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Particle velocity distribution in saltation transport

T. D. Ho,[1] P. Dupont,[2] A. Ould El Moctar,[3] and A. Valance[1]
[1]*Institut de Physique de Rennes, CNRS UMR 6251, Université de Rennes 1, 35042 Rennes cedex, France*
[2]*LGCGM, INSA de Rennes, Campus Beaulieu, 35043 Rennes, France*
[3]*Laboratoire de Thermocinétique, Polytech Nantes, CNRS UMR 6607, 44306 Nantes, France*
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We report on wind-tunnel measurements of particle velocity distribution in aeolian transport. By performing extended statistics, we show that for saltation occurring over an erodible bed the vertical lift-off velocity distributions deviate significantly from a Gaussian law and exhibit a long tail accurately described by a lognormal law. In contrast, saltation over a rigid bed produces Gaussian velocity distributions. These results strongly suggest that the deviation from Gaussian distributions is a consequence of the splash process which is exclusively present in saltation transport over an erodible bed. We further suggest that the non-Gaussian statistics is intimately related to the statistical properties of a single splash event which produces ejection of particles with lift-off velocities distributed according to a lognormal law. This lognormal behavior can be simply inferred from the propagation process of the impact energy through the granular bed which can be viewed as the analog of a fragmentation process. These findings emphasize the crucial role of the splash process in saltation transport.

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Introduction. Sand transport by wind is a complex process which includes air-particle and bed-particle interaction. Saltation motion, in which sand grains are propelled by wind along the sand bed in short jumps, has been identified by Bagnold [1] to be the dominant mode of transport. In the last decades, numerical and theoretical studies on saltation motion have been developed [2–8]. In these models, the grain-bed collision is generally treated through an empirical way; the resulting lift-off velocity distribution of the splashed grains is taken as an input parameter of the model and is assumed to obey an exponential or Gaussian distribution law [3,6,9]. This distribution plays an important role because it affects the calculation of the height and length of the saltating trajectories. It is therefore crucial to have an accurate description of these velocity distributions.

Many efforts have been made to characterize the lift-off velocity distributions either by field measurements or wind tunnel experiments [10–17]. Although most of these investigations reported a deviation from a Gaussian distribution, there is no consensual description with respect to the form and nature of the distribution: a broad spectrum of distributions has been proposed ranging from normal to lognormal distributions and going through Γ, Pearson, or Weibull distributions. The large variety of experimentally found distributions is in general due to a lack of accurate statistics for the determination of the distribution tail which is the most informative part to discriminate between different distribution laws.

In this Brief Report, we give clear evidences through comprehensive wind tunnel measurements that the distribution of lift-off velocities of particles saltating over an erodible bed exhibit a long tail statistic that can be accurately described by a lognormal-type law. Conversely, saltation over a rigid bed (where the splash process is ineffective) reveals that the vertical lift-off velocity distribution obeys a Gaussian law. These results strongly indicate that the splash process plays a key role in the velocity distribution. We suggest that the non-Gaussian statistics is intimately related to the statistical properties of a single splash event. The latter has been shown to produce ejection of particles with lift-off velocities distributed according to a lognormal law [18,19]. We provide in the last part of the article basic arguments explaining the origin of the lognormal law. This lognormal statistics can be simply inferred from the propagation process of the impact energy through the granular bed which can be viewed as the analog of a fragmentation process.

Wind-tunnel facilities and instrumentation. The experiments reported here were performed in a 6 m long wind tunnel with a cross section of dimensions 0.27 m × 0.27 m where the nominal air velocity U_∞ (i.e., the air speed outside from the boundary layer) can be varied between 0 and 20 m/s. The floor of the tunnel is either a sand bed of uniform height or a rigid substrate covered with glued sand grains. To reduce the transient length required to reach steady state saltation, turbulence spires are placed at the tunnel entrance and sand grains are fed from the tunnel roof (see Fig. 1) [6]. We used natural sand with a median diameter $d = 230$ μm and a density $\rho_p = 2470$ kg/m^3.

We employed particle tracking velocimetry techniques to determine the particle velocity of the saltating particles. The scene was illuminated from above with a laser sheet produced by a YAG pulsed laser and the images were captured from the side with a high sensitive camera synchronized with the laser. Image analysis processing was then performed to extract the instantaneous velocity of the saltating particles. Statistics were performed over 250 images and the velocity distributions are obtained on a statistic ensemble of more than 2×10^4 particles.

Particle velocity distribution. We performed experiments over erodible bed for various wind strengths corresponding to Shields parameter S ranging from 0.01 to 0.2 ($S = \rho_{\rm air} u_*^2/\rho_p g d$ where u_* is the air friction velocity, $\rho_{\rm air}$ the air density, ρ_p the particle density, and d the particle diameter). Figure 2 shows the vertical velocity distribution of the saltating particles close to the ground (i.e., at $z \approx 10d$ above the bed

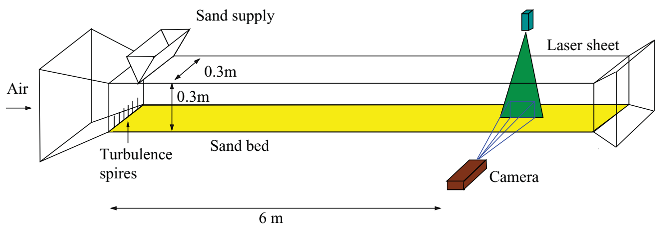

FIG. 1. (Color online) Sketch of the wind-tunnel.

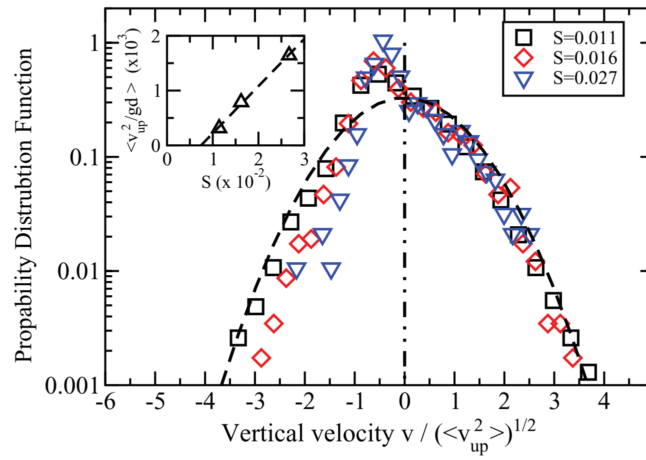

FIG. 3. (Color online) Distribution of the vertical particle velocity close to the ground (i.e., $z \approx 10d$) obtained over a rigid bed for various Shields numbers with a given mass flow rate $Q/\rho_p d\sqrt{gd} \approx 1.7$. Solid line represents a Gaussian distribution that best fits the upward velocities. Inset: Variation of $\langle v_{up}^2/gd \rangle$ with the Shields parameter.

surface within a layer $dz \approx 5d$). These distributions were obtained over a large statistic ensemble and thus provide a unique set of experimental data. Three important features should be emphasized. First, these distributions exhibit unambiguously a deviation from a Gaussian distribution. A long-tail statistic is clearly observed. Second, they do not significantly change with the Shields parameter confirming that the features of the saltating particles are invariant with the wind strength [20]. Third, they are almost symmetrical with respect to the origin indicating that saltating particles experience a negligible drag in the vertical direction during their flight. In other terms, upward and downward velocities have similar features.

Actually, the vertical velocity distribution can be well approximated by a lognormal type distribution of the following form:

$$P(v/\sqrt{gd}) = A\,\frac{e^{-[\ln\sqrt{(v^2+2gz)/gd}-\mu]^2/(2\sigma^2)}}{\sqrt{(v^2+2gz)/gd}}, \qquad (1)$$

where A is a constant of normalization and μ and σ are free parameters of the distribution. z is the altitude where vertical particle velocities were measured and $\sqrt{v^2+2gz}$ corresponds to the particle velocity extrapolated at the bed. This distribution is nothing but a truncated lognormal distribution. It is important to note that due to technical limitation of the particle tracking technique, it is not possible to get accurate measurements at height below $z = 10d$. As a consequence, we miss the low energetic grains of the velocity distribution.

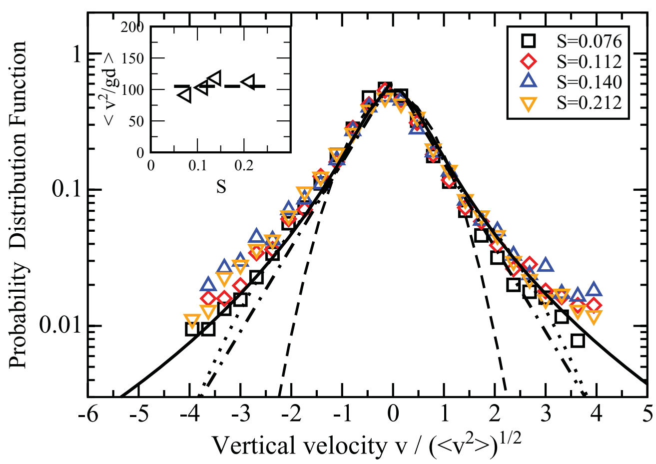

FIG. 2. (Color online) Vertical velocity distribution $p(v)$ of saltating particles located close to the ground (at $z = 10d$ within a layer $dz = 5d$) for various wind strengths measured in terms of the Shields parameter S. Fitted distributions: dashed line—Gaussian law with variance $s^2 = 53\,gd$ (χ^2 test: $\chi^2 = 0.35$), solid line—lognormal law with $\mu = 1.6$ and $\sigma = 1.0$ ($\chi^2 = 0.17$), dot-dashed line—Weibull distribution, $p_W(v) \propto v^{k-1}e^{-(v/\lambda)^k}$, with $k = 1.0$ and $\lambda = 7.8\sqrt{gd}$ ($\chi^2 = 0.20$), dot-dashed line—Γ distribution, $p_G(v) \propto v^{k-1}e^{-v/\lambda}$, $k = 1.1$ and $\theta = 7.9\sqrt{gd}$ ($\chi^2 = 0.20$), dotted line—linear combination of two Gaussian laws with variances $s_1^2 = 25\,gd$ and $s_2^2 = 625\,gd$ ($\chi^2 = 0.17$). Inset: Variation of the variance of the distribution (i.e., $\langle v^2/gd \rangle$) with the Shields parameter S.

The best lognormal fit to the data gives $\mu = 1.6$ and $\sigma = 1.0$. Interestingly, the lognormal distribution is not the only model distribution that can fit to the data with a reasonable agreement: Γ and Weibull distributions are also good candidates (see Fig. 2). However, the lognormal distribution offers several advantages. First, a χ^2 test shows that the latter yields the better agreement, and second, the lognormal behavior can be explained from simple considerations based on the splash process as to be seen later on.

To further investigate the interplay between the particle velocity distribution and the interaction of the saltating particles with the bed, we performed similar measurements over a rigid bed. Vertical velocity distributions obtained over a rigid bed exhibit contrasting features (see Fig. 3). First, the symmetry between upward and downward velocities is broken. This is a manifestation of air drag effect which is no longer negligible because saltating particles over rigid bed experience much higher jump than over an erodible bed [20]. Second, the right part of the distribution corresponding to upward velocities is extremely well described by a Gaussian law, thus contrasting with the lognormal distribution obtained over erodible bed. Third, the distributions over rigid bed are much wider than those over erodible bed and their width increases with increasing wind strength (see inset of Fig. 3). Typically, $\langle v_{up}^2/gd \rangle$ varies from 300 to 1500 for Shields parameter ranging from 0.011 to 0.027. Conversely, for transport over an erodible bed, the velocity variance is on the order of 100 and is invariant with the wind strength.

Gaussian vs non-Gaussian distribution. The above results indicate that there is a marked difference in the velocity distributions obtained respectively over rigid and erodible bed. While the lift-off velocities of grains saltating over a rigid bed obey a Gaussian distribution, those saltating over an erodible bed exhibit a long tail statistics that can be well described by a lognormal type law. The deviation from a Gaussian law can be therefore attributed to the splash process which is solely present in the case of transport over an erodible bed. The identification of the intimate physical grounds is not a trivial issue. Different possible explanations can be put forward. The long tail statistics can be interpreted in terms of a superposition of several Gaussian distributions with different variances producing eventually a seemingly lognormal distribution. It turns out that our data can be captured with a reasonable agreement by the sum of two Gaussian distributions (as shown in Fig. 2). This interpretation is interesting at least in two aspects. First, it offers the advantage to stay within a framework based on Gaussian statistics which is rather convenient for theoretical modeling. Second, it allows the identification of two different populations of grains: weakly energetic grains and highly energetic ones which may be referred, respectively, to as reptating and saltating grains (according to Bagnold classification [1]). This classification between two distinct populations is somehow a simplified representation of the saltation transport. An alternative explanation can be however suggested based on the fundamental characteristics of the Splash process. The long tail statistics of the lift-off velocity distributions of saltating grains is reminiscent of the velocity distribution of the grains ejected from a single splash event. Recent model experiments on the splash collision process [18,19] have indeed shown that the resulting velocity distribution of the splashed grains obeys a lognormal law. This leads us to conjecture that both distributions are intimately connected.

In the remaining part of this Brief Report, we provide simple arguments to understand the physical origin of the lognormal velocity distribution resulting from a splash event and then we argue how to infer the velocity distribution obtained in steady state saltation from that corresponding to a single splash event.

Splash process. A simple and successful model describing the collision process between an impacting particle and a granular bed was recently developed [18]. The key ingredient of the model is the description of the propagation of the energy (resulting from the impact) within the granular bed. In this model, the energy or momentum is described in terms of a succession of binary collisions between neighboring grains of the packing. The energy propagation can be thus seen as a fragmentation process where at each collision the energy is split into two fractions between the "reflected" particle and the "target" one according to a binary collision law characterized by an inelasticity coefficient ε (see [18,21]). A sequence of collisions produced by the impact of one particle onto a granular packing is illustrated in Fig. 4. The result of the propagation process is the creation of a "tree" made of "branches" and nodes, where a branch represents a moving particle and a node a binary collision. The branches of the collision tree correspond to different paths of energy propagation and it results that a branch ending at the surface of the granular packing produces the ejection of a particle.

Within this model, the resulting velocity distribution of the splashed particles can be computed and it was shown that the latter can be well captured by a lognormal law [18]. Here, we provide new insights to understand the origin of the lognormal distribution. In particular, we are able to interpret the lognormal distribution using simple basic arguments based on the energy propagation process through the granular media. Let us call $N(k)$ the average number of paths leading to ejection with a length k. The energy E_k released at the end of a branch of length k simply reads

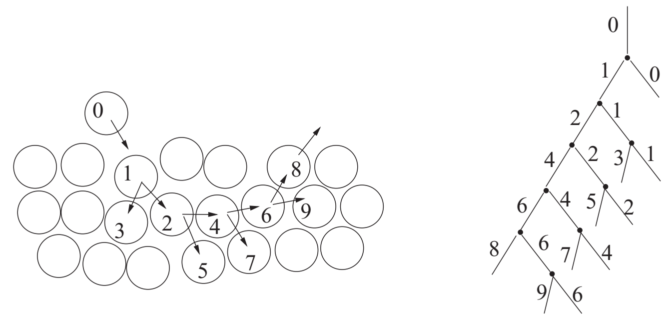

FIG. 4. Left panel: illustration of successive binary collisions produced by an impacting particle and leading to the ejection of a particle. Right panel: construction of the resulting collision tree; from a node (i.e., a collision) results two branches: the right one corresponds to the reflected particle and the left one to the target particle.

$$E_k = u_1 u_2 \dots u_k E_0, \tag{2}$$

where E_0 is the incident energy and u_i $(i = 1,2,\dots,k)$ is the fraction of energy transmitted during the ith collision. When averaged over a large number of configurations, one gets in the elastic limit (i.e., the inelasticity coefficient ε is close to 1):

$$\overline{E_k} \approx \left(\tfrac{1}{2}\right)^k E_0, \tag{3}$$

which yields the following relationship:

$$\ln \overline{V_k^2} \approx 2 \ln \overline{V_k} = 2 \ln V_0 - k \ln 2. \tag{4}$$

It thus turns out that the distribution of the logarithm of velocity V_k, $N(\ln V_k)$, is completely determined by the distribution of path length $N(k)$: $N(\ln V_k) = (\ln 2/2)\, N(k)$.

The distribution of the number of collision $N(k)$ can be computed. The resulting distribution for two different impact velocities V_0 is presented in Fig. 5. The latter can be well captured by a Poisson distribution characterized by a parameter $\lambda \approx 1.25 \ln(V_0^2/gd)$ or equivalently by a Gaussian distribution with a mean $m = \lambda$ and a standard deviation $s = \sqrt{\lambda}$.

This result is strongly reminiscent of outcomes obtained in fragmentation models [22] and it is thus interesting to push further the comparison between the energy propagation process through a granular media and fragmentation processes. In particular, a formal analogy can be made with the discrete sequential fragmentation process, that consists in breaking a unit segment into two pieces of equal length, then in selecting a fragment at random and breaking it into two pieces of equal lengths, and in repeating the process again and again. Within this process, after f fragmentation events, fragments of length $L = 2^{-s}$ $(s = 0,\dots,f)$ are thus produced. The number of

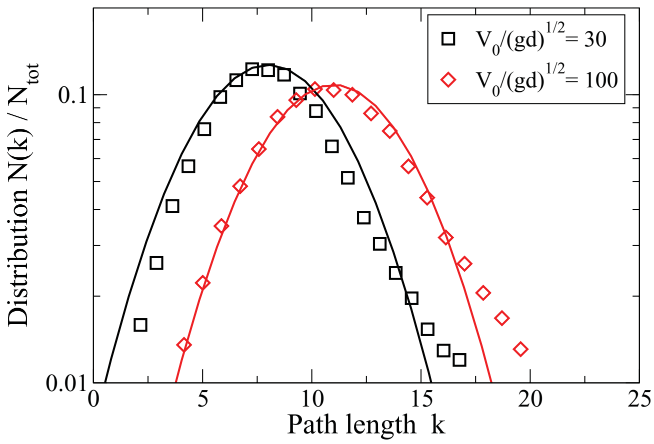

FIG. 5. (Color online) Distribution of path length $N(k)/N_{\mathrm{tot}}$ obtained from the discrete collision model for two different impact velocities $V_0/\sqrt{gd} = 30,100$. Solid lines represent Gaussian fits.

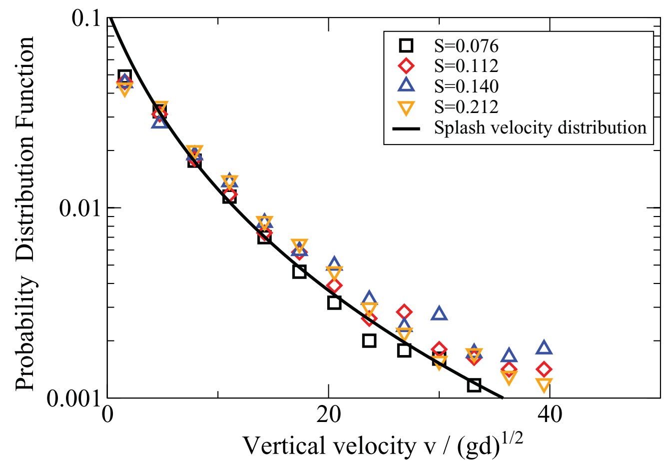

FIG. 6. (Color online) Comparison of the vertical lift-off velocity distribution obtained in steady state saltation over erodible bed and that resulting from a splash event derived from the splash model [cf. Eq. (5)]. Lift-off velocities are evaluated at the height $z = 10d$ within a layer $dz = 5d$.

fragments $n(s)$ of "size" s is the analog of the number of paths $N(k)$ in the energy propagation process within the granular bed, while the length L of a given fragment of size s is equivalent to the energy released E_k at the end of a given path of length k and the total number f of fragmentation events is in correspondence with the impact energy E_0 in the splash process. This mapping is instructive because the simple fragmentation problem is analytically solvable. The distribution of number of fragments $n(s)$ of size s is shown to tend asymptotically to a Poisson distribution with a parameter $\lambda = 2\ln f$. This exact result is very similar to that obtained in the collision model ($\lambda \approx 1.25 \ln E_0/mgd$).

With all the elements given above, it is now straightforward to derive the distribution $N(V_k)$. Taking advantage of the Gaussian feature of the distribution $N(k)$ together with Eq. (4), we arrive at the conclusion that the number $N(V_k)$ of ejected particles of velocity V_k obeys a lognormal distribution:

$$N\left(\frac{V_k}{\sqrt{gd}}\right) = \frac{N_0}{(V_k/\sqrt{gd})\sqrt{2\pi\sigma^2}} \times \exp\left(-\frac{[\ln(V_k/\sqrt{gd}) - \mu]^2}{2\sigma^2}\right), \quad (5)$$

where $\mu = \ln(V_0/\sqrt{gd}) - (\ln 2/2)\lambda$, $\sigma = (\ln 2/2)\sqrt{\lambda}$, and N_0 corresponds to the number of splashed particles. Importantly, the mean and variance of the path length distribution $N(k)$ have a weak (logarithmic) dependence with the incident velocity V_0 (for $V_0 = 100\sqrt{gd}$, $\mu \approx 0.9$, and $\sigma \approx 1.1$). The velocity distribution of a single splash event can be thus considered as invariant with the impact speed, as confirmed experimentally by Beladjine *et al.* [19]. The lift-off velocity distribution measured at the bed in steady state saltation is the result of the sum of many splash events with varying impact velocities (weighted by the number of splashed particles) plus the velocity distribution of the rebound particles. Due to the invariance of the velocity distribution of a single splash, one can argue the sum of many splash events is thus expected to produce the same distribution as that given by a single splash event, that is a lognormal distribution. The final resulting distribution cannot be derived explicitly since we have to combine the splash distribution with that of the rebound particles which is not known *a priori* and depends on the impacting velocity distribution. The latter results from a complex interplay between air flow and saltating particles. However, we are tempted to conjecture that the final lift-off velocity distribution would not differ much from the splash distribution because the statistical weight of the splashed particles is expected to be higher than that of the rebound particles. This conjecture is supported by the remarkable similarity between the lift-off vertical velocity distribution obtained in steady state saltation and that resulting from the splash model (see Fig. 6). In Fig. 6, the lift-off velocities obtained from the splash distribution were evaluated at a height $z = 10d$ as for the experimental data. This amounts to plot $N(V_k')$ with $V_k' = \sqrt{V_k^2 - 2gz}$. The weakly energetic grains of the splashed particles (corresponding to the left part of the lognormal law) are consequently lost in this operation. Note also that we did not adjust any parameter: we used the values of the parameters μ and σ derived from the splash model.

Conclusion. We show in this Brief Report that the lift-off velocity distribution of particles saltating over an erodible bed exhibits a long tail statistics that can be well captured by a lognormal law. We provide strong evidences that the long tail statistics is intimately related to the splash process. First, saltation over rigid bed (where splash is ineffective) is found to produce Gaussian velocity distributions. Second, the lognormal statistics is strongly reminiscent of that found for the velocity of the splashed particles in a single splash event. These findings emphasize the importance of the splash process in the aeolian saltation transport and show that the latter governs the statistics of the saltation layer. Besides, we believe that the long tail statistics of the lift-off velocity distribution is a key feature for a better understanding of the saltation transport.

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