Oscillation death in asymmetrically delay-coupled oscillators

Wei Zou,[1,2,3,*] Yang Tang,[2,3,4] Lixiang Li,[3,5] and Jürgen Kurths[2,3,6]
[1]*School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China*
[2]*Institute of Physics, Humboldt University Berlin, Berlin D-12489, Germany*
[3]*Potsdam Institute for Climate Impact Research, Telegraphenberg, Potsdam D-14415, Germany*
[4]*Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150080, China*
[5]*Information Security Center, Beijing University of Posts and Telecommunications, Beijing 100876, China*
[6]*Institute for Complex Systems and Mathematical Biology, University of Aberdeen, Aberdeen AB24 3FX, United Kingdom*
(Received 19 January 2012; published 9 April 2012)

Symmetrically coupled oscillators represent a limiting case for studying the dynamics of natural systems. Therefore, we here investigate the effect of coupling asymmetry on delay-induced oscillation death (OD) in coupled nonlinear oscillators. It is found that the asymmetrical coupling substantially enlarges the domain of the OD island in the parameter space. Specifically, when the intensity of asymmetry is enhanced by turning down the value of the coupling asymmetry parameter α, the OD island gradually expands along two directions of both the coupling delay and the coupling strength. The expansion behavior of the OD region is well characterized by a power law scaling, $R = \alpha^{\gamma}$ with $\gamma \approx -1.19$. The minimum value of the intrinsic frequency, for which OD is possible, monotonically decreases with decreasing α and saturates around a constant value in the limit of $\alpha \to 0$. The generality of the conducive effect of coupling asymmetry is confirmed in a numerical study of two delay-coupled chaotic Rössler oscillators. Our findings shed an improved light on the understanding of dynamics in asymmetrically delay-coupled systems.

DOI: 10.1103/PhysRevE.85.046206 PACS number(s): 05.45.Xt, 87.10.−e

I. INTRODUCTION

The complex dynamics of a large number of coupled nonlinear oscillators has formed an interesting research topic recently, which aims to efficiently describe the collective activities emerged in nature [1–3]. Oscillation death (OD), an intriguing phenomenon of oscillation quenching due to mutual interactions of oscillatory systems, has attracted growing attention among researchers in the field of nonlinear dynamics. The OD phenomenon has been widely observed in various real systems, such as the Belousov-Zhabotinsky reaction [4], relativistic magnetrons [5], or synthetic genetic networks [6–9]. Initially OD has been analytically proven to be impossible in coupled identical oscillators [10]. In 1998, Reddy *et al.* [11] showed that OD could even appear in coupled identical limit-cycle oscillators if the coupling contained a time delay, which was termed as "death by delay" [12]. Recently researchers have exploited some other coupling mechanisms which could be used to produce a stable OD state in coupled oscillators. Moreover, a series of coupling methods have been introduced such as dynamic coupling [13], conjugate coupling [14,15], nonlinear coupling [16], and indirect coupling [17].

Since the works of Reddy *et al.* on delay-induced OD [11,18], OD has received increasing interest in the past decade. Considerable novel efforts on both experimental and theoretical explorations have been made. For example, in experimental studies, OD induced by a delay was already observed in electronic circuits [19], lasers [20], and thermo-optical oscillators [21]. The theoretical developments were focused mainly on OD induced by various modified forms of delayed coupling in coupled Hopf oscillators such as distributed delays [22], partial time-delay coupling [23], gradient time-delay coupling [24], two long-time delays [25], a time-varied delay [26], integrative time-delay coupling [27], and phase-dependent delayed coupling [28]. In addition, delay-induced OD in coupled chaotic oscillators also has been systematically explored in two oscillators [29,30], a one-way ring network [31], and complex networks [32].

Delay-induced OD has been investigated from various aspects. It is to be noted that the majority of past studies have been hitherto confined only to symmetrical (or homogeneous) coupling. Such a consideration of perfectly symmetrical interaction in coupled oscillators mainly came from the idea that it is convenient for both theoretical investigations and numerical calculations. However, such an ideal assumption is generally considered to be an exception which is often not fulfilled in real coupled-oscillator systems. Indeed, coupling asymmetry arises naturally in various realistic systems. In an ecosystem, the interactions between two coupled predator-prey systems are in general asymmetrical due to their dependence on the density differences of these two population paths [33]. Physiological membranes that selectively diffuse ions reasonably result in asymmetrical diffusion [34]. For a pair of coupled pendula, the coupling asymmetry originates from the dependence of the coupling strength on the mass ratio between both pendula [2].

Previous studies have already indicated that in the presence of coupling asymmetry the dynamics of coupled systems could be in a different situation. In two coupled dynamical systems, a host of novel dynamical effects induced by asymmetrical coupling have been reported so far. For example, the coupling asymmetry is shown to change bifurcation scenarios of desynchronization [35], enhance anomalous phase synchronization [36], and render chaos suppression [37]. Crucial effects of asymmetrical coupling on the synchronization of two interacting, spatially extended chaotic fields have been well addressed by Bragard *et al.* [38–40]. In the aspect of

*zouwei2010@mail.hust.edu.cn

experiments, scaling behaviors of the onset of oscillations in two asymmetrically delay-coupled lasers and optoelectronic oscillators were generically observed [41,42].

To the best of our knowledge, however, delay-induced OD in asymmetrically coupled oscillators has not been studied until now. Therefore, with this simple but not trivial motivation, our intention in this paper is to elucidate the effects of asymmetrical coupling on delay-induced OD in coupled oscillators. At a first glance, one may intuitively conjecture that asymmetrical coupling will inhibit delay-induced OD as the symmetry of the coupling is broken. Nevertheless, via an analytical and numerical approach, our research in this paper gives an intriguing opposite result: the asymmetrical coupling facilitates delay-induced OD.

This paper is outlined as follows. In Sec. II, the studies are first conducted in the context of coupled Stuart-Landau limit-cycle oscillators. The Stuart-Landau model represents a normal form describing dynamics near a supercritical Hopf bifurcation [1]. For this model, both numerical studies and analytical analyses are carried out in detail. Moreover, the derived results are expected to be applicable at least for ensembles of oscillators near a Hopf bifurcation. We find that the asymmetrical coupling is beneficial to delay-induced OD. The presence of asymmetry in delay-coupled systems enhances the onset of the OD regime in the parameter space. A brief numerical study of two delay-coupled chaotic Rössler oscillators further confirms the conducive role of coupling asymmetry in coupled chaotic systems. Finally, in Sec. III we summarize our results and discuss implications of our findings for applications in biological systems. Numerical integrations in this paper are performed under random initial conditions where the fourth-order Runge-Kutta method with an integration step 0.001 is adopted.

II. RESULTS AND OBSERVATIONS

Let us consider the following two delay-coupled Stuart-Landau oscillators:

$$\begin{aligned} \dot{z}_1 &= (1 + iw_1 - |z_1|^2)z_1 + \alpha K[z_2(t-\tau) - z_1(t)], \\ \dot{z}_2 &= (1 + iw_2 - |z_2|^2)z_2 + K[z_1(t-\tau) - z_2(t)], \end{aligned} \tag{1}$$

where z_1 and z_2 are complex amplitudes of the oscillators with natural frequencies of w_1 and w_2, respectively, $K \geqslant 0$ is the coupling strength, τ is the propagation delay, and α ($0 \leqslant \alpha \leqslant 1$) accounts for asymmetry in the coupling. By adjusting α, two extreme cases exist: one is the unidirectional drive-response architecture for $\alpha = 0$, and the other is the bidirectional asymmetrical coupling configure for $\alpha = 1$. The coupling schemes are schematically shown in Fig. 1. It is interesting to change the value of α and probe the dynamics of the coupled system (1).

Without coupling (i.e., $K = 0$) in system (1), each uncoupled oscillator has an unstable focus at the origin ($z_1 = z_2 = 0$) and a stable limit cycle at $|z_1| = |z_2| = 1$ on which it moves with the intrinsic frequencies w_1 and w_2, respectively. When OD occurs, if the coupling is switched on (i.e., $K > 0$), the original limit cycles are completely lost and collapse to the origin. To identify an OD state, the coupled system (1) is linearized at the zero solution ($z_1 = z_2 = 0$). The linearized system reads as follows:

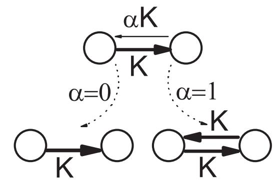

FIG. 1. Schematic of the coupling configuration employed in this paper. The asymmetry parameters $\alpha = 0$ and $\alpha = 1$ recover the unidirectional drive response and the bidirectional symmetrical coupling schemes, respectively.

$$\begin{aligned} \dot{\delta z_1} &= (1 + iw_1)\delta z_1 + \alpha K[\delta z_2(t-\tau) - \delta z_1(t)], \\ \dot{\delta z_2} &= (1 + iw_2)\delta z_2 + K[\delta z_1(t-\tau) - \delta z_2(t)]. \end{aligned} \tag{2}$$

Making the ansatz $\delta z_i \propto e^{\lambda t}$, the following eigenvalue matrix M of Eq. (2) is obtained:

$$M = \begin{pmatrix} 1 + iw_1 - \alpha K & \alpha K e^{-\lambda\tau} \\ K e^{-\lambda\tau} & 1 + iw_2 - K \end{pmatrix}, \tag{3}$$

where λ is the eigenvalue of M. The OD state is asymptotically stable if and only if all the eigenvalues of matrix M have a strictly negative real part. Since Re[Tr(M)]$= 2 - (1+\alpha)K$, a necessary condition for stability is

$$\mathrm{Re}[\mathrm{Tr}(M)] \leqslant 0 \Leftrightarrow K \geqslant \frac{2}{1+\alpha}. \tag{4}$$

All the eigenvalues of matrix M are governed by the characteristic equation as follows:

$$(1 + iw_1 - \alpha K - \lambda)(1 + iw_2 - K - \lambda) - \alpha K^2 e^{-2\lambda\tau} = 0, \tag{5}$$

which has infinitely many roots for $\tau \neq 0$, but only a finite number in any given stripe on the complex plane [43]. The OD stability depends on the sign of the real part of the rightmost root of Eq. (5), which generally cannot be explicitly derived.

For two special cases, the characteristic roots of Eq. (5) have simple forms: (i) For $\alpha = 0$, we have two eigenvalues: $1 + iw_1$ and $1 + iw_2 - K$. Clearly the OD state is unstable for all the values of coupling delay τ and coupling strength K. This can be easily understood from the drive-response coupling scheme for $\alpha = 0$, where the uncoupled drive oscillator always oscillates. (ii) If $w_1 = w_2 = w$ and $\tau = 0$, we get the two eigenvalues: $1 + iw$ and $1 + iw - (1+\alpha)K$. Obviously, the OD state is also unstable for all the values of asymmetry parameter α and coupling strength K, which implies that OD is impossible in asymmetrically coupled identical oscillators without a delay.

To quantitatively discover how asymmetrical coupling affects delay-induced OD, we first consider the case of coupled oscillators with identical frequencies $w_1 = w_2 = w$. This is a more stringent case for the OD stability, as frequency mismatch can induce OD even without any delay, whereas identical frequencies fail to do so [10]. It is easy to verify that $\lambda = 0$ is not a solution of Eq. (5). Thus the stability of the origin may be switched only when the rightmost eigenvalue λ crosses transversally the imaginary axis. The OD state is then achieved via a Hopf bifurcation from a periodic solution.

For the critical situation $\lambda = i\lambda_I$ ($\lambda_I \neq 0$), substituting this into the characteristic equation (5) and separating the real and imaginary parts yields

$$\begin{aligned}(\lambda_I - w)^2 - (1-K)(1-\alpha K) + \alpha K^2 \cos(2\lambda_I \tau) = 0,\\ (\lambda_I - w)[2-(1+\alpha)K] - \alpha K^2 \sin(2\lambda_I \tau) = 0.\end{aligned} \tag{6}$$

After some algebraic manipulations of Eq. (6), we further arrive at the following two equations:

$$\begin{aligned}\lambda_I &= w \pm \sqrt{\frac{-a^2 - b^2 + \sqrt{(a^2-b^2)^2 + c^2}}{2}} \equiv w \pm A,\\ \cos(\lambda_I \tau) &= \sqrt{\frac{1}{2} + \frac{(a+b)^2 - \sqrt{(a^2-b^2)^2+c^2}}{2c}} \equiv B,\end{aligned} \tag{7}$$

where $a = (1-K)$, $b = (1-\alpha K)$ and $c = 2\alpha K^2$. Then by properly choosing the signs in the inversion of cosine function with a similar strategy as in Refs. [11,18], the following two critical curves bounding the OD region are obtained:

$$\tau_1 = \frac{\cos^{-1} B}{w - A}, \qquad \tau_2 = \frac{\pi - \cos^{-1} B}{w + A}, \tag{8}$$

which reduce to the results derived by Reddy *et al.* [11] when $\alpha = 1.0$.

By numerically plotting the two critical curves τ_1 and τ_2 of Eq. (8), the stable regions of OD in the parameter plane of (τ, K) are shown in Fig. 2(a) for $\alpha = 1.0$, 0.5, 0.3, and 0.2, respectively. $w = 10$ is fixed. All the OD regions are independently tested by our numerical integrations of the coupled system (1). Interestingly, from Fig. 2(a), we find that the OD island expands both along the τ and the K directions as the asymmetry parameter α gradually decreases; and the smaller α, the larger the OD island is.

To quantify the size-growing phenomenon depicted in Fig. 2(a), we introduce a normalized size ratio $R = S_\alpha / S_{\alpha=1}$, where S_α denotes the area of the OD island for α and $S_{\alpha=1}$ for $\alpha = 1$. Obviously $R = 0$ for $\alpha = 0$ and $R = 1$ for $\alpha = 1$. For $0 < \alpha < 1$, by counting the number of the data within the OD region, the value of R is numerically calculated and shown in Fig. 2(b). The squares represent the numerical results. We find that R monotonically increases as the asymmetry parameter α decreases from $\alpha = 1$, and grows sharply as α approaches zero. This behavior is well characterized by the power law scaling,

$$R = \alpha^\gamma, \tag{9}$$

where $\gamma \approx -1.19$. The power law relation of Eq. (9), which is plotted by the red line in Fig. 2(b), can be clearly seen from a perfect log-log fit shown in the inset of Fig. 2(b). The above observations are completely distinct from our initial intuition that the system's oscillatory behavior is gradually affected by the asymmetrical coupling as α increases from zero.

The asymmetrical coupling further facilitates a delay-induced OD through the intrinsic frequency w. Equation (8) shows that the frequency w is involved in the OD critical curves. Thus the size of the OD region definitely depends on the value of w. Figure 3(a) displays several OD islands for different values of w, where $\alpha = 0.2$ is fixed. The OD region monotonically decreases with a decreasing of w and vanishes completely below a certain threshold $w_{\min}(\alpha)$. These findings hold for other values of $\alpha > 0$ as well, and have been already reported in the case of symmetrical coupling for $\alpha = 1$. For the asymmetrical coupling case of $\alpha = 1$, the critical frequency $w_{\min}(\alpha = 1)$, numerically found by Reddy *et al.* [18], is 4.182, which has been analytically defined by Song *et al.* [44]. Due to the introduction of coupling asymmetry, the coupled system (1) can experience an OD state even when w is smaller than this value. As seen in Fig. 3(a), the OD island survives even for $w = 4$ if $\alpha = 0.2$, which implies the following relation: $w_{\min}(\alpha = 0.2) < w_{\min}(\alpha = 1)$.

Figure 3(b) further depicts the dependence of $w_{\min}(\alpha)$ on α for $0 < \alpha \leqslant 1$. It can be seen that $w_{\min}(\alpha)$ monotonically decreases as α decreases from $\alpha = 1$, and saturates around 1.57 for $\alpha \to 0$. The black squares stand for the results, which are numerically calculated for both curves of Eq. (8) by decreasing

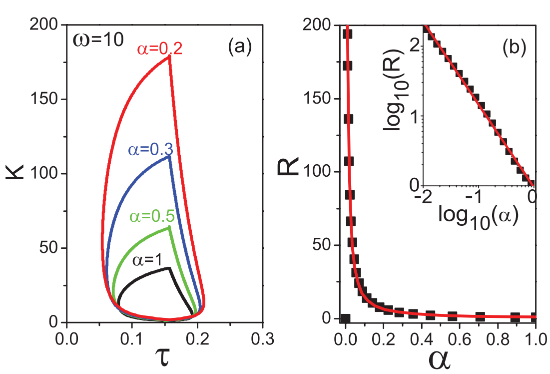

FIG. 2. (Color online) (a) The OD islands of the two coupled identical oscillators of Eq. (1) for the asymmetry parameters $\alpha =$ 1.0, 0.5, 0.3, and 0.2, respectively. $\omega_1 = \omega_2 = \omega = 10$ is fixed. The OD island expands along both the horizontal and vertical directions. (b) The normalized size ratio R vs the asymmetry parameter α, with the data numerically computed from (a) (solid square points) and the fit (red solid line), $R = \alpha^\gamma$ with $\gamma \approx -1.19$. The upper-right inset shows the fit in the log-log plot.

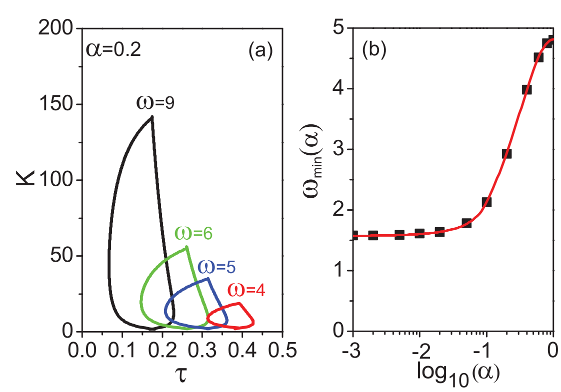

FIG. 3. (Color online) (a) The OD islands of the two coupled identical oscillators of Eq. (1) for $w = 9$, 6, 5, and 4, respectively. $\alpha = 0.2$ is fixed. The OD island gradually shrinks as the frequency w decreases, and completely disappears below a certain threshold $w_{\min}(\alpha)$. (b) The smallest threshold $w_{\min}(\alpha)$ of w, for which OD is possible, monotonically decreases as the asymmetrical ratio α decreases from $\alpha = 1$, and saturates at around 1.57 for a sufficiently small value of α. The solid square points present the numerical result, which is well predicted by Eq. (10) (the red solid line).

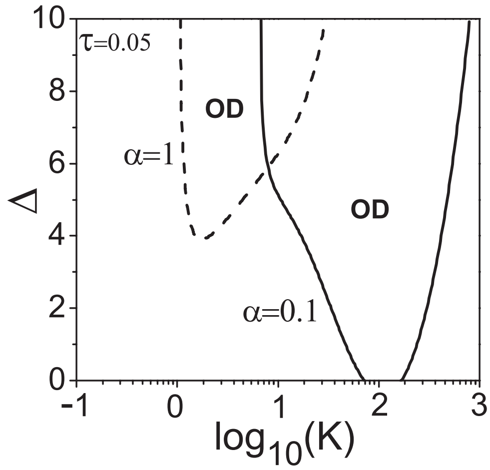

FIG. 4. Stability regions of an OD state in the two coupled nonidentical oscillators of Eq. (1) with frequencies $w_1 = 10 + \Delta/2$ and $w_2 = 10 - \Delta/2$; $\tau = 0.05$ is fixed. The OD region extends downward to $\Delta = 0$ from $\alpha = 1$ (area enclosed by the dashed lines) to $\alpha = 0.1$ (area enclosed by the two solid lines).

w at a fixed value of α until the intersected area is extinct. In fact, from the intersection condition of the two curves in Eq. (8), the value of $w_{\min}(\alpha)$ can be theoretically predicted as

$$w_{\min}(\alpha) = \min\left\{\frac{\pi A}{\pi - 2\cos^{-1} B}, K > \frac{2}{1+\alpha}\right\}, \tag{10}$$

where A and B are given in Eq. (7). For $\alpha = 1$, Eq. (10) degenerates to the same form as that in Ref. [44]. This prediction is plotted with the red solid line in Fig. 3(b), which shows a good agreement with the previous numerical calculations.

The qualitatively similar situation holds even if the intrinsic frequencies are not identical ($w_1 \neq w_2$). To illustrate the beneficial effect of asymmetrical coupling on delay-induced OD in coupled nonidentical oscillators, Fig. 4 compares the stable OD regions for $\alpha = 1$ and $\alpha = 0.1$ on the $[\log_{10}(K), \Delta]$ panel, respectively. The value of Δ describes the mismatch of the frequencies as $w_1 = 10 + \Delta/2$ and $w_2 = 10 - \Delta/2$. $\tau = 0.05$ is chosen. We find that the OD region grows and extends toward the $\Delta = 0$ axis for a finite range of the coupling strength K. This demonstrates that the asymmetrically coupled system suffers OD for a small or even zero mismatch of the frequencies with a smaller time delay τ. The above observation is also consistent with previous results for the case of identical frequencies in Fig. 2(a), where the OD island is clearly shown to expand along the τ direction. The OD regions in Fig. 4 are obtained by numerically computing the rightmost root of Eq. (5) with a negative real part, which is independently confirmed by the numerical integrations of the coupled system (1).

The beneficial influence of asymmetrical coupling on delay-induced OD is not limited to just a limit-cycle model but also can extend to chaotic oscillators. For example, here we study two delay-coupled chaotic Rössler oscillators:

$$\begin{aligned}
\dot{x}_1 &= -y_1 - z_1,\\
\dot{y}_1 &= x_1 + a y_1 + \alpha K[y_2(t-\tau) - y_1(t)],\\
\dot{z}_1 &= b + z_1(x_1 - c),
\end{aligned}$$

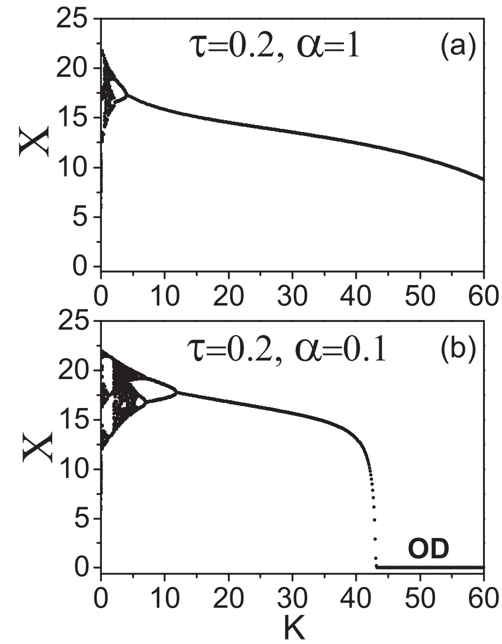

FIG. 5. Bifurcation diagrams obtained by plotting the local maxima of $X = \frac{x_1+x_2}{2}$ for the two coupled chaotic Rössler oscillators in Eq. (11). With increasing the coupling strength K, the coupled system (11) experiences a reverse period-doubling bifurcation from chaos to (a) one cycle if $K > 4.2$ for $\alpha = 1.0$, and (b) an OD state if $K > 43.5$ for $\alpha = 0.1$. $\tau = 0.2$ is fixed.

$$\begin{aligned}
\dot{x}_2 &= -y_2 - z_2,\\
\dot{y}_2 &= x_2 + a y_2 + K[y_1(t-\tau) - y_2(t)],\\
\dot{z}_2 &= b + z_2(x_2 - c),
\end{aligned} \tag{11}$$

where $a = b = 0.1$ and $c = 14$. With these parameters, each uncoupled oscillator evolves chaotically and has an unstable focus near the origin, given as $P = (x^*, y^*, z^*)$ with $x^* = -a y^*$, $y^* = -z^*$, and $z^* = \frac{c - \sqrt{c^2 - 4ab}}{2a}$. The occurrence of OD in the coupled system (11) refers to the fact that the unstable fixed point P is stabilized by the delayed interaction. The case of a symmetrically coupled system (11) ($\alpha = 1$) has already been numerically studied by Prasad [29], who found that the phenomenon of delay-induced OD in coupled chaotic oscillators is quite general. To highlight the superiority of asymmetrical coupling on delay-induced OD in coupled chaotic oscillators, we intentionally set $\tau = 0.2$. Note that here OD is impossible when $\alpha = 1$ [29]. Figure 5(a) plots the bifurcation diagram of the coupled system (11) for $\alpha = 1$, which shows that with increasing the coupling strength K, the system (11) undergoes a reverse period-doubling sequence, leading to one cycle for $K > 4.2$. For the asymmetrical coupling with $\alpha = 0.1$, we find that after a reverse period-doubling cascade from chaos to one cycle at $K \approx 11.8$, with a further increase in K the coupled system (11) achieves OD via a Hopf bifurcation at $K \approx 43.5$ [see Fig. 5(b)]. This observation clearly demonstrates that the asymmetrical coupling facilitates delay-induced OD in coupled chaotic oscillators.

It is notable that qualitatively similar results are obtained in our numerical experiments with other system parameters, coupling forms, and other chaotic oscillator models such as the Lorenz oscillator and Chua's circuit. The advantage of asymmetrical coupling on delay-induced OD is thus postulated to be generic in coupled chaotic oscillators.

III. DISCUSSION AND CONCLUSION

In summary, by adjusting the coupling asymmetry parameter α ($0 \leqslant \alpha \leqslant 1$), we have addressed the influence of coupling asymmetry on delay-induced OD in coupled oscillators. By combining theoretical analyses with numerical methods, it is found that the asymmetrical coupling is beneficial to the occurrence of delay-induced OD. The asymmetrically coupled system experiences OD for a larger set of parameter values compared with the symmetrical coupling one. We found somewhat counterintuitively that as the strength of the coupling asymmetry is increased by decreasing the value of α from $\alpha = 1$ to $\alpha \to 0$, the OD island gradually expands along two directions of both the coupling delay and the coupling strength in the parameter space. The smaller the value of α ($\alpha > 0$), i.e., the stronger the coupling asymmetry, the larger the OD island that forms. The expansion well obeys a power law scaling, $R = \alpha^{\gamma}$ with $\gamma \approx -1.19$. The threshold of the intrinsic frequency, beyond which OD is possible, decreases as the asymmetry parameter α deceases, and approaches a constant value for an infinitesimally small value of α. The beneficial effect of asymmetrical coupling on delay-induced OD is numerically shown to be generic in coupled nonidentical oscillators and coupled chaotic oscillators as well. The presented results are illustrated by consideration of a system of only two delay-coupled nonlinear oscillators. In fact, quite similar phenomena are also observed in our numerical experiments of an array of asymmetrically delay-coupled Stuart-Landau oscillators.

The phenomenon of OD suggests an effective scheme to rapidly terminate some undesirable oscillations, and has been shown to gain broad implications and wide applications for biological systems. For instance, an OD state in coupled synthetic genetic networks is responsible for a stable production of protein concentration in interacting cellular populations [6–9]. Therefore, it is of practical value to develop coupling techniques which can effectively induce a stable OD state. Our study in this paper clearly reveals that the presence of asymmetry in the coupling may serve as an excellent candidate for producing OD with a remarkably high efficiency. On the contrary, in some real-life systems if an OD state is assumed to do harm to their normal functions, such as in the onset of cessation of rhythmic activity in neuronal disorder associated with Parkinson disease [45], then the coupling asymmetry should be properly avoided to maintain a rhythmical oscillation.

Finally, the beneficial effect of asymmetrical coupling on delay-induced OD is presumed to be easily implemented in experiments of coupled lasers and circuits. The numerical and theoretical findings in this paper are expected to be connected with experimental data in the future. The studies provide a valuable guideline for further (both experimental and theoretical) investigations of collective states in asymmetrically delay-coupled systems. Since both the coupling asymmetry and the coupling delay are ubiquitous in natural systems, the presented results may be of relevance for various potential applications in biology, ecology, and neuroscience.

ACKNOWLEDGMENTS

This work was supported by the Alexander von Humboldt Foundation of Germany, the National Natural Science Foundation of China under Grant No. 11147179, the Fundamental Research Funds for the Central Universities of China under Grant No. 2011QN161, the Foundation for the Author of National Excellent Doctoral Dissertation of PR China (FANEDD) under Grant No. 200951, the SUMO (EU), and the ECONS (WGL).

[1] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984).
[2] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, England, 2003).
[3] M. Lakshmanan and D. V. Senthilkumar, *Dynamics of Nonlinear Time-Delay Systems* (Springer, Berlin, 2010).
[4] M. F. Crowley and I. R. Epstein, J. Phys. Chem. **93**, 2496 (1989).
[5] J. Benford, H. Sze, W. Woo, R. R. Smith, and B. Harteneck, Phys. Rev. Lett. **62**, 969 (1989).
[6] E. Ullner, A. Zaikin, E. I. Volkov, and J. García-Ojalvo, Phys. Rev. Lett. **99**, 148103 (2007).
[7] A. Koseska, E. Volkov, and J. Kurths, Europhys. Lett. **85**, 28002 (2009).
[8] A. Koseska, E. Volkov, and J. Kurths, Chaos **20**, 023132 (2010).
[9] A. Koseska, E. Volkov, and J. Kurths, Phys. Scr. **84**, 045007 (2011).
[10] D. G. Aronson, G. B. Ermentrout, and N. Kopell, Physica D **41**, 403 (1990).
[11] D. V. Ramana Reddy, A. Sen, and G. L. Johnston, Phys. Rev. Lett. **80**, 5109 (1998).
[12] S. H. Strogatz, Nature (London) **394**, 316 (1998).
[13] K. Konishi, Phys. Rev. E **68**, 067202 (2003).
[14] R. Karnatak, R. Ramaswamy, and A. Prasad, Phys. Rev. E **76**, 035201(R) (2007).
[15] W. Zou, X. G. Wang, Q. Zhao, and M. Zhan, Fron. Phys. China **4**, 97 (2009).
[16] A. Prasad, M. Dhamala, B. M. Adhikari, and R. Ramaswamy, Phys. Rev. E **81**, 027201 (2010).
[17] V. Resmi, G. Ambika, and R. E. Amritkar, Phys. Rev. E **84**, 046212 (2011).
[18] D. V. Ramana Reddy, A. Sen, and G. L. Johnston, Physica D **129**, 15 (1999).
[19] D. V. Ramana Reddy, A. Sen, and G. L. Johnston, Phys. Rev. Lett. **85**, 3381 (2000).
[20] A. Prasad, Y. C. Lai, A. Gavrielides, and V. Kovanis, Phys. Lett. A **318**, 71 (2003).
[21] R. Herrero, M. Figueras, J. Rius, F. Pi, and G. Orriols, Phys. Rev. Lett. **84**, 5312 (2000).

[22] F. M. Atay, Phys. Rev. Lett. **91**, 094101 (2003).
[23] W. Zou and M. Zhan, Phys. Rev. E **80**, 065204(R) (2009).
[24] W. Zou, C. G. Yao, and M. Zhan, Phys. Rev. E **82**, 056203 (2010).
[25] K. Konishi, H. Kokame, and N. Hara, Phys. Rev. E **81**, 016201 (2010).
[26] K. Konishi, H. Kokame, and N. Hara, Phys. Lett. A **374**, 733 (2010).
[27] G. Saxena, A. Prasad, and R. Ramaswamy, Phys. Rev. E **82**, 017201 (2010).
[28] W. Zou, J. Q. Lu, Y. Tang, C. J. Zhang, and J. Kurths, Phys. Rev. E **84**, 066208 (2011).
[29] A. Prasad, Phys. Rev. E **72**, 056204 (2005).
[30] A. Prasad, J. Kurths, S. K. Dana, and R. Ramaswamy, Phys. Rev. E **74**, 035204(R) (2006).
[31] K. Konishi, Phys. Rev. E **70**, 066201 (2004).
[32] W. Zou, X. Zheng, and M. Zhan, Chaos **21**, 023130 (2011).
[33] M. Begon, J. L. Harper, and C. R. Townsend, *Ecology: Individuals, Populations and Communities* (Blackwell Scientific Publications, Inc., Oxford, UK, 1988).
[34] P. Nelson, *Biological Physics: Energy, Information, Life* (Freeman, New York, 2003).
[35] S.-Y. Kim and W. Lim, Phys. Rev. E **64**, 016211 (2001).
[36] B. Blasius, Phys. Rev. E **72**, 066216 (2005).
[37] J. Bragard, G. Vidal, H. Mancini, C. Mendoza, and S. Boccaletti, Chaos **17**, 043107 (2007).
[38] J. Bragard, S. Boccaletti, and H. Mancini, Phys. Rev. Lett. **91**, 064103 (2003).
[39] J. Bragard, S. Boccaletti, C. Mendoza, H. G. E. Hentschel, and H. Mancini, Phys. Rev. E **70**, 036219 (2004).
[40] J. Bragard, E. Montbrió, C. Mendoza, S. Boccaletti, and B. Blasius, Phys. Rev. E **71**, 025201(R) (2005).
[41] M.-Y. Kim, R. Roy, J. L. Aron, T. W. Carr, and I. B. Schwartz, Phys. Rev. Lett. **94**, 088101 (2005).
[42] L. Illing, G. Hoth, L. Shareshian, and C. May, Phys. Rev. E **83**, 026107 (2011).
[43] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations* (Springer, New York, 1993).
[44] Y. Song, J. Wei, and Y. Yuan, J. Nonlinear Sci. **17**, 145 (2007).
[45] B. Caughey and P. T. Lansbury, Jr., Annu. Rev. Neurosci. **26**, 267 (2003).