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# Phenomenology of aging in the Kardar-Parisi-Zhang equation

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We study aging during surface growth processes described by the one-dimensional Kardar-Parisi-Zhang equation. Starting from a flat initial state, the systems undergo simple aging in both correlators and linear responses, and its dynamical scaling is characterized by the aging exponents $a = -1/3, b = -2/3, \lambda_C = \lambda_R = 1$, and $z = 3/2$. The form of the autoresponse scaling function is well described by the recently constructed logarithmic extension of local scale invariance.

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The study of the motion of interfaces continues as a widely interesting topic of statistical physics. One particularly intensively studied case is nonequilibrium growth processes, which are governed by local rules. Of these, the model equation proposed by Kardar, Parisi, and Zhang (KPZ) [1] continues to play a paradigmatic role in the investigation of the dynamical scaling of such interfaces, with an extensive range of applications, including Burgers turbulence, directed polymers in a random medium, glasses and vortex lines, and domain walls, and biophysics—see Refs. [2–9] for reviews. Remarkably, in one dimension (1D) the height distribution can be shown to converge for large times toward the Gaussian Tracy-Widom distribution [10,11]. A particularly clean experimental realization of this universality class has been found recently in the growing interfaces of turbulent liquid crystals [12].

Insight into the nonequilibrium properties of many-body systems comes from an analysis of the *aging properties*, which are realized if the system is rapidly brought out of equilibrium by a change of one of its state variables [13,14]. By definition, an aging system (i) undergoes a slow, nonexponential relaxation toward its stationary state(s), (ii) does not satisfy time-translation invariance, and (iii) shows dynamical scaling. Studies of aging require the analysis of both correlators $C$ and responses $R$ to be complete and also go beyond the study of dynamics in analyzing at least two-time observables. Let $s$ denote the waiting time and $t > s$ the observation time. For *simple aging*, one expects in the aging regime $s \gg \tau_{\rm micro}$ and $t - s \gg \tau_{\rm micro}$, where $\tau_{\rm micro}$ is a microscopic time scale, a single relevant length scale $L(t) \sim t^{1/z}$ such that

$$\begin{aligned} C(t,s) &= \langle\phi(t)\phi(s)\rangle - \langle\phi(t)\rangle\langle\phi(s)\rangle = s^{-b} f_C\left(\frac{t}{s}\right), \\ R(t,s) &= \frac{\delta\langle\phi(t)\rangle}{\delta j(s)}\Big|_{j=0} = \langle\phi(t)\widetilde{\phi}(s)\rangle = s^{-1-a} f_R\left(\frac{t}{s}\right), \end{aligned} \tag{1}$$

where $j$ is the external field conjugate to $\phi$. This defines the aging exponents $a,b$ and the scaling functions, from whose asymptotic behavior $f_{C,R}(y) \sim y^{-\lambda_{C,R}/z}$ as $y \to \infty$ one has the autocorrelation and autoresponse exponents $\lambda_{C,R}$ where $z$ is the dynamical exponent. In the context of Janssen–de Dominicis theory, $\widetilde{\phi}(t)$ is the response field conjugate to the order parameter $\phi(t)$.

For example, simple aging is found in nondisordered, unfrustrated magnets, quenched from an initial disordered state to a temperature $T \leqslant T_c$ at or below its critical temperature $T_c$ (see Ref. [14] and references therein) or else in microscopically irreversible systems with a nonequilibrium stationary state [15–18]. Generically, one finds $\lambda_C = \lambda_R$, but the values of $a,b$ depend more sensitively on the kind of aging investigated (for reversible systems on the type of quench and for irreversible ones on the specific type of dynamics).

Here, we shall study what kind of aging phenomena can arise in the growth of interfaces. A typical system is formulated in terms of a height variable $h = h_i(t) = h(t,\mathbf{r}_i)$, defined over a substrate in $d$ dimensions. A local, microscopic rule indicates how single particles are added to the surface. One of the main quantities studied is the surface roughness,

$$w^2(t;L) = \frac{1}{L^d}\sum_{i=1}^{L^d}\langle(h_i(t) - \overline{h}(t))^2\rangle, \tag{2}$$

on a lattice with $L^d$ sites and average height $\overline{h}(t) = L^{-d}\sum_i h_i(t)$. It obeys Family-Vicsek scaling [19]

$$w^2(t;L) = L^{2\zeta} f(tL^{-z}), \quad f(u) \sim \begin{cases} u^{2\beta}, & \text{if } u \ll 1, \\ \text{const}, & \text{if } u \gg 1, \end{cases} \tag{3}$$

where $\beta$ is the growth exponent and $\zeta = \beta z$ is the roughness exponent. For an infinite system, the width grows for large times as $w^2(t;\infty) \sim t^{2\beta}$.

The generic universality class for growth phenomena is given by the KPZ equation [1]

$$\frac{\partial h}{\partial t} = \nu\frac{\partial^2 h}{\partial \mathbf{r}^2} + \frac{\mu}{2}\left(\frac{\partial h}{\partial \mathbf{r}}\right)^2 + \eta, \tag{4}$$

where $\eta(t,\mathbf{r})$ is a white noise with zero mean and variance $\langle\eta(t,\mathbf{r})\eta(t',\mathbf{r}')\rangle = 2\nu T\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')$ and $\mu,\nu,T$ are material-dependent constants. For comparison, we introduce two more universality classes of surface growth: Elimination of the nonlinear term in (4) by setting $\mu = 0$ gives the Edwards-Wilkinson (EW) universality class [20]. The Mullins-Herrings (MH) universality class is given by $\partial_t h = -\nu\partial_r^4 h + \eta$ [21].

 

TABLE I. Some dynamical, aging, and growth exponents of several universality classes in $d = 1$ dimension.

| Model | $z$ | $a$ | $b$ | $\lambda_R = \lambda_C$ | $\beta$ | $\zeta$ |
|---|---|---|---|---|---|---|
| KPZ | 3/2 | −1/3 | −2/3 | 1 | 1/3 | 1/2 |
| EW | 2 | −1/2 | −1/2 | 1 | 1/4 | 1/2 |
| MH | 4 | −3/4 | −3/4 | 1 | 3/8 | 3/2 |

For both EW and MH classes, the aging scaling forms (1) for $C$ and $R$ have been explicitly confirmed [22]. Values of some growth and aging exponents in 1D are listed in Table I. For the 1D KPZ class, the exponents $z,\beta$ are exactly known [1], whereas the relation $b = -2\zeta/z = -2\beta$ follows from dynamical scaling [23,24]. Also, evidence for a growing length $L(t) \sim t^{1/z}$ [25,26] and estimates of $\lambda_C$ [23,24] have been reported [27]. However, to the best of our knowledge, no systematic test of the aging scaling has been reported for the space-time correlation function, and no information exists at all for the response function $R$. These will be provided now.

Our numerical simulations in the 1D KPZ class either use the discretized KPZ equation (4) in the strong coupling limit [28] (we checked that our results do not depend on the chosen discretization scheme) or else the Kim-Kosterlitz (KK) model [29]. This model uses a height variable $h_i(t) \in \mathbb{Z}$ attached to the sites of a chain with $L$ sites and subject to the constraints $|h_i(t) - h_{i\pm 1}(t)| = 0,1$, at all sites $i$. From a flat initial condition, that is, $h_i(0) = 0$, the dynamics of the model is as follows: At each time step, select randomly a site $i$ and deposit a particle with probability $p$ or else eliminate a particle with probability $1 - p$. $L$ such deposition attempts make up a Monte Carlo step. It is well known that this model is in the KPZ universality class. The choice of the value of $p$ is a practical matter. In order to avoid metastable states, we have chosen $p = 0.98$. In simulations, we have taken $L = 2^{17}$ and all the data have been averaged over $10^4$ samples. For the discretized KPZ equation we considered systems of size $L = 10^4$ and averaged over typically $10^5$ samples.

In studying the aging behavior, we shall consider the two-time spatio-temporal correlator

$$\begin{aligned} C(t,s;\mathbf{r}) &= \langle (h(t,\mathbf{r}+\mathbf{r}_0) - \langle \overline{h}(t)\rangle)(h(s,\mathbf{r}_0) - \langle \overline{h}(s)\rangle)\rangle \\ &= \langle h(t,\mathbf{r}+\mathbf{r}_0)h(s,\mathbf{r}_0)\rangle - \langle \overline{h}(t)\rangle\langle \overline{h}(s)\rangle \\ &= s^{-b}F_C\left(\frac{t}{s},\frac{|\mathbf{r}|^z}{s}\right), \end{aligned} \tag{5}$$

along with the extended Family-Vicsek scaling in the $L \to \infty$ limit and where the definition of the exponents is analogous to the usual one for simple aging. The autocorrelation exponent can be found from $f_C(y) = F_C(y,0) \sim y^{-\lambda_C/z}$ as $y \to \infty$. We also have $b = -2\beta$, since the width $w^2(t;\infty) = C(t,t;\mathbf{0}) = t^{-b}F_C(1,0)$. This is justified since the initial conditions in the 1D KPZ do not generate additional, independent renormalizations [30].

In Fig. 1, we show data for the autocorrelator $C(t,s) = C(t,s;\mathbf{0})$ obtained from the KK model. A clear data collapse is seen, and for large values of the scaling variable $y = t/s$, an effective power-law behavior with an exponent $\lambda_C/z \approx \frac{2}{3}$ is found. The data are fully compatible with a numerical solution of the KPZ equation and directly test simple aging (1) in

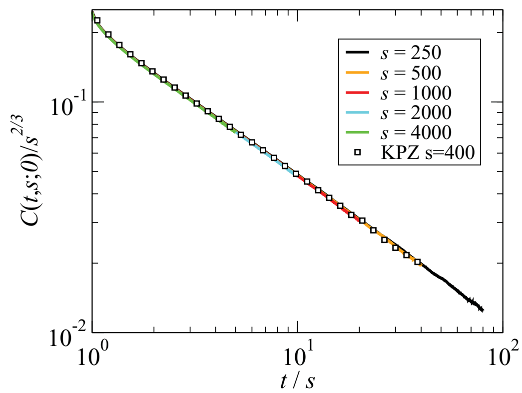


FIG. 1. (Color online) Scaling of the two-time autocorrelator $C(t,s)$ from the KK model and several values of $s$, and the discretized KPZ equation, with $s = 400$, rescaled by a factor 2.79.

the 1D KPZ class. All this, completely analogous to the EW and MH classes, confirms and strengthens earlier conclusions [23–26,30].

In order to define a response, we appeal to the procedures used in irreversible systems [15,17,18,31] where the external field is related to a local change of rates. In the KK model, we consider a space-dependent deposition rate $p_i = p_0 + a_i\varepsilon/2$ with $a_i = \pm 1$ and $\varepsilon = 0.005$ a small parameter. Then consider, with the *same* stochastic noise $\eta$, two realizations: System A evolves, up to the waiting time $s$, with the site-dependent deposition rate $p_i$ and, afterward, with the uniform deposition rate $p_0$. System B evolves always with the uniform deposition rate $p_i = p_0$. Of course, the evaporation rate $q_i = 1 - p_i$. Then, the time-integrated response function is

$$\begin{aligned} \chi(t,s;\mathbf{r}) &= \int_0^s du\; R(t,u;\mathbf{r}) \\ &= \frac{1}{L}\sum_{i=1}^{L}\left\langle \frac{h_{i+r}^{(A)}(t;s) - h_{i+r}^{(B)}(t)}{\varepsilon a_i}\right\rangle \\ &= s^{-a}F_\chi\left(\frac{t}{s},\frac{|\mathbf{r}|^z}{s}\right), \end{aligned} \tag{6}$$

together with the expected scaling. The time-integrated autoresponse $\chi(t,s) = \chi(t,s;\mathbf{0})$ plays the same role as the thermoremanent integrated response of magnetic systems [14]. The autoresponse exponent is read off from $f_\chi(y) = F_\chi(y,0) \sim y^{-\lambda_R/z}$ for $y \to \infty$. For the discretized KPZ equation we realize the perturbation by adding a spatially random force, of strength $\pm f_0 = \pm 0.3$, up to the waiting time $s$.

In Fig. 2, data for the integrated autoresponse $\chi(t,s)$ coming from the KK model are shown. An excellent collapse is found for $a = -\frac{1}{3}$. The effective power law, for $y = t/s$ large, reproduces well the expected $\lambda_R/z \approx \frac{2}{3}$. Indeed, from the exact fluctuation-dissipation theorem $T R(t,s;r) = -\partial_r^2 C(t,s;r)$, valid in the 1D KPZ universality class (because of time-reversal invariance) [32–35], we obtain the predictions $1 + a = b + 2/z$ and $\lambda_C = \lambda_R$, in agreement with our data. The data are essentially identical to those obtained from the KPZ equation, in agreement with universality. The aging form (1) of the linear response is confirmed in a nonlinear growth model. In contrast with the EW and MH classes, $a$

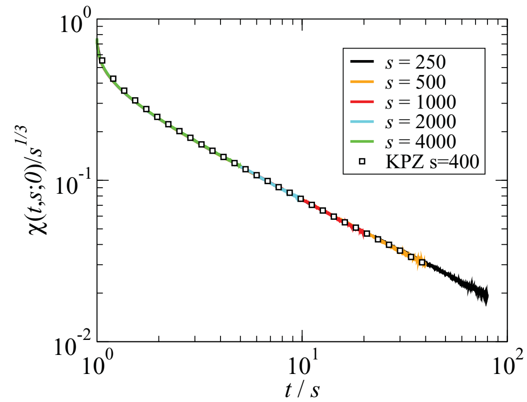


FIG. 2. (Color online) Scaling of the two-time integrated autoresponse $\chi(t,s)$ from the KK model for several values of $s$, and from the KPZ equation with $s = 400$, rescaled by a factor 7.25.

and $b$ are different, a feature commonly seen in irreversible systems [14,36].

Next, in Fig. 3, we illustrate the space-dependent scaling of both the correlator and the integrated response. For several values of the scaling variable $y = t/s$, the dependence on the second argument in the scaling forms (5) and (6) is illustrated. An excellent data collapse is found, which further confirms the conclusions already drawn from the autocorrelator and the autoresponse and also confirms the exactly known dynamical exponent $z = \frac{3}{2}$ in the 1D KPZ universality class. The shape of the scaling functions changes notably when $y$ is varied.

We now turn to an analysis of the *form* of the autoresponse scaling function $f_\chi(y)$. For aging simple magnets (i.e., nondisordered and unfrustrated), it has been proposed to generalize dynamical scaling to a larger set of local scale transformations [37], which includes the transformation $t \mapsto t/(1+\gamma t)$. This hypothesis of local scale invariance (LSI) indeed reproduces precisely the universal shapes of responses and correlators in a large variety of models, as reviewed in detail in Ref. [14]. Analogous evidence exists in some irreversible models [14,15,17,36]. Similarly, the responses and correlators in the EW and MH classes, with the local height variable $h(t,\mathbf{r}) - \overline{h}(t)$ acting as a quasiprimary scaling operator, are described by LSI [22,38]. *Is LSI also realized in the 1D KPZ class, with the local height as a quasiprimary operator?*

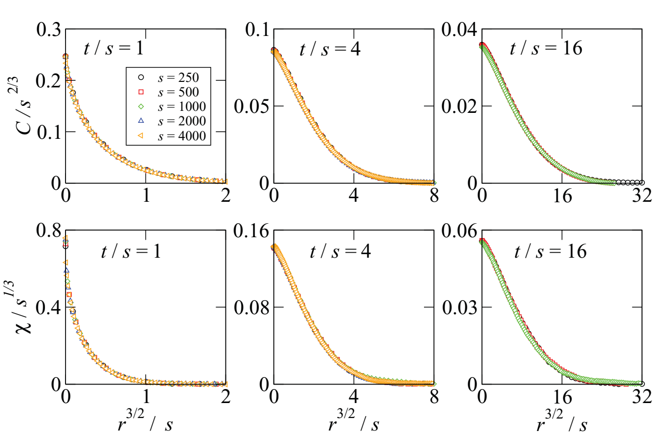


FIG. 3. (Color online) Space-dependent scaling of the two-time correlator $C(t,s)$ (upper row) and the integrated response $\chi(t,s)$ (lower row), for several values of $s$ and the scaling variable $y = t/s$.

We concentrate on the autoresponse function and shall restrict attention to the transformations in time. The transformation $\delta\phi = \varepsilon X_n \phi$ of the quasiprimary operators under local scale transformations is given by the infinitesimal generators $X_n$, which read [39]

$$X_n = -t^{n+1}\partial_t - (n+1)\frac{x}{z}t^n - n\frac{2\xi}{z}t^n, \quad n \geqslant 0, \tag{7}$$

and satisfy the commutator $[X_n, X_m] = (n-m)X_{n+m}$. We merely look at the finite-dimensional subalgebra spanned by the dilatations $X_0$ and the special transformations $X_1$. Since time translations (generated by $X_{-1}$) are absent, we have two *distinct* scaling dimensions $x$ and $\xi$, which together give the shape of the autoresponse function (see below). Now, consider a possible extension to so-called logarithmic form, where a primary operator $\phi$ is replaced by a doublet $\binom{\phi}{\psi}$. In analogy with logarithmic conformal invariance [40,41], this extension is formally carried out by replacing the scaling dimensions $x,\xi$ by matrices (restricted to the $2 \times 2$ case) [42]

$$x \mapsto \begin{pmatrix} x & x' \\ 0 & x \end{pmatrix}, \quad \xi \mapsto \begin{pmatrix} \xi & \xi' \\ \xi'' & \xi \end{pmatrix}, \tag{8}$$

where the first scaling dimension is immediately taken in a Jordan form. Consistency with the commutators then leads to $\xi'' = 0$ [42]. Recalling (1), consider the following quasiprimary two-point functions, with $y = t/s$

$$\begin{aligned}
\langle \phi(t)\widetilde{\phi}(s)\rangle &= s^{-(x+\widetilde{x})/z}\,\mathcal{F}(y) f_0,\\
\langle \phi(t)\widetilde{\psi}(s)\rangle &= s^{-(x+\widetilde{x})/z}\,\mathcal{F}(y)(g_{12}(y) + \gamma_{12}\ln s),\\
\langle \psi(t)\widetilde{\phi}(s)\rangle &= s^{-(x+\widetilde{x})/z}\,\mathcal{F}(y)(g_{21}(y) + \gamma_{21}\ln s),\\
\langle \psi(t)\widetilde{\psi}(s)\rangle &= s^{-(x+\widetilde{x})/z}\,\mathcal{F}(y)\sum_{j=0}^{2} h_j(y)\ln^j s,
\end{aligned} \tag{9}$$

where $\mathcal{F}(y) = y^{(2\widetilde{\xi}+\widetilde{x}-x)/z}(y-1)^{-(x+\widetilde{x}+2\xi+2\widetilde{\xi})/z}$ and explicitly known scaling functions [42]. In contrast to logarithmic conformal invariance, logarithmic corrections to scaling are absent if $x' = \widetilde{x}' = 0$ and there are no logarithmic factors for $y \to \infty$ if furthermore $\xi' = 0$. If we take $R(t,s) = \langle \psi(t)\widetilde{\psi}(s)\rangle = s^{-1-a} f_R(t/s)$, we find

$$\begin{aligned}
f_R(y) = {} & y^{-\lambda_R/z}(1-y^{-1})^{-1-a'}\\
& \times \left[h_0 - g_0\ln(1-y^{-1}) - \tfrac{1}{2} f_0 \ln^2(1-y^{-1})\right],
\end{aligned} \tag{10}$$

with the exponents $1 + a = (x+\widetilde{x})/z$, $a' - a = \frac{2}{z}(\xi + \widetilde{\xi})$, $\lambda_R/z = x + \xi$, and the normalization constants $h_0, g_0, f_0$.

The integrated autoresponse $\chi(t,s) = s^{-a} f_\chi(t/s)$ is found from (10) by using the specific value $\lambda_R/z - a = 1$ which holds true for the 1D KPZ. We find

$$\begin{aligned}
f_\chi(y) = {} & y^{+1/3}\{A_0[1 - (1-y^{-1})^{-a'}]\\
& + (1-y^{-1})^{-a'}[A_1\ln(1-y^{-1}) + A_2\ln^2(1-y^{-1})]\},
\end{aligned} \tag{11}$$

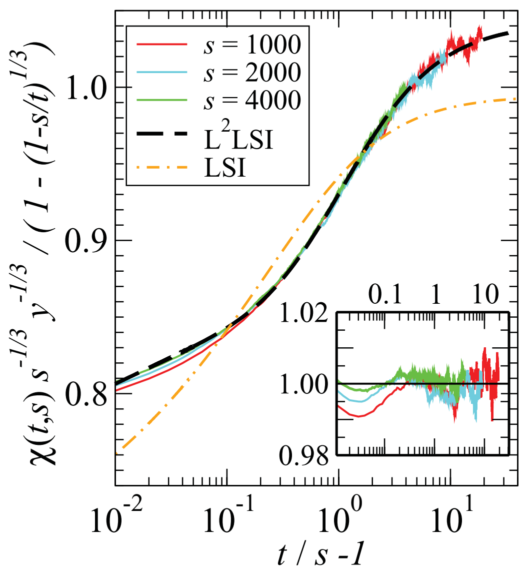


FIG. 4. (Color online) Comparison of the reduced scaling function $f_{\rm red}(y) = f_\chi(y)y^{-1/3}(1-(1-1/y)^{1/3})^{-1}$ of $\chi(t,s) = s^{1/3} f_\chi(t/s)$ with logarithmic local scale invariance. Nonlogarithmic LSI gives the dashed-dotted curve labeled LSI and full logarithmic LSI (11) gives the dashed curve labeled L$^2$LSI. The inset shows the ratio $\chi(t,s)/\chi_{\rm L^2LSI}(t,s)$ over $t/s-1$.

where $A_{0,1,2}$ are normalizations related to $f_0, g_0, h_0$. The nonlogarithmic case is recovered if $A_1 = A_2 = 0$. Indeed, for $y \gg 1$, one has $f_\chi(y) \sim y^{-2/3}$, as it should be.

In Fig. 4, which gives a more fine appreciation of the shape of $f_\chi(y)$ than Fig. 2, we compare data for the reduced scaling function $f_{\rm red}(y) = f_\chi(y)y^{-1/3}[1-(1-y^{-1})^{1/3}]^{-1}$ with the predicted form (11). Data with $s < 10^3$ are not yet fully in the scaling regime. If one tries to fit the data with a nonlogarithmic LSI (then $R = \langle\phi\widetilde{\phi}\rangle$ or $\langle\psi\widetilde{\phi}\rangle$), one obtains an agreement with the data, with a numerical precision of about 5%. An attempt to fit only with the first-order logarithmic terms (then $R = \langle\phi\widetilde{\psi}\rangle$) with $A_2 = 0$ assumed gives back the same result—see Table II. Only if one uses the full structure of logarithmic LSI, an excellent representation of the data is found, to an accuracy better than 0.1% over the range of data available. In the inset the ratio $\chi(t,s)/\chi_{\rm L^2LSI}(t,s)$ is shown, and we see that at least down to $t/s \approx 1.03$, the data collapse indicating dynamical scaling holds true, within the accuracy limits set by the stochastic noise, within ≈0.5%. For the largest waiting time $s = 4000$, this observation extends over the entire range of values of $t/s$ considered. This indicates that the local height $h$ and its response field $\widetilde{h}$ of the 1D KPZ equation could be tentatively identified with the logarithmic quasiprimary operators $\psi, \widetilde{\psi}$, which slightly generalizes the findings for the EW and MH classes, which obey nonlogarithmic LSI. It is an open question whether the approach outlined here just generates the first two terms of an infinite logarithmic series in $R(t,s)$.

TABLE II. Fitted parameters $A_0$, $A_1$, $A_2$, and $a'$ used in Fig. 4.

| | Parameters | | | |
|---|---|---|---|---|
| $R$ | $a'$ | $A_0$ | $A_1$ | $A_2$ |
| $\langle\phi\widetilde{\phi}\rangle$–LSI | −0.500 | 0.662 | 0 | 0 |
| $\langle\phi\widetilde{\psi}\rangle$–L$^1$LSI | −0.500 | 0.663 | $-6\times10^{-4}$ | 0 |
| $\langle\psi\widetilde{\psi}\rangle$–L$^2$LSI | −0.8206 | 0.7187 | 0.2424 | −0.090 87 |

A systematic analysis of the invariance properties of the dynamical functionals studied, for instance, in Refs. [30,33,34], or the alternate form derived in Ref. [43], would be of interest, following the lines of study for the analysis of dynamical symmetries in phase-ordering kinetics—see Ref. [14] and references therein.

Summarizing, we tested the full scaling behavior of simple aging, both for correlators and responses, of systems in the 1D KPZ universality class. This is an example of a growth process described by a nonlinear equation which is shown to satisfy simple aging scaling for space- and time-dependent quantities. It is nontrivial that the values of the growth and dynamical exponents, previously known from the study of the *stationary state*, are confirmed *far* from stationarity. It would be interesting to measure it also experimentally. Performing a numerical experiment, we find the form of the autoresponse scaling function to be very well described by the recently constructed logarithmic extension of local scale invariance, with a natural identification of the leading quasiprimary operators. In view of important recent progress in the exact solution of the 1D KPZ equation (see Refs. [7,10,11]), one may expect that the question of a logarithmic dynamical scaling can be addressed and its further consequences explored.

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