Fluid-model analysis of electron swarms in a space-varying field: Nonlocality and resonance phenomena

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The physically based, benchmarked fluid model developed by Robson *et al.* [R. E. Robson, R. D. White, and Z. Lj. Petrovic, Rev. Mod. Phys. **77**, 1303 (2005)] and extended to analyze electron swarms in a spatially homogeneous electric field under conditions corresponding to the Franck-Hertz experiment [P. Nicoletopoulos and R. E. Robson, Phys. Rev. Lett. **100**, 124502 (2008)] is generalized to investigate the nonlocal response and resonance phenomena associated with electrons subject to an externally prescribed, spatially varying electrostatic field. Analytic expressions are first derived for the mean velocity, energy, and heat flux of electrons in a harmonically varying field, and expressions are then given for fields with more general spatial dependences. Numerical examples are given for both benchmark model cross sections and a real gas.

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I. INTRODUCTION

The need for physically based, benchmarked fluid modeling of low-temperature plasmas and swarms was emphasized in [1] as an alternative to the plethora of *ad hoc* empirical models which plague plasma physics, and which have little chance of leading to either a correct qualitative or quantitative understanding of many naturally occurring or laboratory-based phenomena. This model has been recently generalized and applied to analysis of electron swarms in a spatially homogeneous electric field E_0, under conditions corresponding to the Franck-Hertz experiment [2]. The same periodic structures observed in more rigorous Boltzmann equation treatments were reproduced, though with greater computational economy and yielding considerably more physical insight. This same fluid model is further extended in the present article to investigate the nonlocal response and resonance phenomena associated with electrons subject to an externally prescribed, spatially varying electrostatic field, a problem of much greater complexity than dealt with in Ref. [2]. Our results are consistent with more rigorous solutions of Boltzmann equations for electron swarms in spatially varying fields, and in addition, offer the physical insight which these purely numerical treatments lack. This remark applies in particular to the two-way synergy which develops between nonlocality and resonances with the "natural" Franck-Hertz oscillatory modes.

Although the response of single-species, electron *swarms* to external fields is a significant problem in its own right, and indeed, forms the scope of the present paper, it is worth noting that the Boltzmann treatments referred to above have been largely motivated by the need to better understand *striations* in *plasmas*. A brief discussion of the phenomenon is warranted.

Striations are ubiquitous, spatially periodic electron structures in gaseous electronics [3–8], first reported by Abria over 150 years ago [3] and perhaps observed even earlier by Michael Faraday. They are sometimes considered to have been the prime motivation for 19th-century scientists to study gas discharges, which in turn laid the foundations of modern physics in the famous experiments of Franck and Hertz of the early 20th century [4]. Ironically, however, while tremendous advances in science have resulted, the basic physics of striations remain at best incompletely understood. One has to draw a sharp distinction between striations in plasmas and those periodic structures associated with the Franck-Hertz experiment [4], which, like the closely related Holst-Oosterhuis luminous layers [9,10] or the periodic structures observed in the steady-state Townsend experiment [10], constitute the "natural" or "free" oscillations originating from inelastic collisions between electrons and gas atoms, and driven by an external, spatially homogeneous field [11,12]. Such oscillations, which have themselves only recently become well understood [2], differ in origin qualitatively and quantitatively from striations which derive from *internal*, self-consistent, spatially varying fields in a plasma consisting of both electrons and ions. Often these striations are modeled in terms of the "forced" oscillations of an electron swarm in an externally prescribed, spatially varying external field [6,13], which in some way is hoped to mimic the internal field in a plasma. Regardless of the validity of this idea, it is nevertheless true to say [1] that a theoretical framework for electron swarms, be it kinetic or fluid in nature, should be adaptable to the electron component of a plasma and, when coupled with a similar kinetic or fluid-based model of the ion component, should be capable of handling the internal, space-charge fields responsible for formation of striations. While a full kinetic theoretical treatment remains a formidable task, a fluid approach offers the possibility of a computationally economical, physically tenable analysis of a plasma. Thus, although striations *per se* are not directly within the scope of the present paper, the potential for wider application of the electron fluid model presented here is clear.

In this investigation we present a discussion (Sec. II) and fluid analysis (Sec. III) of the nonlocal response and resonances of electron properties in a gas subject to an externally prescribed spatially varying electrostatic field. By considering the latter to be a small perturbation superimposed upon a larger dc field, we obtain analytic expressions for the

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mean electron velocity, energy, and heat flux in a harmonically varying field [Secs. III C 1–III C 3], and give formulas for fields with more general spatial dependences [Sec. III D]. Illustrative numerical examples are given, highlighting nonlocality and resonance conditions.

II. NONLOCAL TRANSPORT, RESONANCES, AND SYNERGIES

Before outlining the fluid approach in detail, it is useful to discuss key physical phenomena from a physical perspective.

A. Nonlocal transport

Generally speaking, if the dimension l of the transfer mechanism—mean free path in collisional transport, or eddy size for turbulent transport [14]—is comparable with the scale length λ of the spatial variations of an externally imposed force field $F_z = -\partial_z\varphi(z)$ (taking only one spatial dimension z for simplicity), the response of the system may be *nonlocal*, with the flux of some physical property at point z linked to the gradient in concentration or potential φ at *different* points z', through a generalized transport relation

$$J(z) = -\int_{-\infty}^{\infty} K(z-z')\,\partial_z\varphi(z')dz', \tag{1}$$

where $K(z-z')$ is a transport kernel [14,15]. On the other hand, if $l \ll \lambda$, then $K(z-z') = K\delta(z-z')$ and Eq. (1) reduces to the familiar *local* transport relation, $J(z) = -K\partial_z\varphi$, where K is a constant transport coefficient. However, unusual, sometimes counterintuitive phenomena, may arise due to nonlocality, e.g., in boundary layer meteorology, the turbulent flux of an atmospheric property can be directed *up* the gradient ("countergradient" flow), from low to high concentrations, for large eddy turbulent transport within a forest canopy [14]. In partially ionized plasmas, the properties of the lighter *electron* component are actually determined by the mean free path for *energy transfer* l_ε, a somewhat larger quantity than l [see Eq. (8) below]. Hence if the plasma is subject to a sinusoidal electrostatic field $E_z = -\partial_z\varphi$ of wavelength $\lambda \sim l_\varepsilon > l$, electrons respond nonlocally while the heavier *ion* component, for which there is little distinction between the two types of mean free path, can be expected to respond locally.

B. Franck-Hertz oscillations

In the idealized form of the Franck-Hertz experiment, a prescribed *uniform* electric field E_0 drives a swarm of electrons from a localized plane source (the cathode) through an atomic gas of number density N. The quantized nature of the atoms leads to a discrete loss of energy ε_i in inelastic collisions (taking only one quantized level for simplicity), reflected in spatial oscillations in macroscopic properties, which occur only in a certain "window" of reduced fields E_0/N [2]. The wavelength $l_{FH} \approx \varepsilon_i/eE$ of this "natural" or "free" mode of vibration can be measured indirectly, as an I-V characteristic in the traditional form of the experiment [16], or directly using the photon flux technique [10]. Note that no such periodicity is expected to occur for ions because of their much greater mass.

C. Synergies

Whereas nonlocality may become important if the external field wavelength is close to the electron mean free path for energy exchange, i.e., $\lambda \sim l_\varepsilon$, *resonances* can be expected [6] if λ matches the natural frequency of oscillation of the system, i.e., $\lambda \sim l_{FH}$. In the interesting case when the three length scales are comparable,

$$\lambda \sim l_\varepsilon \sim l_{FH}, \tag{2}$$

there is clearly a possibility of a *synergy* between resonance and nonlocal effects, something which has attracted considerable interest in recent times [7,13].

III. FLUID EQUATIONS: FORMULATION AND SOLUTION FOR SPACE-VARYING FIELDS

A. General remarks on fluid equations

Physically based fluid equations (as distinct from the numerous *ad hoc* models which plague plasma physics [1]) have been formulated over many years for both ions and electrons [17], with closure representing the main obstacle to practical application. Recently a physically based, benchmarked heat flux ansatz has enabled satisfactory closure of the electron fluid equations [1]. We therefore begin by generalizing our earlier fluid treatment of electron swarms in a constant electric field [2] to space-dependent fields.

B. Electron fluid model

Consider for simplicity an infinite system of electrons of charge $-e$ whose properties vary along the z axis only and undergo neither ionization nor attachment. (The latter condition will be relaxed in subsequent articles.) In the steady state the equations of continuity, momentum, and energy balance are

$$\frac{\partial\Gamma_z}{\partial z} = 0, \tag{3a}$$

$$\frac{2}{3}\frac{\partial(n\varepsilon)}{\partial z} = -neE_z - n\,m\,\nu_m(\varepsilon)v_z, \tag{3b}$$

$$-\frac{1}{\nu_e}\left[v_z\frac{\partial\varepsilon}{\partial z} + \frac{2\varepsilon}{3}\frac{\partial v_z}{\partial z} + \frac{1}{n}\frac{\partial J_z}{\partial z}\right] = \varepsilon - \frac{3}{2}k_B T_g - \frac{1}{2}Mv_z^2 + \Omega(\varepsilon), \tag{3c}$$

respectively, where the meaning of the symbols is as follows: M and T_g are the molecular mass and temperature of the gas; k_B is Boltzmann's constant; n, v_z, and ε are the electron number density, average velocity in the z direction, and average energy; Γ_z and J_z are the electron particle and heat fluxes in the z direction; while the quantity $\Omega = \sum_i \varepsilon_i(\vec{\nu}_i - \overleftarrow{\nu}_i)/\nu_e$ accounts for inelastic processes i with threshold energies ε_i, and average collision frequencies for inelastic and superelastic collisions $\vec{\nu}_i$, $\overleftarrow{\nu}_i$, respectively. The latter may be neglected if $\varepsilon_i \gg k_B T_g$, as is the case for a noble gas. For the same reason all processes i may be considered as excitations of the ground state of the neutral atoms. The average elastic collision frequencies for momentum and energy transfer may be taken as simply $\nu_m(\varepsilon) \equiv N(2\varepsilon/m)^{1/2}\sigma_m(\varepsilon)$ and $\nu_e = (2m/M)\nu_m$, respectively, where $\sigma_m(\varepsilon)$ is the momentum transfer cross section evaluated at the average electron energy ε. However, the connection between $\vec{\nu}_i$ and inelastic cross sections is not so straightforward [2].

Equations (3a)–(3c) are closed using the established *ansatz* for heat flux [1,2]:

$$J_z = -\frac{23}{m}\frac{\partial}{\partial z}\left[\frac{n\xi(\varepsilon)}{\nu_m(\varepsilon)}\right] - \frac{(5-2p)}{3}\frac{neE_z\varepsilon}{m\nu_m(\varepsilon)} - \frac{5}{3}\Gamma_z\varepsilon, \quad (4)$$

in which

$$p = d\ln\nu_m/d\ln\varepsilon, \quad (5)$$

$$\xi = \alpha_0\varepsilon^2\left[1 + \frac{\Omega(\varepsilon)}{\varepsilon}\right]^{-r}, \quad (6)$$

where α_0 and r are adjustable parameters [2].

1. Unperturbed state

Initially the electrons are uniformly distributed in $-\infty < z < \infty$ and subject to a constant applied field, $E_z = -E_0$. The corresponding momentum and energy balance equations,

$$eE_0 = m\nu_m(\varepsilon_0)v_0, \quad (7a)$$

$$\varepsilon_0 = \frac{3}{2}kT_g + \frac{1}{2}Mv_0^2 - \Omega(\varepsilon_0), \quad (7b)$$

may be solved for the unperturbed mean velocity and energy, v_0 and ε_0, as functions of E_0/N for a given set of cross sections, without any reference to a heat flux ansatz. It can then be shown that $\frac{\varepsilon_0}{eE_0} = \frac{l_\varepsilon}{\sqrt{2}}$, where

$$l_\varepsilon \equiv \sqrt{\frac{M}{2m}}\beta\frac{1}{N\sigma_m} \quad (8)$$

is representative of the mean free path for energy transfer, and $\beta \equiv \sqrt{(1 - \frac{2\Omega(\varepsilon_0)}{Mv_0^2})}$.

Equations (7a) and (7b) lead to the following expressions for differential mobility and differential energy:

$$\mu_d \equiv \frac{\partial v_0}{\partial E_0} = \frac{\gamma}{\gamma + 2p}\frac{v_0}{E_0}, \quad \varepsilon_d \equiv \frac{\partial\varepsilon_0}{\partial E_0} = \frac{2}{\gamma + 2p}\frac{\varepsilon_0}{E_0}, \quad (9)$$

where the parameter

$$\gamma \equiv \frac{(1+\Omega')\varepsilon_0}{\varepsilon_0 + \Omega - \frac{3}{2}k_BT_g} \quad (10)$$

plays a key role in the description of periodic structures. (The quantity Ω' is the derivative of Ω with respect to the mean energy ε.) It may be evaluated from Eq. (9) using experimental drift velocity data according to the procedure outlined in [1], obviating the need to assume an empirical form of Ω, as has hitherto been the case [2], and greatly enhancing the accuracy of the fluid model.

2. Perturbed state

To facilitate physical understanding through analytic solution/expressions, we consider the case where a small-amplitude space-varying field $E_1(z)$ is now switched on, also in the z direction, giving a total field

$$E_z = -E_0 - E_1(z), \quad (11)$$

leading to a new steady state with perturbed mean velocity,

$$v_z(z) = v_0 + v_1(z), \quad (12)$$

where $|v_1(z)| \ll v_0$. Similarly, number density, mean energy, and heat flux are perturbed from their spatially uniform values $n_0,\ \varepsilon_0,\ J_0$ by small amounts $n_1(z),\ \varepsilon_1(z),\ \ J_1(z)$, respectively. However, the particle flux $\Gamma_z = n_0v_0$ remains constant, by virtue of Eq. (3a). The fluid equations (3a)–(3c) are then linearized in these small quantities and solved for the prescribed form of $E_1(z)$.

C. Solution of the fluid equations: Harmonic field

1. Sinusoidal perturbations

We first consider a sinusoidal field of wavelength λ and wave number $k = 2\pi\ /\lambda$, with corresponding sinusoidal perturbations in electron properties, i.e.,

$$E_1(z) = E_1\ e^{ikz},\ v_1(z) = v_1\ e^{ikz},\ \text{etc.} \quad (13)$$

The linearized equations for the complex amplitudes n_1, v_1, and ε_1 are

$$\frac{n_1}{n_0} = -\frac{v_1}{v_0}, \quad (14a)$$

$$(i\kappa + p)\tfrac{\varepsilon_1}{\varepsilon_0} + (1 - i\kappa)\tfrac{v_1}{v_0} = \tfrac{E_1}{E_0}, \quad (14b)$$

$$\tfrac{3}{2}i\kappa\left[\tfrac{\varepsilon_1}{\varepsilon_0} + \tfrac{2v_1}{3v_0} + \tfrac{J_1}{\Gamma\varepsilon_0}\right] + \gamma\tfrac{\varepsilon_1}{\varepsilon_0} - \tfrac{2v_1}{v_0} = 0, \quad (14c)$$

where

$$\kappa \equiv \frac{2}{3}\frac{\varepsilon_0}{eE_0}k = \frac{4\pi}{3\sqrt{2}}\frac{l_\varepsilon}{\lambda} \quad (15)$$

is a dimensionless quantity whose magnitude characterizes nonlocal effects. The heat flux perturbation is given by

$$\frac{J_1}{\Gamma_z\varepsilon_0} = Q(i\kappa)\frac{\varepsilon_1}{\varepsilon_0} + \bar{Q}(i\kappa)\frac{E_1}{E_0}, \quad (16)$$

where the "spectral heat fluxes" are defined as follows:

$$Q(i\kappa) = \frac{A + iB\kappa + C(i\kappa)}{i\kappa - 1} \quad (17)$$

and

$$\bar{Q}(i\kappa) = \frac{(5 - 2p - 3\alpha)i\kappa}{3(i\kappa - 1)}, \quad (18)$$

respectively. Other parameters were introduced in previous work [1,2]:

$$A = \frac{2p}{3}, \quad B = \alpha\bar{p} - \frac{5}{3}\left(1 + p - \frac{2p^2}{5}\right), \qquad C = -\alpha(\bar{p} - p - 1), \quad (19)$$

and

$$\bar{p} = \frac{\varepsilon_0\xi'(\varepsilon_0)}{\xi(\varepsilon_0)}, \quad \alpha = \frac{\xi(\varepsilon_0)}{\varepsilon_0^2}. \quad (20)$$

Equations (14a)–(14c) and (16) may be solved simultaneously to give

$$v_1(z) = \frac{-1 - \frac{3}{2\gamma} i\kappa[1 + Q(i\kappa) + (p + i\kappa)\bar{Q}(i\kappa)]}{D(i\kappa)} \mu_d E_1 e^{ikz} \equiv \bar{t}(i\kappa)\mu_d E_1 e^{ikz}, \tag{21}$$

$$\varepsilon_1(z) = \frac{-1 + [2 + 3\bar{Q}(i\kappa)]i\kappa + \frac{3}{4}\kappa^2 \bar{Q}(i\kappa)}{D(i\kappa)} \varepsilon_d E_1 e^{ikz} \equiv \bar{s}(i\kappa)\varepsilon_d E_1 e^{ikz}, \tag{22}$$

$$J_1(z) = \frac{Q(i\kappa)(i\kappa - 2) + \bar{Q}(i\kappa)[p(i\kappa - 2) + \gamma(i\kappa - 1) + \frac{i\kappa}{2}(5i\kappa - 7)]}{D(i\kappa)} \frac{\Gamma_z \varepsilon_d}{2} E_1 e^{ikz}, \tag{23}$$

where

$$D(i\kappa) = -1 + \frac{i\kappa(p+\gamma)}{2p+\gamma} + \frac{i\kappa}{2\,(2p+\gamma)}[-7 + 3Q(i\kappa)\,(i\kappa - 1) + 5i\kappa]. \tag{24}$$

The functions $\bar{t}(i\kappa)$, $\bar{s}(i\kappa)$, and $\bar{h}(i\kappa)$ are effectively defined by (21)–(23). We focus on velocity perturbations and explore the properties of the "transmission function" $\bar{t}(i\kappa)$.

The quantity $D(i\kappa)$ in the denominator of the terms on the right-hand side of (21)–(23) has appeared previously in the fluid model of the *uniform* field E_0, idealized Franck-Hertz experiment [2], where spatial variations are produced by a localized source (the cathode). In that case, the argument of D has both real and imaginary parts κ_r and κ_i, respectively, which are determined by the solvability condition

$$D(i\kappa_i + \kappa_r) = 0. \tag{25}$$

Furthermore, it can be shown [2] that there is a range ("window") of E_0/N for which the real part κ_r of the solution of (25) is small and negative, and at the same time the imaginary part of the zero has the property $\frac{3}{2}\frac{eE_0}{\varepsilon_0}\kappa_i \approx \frac{2\pi}{l_{FH}}$, where $l_{FH} = \varepsilon_i/eE_0$ is the Franck-Hertz wavelength. Hence it follows that

$$D(i\kappa_i) \approx 0 \tag{26}$$

for E_0/N inside the window.

2. Resonances and nonlocality

For the present source-free problem in unbounded space, where κ is a *prescribed, real* quantity, the argument of $D(i\kappa)$ is purely imaginary. Hence while $D(i\kappa)$ can never vanish *exactly*, by virtue of (26) it can be small if $\kappa \approx \kappa_i$, and in that case

$$k = \frac{2\pi}{\lambda} = \frac{3}{2}\frac{eE_0}{\varepsilon_0}\kappa \approx \frac{3}{2}\frac{eE_0}{\varepsilon_0}\kappa_i \approx \frac{2\pi}{l_{FH}}. \tag{27}$$

In other words, the denominators of the terms on right-hand sides of Eqs. (21)–(23) are all small if

$$\lambda \approx l_{FH}, \tag{28}$$

that is, when the wavelength of the applied field matches the system's natural "Franck-Hertz" wavelength, the quantities defined by Eqs. (21)–(23) are all very large and a *resonance* situation prevails. It is to be emphasized that there is only *one* natural frequency and consequently, there can be only *one* resonance condition (28) in any *linear* treatment. This point is discussed further below.

If $\kappa \neq 0$, $\mathrm{Im}[\bar{t}(i\kappa)] \neq 0$, and Eq. (21) shows that $v_1(z)$ generally differs in phase from $E_1(z)$. Thus cause (the field) and effect (the velocity) are displaced in space, and this phase shift can be taken as a measure of nonlocality. One expects that the phase shift and nonlocality will be appreciable if $\kappa \approx l_\varepsilon/\lambda \sim 1$. However, in the limit $\kappa \approx l_\varepsilon/\lambda \ll 1$, it is clear from Eqs. (21) and (24) that $\bar{t}(i\kappa) \approx 1$, $\mathrm{Im}[\bar{t}(i\kappa)] \approx 0$, and hence Eq. (21) reduces to the expected *local* relation $v_1 \approx \mu_d E_1$.

Similar comments apply to the mean energy and heat flux, Eqs. (22) and (23), respectively.

3. Numerical examples

It is clear from the above discussion that for wavelengths satisfying Eq. (2), both resonant and nonlocal effects can occur simultaneously, and Figs. 1(a) and 1(b) show just such a situation. Figure 1(a) illustrates the situation using the benchmark model of Ref. [2], with $M = 4$ amu, $T_g = 293$ K, and constant elastic and inelastic cross sections $\sigma_m = 6$ Å^2 and $\sigma_i = 0.1$ Å^2, respectively, the latter having a threshold energy of $\varepsilon_i = 2$ eV. Figure 1(b) is for neon gas at $T_g = 293$ K, using elastic, inelastic, and ionization cross sections provided by Hayashi [18], with ionization treated approximately as just another inelastic event. (A more accurate treatment of ionization would require reformulation of the fluid equations, something that is left for future work.) Differential mobility and differential energy in each case were evaluated from drift velocity and energy swarm data calculated using a numerical solution of the spatially homogeneous Boltzmann equation (see [19] for details). We could also have used measured swarm transport data to evaluate these differential properties for neon. Figures 2(a) and 2(b) show how resonances occur within the Franck-Hertz "window" for the same model.

D. Solution of the fluid equations: General spatially dependent fields

Equations (13) and (14) effectively constitute a Fourier transformation of the fluid equations, and hence the corresponding solutions for the general z-dependent field $E_1(z)$ can be found from the results for a sinusoidal field, simply by

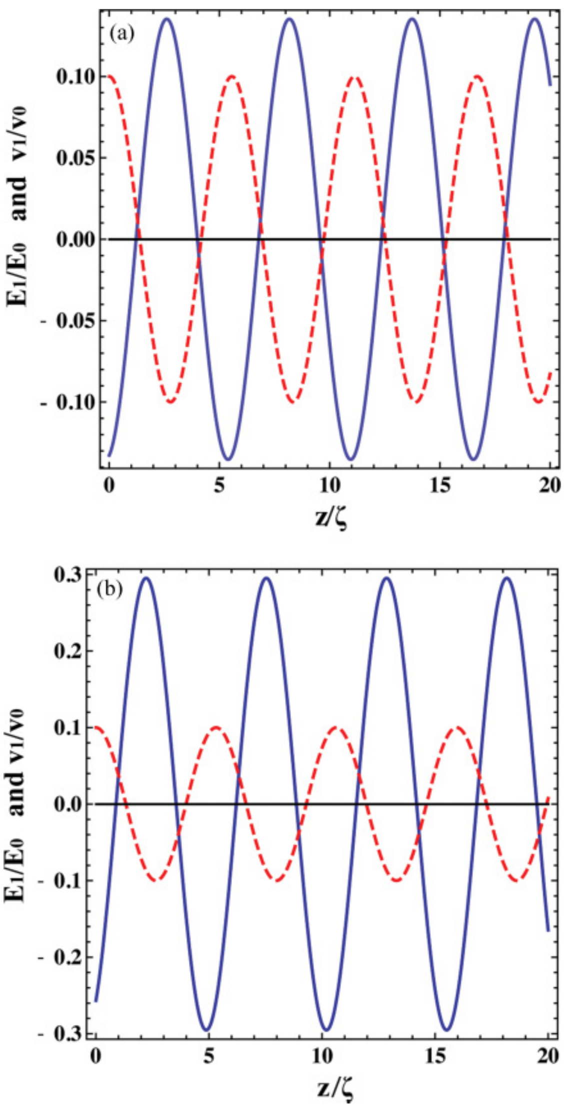

FIG. 1. (Color online) (a) The mean electron velocity (21) (solid curve) for a sinusoidal perturbing field (dashed curve) of amplitude 0.5 Td, superimposed on a constant uniform field of $E_0/N = 5$ Td, with the resonant wavelength $\sim$5.5 ζ, where $\zeta = (2^{1/2}N\sigma_0)^{-1}$ and $\sigma_0 = 10^{-20}$ m^2. The collision model is the same as in Ref. [2], and is described in the text (1 Td = 1 townsend = 10^{-21} V m^2). (b) The mean electron velocity (21) (solid curve) for a sinusoidal perturbing field (dashed line) of amplitude 6 Td, superimposed on a constant uniform field of $E_0/N = 60$ Td, with the resonant wavelength $\sim$5.5 ζ. The collision model is for neon gas as described in the text.

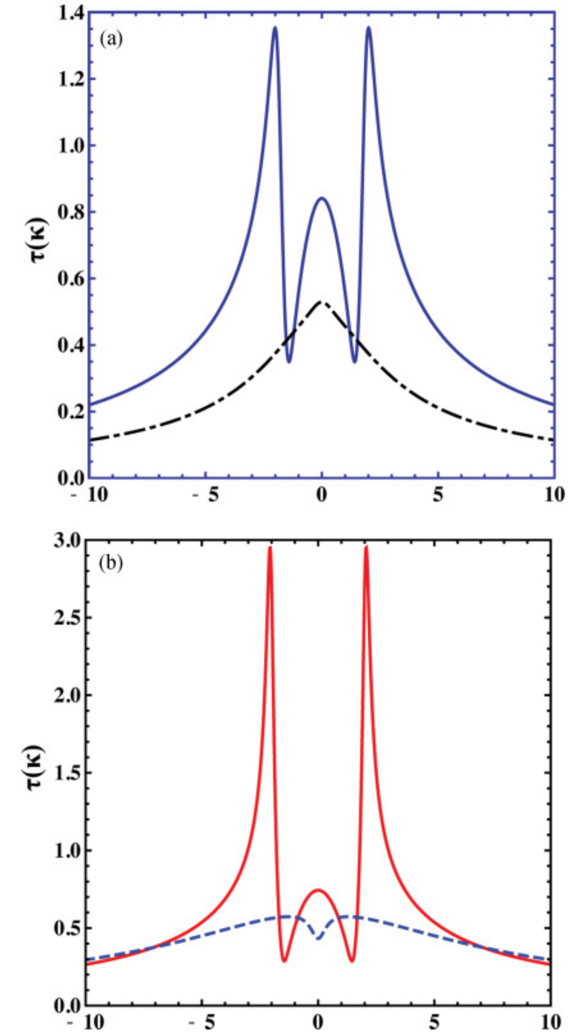

FIG. 2. (Color online) (a) The function $\tau(\kappa) = \frac{\gamma}{\gamma+2p}|\bar{t}(i\kappa)|$ for values of E_0 /N lying below (0.5 Td, dashed line) and within (5 Td, solid line) the Franck-Hertz "window" corresponding to the collision model of Fig. 1(a). (b) The function $\tau(\kappa) = \frac{\gamma}{\gamma+2p}|\bar{t}(i\kappa)|$ for values of E_0 /N lying below (0.5 Td, dashed curve) and within (60 Td, solid curve) the Franck-Hertz "window" corresponding to the collision model for neon of Fig. 1(b). Resonance occurs when the wavelength of the field matches the Franck-Hertz wavelength, at the values of κ given by Eq. (15).

replacing the constant E_1 with

$$E_1(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_1(z)e^{-ikz}dz, \tag{29}$$

and Eqs. (21)–(23) by integrals over k, e.g.,

$$v_1(z) = \mu_d \int_{-\infty}^{\infty} \bar{t}\left(\frac{2}{3}\frac{\varepsilon_0}{eE_0}ik\right) E_1(k)e^{ikz}dk. \tag{30}$$

Using the convolution theorem, this can be written in the form of a general nonlocal transport relation (1),

$$v_1(z) = \int_{-\infty}^{\infty} K(z-z')E_1(z')dz', \tag{31}$$

where

$$K(z) \equiv \frac{1}{2\pi}\mu_d \int_{-\infty}^{\infty} \bar{t}\left(\frac{2}{3}\frac{\varepsilon_0}{eE_0}i\kappa\right) e^{ikz}dk. \tag{32}$$

A space-dependent field generally produces nonlocal effects arising from phase shifts associated with its various Fourier components (29). Figures 3(a) and 3(b) show the situation for a perturbing Gaussian disturbance $E_1(z)$ using the same collision models as in Figs. 1(a) and 1(b). The picture is similar to that obtained by Sigeneger and Winkler [13]

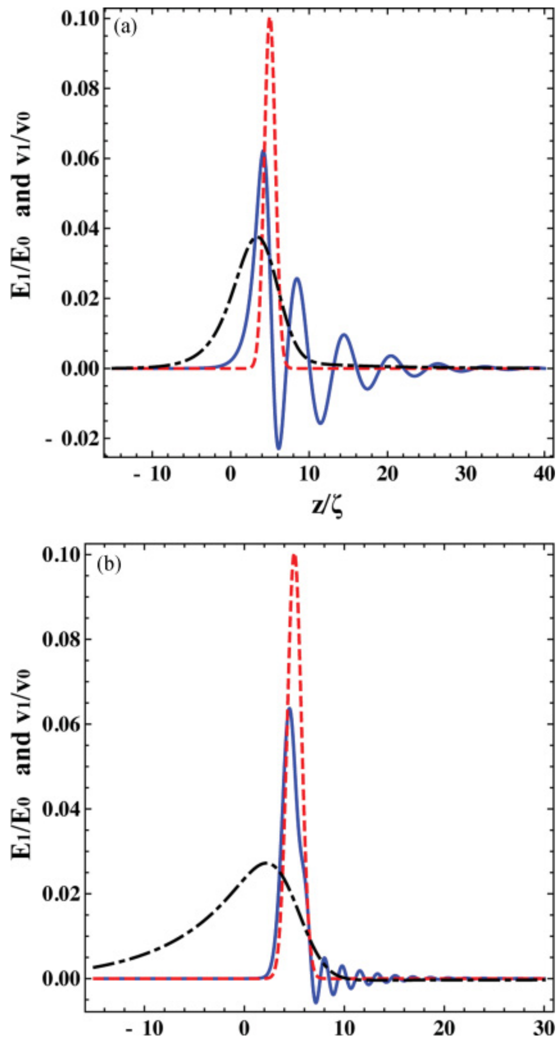

FIG. 3. (Color online) (a) A narrow Gaussian perturbation in the field centered at $z = 5\zeta$ produces pronounced nonlocal and resonant effects if $E_0\ /N$ lies within the "Franck-Hertz window" for the collision model of (a) Fig. 1(a) and (b) Fig. 1(b). Similar results for neon have been obtained by Sigeneger and Winkler [13] via solution of the Boltzmann equation for an arbitrarily large Gaussian pulse. The chain curve is the response for $E_0\ /N$ below the window.

in a rigorous numerical solution of Boltzmann's equation for electrons in a large-amplitude Gaussian pulsed field $E(z)$. In the special case where the dominant contribution is from wave numbers such that $\kappa = \frac{2}{3}\frac{\varepsilon_0}{eE_0}k = \frac{l_\varepsilon}{\lambda} \ll 1$, Eq. (21) shows that $\bar{t} \approx 1$ and hence Eq. (32) gives $K(z) \approx \frac{1}{2\pi}\mu_d \int_{-\infty}^{\infty} e^{ikz}dk = \mu_d\ \delta(z)$. Equation (31) then reduces to the local relation $v_1(z) = \mu_d E_1(z)$, as expected. Similarly, if the width of the Gaussian pulse is increased sufficiently we obtain local behavior, as has been verified by numerical calculations.

IV. CONCLUDING REMARKS

In this paper we have generalized a physically based and benchmarked fluid model to consider electron swarms under the influence of perturbative nonuniform electric fields $E_1(z)$. The key results arising from this paper are:

(1) Analytic expressions have been derived for physical quantities in the case of harmonically varying fields, in which the origin of nonlocality and resonances, and the interplay between the two effects, is now transparent.

(2) This formalism for harmonic fields enables us to generalize the expressions to the case of arbitrary space-varying fields. Results of numerical calculations have been given for Gaussian space dependences for model and real gases.

These results are consistent with the previous Boltzmann equation analyses, though the fluid model offers the distinct advantage of offering more physical insight with far less computational effort.

Following the same rationale of the previous kinetic work, the next logical phase would be to adapt the current formalism of this paper to the electron component of a *plasma* and consider $E_1(z)$ to be now the *internal*, self-consistent space-charge field, to be found through simultaneous solutions of Poisson's equation and the fluid equations for *both* electrons and ions. Nonlocality and resonance effects will then be accompanied by screening effects, as characterized by an additional intrinsic scaling parameter, the Debye length l_D. However, further progress depends upon development of a corresponding set of physically based and benchmarked ion fluid equations, suitable for practical application to plasmas. Reference [1] sets out such general equations, which, however, need to be adapted to a form suitable for practical application to plasmas.

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