany*
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The dust-modified ion-acoustic waves of Shukla and Silin are revisited within the theoretical framework of the Tsallis statistical mechanics. Nonextensivity may originate from correlation or long-range plasma interactions. Interestingly, we find that owing to electron nonextensivity, dust ion-acoustic (DIA) solitary waves may exhibit either compression or rarefaction. Our analysis is then extended to include self-consistent dust charge fluctuation. In this connection, the correct nonextensive electron charging current is rederived. The Korteweg–de Vries equation, as well as the Korteweg–de Vries–Burgers equation, is obtained, making use of the reductive perturbation method. The DIA waves are then analyzed for parameters corresponding to space dusty plasma situations.

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I. INTRODUCTION

Complex or dusty plasmas—a mixture of ions, electrons, and highly charged microparticles and nanoparticles—are currently being considered as a major interdisciplinary research field [1–3]. Linear as well as nonlinear collective processes in dusty plasmas have received special attention in the past decade mainly due to the realization of their occurrence in both the laboratory and space environments [4–8]. Examples include cometary comae and tails, planetary rings, the interstellar medium, the lower ionosphere, plasma processing devices, limiter regions of fusion plasmas, etc. The dust particles are many orders of magnitude heavier than ions, they are a source of ionization and recombination for electrons, and their charge is not fixed, but depends on local plasma parameters. Wave propagation in such complex systems is therefore expected to be substantially different from the ordinary two component plasmas, and the presence of charged dust can have a strong influence on the characteristics of the usual plasma wave modes, even at frequencies where the dust grains do not participate in the wave motion. It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra [9]. On the other hand, it has been shown that the dust dynamics introduces new eigenmodes, such as the dust-acoustic mode [10] (weak-coupling regime), the dust-lattice mode [11] (strong-coupling regime), dust Bernstein-Greene-Kruskal modes [12], etc. Among the host of modified dusty modes discussed in the literature, the dust ion-acoustic wave [13] (DIAW) has received wide attention as well as experimental confirmation in several low-temperature dusty plasma devices [14].

During the past two decades, it has been proven that systems which present long-range interactions, long-time memory, fractality of the corresponding space time and phase space, or intrinsic inhomogeneity are intractable within the conventional Boltzmann-Gibbs (BG) statistics [15]. The main reason for this failure is that BG statistics is an additive or extensive formalism. In dealing with the statistical properties of systems with long-range correlations, Tsallis [16] consistently extended BG thermodynamics by generalizing the concept of entropy to nonextensive regimes. To this end, Tsallis modeled nonextensivity by assuming a composition law in the sense that the entropy of the composition $(A + B)$ of two independent systems A and B is equal to $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1 - q)S_q^{(A)}S_q^{(B)}$, where the parameter q that underpins the generalized entropy is linked to the underlying dynamics of the system and provides a measure of the degree of its correlation. A growing body of evidence suggests that the q entropy may provide a convenient frame for the analysis of many astrophysical scenarios, such as stellar polytropes, the solar neutrino problem, the peculiar velocity distribution of galaxy clusters, etc. [17–28]. It has been shown that the experimental results, for electrostatic plane-wave propagation in a collisionless thermal plasma, point to a class of Tsallis's velocity distribution described by a nonextensive q parameter smaller than unity [29]. Liu and Goree [30] detected experimentally anomalous diffusion and non-Gaussian statistics fitting a Tsallis distribution, in a two-dimensional driven-dissipative system. A dusty plasma suspension with a Yukawa interaction has been heated to yield a structure with liquid ordering. Tsallis thermostatistics should therefore be applied to plasma systems which may be rightly viewed as systems endowed with long-range interactions and (most importantly) where nonequilibrium stationary states may exist. This kind of state is generally described by the *ad hoc* Cairns and κ distributions, which suffer from a lack of formal derivation. However, one should emphasize that (in the light of present understanding) it is still unclear which class of systems requires Tsallis statistics. Nevertheless, Wilk and Wlodarczyk [31] showed that the nonextensivity parameter q may be given (in the $q > 1$ case) entirely by the fluctuations of the parameters of the usual exponential distribution. Moreover, Almeida [32] claimed that the canonical distribution function of a system is Tsallis distribution if the relation

$$1 - q = \frac{dT}{dE} \tag{1}$$

is satisfied, where E is the energy of the environment of the system (or the "heat bath") and T is the equilibrium temperature in energy units. Alternatively, one could use numerical tools to determine (in some specific situations) whether the electrons and/or ions do roughly behave like q nonextensive.

To complement and provide new insights into previously published results, we propose here to extend the analysis of Shukla and Silin [13], done under the assumption that the electrons are Maxwell-Boltzmann distributed, to situations where the electrons may exhibit nonextensive effects.

II. THEORETICAL MODEL

Let us consider an unmagnetized dusty plasma composed of nonextensive electrons, inertial warm ions, and immobile negatively charged dust grains of density n_e, n_i, and n_d, respectively. We assume for simplicity that all the grains have the same charge, equal to $q_d = -eZ_d$, with Z_d positive for negatively charged dust and negative for positively charged dust. To model the effects of electron nonextensivity, we refer to the following q-distribution function [33]:

$$f_e(v) = C_q \left\{1 - (q-1)\left[\frac{m_e v^2}{2T_e} - \frac{e\phi}{T_e}\right]\right\}^{1/(q-1)}. \quad (2)$$

The constant of normalization is

$$C_q = \begin{cases} n_{e0}\frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{1}{2})}\sqrt{\frac{m_e(1-q)}{2\pi T_e}}, & \text{for } -1 < q < 1 \\ n_{e0}\left(\frac{1+q}{2}\right)\frac{\Gamma(\frac{1}{q-1}+\frac{1}{2})}{\Gamma(\frac{1}{q-1})}\sqrt{\frac{m_e(q-1)}{2\pi T_e}}, & \text{for } q > 1. \end{cases} \quad (3)$$

Here, the parameter q stands for the strength of electrons nonextensivity and the quantity Γ for the standard γ function. It may be useful to note that for $q < -1$, the q distribution (2) is unnormalizable. In the extensive limiting case ($q = 1$), distribution (2) reduces to the well-known Maxwell-Boltzmann velocity distribution. Integrating $f_e(v_x)$ over the velocity space and noting that for $q > 1$, the distribution function (2) exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles, given by

$$v_{\max} = \sqrt{\frac{2T_e}{m_e(q-1)} - \frac{2e\phi}{m_e}}, \quad (4)$$

we get

$$n_e(\phi) = \begin{cases} \int_{-\infty}^{+\infty} f_e(v)dv, & \text{for } -1 < q < 1 \\ \int_{-v_{\max}}^{+v_{\max}} f_e(v)dv, & \text{for } q > 1 \end{cases}$$
$$= n_{e0}\left\{1 + (q-1)\frac{e\phi}{T_e}\right\}^{\frac{1}{q-1}+\frac{1}{2}}. \quad (5)$$

The basic equations for one-dimensional dust ion-acoustic waves can be expressed in terms of normalized variables as

$$\frac{\partial N_i}{\partial t} + \frac{\partial (N_i V_i)}{\partial x} = 0, \quad (6)$$

$$\frac{\partial V_i}{\partial t} + V_i\frac{\partial V_i}{\partial x} = -\frac{\partial \Psi}{\partial x}, \quad (7)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = (1-\delta Z_d)N_e - N_i + \delta Z_d, \quad (8)$$

where $\delta = n_{d0}/n_{i0}$. The subscript 0 stands for equilibrium values. The electrostatic potential Ψ, the ion fluid velocity V_i, the electron number density N_e, and the ion number density N_i are normalized by T_e/e, $C_i = (T_e/m_i)^{1/2}$, n_{e0}, and n_{i0}, respectively. The time t and the space variable x are in units of the ion plasma period $\omega_{pi}^{-1} = (m_i/4\pi n_{i0}e^2)^{1/2}$ and the Debye length $\lambda_i = (T_e/4\pi n_{i0}e^2)^{1/2}$, respectively. $q_{j=e,i} = \mp e$ are the charges, m_j are the masses, and T_j the temperatures. Charge neutrality at equilibrium requires $\delta Z_d = 1 - n_{e0}/n_{i0}$, where δZ_d represents the fraction of the negative charge in the plasma which resides on the dust grains. To study the time-independent arbitrary amplitude DIA solitary waves, we assume that all the dependent variables in Eqs. (6)–(8) depend only on a single variable $\xi = x - Mt$ (where again ξ is normalized by λ_i and $M =$ solitary wave speed$/C_i$). Now, under the appropriate boundary conditions, viz., $\Psi \longrightarrow 0$, $V_i \longrightarrow 0$, and $N_i \longrightarrow 1$ at $\xi \longrightarrow \pm\infty$, Eqs. (6) and (7) can be integrated to give

$$N_i = \frac{1}{(1 - 2\Psi/M^2)^{1/2}}. \quad (9)$$

Substituting for N_i from (9) into Poisson's equation (8), and multiplying both sides of the resulting equation by $d\Psi/d\xi$, integrating once, and imposing the appropriate boundary conditions for localized solutions, namely, $\Psi \longrightarrow 0$ and $d\Psi/d\xi \longrightarrow 0$, at $\xi \longrightarrow \pm\infty$, we obtain the quadrature

$$\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0, \quad (10)$$

where the Sagdeev potential [34] for our purposes reads as

$$V(\Psi) = \frac{2(1-\delta Z_d)}{3q-1}\{1 - [1 + (q-1)\Psi]^{\frac{3q-1}{2(q-1)}}\}$$
$$+ M^2\left\{1 - \left(1 - \frac{2\Psi}{M^2}\right)^{1/2}\right\} - \delta Z_d \Psi. \quad (11)$$

Equation (11) can be regarded as an "energy integral" of an oscillating particle of unit mass, with a velocity $d\Psi/d\xi$ and position Ψ in a potential $V(\Psi)$. It is clear from (11) that $V(\Psi) = 0$ and $dV(\Psi)/d\Psi = 0$ at $\Psi = 0$. Solitary wave solutions of (10) exist if (i) $(d^2V/d\Psi^2)_{\Psi=0} < 0$, so that the fixed point at the origin is unstable; (ii) there exists a nonzero Ψ_m at which $V(\Psi_m) = 0$; and (iii) $V(\Psi) < 0$ when Ψ lies between 0 and Ψ_m. The second condition simply means that a quasiparticle of zero total energy will be reflected at the position $\Psi = \Psi_m$. The third condition means that V has to be a potential trough in which the quasiparticle can be trapped and experience oscillations. Condition (i) for the existence of localized structures requires the Mach number to satisfy

$$M^2 > \frac{2}{(1-\delta Z_d)(q+1)}. \quad (12)$$

It follows that for $q > 1$ $(-1 < q < 1)$, the lower limit $M_{\min} = [\frac{2}{(1-\delta Z_d)(q+1)}]^{1/2}$ is smaller (greater) than its Boltzmannian counterpart ($q = 1$), $M_{\min} = \frac{1}{(1-\delta Z_d)^{1/2}}$. For the sake of comparison, we have plotted the variation of $M_{\min}$ with the nonextensive parameter q for different values of $\delta Z_d = 0.1$ (solid line), 0.4 (dashed line), and 0.7 (dash-dotted line); see Fig. 1. It can be seen that as q increases, the lower limit $M_{\min}$ decreases and becomes less than unity beyond $q = \frac{1+\delta Z_d}{1-\delta Z_d}$. Furthermore, the lower limit $M_{\min}$ is shifted toward higher values as the fraction of the plasma negative charge residing on the dust grains, δZ_d, increases. It turns out that a strong

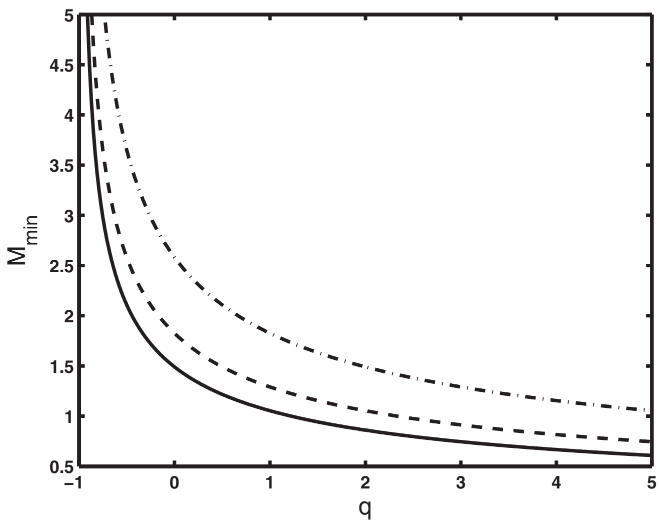

FIG. 1. Variation of the lower limit $M_{\rm min}$ with the nonextensive parameter q for different values of $\mu Z_d = 0.1$ (solid line), 0.4 (dashed line), and 0.7 (dash-dotted line).

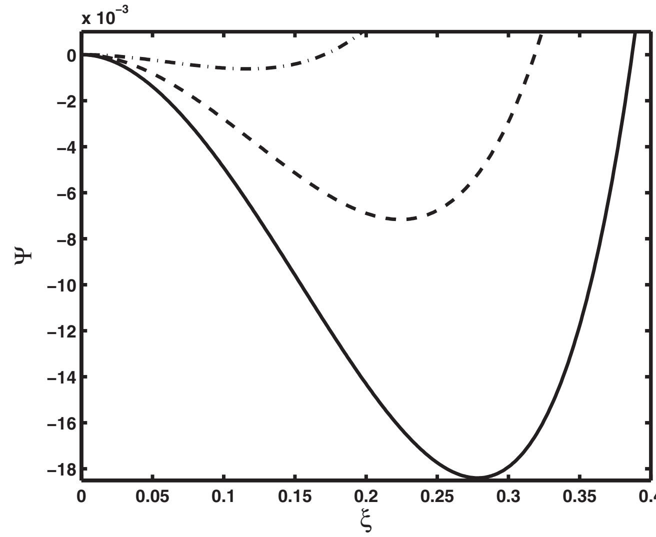

FIG. 2. Pseudopotential associated to compressive DIA solitary waves for different values of $q = 8$ (solid line), 6 (dashed line), and 4 (dash-dotted line), with $\delta = 0.5$ and $M = 1$.

electron depletion process (on account of the attachment of the background plasma electrons on the surface of the dust grains) may not favor the propagation of the DIA waves. Note that the upper limit of M, $M_{\rm max}$, can be found by the condition $V(\Psi_c) \geqslant 0$, where $\Psi_c = M_{\rm max}^2/2$ is the maximum value of Ψ for which the cold ion density N_i is real. Thus, we have

$$\frac{2(1-\delta Z_d)}{3q-1}\left\{1-\left[1+(q-1)\frac{M_{\rm max}^2}{2}\right]^{\frac{3q-1}{2(q-1)}}\right\} + M_{\rm max}^2\left(1-\frac{\delta Z_d}{2}\right) \geqslant 0. \quad (13)$$

Next, keeping δZ_d at a constant value equal to 0.5, the nature of these solitary waves is investigated by analyzing the Sagdeev potential (11) [note that the method which consists in expanding $V(\Psi)$ to third order in a Taylor series in Ψ is not an accurate one]. Interestingly, we found that owing to electron nonextensivity, our plasma model may admit compressive (Fig. 2) as well as rarefactive (Fig. 3) nonextensive DIA solitary waves. Note that larger values of q favor the development of compressive DIA solitary waves, whereas smaller values of q are required for the existence of rarefactive ones.

III. VARIABLE DUST CHARGE CASE

Let us now extend our analysis to the case where dust grains exhibit self-consistent charge variation. We will go parallel to that done in Ref. [35] and adopt their notation. For a nonisothermal electron distribution function, such as that given by Eq. (2), one should first rederive the electron current by using the orbit limited theory. The latter requires an effective collisional cross section $\sigma_e(v_e,q_d) = \pi r_d^2(1 + 2eq_d/m_eCv_e^2)$ for the electrons impacting onto a dust grain surface over the electron distribution, where $C \simeq r_d$ is the effective dust grain capacitance and r_d the grain radius. For the electron current we thus have

$$I_e = -e\iiint f_e\sigma_e v_e d^3v_e = \begin{cases}-4\pi e\int_{v_{\rm min}}^{\infty} v_e^3\sigma_e f_e\, dv_e, & \text{for } -1<q<1\\ -4\pi e\int_{v_{\rm min}}^{v_{\rm max}} v_e^3\sigma_e f_e\, dv_e, & \text{for } q>1,\end{cases} \quad (14)$$

where $v_{\rm min} = (-2eq_d/m_er_d)^{1/2}$ is the minimum value of the electron velocity for which the electrons are collected by a dust particle. After performing the velocity integration in Eq. (9), we obtain for the electron current

$$I_e = -\pi r_d^2 e n_{e0}\sqrt{\frac{8T_e}{\pi m_e}}B_q\left[1+(q-1)\left(\frac{eq_d}{T_er_d}+\frac{e\phi}{T_e}\right)\right]^{\frac{1}{q-1}+2}, \quad (15)$$

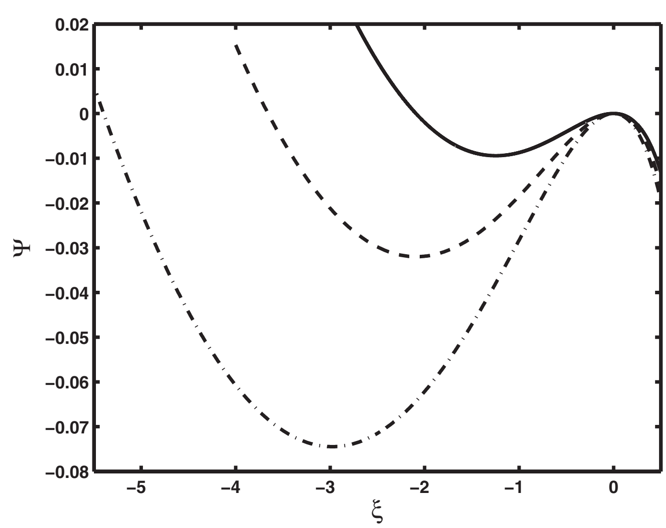

FIG. 3. Pseudopotential associated to rarefactive DIA solitary waves for different values of $q = 0.4$ (solid line), 0.5 (dashed line), and 0.6 (dash-dotted line), with $\delta = 0.5$ and $M = 1.85$.

where

$$B_q = \begin{cases} \frac{(1-q)^{3/2}}{q(2q-1)} \frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{3}{2})}, & \text{for} \quad -1 < q < 1 \\ \frac{(3q-1)}{2} \frac{(q-1)^{3/2}}{q(2q-1)} \frac{\Gamma(\frac{1}{q-1}+\frac{3}{2})}{\Gamma(\frac{1}{q-1})}, & \text{for } q > 1. \end{cases} \tag{16}$$

Equation (15) reproduces the well-known thermal electron charging current [4] when $q \to 1$. The nonlinear dynamics of DIA waves is then governed by

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x}(N_i U_i), \tag{17}$$

$$\frac{\partial U_i}{\partial t} + U_i \frac{\partial U_i}{\partial x} = -\frac{\partial \Psi}{\partial x}, \tag{18}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \mu[1 + (q-1)\Psi]^{\frac{1}{q-1}+\frac{3}{2}} - N_i + (1-\mu)Z_d, \tag{19}$$

where $\mu = n_{e0}/n_{i0}$. N_i is the ion number density normalized by its equilibrium value n_{i0}, U_i is the ion fluid velocity normalized by $C_i = (T_e/m_i)^{1/2}$, Ψ is the electrostatic wave potential normalized by T_e/e, and Z_d is the number of electrons residing onto the dust grain surface normalized by its equilibrium value Z_{d0}. The time and space variables are in units of the ion plasma period $\omega_{pi}^{-1} = (m_i/4\pi n_{i0} e^2)^{1/2}$ and the Debye radius $\lambda_D = (T_e/4\pi n_{i0} e^2)^{1/2}$, respectively. We note that Z_d varies with space and time. Thus, the normalized dust grain charging equation is given by

$$\eta \frac{\partial Z_d}{\partial t} = \mu\beta B_q[1 + (q-1)(\Psi - \alpha Z_d)]^{\frac{1}{q-1}+2} - \beta_i N_i U_i \left(1 + \frac{2\alpha Z_d}{U_i^2}\right), \tag{20}$$

where

$$B_q = \begin{cases} \frac{(1-q)^{3/2}}{q(2q-1)} \frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{3}{2})}, & \text{for} \quad -1 < q < 1 \\ \frac{(3q-1)}{2} \frac{(q-1)^{3/2}}{q(2q-1)} \frac{\Gamma(\frac{1}{q-1}+\frac{3}{2})}{\Gamma(\frac{1}{q-1})}, & \text{for} \quad q > 1. \end{cases} \tag{21}$$

Here, $\eta = \sqrt{\alpha m_e(1-\mu)/2m_i}$, $\beta = (r_d/a)^{3/2}$, $\beta_i = \beta\sqrt{\pi m_e/8m_i}$, $\alpha = Z_{d0}e^2/k_B T_e r_d$, and $a = n_{d0}^{-1/3}$. We note that at equilibrium, we have $\mu\beta B_q[1 - (q-1)\alpha] = \beta_i U_0(1 + 2\alpha/U_0^2)$, where U_0 is the ion streaming speed normalized by C_i.

A. DIA Solitary Waves

To investigate the small but finite amplitude DIA solitary wave, we introduce the stretched coordinates [36] $\xi = \varepsilon^{1/2}(x - V_0 t)$ and $\tau = \varepsilon^{3/2} t$, where ε is an expansion parameter characterizing the strength of the nonlinearity and V_0 is the phase velocity of the DIA soliton. N_i, U_i, and Z_d are then expanded in power series of ε as

$$N_i = 1 + \varepsilon N_i^{(1)} + \varepsilon^2 N_i^{(2)} + \cdots, \tag{22}$$

$$U_i = U_0 + \varepsilon U_i^{(1)} + \varepsilon^2 U_i^{(2)} + \cdots, \tag{23}$$

$$\Psi = \varepsilon \Psi^{(1)} + \varepsilon^2 \Psi^{(2)} + \cdots, \tag{24}$$

$$Z_d = 1 + \varepsilon Z_d^{(1)} + \varepsilon^2 Z_d^{(2)} + \cdots. \tag{25}$$

Introducing (22)–(25) into (17)–(20) and equating the terms in lowest power of ε, we obtain

$$\omega_0 N_i^{(1)} = U_i^{(1)}, \tag{26}$$

$$\omega_0 U_i^{(1)} = \Psi^{(1)}, \tag{27}$$

$$\frac{(3q-1)\mu}{2}\Psi^{(1)} - N_i^{(1)} + (1-\mu)Z_d^{(1)} = 0, \tag{28}$$

$$\beta_e \Psi^{(1)} - \alpha u_\beta Z_d^{(1)} - \beta_i u_1 U_i^{(1)} - U_0 \beta_i u_2 N_i^{(1)} = 0, \tag{29}$$

where $\omega_0 = V_0 - U_0$, $u_1 = 1 - 2\alpha/U_0^2$, $u_2 = 1 + 2\alpha/U_0^2$, $u_\beta = \beta_e + 2\beta_i/U_0$, and $\beta_e = 2\mu\beta B_q(q - 1/2)(1 - \alpha q)$. Now, substituting $N_i^{(1)}$, $U_i^{(1)}$, and $Z_d^{(1)}$ from Eqs. (26)–(29), we obtain the dispersion relation

$$a\omega_0^2 - b\omega_0 - c = 0, \tag{30}$$

where

$$a = \frac{(3q-1)\mu}{2} + \frac{\beta_e(1-\mu)}{\alpha u_\beta}, \tag{31}$$

$$b = \frac{u_1 \beta_i (1-\mu)}{\alpha u_\beta}, \tag{32}$$

$$c = 1 + \frac{u_2 \beta_i U_0 (1-\mu)}{\alpha u_\beta}. \tag{33}$$

Equating the terms containing the next higher order in ε, we get from (17)–(20)

$$\frac{1}{\omega_0^2}\frac{\partial \Psi^{(1)}}{\partial \tau} - \omega_0 \frac{\partial N_i^{(2)}}{\partial \xi} + \frac{\partial U_i^{(2)}}{\partial \xi} + \frac{2}{\omega_0^3}\Psi^{(1)}\frac{\partial \Psi^{(1)}}{\partial \xi} = 0, \tag{34}$$

$$\frac{1}{\omega_0}\frac{\partial \Psi^{(1)}}{\partial \tau} - \omega_0 \frac{\partial U_i^{(2)}}{\partial \xi} + \frac{1}{\omega_0^2}\Psi^{(1)}\frac{\partial \Psi^{(1)}}{\partial \xi} = -\frac{\partial \Psi^{(2)}}{\partial \xi}, \tag{35}$$

$$\frac{\partial^2 \Psi^{(1)}}{\partial \xi^2} = \frac{(3q-1)\mu}{2}\Psi^{(2)} + \frac{(3q-1)(q+1)\mu}{8}[\Psi^{(1)}]^2 - N_i^{(2)} + (1-\mu)Z_d^{(2)}, \tag{36}$$

$$\beta_e \Psi^{(2)} - \alpha u_\beta Z_d^{(2)} - \beta_i u_1 U_i^{(2)} - U_0 \beta_i u_2 N_i^{(2)} + \beta_1 [\Psi^{(1)}]^2 = 0, \tag{37}$$

where

$$\beta_1 = \frac{\beta_e}{2(1-\alpha q)}[1 + \alpha\beta_0(\alpha\beta_0 - 2)] - \frac{2\beta_i}{\omega_0^3}\left[1 + \frac{\omega_\alpha}{U_0^3}\right], \tag{38}$$

$$\beta_0 = \frac{1 - \frac{(3q-1)}{2}\omega_0^2 \mu}{\omega_0^2(1-\mu)}, \tag{39}$$

and $\omega_\alpha = \alpha\omega_0\omega_1(1 - \omega_0\beta_0/\omega_1)$, $\omega_1 = 1 - U_0/\omega_0$. Now, combining (34)–(37), we obtain the following Korteweg–de Vries (KdV) equation:

$$\frac{\partial \Psi^{(1)}}{\partial \tau} + A\Psi^{(1)}\frac{\partial \Psi^{(1)}}{\partial \xi} + B\frac{\partial^3 \Psi^{(1)}}{\partial \xi^3} = 0, \tag{40}$$

where

$$A = \frac{b\omega_0 + 3c - \beta_2 \omega_0^4}{\omega_0(2c + b\omega_0)}, \tag{41}$$

$$B = \frac{\omega_0^3}{2c + b\omega_0}, \tag{42}$$

and

$$\beta_2 = \frac{(3q-1)(q+1)}{4}\mu + 2\beta_1 \frac{(1-\mu)}{\alpha u_\beta}. \tag{43}$$

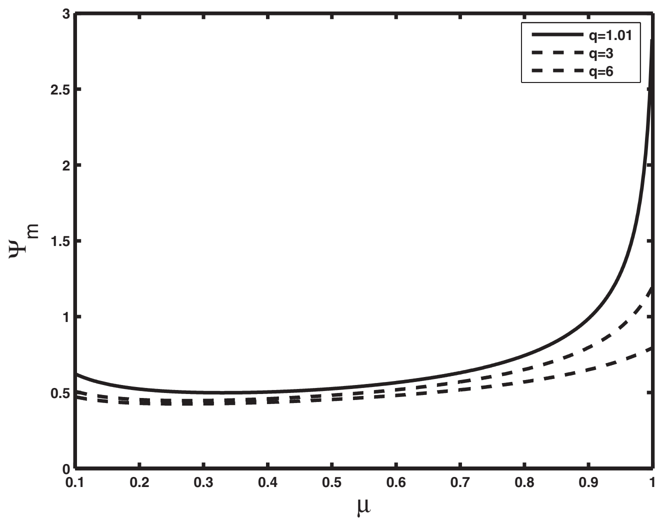

FIG. 4. Variation of the amplitude Ψ_m of the solitary wave with μ for different values of the nonextensive electron parameter $q = 1.01$, 3, and 6, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, and $U_0 = 1$.

The stationary solution of the KdV equation (40) is obtained by transforming the independent variable ξ to $\zeta = \xi - W_0\tau$, where W_0 is a constant speed normalized by C_i and imposing the appropriate boundary conditions for localized perturbations, viz. $\Psi^{(1)} \to 0$, $d\Psi^{(1)}/d\zeta \to 0$, $d^2\Psi^{(1)}/d\zeta^2 \to 0$ at $\zeta \to \pm\infty$. Accordingly, the stationary solitary wave solution of (40) is

$$\Psi^{(1)} = \Psi_m \sec h^2 \left[\frac{\xi - W_0\tau}{\Delta}\right], \tag{44}$$

where $\Psi_m = 3W_0/A$ and $\Delta = \sqrt{4B/W_0}$ represent the amplitude and the width of the solitary waves, respectively. It is obvious from (44) that there exist compressive (rarefactive) solitary waves if $A > 0\,(A < 0)$. We have numerically analyzed Ψ_m over a wide range of electron nonextensivity and for parameters corresponding to space dusty plasma situations. The following parameters $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, and $U_0 = 1$ (corresponding to space dusty plasma parameters with $T_e = 50$ eV, $n_{d0} = 10^{-7}\,\text{cm}^{-3}$, $r_d = 1\,\mu$m, and $Z_{d0} = 10^3$, see Refs. [4,9,35]) have been chosen. The variation of Ψ_m with μ for different values of the nonextensive electron parameter is shown in Fig. 4. In the limiting case, $q \to 1$ (Maxwell-Boltzmann–distributed electrons), similar results to those of Mamun and Shukla [35] are obtained. Figure 4 indicates that as q $(q > 1)$ increases, the pulse amplitude decreases and remains positive. However, for $q < 1$, Fig. 5 shows that an increasing nonextensivity yields a different (opposite) result than for $q > 1$. Moreover, the influence of the electron nonextensivity is more noticeable for larger (lower) values of μ in the case $q > 1$ $(q < 1)$. Figures 6 and 7 show the μ dependence of the potential structure width Δ for a given nonextensive parameter q. We see that for $q > 1$ $(q < 1)$ the structure narrows (enlarges) as the nonextensivity increases.

B. DIA Shock Waves

In the preceding subsection, the parameter η does not play any role because of the scaling that we have used. Let now consider a situation in which we can scale the parameter η as $\eta = \varepsilon^{1/2}\eta_0$. Therefore, with this additional scaling, Eq. (37) can be rewritten as

$$-\eta_0 V_0 \frac{\partial Z_d^{(1)}}{\partial \xi} = \beta_e \Psi^{(2)} - \alpha u_\beta Z_d^{(2)} - \beta_i u_1 U_i^{(2)} - U_0 \beta_i u_2 N_i^{(2)} + \beta_1 [\Psi^{(1)}]^2. \tag{45}$$

Replacing (37) by (45) and performing all mathematical steps as we did in the preceding subsection, we obtain the following KdV–Burger equation:

$$\frac{\partial \Psi^{(1)}}{\partial \tau} + A\Psi^{(1)}\frac{\partial \Psi^{(1)}}{\partial \xi} + B\frac{\partial^3 \Psi^{(1)}}{\partial \xi^3} = C\frac{\partial^2 \Psi^{(1)}}{\partial \xi^2}, \tag{46}$$

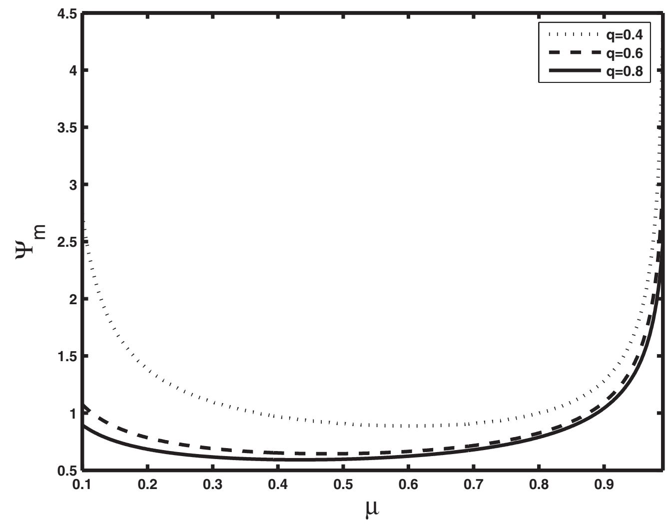

FIG. 5. Variation of the amplitude Ψ_m of the solitary wave with μ for different values of the nonextensive electron parameter $q = 0.4$, 0.6, and 0.8, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, and $U_0 = 1$.

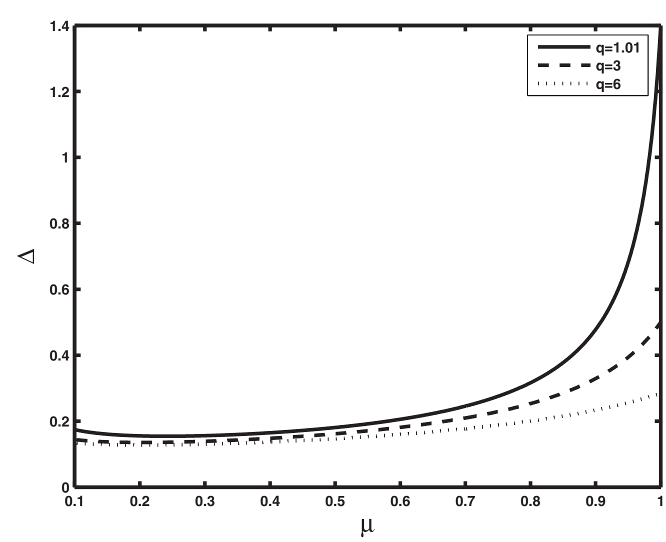

FIG. 6. Variation of the width Δ of the solitary wave with μ for different values of the nonextensive electron parameter $q = 1.01$, 3, and 6, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, and $U_0 = 1$.

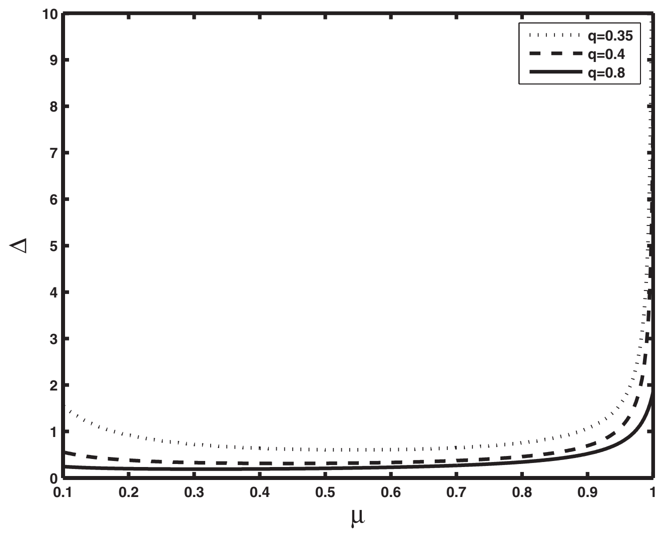

FIG. 7. Variation of the width Δ of the solitary wave with μ for different values of the nonextensive electron parameter $q = 0.35, 0.4$, and 0.8, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, and $U_0 = 1$.

where

$$C = \frac{B V_0 \eta_0 \beta_0 (1-\mu)}{\alpha\left(\beta_e + \frac{2\beta_i}{U_0}\right)}. \tag{47}$$

An exact analytical solution of (46) is not possible. Transforming the independent variables ξ to $\zeta = \xi - W_0$, one can find, under steady-state condition, a third-order ordinary differential equation for $\varphi = \Psi^{(1)}(\zeta)$. A first integration of the latter gives

$$B\frac{d^2\varphi}{d\zeta^2} - C\frac{d\varphi}{d\zeta} + \frac{A}{2}\varphi^2 - W_0\varphi = 0, \tag{48}$$

where we have imposed the appropriate boundary conditions, viz. $\varphi \longrightarrow 0$, $d\varphi/d\zeta \longrightarrow 0$, $d^2\varphi/d\zeta^2 \longrightarrow 0$ at $\zeta \longrightarrow \infty$. Multiplying both sides of Eq. (48) by $d\varphi/d\zeta$, and integrating

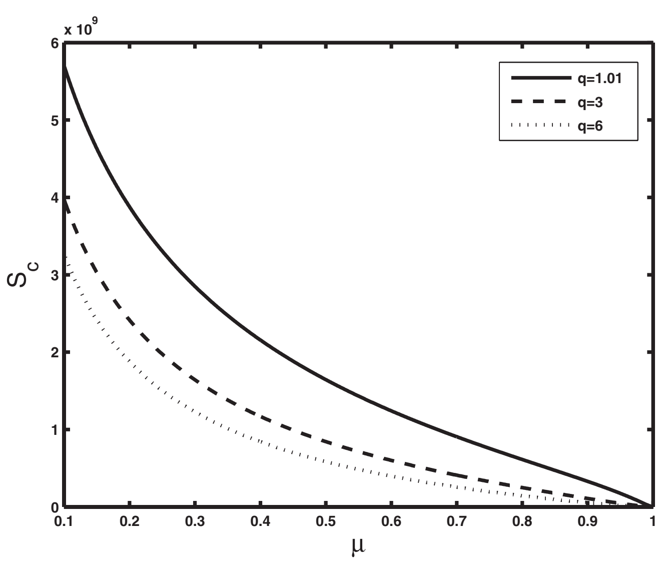

FIG. 8. Variation of S_c with μ for different values of the nonextensive electron parameter $q = 1.01, 3$, and 6, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, $U_0 = 1$, and $\varepsilon = 10^{-2}$.

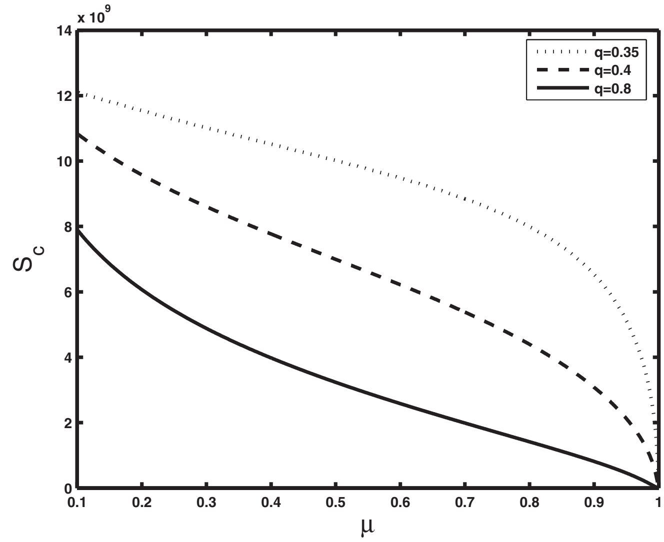

FIG. 9. Variation of S_c with μ for different values of the nonextensive electron parameter $q = 0.35, 0.4$, and 0.8, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, $U_0 = 1$, and $\varepsilon = 10^{-2}$.

once, we obtain the quadrature

$$\frac{B}{2}\left(\frac{d\varphi}{d\zeta}\right)^2 + V(\varphi) = 0, \tag{49}$$

where

$$V(\varphi) = \frac{A}{6}\varphi^3 - \frac{W_0}{2}\varphi^2 - C\int\left(\frac{d\varphi}{d\zeta}\right)^2 d\zeta. \tag{50}$$

Equation (49) can be regarded as an "energy integral" of an oscillating particle of mass B, with a velocity $d\varphi/d\zeta$ and position φ in a potential $V(\varphi)$. The quasiparticle suffers a frictional force with the coefficient C leading to the development of collisionless shocklike [37–41] wave in the sense that no viscous or damping effects resulting from collisions between dust and plasma particles are involved

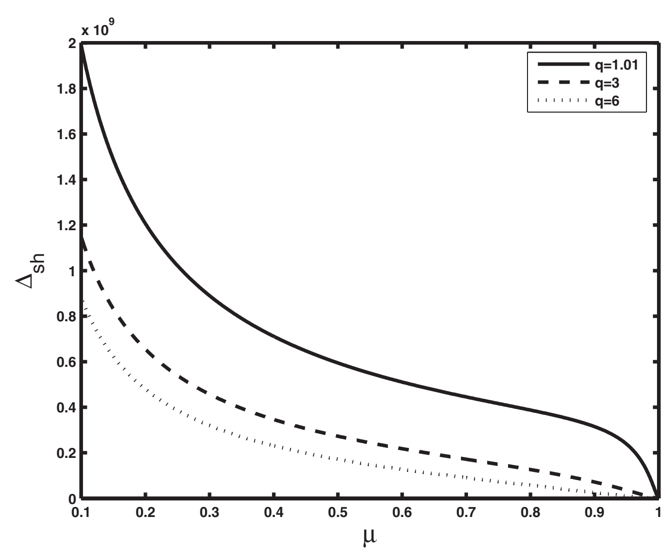

FIG. 10. Variation of the shock wave width $\Delta_{\rm sh}$ with μ for different values of the nonextensive electron parameter $q = 1.01$, 3, and 6, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, $U_0 = 1$, and $\varepsilon = 10^{-2}$.

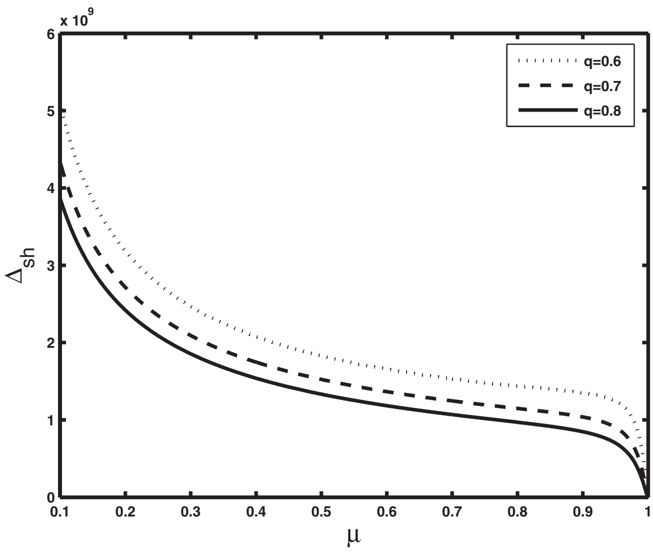

FIG. 11. Variation of the shock wave width Δ_{sh} with μ for different values of the nonextensive electron parameter $q = 0.6, 0.7$, and 0.8, with $\alpha = 0.0288$, $\beta = 3 \times 10^{-10}$, $U_0 = 1$, and $\varepsilon = 10^{-2}$.

[42,43]. As demonstrated in Ref. [35] and elsewhere [44], the solution of (48) describes a shock wave whose velocity W_0 is related to the extreme values by $\varphi(\zeta = -\infty) - \varphi(\zeta = \infty) = 2W_0/A$. The nature of these shock structures depends on the strength of dissipation as well as on the relative values between the dispersive and dissipative terms B and C. Let us then determine the condition for monotonic as well as for oscillating shock profiles by investigating the asymptotic behavior of the solutions of Eq. (48) for $\zeta \longrightarrow -\infty$. We first substitute $\varphi(\zeta) = \varphi_0 + \psi$, where $\psi \ll \varphi_0$, into Eq. (48) and then linearize it with respect to ψ. But then instead of (48) we obtain

$$B\frac{d^2\psi}{d\zeta^2} - C\frac{d\psi}{d\zeta} + W_0\psi = 0. \tag{51}$$

The solutions of (51) are proportional to $\exp(p\zeta)$, where p is given by $p = C \pm \sqrt{C^2 - 4BW_0}/2B$. It turns out that the shock wave has a monotonic profile for $S_c = C/2\sqrt{BW_0} > 1$, and an oscillating profile for $S_c < 1$. We have numerically analyzed S_c for space dusty plasma parameters. The variation of S_c with μ for different values of q is shown in Figs. 8 ($q > 1$) and 9 ($q < 1$). It can be seen that $S_c \gg 1$ is always valid for $0 < \mu < 1$ indicating therefore that only monotonic weak DIA shock waves are admitted. Let now consider a situation where we can neglect the dispersive term. In this limiting case, Eq. (48) can be rewritten as

$$\left(\varphi - \frac{W_0}{A}\right)\frac{d\varphi}{d\zeta} = \frac{C}{A}\frac{d^2\varphi}{d\zeta^2}. \tag{52}$$

The latter can be integrated, making use of the condition that φ is bounded as $\zeta \longrightarrow \pm\infty$, to yield

$$\Psi^{(1)} = \Psi_{sh}\left\{1 - \tanh\left(\frac{\xi - W_0\tau}{\Delta_{sh}}\right)\right\}, \tag{53}$$

where $\Psi_{sh} = W_0/A$ and $\Delta_{sh} = 2C/W_0$. Equation (53) represents a monotonic shocklike solution with the shock speed, the shock height, and the shock thickness given by W_0, Ψ_{sh}, and Δ_{sh}, respectively. Figures 10 and 11 show the μ dependence of the shock width Δ_{sh} for a given nonextensive parameter q. We see that for $q > 1$ ($q < 1$) the dissipative structure narrows (enlarges) as the electron nonextensivity strengthens.

IV. CONCLUSION

To conclude, we have revisited the DIA waves of Shukla and Silin [13] within the theoretical framework of the Tsallis statistical mechanics, and thereby shown that under certain conditions the effect of electron nonextensivity can be quite important. In particular, it may be noted that because of electron nonextensivity the DIA solitary wave may exhibit either a compression or a rarefaction. The lower limit of the Mach number for the existence of DIA solitary waves is smaller (greater) than its Boltzmannian counterpart ($q \to 1$) for $q > 1$ ($-1 < q < 1$). This lower limit decreases and becomes less than unity, as the nonextensive parameter q increases. However, dust grains immersed in a plasma may exhibit charge fluctuations in response to plasma charging currents flowing onto them. Our analysis has then been extended to include self-consistent dust charge fluctuation. In this connection, the correct nonextensive electron charging current is rederived. The Korteweg–de Vries equation, as well as the Korteweg–de Vries–Burgers equation, are obtained making use of the reductive perturbation method. The DIA waves are then analyzed for parameters corresponding to space dusty plasma situations. Considering the wide relevance of nonlinear oscillations, our results should help to understand the salient features of coherent nonlinear structures that may occur in dusty plasmas with nonextensive particles.

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