Spectra of ion density and potential fluctuations in weakly ionized plasmas in the presence of dust grains

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The spectral densities of ion density and electrostatic potential fluctuations are derived in the framework of a self-consistent kinetic model of partially ionized dusty plasmas in the low-frequency regime. Neutral gas density can be responsible for significant modifications of the fluctuation level, hence the inclusion of the effect of neutrals is essential for a more realistic comparison with experiments, especially if spectral measurements are intended for dust diagnostic purposes. Comparison with the multicomponent model, attractive due to its simplicity as compared to the self-consistent one, is carried out to establish its limits of validity. Numerical calculations are performed for parameters typical of low-temperature plasma discharges. A criterion is derived for the omission of plasma discreteness in the low-frequency regime.

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I. INTRODUCTION

Fluctuations in plasmas are omnipresent, even in thermodynamic equilibrium, due to the discrete nature of electrons and ions [1,2]. In the 1960s it was suggested by Ichimaru that the spectral densities of fluctuating plasma observables, if measured, could provide vast information on fundamental plasma parameters [3]. In ordinary plasmas the level of fluctuations is low and usually can be detected only in quiescent environments. Examples are specially devised plasma configurations [4–9] and space plasmas where Langmuir probes and dipole antennas were used for measurements of the fluctuating plasma density [10] and electric field [11,12], respectively.

In plasmas seeded with dust particles the spectral densities of plasma fluctuations can be strongly modified due to the large number of elementary charges residing on the dust surface and the dissipative nature of inelastic charging collisions between dust and plasma particles. Recently it has been pointed out that such modifications, if detected, can serve both as an experimental verification of the kinetic model as well as an *in situ* dust diagnostic [13–15]. This is due to the fact that (i) changes of the plasma spectra are dependent on the dust number density and size (charge) and (ii) the fluctuation level can be drastically enhanced compared to dust-free plasmas, as also supported by first experimental tests [14,15].

It has been shown that the contribution of dust to the spectral densities of plasma quantities, e.g., ion density fluctuations, is most pronounced in the low-frequency regime, well below the ion acoustic mode [15]. This is due to the fact that the dominant frequency dependence is an exponential decay of the type $\propto \exp(-\frac{\omega^2}{2k^2 v_{T\gamma}^2})$, where $\gamma = \{i,e,d\}$ refers to both plasma and dust species, and that for the ion and dust thermal velocities the inequality $v_{Ti} \gg v_{Td}$ holds. Such a scaling with the dust thermal velocity implies that the smaller the dust size, the more extended the range of frequencies where the enhancement takes place. This can be exploited in experiments with *in situ* produced or naturally occurring submicron dust.

In light of the above, targeting regions of strong spectral modification which can be experimentally assessed justifies the low-frequency approximation. In such a regime one can neglect electron and ion natural fluctuations compared to dust natural fluctuations, and hence treat only electron and ion fluctuations induced by dust discreteness [16]. This allows one to derive expressions for evaluation of spectral densities of plasma fluctuations and to simplify the computations significantly.

So far theoretical works on the spectral densities have focused on fully ionized plasmas and constant nonfluctuating sources. The spectral densities have been explored within the "full" kinetic model on the low-frequency regime [13], which treats charge fluctuations and absorption of plasma fluxes on dust self-consistently, and within the multicomponent model [15,17], where dust is treated as an additional massive plasma species with fixed charge over mass ratio. While the latter has been used for comparison with experiments, the full model has not been exploited for this purpose. However, its development is important for more appropriate comparison with experiments and to establish the limits of validity of the simplified multicomponent.

In typical low-temperature laboratory plasmas, the effect of neutrals cannot be neglected. Here we derive the spectral densities of fluctuating plasma quantities employing a kinetic model of partially ionized dusty plasmas in the low-frequency regime which takes into account in a self-consistent manner [18,19]; (i) the continuous absorption of plasma fluxes on dust grains, (ii) the dust charge fluctuations, (iii) the effect of neutrals via the Bhatnagar-Gross-Krook (BGK) formalism [20] and their effect in grain charging, (iv) electron impact ionization of neutrals. Taking into account all these effects results in very cumbersome expressions and it is reasonable to expect that in some parameter regime, e.g., "low" dust density and "small" dust size, some of the effects are negligible and a multicomponent model can be used. Therefore here the spectral densities of the multicomponent model, properly extended to include the presence of neutrals [19], are found to elucidate its applicability limits.

In addition, the effect of gas pressure and electron impact ionization is explored for the first time. The spectra of ion density and electrostatic potential fluctuations derived here are valid for both laboratory and space plasmas; however, they are evaluated numerically for plasma conditions of laboratory low-temperature discharges. Finally, a condition for neglecting plasma discreteness is derived which defines the low-frequency regime (see the Appendix), previously roughly identified as $\omega \ll k v_{Ti}$.

II. SPECTRAL DENSITIES OF PLASMA FLUCTUATIONS

A. Fluctuating plasma quantities

In weakly coupled plasmas, temporal and spatial scale considerations enable the decomposition of the distribution function $f_{\boldsymbol{p}}^{\alpha}$ into an average continuous part $\Phi_{\boldsymbol{p}}^{\alpha} = \langle f_{\boldsymbol{p}}^{\alpha}\rangle$ and a fluctuating part $\delta f_{\boldsymbol{p}}^{\alpha} = f_{\boldsymbol{p}}^{\alpha} - \langle \Phi_{\boldsymbol{p}}^{\alpha}\rangle$, and the evolution equations for both $\Phi_{\boldsymbol{p}}^{\alpha}$ and $\delta f_{\boldsymbol{p}}^{\alpha}$ can then be found [1,2]. The latter are the sum of "natural" fluctuations $\delta f_{\boldsymbol{p}}^{\alpha,(0)}$, related to particle discreteness, and "induced" fluctuations $\delta f_{\boldsymbol{p}}^{\alpha,(\mathrm{ind})}$ in the presence of the fluctuating field $\delta \mathbf{E}(\mathbf{r},t)$. Due to the variability of the dust charge, q can be considered as a new phase variable, hence the Hamiltonian phase-space $(\boldsymbol{r},\boldsymbol{p})$ is extended to a seven-dimensional phase-space $(\boldsymbol{r},\boldsymbol{p},q)$, and the distribution function of the dust particles is also a function of q, that is, $f_{\boldsymbol{p}'}^{d}(q,\mathbf{r},t)$ [16]. In the low-frequency regime, where $\frac{\omega}{k v_{Td}} < \Lambda_{i,e}$ (with typical $\Lambda_{i,e}$ of the order of few, see Appendix), the natural fluctuations of electrons and ions can be neglected, and the fluctuating densities of electrons and ions are then

$$\delta n^{\alpha}(\mathbf{r},t) = \int \delta f_{\boldsymbol{p}}^{\alpha,(\mathrm{ind})}(\mathbf{r},t)\frac{d\mathbf{p}}{(2\pi)^3}, \tag{1}$$

only induced by dust discreteness. The fluctuating dust density, on the other hand, is $\delta n^{d}(\mathbf{r},t) = \delta n^{d,(0)}(\mathbf{r},t) + \delta n^{d,(\mathrm{ind})}(\mathbf{r},t)$, where

$$\delta n^{d,(0)}(\mathbf{r},t) = \int\int \delta f_{\boldsymbol{p}'}^{d,(0)}(q,\mathbf{r},t)\frac{dq\, d^3p'}{(2\pi)^3} \tag{2}$$

due to dust discreteness (free streaming particles) starts the process, and

$$\delta n^{d,(\mathrm{ind})}(\mathbf{r},t) = \int\int \delta f_{\boldsymbol{p}'}^{d,(\mathrm{ind})}(q,\mathbf{r},t)\frac{dq\, d^3p'}{(2\pi)^3} \tag{3}$$

is induced by the fluctuating field generated in the system by the fluctuating parts of the distributions via the Poisson equation,

$$\nabla\cdot\delta\boldsymbol{E}(\boldsymbol{r},t) = 4\pi\left(\sum_{\alpha} e_{\alpha}\delta n^{\alpha}(\mathbf{r},t) + \int q'\delta f_{\boldsymbol{p}'}^{d}(q')\frac{dq' d^3\boldsymbol{p}'}{(2\pi)^3}\right). \tag{4}$$

The spectral densities of all fluctuating quantities are calculated as statistical averages of products of fluctuations: $S_{\boldsymbol{k},\omega}^{\alpha} = \langle \delta n_{\boldsymbol{k},\omega}^{\alpha}\delta n_{\boldsymbol{k},\omega}^{\alpha *}\rangle$ for the density fluctuations, $S_{\boldsymbol{k},\omega}^{E} = \langle \delta E_{\boldsymbol{k},\omega}\delta E_{\boldsymbol{k},\omega}^{*}\rangle$ for the electric field fluctuations, and $S_{\boldsymbol{k},\omega}^{\phi} = \langle \delta\phi_{\boldsymbol{k},\omega}\delta\phi_{\boldsymbol{k},\omega}^{*}\rangle$ for the electrostatic potential fluctuations. In the present low-frequency regime the statistical averages are taken over the dust ensemble, i.e., all fluctuations are induced only by dust discreteness, and the averages are done using the dust natural statistical correlator, given by [2,16]

$$\begin{aligned}\langle \delta f_{\boldsymbol{p},\boldsymbol{k},\omega}^{d,(0)}(q)\delta f_{\boldsymbol{p}',\boldsymbol{k}',\omega'}^{d,(0)}(q')\rangle &= (2\pi)^4\Phi_{\boldsymbol{p}}^{d}(q)\delta(\boldsymbol{p}-\boldsymbol{p}')\delta(\omega+\omega')\\ &\quad\times\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})\delta(q-q')\delta(q-q_{\mathrm{eq}}).\end{aligned} \tag{5}$$

Note that here, in contrast to earlier works, the Fourier normalization factor $(2\pi)^{-4}$ appears in the inverse transform following the convention of the textbooks [17].

In this section the spectral densities are derived for the full and multicomponent models following the methodology outlined below [1,2,16]: (i) solution of the Klimontovich equations and the Poisson equation for the fluctuating quantities, (ii) expression of all fluctuating quantities in terms of the natural dust fluctuations $\delta f_{\boldsymbol{p},\boldsymbol{k},\omega}^{d,(0)}(q)$, and (iii) statistical averaging with use of the natural dust correlator, given by Eq. (5).

The Klimontovich equations for both models have been formulated in previous works [18,19]. Hence, below we only provide final expressions for the fluctuating quantities of interest and their spectral densities.

B. Spectral densities for the full kinetic model

The definitions of the auxiliary $[a_{\boldsymbol{k},\omega}^{\mathrm{nd}},\gamma_{\boldsymbol{k},\omega}(q,q'),\beta_{\boldsymbol{k},\omega}(q),\chi_{\boldsymbol{k},\omega}^{\mathrm{nd}}]$ and low-frequency responses $[\chi_{\boldsymbol{k},\omega}^{d,\mathrm{eq}},\chi_{\boldsymbol{k},\omega}^{d,\mathrm{ch}},\chi_{\boldsymbol{k},\omega}^{i},G_{\boldsymbol{k},\omega},\widetilde{q}_{\boldsymbol{k},\omega}^{i}(q),\widetilde{\beta}_{\boldsymbol{k},\omega}^{i}(q)]$ and their expressions for Maxwellian distributions, and also the description of collisions with neutrals ($\nu_{n,\alpha}$ for the collision frequency where $\alpha=\{i,d\}$), electron impact ionization [$\nu^{I}(v)$ for the instantaneous ionization frequency] and inelastic absorption of plasma on dust [$\nu_{d,\alpha} = n_d v\sigma_{\alpha}(q,v)$ for the collisional frequency with $\sigma_{\alpha}(q,v)$ the relevant cross section] are provided in Refs. [18,19].

The solution for the fluctuating ion distribution, Fourier transformed in space and time, is found in terms of the field and dust fluctuations, and integrating over the momentum space of the ions gives the ion density fluctuations

$$\begin{aligned}\delta n_{\boldsymbol{k},\omega}^{i} &= -\left(\frac{\imath k}{4\pi e}\frac{\chi_{\boldsymbol{k},\omega}^{i}}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}} + \frac{\imath e n_e\nu_e}{T_e k}\frac{G_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}\right)\\ &\quad\times\delta E_{\boldsymbol{k},\omega} + \frac{1}{e}\frac{1}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}\int\widetilde{q}_{\boldsymbol{k},\omega}^{i}(q)\\ &\quad\times\left[\delta f_{\boldsymbol{p}',\boldsymbol{k},\omega}^{d,(\mathrm{ind})}(q) + \delta f_{\boldsymbol{p}',\boldsymbol{k},\omega}^{d,(0)}(q)\right]\frac{d^3p'dq}{(2\pi)^3},\end{aligned} \tag{6}$$

where $\nu_e = \frac{1}{n_e}\int \nu^{I}(v)\Phi_{\boldsymbol{p}}^{e}\frac{d^3p}{(2\pi)^3}$ is the average part of the ionization frequency and $\nu_{n,i} = n_n v_{Ti}\sigma_{n,i}$ is the average velocity-independent collisional frequency of ions with neutrals.

The equation for the fluctuating part of the dust distribution function is a first-order inhomogeneous differential equation with respect to the additional charge phase-space variable. Knowledge of Green's function of the equation, under the assumption of small deviations from the equilibrium charge, i.e, $|\frac{q-q_{\mathrm{eq}}}{q_{\mathrm{eq}}}| \ll 1$, so that the average dust distribution function is assumed to play the role of a δ function in charge and for any function $F(q)$ we have $\int F(q)\Phi_{\boldsymbol{p}'}^{d}(q)dq \simeq F(q_{\mathrm{eq}})\int \Phi_{\boldsymbol{p}'}^{d}(q)dq = F(q_{\mathrm{eq}})\Phi_{\boldsymbol{p}'}^{d}$ [21], allows the calculation of the integrals over the dust-induced fluctuations appearing in the Poisson equation and in the equation for the ion density fluctuations. Here, $q_{\rm eq} = -Z_d e$, where Z_d is an integer that can be either positive or negative.

Thus the induced field fluctuations are expressed in terms of only the natural dust fluctuations $\delta f^{d,(0)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)$:

$$\delta E_{\boldsymbol{k},\omega} = \frac{4\pi}{\imath k \epsilon_{\boldsymbol{k},\omega}} \int q^{\rm eff}_{\boldsymbol{k},\omega}(q)\delta f^{d,(0)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)\,\frac{d^3p'dq}{(2\pi)^3}\,. \quad (7)$$

The permittivity and the effective dust charge, defined by Eq. (7), are given by

$$q^{\rm eff}_{\boldsymbol{k},\omega}(q) = q - q_{\rm eq}\chi^{\rm nd}_{\boldsymbol{k},\omega} + \chi^{d,\rm ch}_{\boldsymbol{k},\omega}\gamma_{\boldsymbol{k},\omega}(q_{\rm eq},q_{\rm eq}) + \frac{1}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}} \times\left[\widetilde{q}^{i}_{\boldsymbol{k},\omega}(q) + \chi^{d,\rm ch}_{\boldsymbol{k},\omega}\gamma_{\boldsymbol{k},\omega}(q_{\rm eq},q_{\rm eq})\widetilde{\beta}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})\right], \quad (8)$$

$$\epsilon_{\boldsymbol{k},\omega} = \epsilon^{p}_{\boldsymbol{k},\omega} + a^{\rm nd}_{\boldsymbol{k},\omega}\chi^{d,\rm eq}_{\boldsymbol{k},\omega}\left[1 + \frac{1}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}\,\frac{\widetilde{q}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})}{q_{\rm eq}}\right], \quad (9)$$

where

$$\epsilon^{p}_{\boldsymbol{k},\omega} = 1 + \frac{1}{k^2\lambda^2_{De}}\left[1 + \frac{\nu_e G_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}\right] + \frac{\chi^{i}_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}} + \frac{4\pi\imath}{k}\,\chi^{d,\rm ch}_{\boldsymbol{k},\omega}\beta_{\boldsymbol{k},\omega}(q_{\rm eq})\left[1 + \frac{\widetilde{\beta}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}\right]. \quad (10)$$

We also define $\epsilon^{\rm eff}_{\boldsymbol{k},\omega}(q) = \frac{q_{\rm eq}\epsilon_{\boldsymbol{k},\omega}}{q^{\rm eff}_{\boldsymbol{k},\omega}(q)}$ as the effective permittivity.

Finally, also the ion density fluctuations can be expressed in terms of $\delta f^{d,(0)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)$. Evaluating the integral $\int \widetilde{q}^{i}_{\boldsymbol{k},\omega}\delta f^{d,(\rm ind)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)\frac{d^3p'dq}{(2\pi)^3}$ with Green's function and substituting for the electric field fluctuations from Eq. (7), the result is

$$\delta n^{i}_{\boldsymbol{k},\omega} = \int N^{i}_{\boldsymbol{k},\omega}(q)\delta f^{d,(0)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)\frac{d^3p'dq}{(2\pi)^3}, \quad (11)$$

where $N^{i}_{\boldsymbol{k},\omega}(q) = \frac{1}{1+\nu_{n,i}G_{\boldsymbol{k},\omega}}(\frac{Z_d}{\epsilon^{\rm eff}_{\boldsymbol{k},\omega}(q)}M^{i}_{\boldsymbol{k},\omega}(q) + \Lambda^{i}_{\boldsymbol{k},\omega}(q))$ with

$$M^{i}_{\boldsymbol{k},\omega}(q) = \chi^{i}_{\boldsymbol{k},\omega} + \frac{\nu_e}{k^2\lambda^2_{De}}\,G_{\boldsymbol{k},\omega} + \frac{\widetilde{q}^{i}_{\boldsymbol{k},\omega}}{q_{\rm eq}}\,a^{\rm nd}_{\boldsymbol{k},\omega}\chi^{d,\rm eq}_{\boldsymbol{k},\omega} + \frac{4\pi\imath}{k}\,\widetilde{\beta}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})\beta_{\boldsymbol{k},\omega}(q_{\rm eq})\chi^{d,\rm ch}_{\boldsymbol{k},\omega}\,, \quad (12)$$

$$\Lambda^{i}_{\boldsymbol{k},\omega}(q) = \frac{\widetilde{q}^{i}_{\boldsymbol{k},\omega}(q)}{e} + \frac{\widetilde{\beta}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})\gamma_{\boldsymbol{k},\omega}(q_{\rm eq},q_{\rm eq})\chi^{d,\rm ch}_{\boldsymbol{k},\omega}}{e} + \imath\,\nu_{n,d}a^{\rm nd}_{\boldsymbol{k},\omega}d^{\rm eq}_{\boldsymbol{k},\omega}\,\frac{\widetilde{q}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})}{e}\,. \quad (13)$$

Due to the use of the adiabatic assumption, the relation for the electron density fluctuations is quite straightforward and is found to be

$$\delta n^{e}_{\boldsymbol{k},\omega} = \int N^{e}_{\boldsymbol{k},\omega}(q)\delta f^{d,(0)}_{\boldsymbol{p}',\boldsymbol{k},\omega}(q)\frac{d^3p'dq}{(2\pi)^3}, \quad (14)$$

with the response $N^{e}_{\boldsymbol{k},\omega}(q)$ defined by

$$N^{e}_{\boldsymbol{k},\omega}(q) = -\frac{Z_d}{k^2\lambda^2_{De}}\,\frac{1}{\epsilon^{\rm eff}_{\boldsymbol{k},\omega}(q)}\,. \quad (15)$$

The above expressions with the aid of the natural correlator for the dust species can be used to obtain relations for the spectral densities of the electrostatic potential fluctuations ($S^{\phi}_{\boldsymbol{k},\omega}$) and the ion density fluctuations ($S^{i}_{\boldsymbol{k},\omega}$),

$$S^{\phi}_{\boldsymbol{k},\omega} = \langle\delta\phi_{\boldsymbol{k},\omega}\delta\phi^{*}_{\boldsymbol{k},\omega}\rangle = \frac{16\pi^2e^2}{k^4|\epsilon^{\rm eff}_{\boldsymbol{k},\omega}(q_{\rm eq})|^2}\,Z^2_d S^{d,(0)}_{\boldsymbol{k},\omega}\,, \quad (16)$$

$$S^{i}_{\boldsymbol{k},\omega} = \langle\delta n^{i}_{\boldsymbol{k},\omega}\delta n^{i*}_{\boldsymbol{k},\omega}\rangle = |N^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})|^2 S^{d,(0)}_{\boldsymbol{k},\omega}\,. \quad (17)$$

Here, the spectral density of the natural dust density fluctuations is defined as

$$S^{d,(0)}_{\boldsymbol{k},\omega} = \langle\delta n^{d,(0)}_{\boldsymbol{k},\omega}\delta n^{d,(0)*}_{\boldsymbol{k},\omega}\rangle = 2\pi\int\Phi^{d}(\boldsymbol{v})\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})d^3v\,, \quad (18)$$

which for the Maxwellian distribution becomes

$$S^{d,(0)}_{\boldsymbol{k},\omega} = (2\pi)^{1/2}\,\frac{n_d}{kv_{Td}}\,\exp\left\{-\left(\frac{\omega^2}{2k^2v^2_{Td}}\right)\right\}. \quad (19)$$

It is important to note the dependence of $S^{\phi}_{\boldsymbol{k},\omega}$ on Z^2_d. The same dependence appears also for $S^{i}_{\boldsymbol{k},\omega}$, since $M^{i}_{\boldsymbol{k},\omega} \propto Z_d$ and $\Lambda^{i}_{\boldsymbol{k},\omega} \propto Z_d$, the latter through $\widetilde{q}^{i}_{\boldsymbol{k},\omega}(q_{\rm eq})/e \propto Z_d$ and $\gamma_{\boldsymbol{k},\omega}(q_{\rm eq},q_{\rm eq})/e \propto Z_d$ (see the Appendixes of Ref. [19]). Such a proportionality is also valid in the multicomponent model, to be analyzed in the next subsection.

C. Spectral densities for the multicomponent kinetic model

In the multicomponent model the dust species is treated as a massive ion species; the dust charge is fixed at the equilibrium value $q_{\rm eq}$, absorption of plasma fluxes on the grains is not considered and hence there is no need for a plasma source. For the electron species the adiabatic assumption is used again, while the fluctuating part of the Poisson equation will now read as

$$\nabla\cdot\delta\boldsymbol{E} = 4\pi(e\delta n_i - e\delta n_e + q_{\rm eq}\delta n_d)\,. \quad (20)$$

After the standard decomposition, the permittivity and the effective charge defined through $\delta E_{\boldsymbol{k},\omega} = \frac{4\pi q^{\rm eff,MC}_{\boldsymbol{k},\omega}}{\imath k\epsilon^{\rm MC}_{\boldsymbol{k},\omega}}\delta n^{d,(0)}_{\boldsymbol{k},\omega}$ are found:

$$q^{\rm eff,MC}_{\boldsymbol{k},\omega} = q_{\rm eq}a^{\rm nd}_{\boldsymbol{k},\omega}, \qquad \epsilon^{\rm MC}_{\boldsymbol{k},\omega} = 1 + \frac{1}{k^2\lambda^2_{De}} + \frac{\chi^{i,\rm MC}_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},\omega}} + a^{\rm nd}_{\boldsymbol{k},\omega}\chi^{d,\rm eq}_{\boldsymbol{k},\omega}\,, \quad (21)$$

where the responses $\chi^{i,\rm MC}_{\boldsymbol{k},\omega}$, $G^{\rm MC}_{\boldsymbol{k},\omega}$ are altered from the full kinetic model ones due to the absence of the $\nu_{d,i}$ term in the denominator.

The fluctuating parts of the densities will be

$$\delta n^{i}_{\boldsymbol{k},\omega} = -\frac{\imath k}{4\pi e}\left[\frac{\chi^{i,\rm MC}_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},\omega}}\right]\delta E_{\boldsymbol{k},\omega},$$
$$\delta n^{d}_{\boldsymbol{k},\omega} = -\frac{\imath k}{4\pi q_{\rm eq}}\,\chi^{d,\rm eq}_{\boldsymbol{k},\omega}a^{\rm nd}_{\boldsymbol{k},\omega}\delta E_{\boldsymbol{k},\omega} + a^{\rm nd}_{\boldsymbol{k},\omega}\delta n^{d,(0)}_{\boldsymbol{k},\omega}\,, \quad (22)$$
$$\delta n^{e}_{\boldsymbol{k},\omega} = \frac{\imath k}{4\pi e}\,\frac{1}{k^2\lambda^2_{De}}\,\delta E_{\boldsymbol{k},\omega},$$

which result in expressions of all the density fluctuations as linear functions of the natural dust fluctuations,

$$\delta n^i_{\boldsymbol{k},\omega} = \left\{ Z_d \frac{a^{\rm nd}_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},\omega}} \frac{\chi^{i,\rm MC}_{\boldsymbol{k},\omega}}{\epsilon^{\rm MC}_{\boldsymbol{k},\omega}} \right\} \delta n^{d,(0)}_{\boldsymbol{k},\omega},$$
$$\delta n^d_{\boldsymbol{k},\omega} = \left\{ a^{\rm nd}_{\boldsymbol{k},\omega} \left[1 - \frac{a^{\rm nd}_{\boldsymbol{k},\omega}\chi^{d,\rm eq}_{\boldsymbol{k},\omega}}{\epsilon^{\rm MC}_{\boldsymbol{k},\omega}} \right] \right\} \delta n^{d,(0)}_{\boldsymbol{k},\omega}, \tag{23}$$
$$\delta n^e_{\boldsymbol{k},\omega} = \left\{ -\frac{Z_d}{k^2\lambda^2_{De}} \frac{a^{\rm nd}_{\boldsymbol{k},\omega}}{\epsilon^{\rm MC}_{\boldsymbol{k},\omega}} \right\} \delta n^{d,(0)}_{\boldsymbol{k},\omega}.$$

The above relations enable us to find the spectral densities of all fluctuating quantities as a linear function of the spectral density of the natural dust density fluctuations. For the quantities of interest we obtain

$$S^{\phi,\rm MC}_{\boldsymbol{k},\omega} = \langle \delta\phi_{\boldsymbol{k},\omega}\delta\phi^*_{\boldsymbol{k},\omega}\rangle = \frac{16\pi^2e^2}{k^4}\left|\frac{a^{\rm nd}_{\boldsymbol{k},\omega}}{\epsilon^{\rm MC}_{\boldsymbol{k},\omega}}\right|^2 Z_d^2 S^{d,(0)}_{\boldsymbol{k},\omega}, \tag{24}$$

$$S^{i,\rm MC}_{\boldsymbol{k},\omega} = \langle \delta n^i_{\boldsymbol{k},\omega}\delta n^{i*}_{\boldsymbol{k},\omega}\rangle = Z_d^2\left|\frac{a^{\rm nd}_{\boldsymbol{k},\omega}}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},\omega}}\right|^2 \left|\frac{\chi^{i,\rm MC}_{\boldsymbol{k},\omega}}{\epsilon^{\rm MC}_{\boldsymbol{k},\omega}}\right|^2 S^{d,(0)}_{\boldsymbol{k},\omega}. \tag{25}$$

Note that apart from the changes neutrals inflict in $\chi^{i,\rm MC}_{\boldsymbol{k},\omega}$, $\epsilon^{\rm MC}_{\boldsymbol{k},\omega}$, and Z_d, their effect stems mainly from the explicit terms $a^{\rm nd}_{\boldsymbol{k},\omega}$ and $\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},\omega}$ that tend to unity and zero, respectively, for a fully ionized plasma.

D. Spectral densities in absence of dust particles

To quantify the spectral modifications due to the presence of dust, a comparison with theoretical results for ordinary plasmas is necessary. Several simplifications can be made here for the low-frequency regime of interest: (i) ions are considered as the only source of discreteness, i.e., $\delta n^i(\boldsymbol{r},t) = \delta n^{i,(0)}(\boldsymbol{r},t) + \delta n^{i,(\rm ind)}(\boldsymbol{r},t)$, (ii) electrons are described as continuous Vlasov fluids under the adiabatic assumption, and (iii) collisions with neutrals are treated under the BGK description.

Following the aforementioned methodology, but now expressing all fluctuating quantities via $\delta f^{i,(0)}_{\boldsymbol{p},\boldsymbol{k},\omega}$, which is the only source of discreteness, the fluctuating parts of the densities are

$$\delta n^i_{\boldsymbol{k},\omega} = \frac{1}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},0}}\left(1-\frac{\chi^{i,\rm MC}_{\boldsymbol{k},0}}{\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}}\right)\delta n^{i,(0)}_{\boldsymbol{k},\omega}, \tag{26}$$

$$\delta n^e_{\boldsymbol{k},\omega} = \frac{1}{k^2\lambda^2_{De}}\frac{1}{\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}}\delta n^{i,(0)}_{\boldsymbol{k},\omega}, \tag{27}$$

where the effective permittivity is given by $\epsilon^{{\rm eff},i}_{\boldsymbol{k},0} = \frac{e\epsilon^i_{\boldsymbol{k},0}}{q^{{\rm eff},i}_{\boldsymbol{k},0}}$ and defined through $\delta E_{\boldsymbol{k},\omega} = \frac{4\pi e}{\imath k\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}}\delta n^{i,(0)}_{\boldsymbol{k},\omega}$, with the permittivity given by $\epsilon^i_{\boldsymbol{k},0} = 1 + \frac{1}{k^2\lambda^2_{De}} + \frac{\chi^{i,\rm MC}_{\boldsymbol{k},0}}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},0}}$ and the effective charge by $q^{{\rm eff},i}_{\boldsymbol{k},0} = \frac{e}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},0}}$. Note that owing to $\omega \ll kv_{Ti}$, the frequency can be neglected in all the low-frequency responses.

The spectral densities of interest will be

$$S^\phi_{\boldsymbol{k},\omega} = \frac{16\pi^2e^2}{k^4}\frac{1}{|\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}|^2}S^{i,(0)}_{\boldsymbol{k},\omega},$$
$$S^i_{\boldsymbol{k},\omega} = \left|\frac{1}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},0}}\right|^2\left|1-\frac{\chi^{i,\rm MC}_{\boldsymbol{k},0}}{\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}}\right|^2 S^{i,(0)}_{\boldsymbol{k},\omega}.$$

Finally, $S^{i,(0)}_{\boldsymbol{k},\omega} = (2\pi)^{1/2}\frac{n_i}{kv_{Ti}}\exp\{-(\frac{\omega^2}{2k^2v^2_{Ti}})\} \simeq (2\pi)^{1/2}\frac{n_i}{kv_{Ti}}$, for $\omega \ll kv_{Ti}$, which yields the frequency-independent expressions

$$S^\phi_{\boldsymbol{k},0} = \frac{16\sqrt{2}\pi^{5/2}e^2n_i}{k^5v_{Ti}}\frac{1}{|\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}|^2}, \tag{28}$$

$$S^i_{\boldsymbol{k},0} = (2\pi)^{1/2}\frac{n_i}{kv_{Ti}}\left|\frac{1}{1+\nu_{n,i}G^{\rm MC}_{\boldsymbol{k},0}}\right|^2\left|1-\frac{\chi^{i,\rm MC}_{\boldsymbol{k},0}}{\epsilon^{{\rm eff},i}_{\boldsymbol{k},0}}\right|^2. \tag{29}$$

III. NUMERICAL RESULTS

Quiescent laboratory plasmas are characterized by large ion densities, e.g., for the cusp device [9] $n_i \sim 10^{11}$ cm^{-3}, and for the brush cathode [4] and its variants $n_i \sim 10^{10}$–10^{12} cm^{-3}. The electron temperatures can vary substantially from $T_e \sim 3$ eV (cusp device, reflex brush cathode [5]) to $T_e \sim 0.1$ eV (brush cathode, inverse brush cathode [6], large V-groove cathode discharge [7]). The gas pressure can vary up to $P_n \sim 100$ Pa.

In the calculations below we will use the typical set of parameters: $n_i = 10^{11}$ cm^{-3}, $T_i \sim T_n \sim T_d \sim 0.03$ eV, $T_e \sim 3$ eV, $P_n = 10$ Pa, and argon as the operating gas. In addition the electron temperature and pressure dependence are investigated. As a typical *in situ* dust size we consider $a = 50$ nm (in accordance with a commonly observed value of the dust radius after the agglomeration phase) and for the dust density $n_d \sim 10^6$ cm^{-3}, for these values the dust density parameter is $P = n_dZ_d/n_e \sim 0.01$.

In the numerical results presented below the normalized wave numbers $k\lambda_{Di}$ and the normalized frequencies ω/kv_{Td} are used. The condition for the omission of plasma discreetness (see Appendix) is $\omega/kv_{Td} < \Lambda_i,\Lambda_e$, thus with the latter normalization the same frequency interval can be used for all the parameters investigated. The spectral densities of ion fluctuations are plotted as $\frac{S^i_{\boldsymbol{k},\omega}}{n_i}$ (s) and the spectral densities of electrostatic potential fluctuations as $\frac{S^\phi_{\boldsymbol{k},\omega}}{e^2}$ (cm s). Moreover, the level of fluctuations in dust-free plasmas is indicated for every set of parameters in order to manifest the spectral enhancement due to the presence of dust.

To motivate the choice of wave numbers we point out that due to the finite probe size, not all k's are resolved [15]. The density measured at the probe position is the uniform average of the density over its collecting volume [22]. Therefore, wavelengths shorter than the probe's largest dimension will be averaged out. For a cylindrical probe (with length L, diameter d, and $L \ll d$), typical lengths are 1–0.1 mm and the wave numbers of interest will be $k\lambda_{Di} < \frac{2\pi}{L}\lambda_{Di}$ or $k\lambda_{Di} < 0.1$.

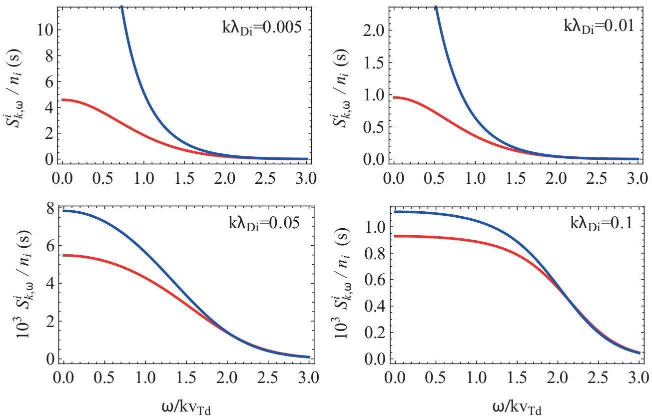

FIG. 1. (Color online) Spectral density of the ion density fluctuations as a function of the normalized frequency ω/kv_{Td} for the full (red bottom line) and the multicomponent (blue top line) kinetic models for different wave numbers $k\lambda_{Di}$. The plasma and dust parameters are $n_i = 10^{11}$ cm^{-3}, $P_n = 10$ Pa, $T_e = 3$ eV, $a = 50$ nm, and $P = 0.01$. The levels of fluctuations in a dust-free plasma for increasing $k\lambda_{Di}$ are $S^i_{\boldsymbol{k},\omega}/n_i = 1.5 \times 10^{-7}$, 1×10^{-8}, 2.6×10^{-10}, and 2.2×10^{-10} s, respectively.

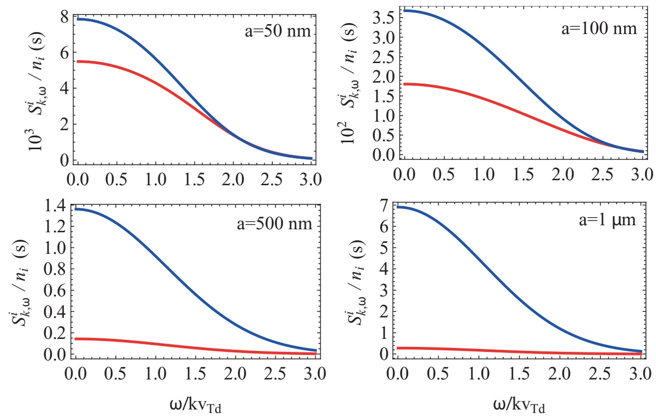

FIG. 3. (Color online) Spectral density of the ion density fluctuations as a function of the normalized frequency ω/kv_{Td} for the full (red bottom line) and the multicomponent (blue top line) kinetic models for varying dust radii. The parameters are $n_i = 10^{11}$ cm^{-3}, $P_n = 10$ Pa, $T_e = 3$ eV, $P = 0.01$, and $k\lambda_{Di} = 0.05$. For increasing radius the dust densities are $n_d = 4.8 \times 10^6$, 2.3×10^6, 4.3×10^5, and 2.1×10^5 cm^{-3}. The level of fluctuations in a dust-free plasma is $S^i_{\boldsymbol{k},\omega}/n_i = 2.6 \times 10^{-10}$ s.

A. Spectral densities for varying wave numbers

In Figs. 1 and 2 the spectral densities of the ion density fluctuations and the electrostatic potential fluctuations are plotted for a range of normalized wave numbers $k\lambda_{Di}$. The spectral densities are both proportional to the spectral density of natural dust fluctuations $S^{d,(0)}_{\boldsymbol{k},\omega}$, which bears an exponential $\propto \exp(-\frac{\omega^2}{2k^2 v_{Td}})$ dependence and decays much faster than all the other responses (that are not purely exponential). Thus, the spectral densities are rapidly decaying functions of the normalized frequency, especially in the $\omega/kv_{Td} > 1$ range. When examining the dependence of the spectral density magnitude on the wave number, we notice that they decrease as $k\lambda_{Di}$ increases. This is due to the scaling of the normalized frequency with k.

In the range $k\lambda_{Di} < 0.01$ the deviations between the models are always very large, even when considering low densities of nano-dust. Such spatial scales are not only relevant to absorption length scales, $k_{\text{abs}} = \bar{\nu}_{di}/v_{Ti}$ where $\bar{\nu}_{di}$ is the average absorption frequency, but also to the low-frequency roots of the permittivity. As $k\lambda_{Di}$ increases, the deviation between the models decreases and in the short wavelength limit they eventually overlap. Similar behavior has been observed in the real parts of the permittivity and the static permittivity of the models [19]. In light of the above, a length scale $k\lambda_{Di} \sim 0.05$ can be regarded as representative of the deviations between the models and will be thereafter used in Figs. 3–6.

Finally, we notice that for $S^i_{\boldsymbol{k},\omega}$ the full model predicts lower values than the multicomponent one, whereas the behavior is inverse for $S^\phi_{\boldsymbol{k},\omega}$; also, the deviations are always larger for $S^i_{\boldsymbol{k},\omega}$. While the formulas for $S^\phi_{\boldsymbol{k},\omega}$ differ only in the effective permittivities in the denominator [compare Eqs. (16) and (24)], the formulas for $S^i_{\boldsymbol{k},\omega}$ are completely different [compare Eqs. (17) and (25)].

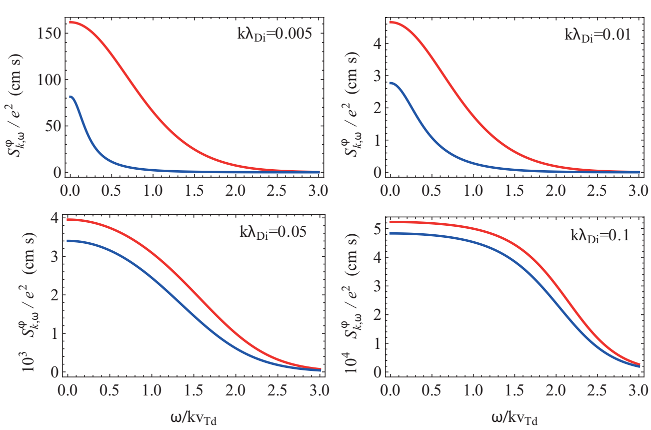

FIG. 2. (Color online) Spectral density of the electrostatic potential fluctuations as a function of the normalized frequency ω/kv_{Td} for the full (red top line) and the multicomponent (blue bottom line) kinetic models for different wave numbers $k\lambda_{Di}$ and the parameters of Fig. 1. The levels of fluctuations in a dust-free plasma for increasing $k\lambda_{Di}$ are $S^\phi_{\boldsymbol{k},\omega}/e^2 = 6 \times 10^{-4}$, 4×10^{-5}, 7×10^{-7}, and 2.4×10^{-7} cm s, respectively.

B. Ion spectral density for varying dust parameters

In Fig. 3, the ion fluctuation spectral densities of the full kinetic model are compared to the multicomponent kinetic model for varying grain radii from 50 nm to 1 μm. For the comparison, the dust density parameter P is kept constant and hence the dust density is varying. The reason for this choice is that in order to isolate the effect of plasma absorption, the effect of dust in the quasineutrality condition (the physical meaning of P) should be kept constant. Moreover, this leads to more realistic dust densities, i.e., $\sim 10^6$ cm^{-3} for small nano-dust and $\sim 10^5$ cm^{-3} for larger dust. Nevertheless, the conclusions below are valid also for n_d constant.

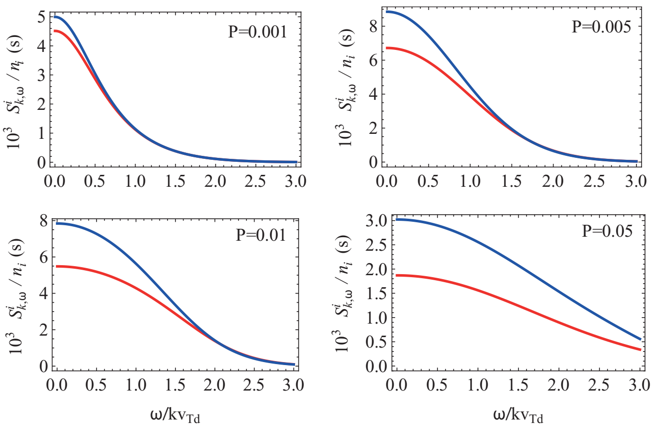

FIG. 4. (Color online) Spectral density of the ion density fluctuations as a function of the normalized frequency ω/kv_{Td} for the full (red bottom line) and the multicomponent (blue top line) kinetic models for varying dust densities. The parameters are $n_i = 10^{11}$ cm^{-3}, $P_n = 10$ Pa, $T_e = 3$ eV, $a = 50$ nm, and $k\lambda_{Di} = 0.05$. For increasing P the dust densities are $n_d = 4.8 \times 10^5, 2.4 \times 10^6, 4.8 \times 10^6$, and 2.3×10^7 cm^{-3}. The level of fluctuations in a dust-free plasma is $S^i_{\boldsymbol{k},\omega}/n_i = 2.6 \times 10^{-10}$ s.

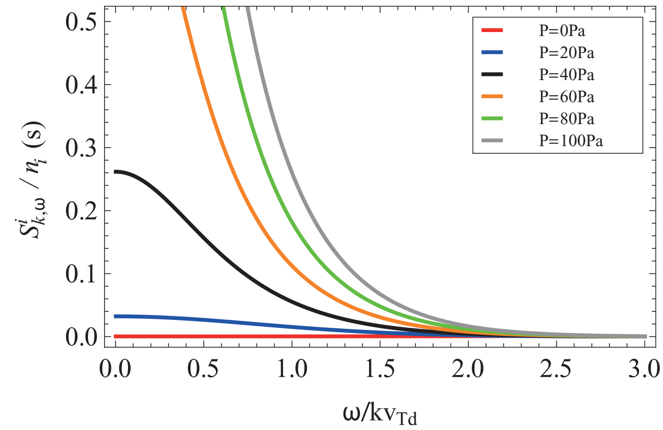

FIG. 5. (Color online) Spectral density of the ion density fluctuations as a function of the normalized frequency ω/kv_{Td} for the full kinetic model for varying pressure. The parameters are $n_i = 10^{11}$ cm^{-3}, $T_e = 3$ eV, $a = 50$ nm, $P = 0.01$, and $k\lambda_{Di} = 0.05$. The level of fluctuations in a dust-free plasma for increasing pressure is $S^i_{\boldsymbol{k},\omega}/n_i = 1.2 \times 10^{-10}, 5.3 \times 10^{-10}, 1.8 \times 10^{-9}, 5 \times 10^{-9}, 1 \times 10^{-8}$, and 2.3×10^{-8} s, respectively.

As the radius increases, (i) absorption of plasma fluxes on the grains becomes more significant and dust cannot be treated adequately as an additional ion species, and the differences between the models increases monotonically from 20% to orders of magnitude, (ii) for both models the spectral densities are greatly increasing in magnitude, and (iii) the dust thermal velocity is decreasing, hence the (ω) frequency interval of significant enhancement noticeably reduces.

In Fig. 4, the comparison of the ion fluctuation spectral densities is carried out for varying dust density (through control of the dust density parameter P). For both kinetic models, $S^i_{\boldsymbol{k},\omega}$ is not a monotonic growing function of P. Increase of the dust density initially leads to an increase of $S^i_{\boldsymbol{k},\omega}$, which reaches a maximum and then decreases. This behavior has been discussed and analytically investigated in Ref. [15]. We also notice that as P increases, the deviation becomes more significant. This is to be expected since larger n_d implies larger absorption frequencies on dust.

C. Effect of neutrals and electron temperature

The effect of neutrals can be very important, especially for low-temperature discharges operating at elevated pressures. The presence of neutrals affects the permittivity and density fluctuations due to (i) additional fluctuations induced by collisions with neutrals, (ii) electron impact ionization and the fluctuations it induces, and (iii) the change in absorption cross sections of ions on dust (which also affect the equilibrium dust charge).

In Fig. 5, we investigate the sensitivity of the full kinetic model $S^i_{\boldsymbol{k},\omega}$ on pressure. Notice that increase of the pressure leads to a significant monotonic enhancement of the spectral density by orders of magnitude, despite the monotonic dust charge depletion.

The electron temperature variation can be of major importance in partially ionized dusty plasmas, since T_e controls the value of the dust equilibrium charge and also the strength of the ionization frequency. In Fig. 6, $S^i_{\boldsymbol{k},\omega}$ is plotted for different electron temperatures. For $T_e < 3$ eV electron impact ionization is not important; as T_e increases, more electrons obtain kinetic energies large enough to overcome the repulsive potential barrier of the dust grains and therefore the dust charge number also increases, $Z_d = \frac{zaT_e}{e^2}$ with z weakly dependent on T_e. Taking into account the rough proportionality of both spectral densities on Z_d^2 [see Eqs. (17) and (25)], it is expected that increase of T_e will lead to an increase of the spectral densities in all frequency ranges. For $T_e \geqslant 3$ eV electron impact ionization becomes important; as T_e increases, more electrons from the tail of the Maxwellian distribution obtain enough

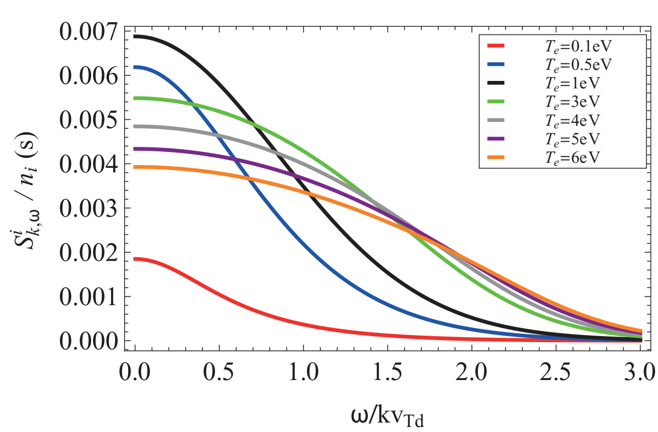

FIG. 6. (Color online) Spectral density of the ion density fluctuations as a function of the normalized frequency ω/kv_{Td} for the full kinetic model for varying electron temperature. The parameters are $n_i = 10^{11}$ cm^{-3}, $P_n = 10$ Pa, $a = 50$ nm, $P = 0.01$, and $k\lambda_{Di} = 0.05$. For increasing T_e the dust charge number is $Z_d = 12, 50, 87, 207, 257, 303$, and 346. The level of fluctuations in a dust-free plasma for increasing T_e is $S^i_{\boldsymbol{k},\omega}/n_i = 9 \times 10^{-8}, 6 \times 10^{-9}, 1.7 \times 10^{-9}, 2.6 \times 10^{-10}, 1.6 \times 10^{-10}, 1.2 \times 10^{-10}$, and 9×10^{-11} s, respectively.

kinetic energy to ionize the argon gas ($U_{iz} = 15.76$ eV), the ionization related fluctuating quantities increase (being directly proportional to the average ionization frequency ν_e), and the spectral densities decrease in most frequency ranges. This picture is confirmed by the results of Fig. 6. When comparing the cold ($T_e \sim 0.1$ eV) electrons with the typical $T_e \sim 3$ eV the differences are of a factor of 5 over the whole frequency range.

IV. CONCLUSIONS

(1) The spectra of ion density and electrostatic potential fluctuations are derived and evaluated numerically for plasma conditions of laboratory low-temperature discharges. The measurement of two independent fluctuating quantities can prove useful for the verification of the theoretical predictions and the development of the dust diagnostic.

(2) The results of both kinetic models taking into account the presence of neutrals reveal orders of magnitude enhancement (five to ten orders of magnitude) of the fluctuation level due to the presence of dust, similar to previously reported spectral changes for fully ionized plasmas.

(3) Neutral gas density (pressure) can be responsible for significant modifications of spectral density magnitudes, despite the opposite behavior of the equilibrium dust charge number Z_d. Hence the inclusion of the effect of neutrals is essential for a more realistic comparison with experiments, especially if spectral measurements are intended for dust diagnostic purposes.

(4) The full self-consistent model deviates from the multicomponent model significantly (above typical experimental errors) already for typical dust densities $n_d \sim 10^5$ cm^{-3} and dust radii $a > 100$ nm, which is common for *in situ* produced dust.

(5) The nonmonotonic effect of electron temperature variation is attributed to the strength of electron impact ionization.

We also mention that for some space environments, plasma fluxes emitted by the grain might be of importance in dust charging, e.g., photoelectric emission in presence of a strong radiation source [23–27]. In such cases an additional term $I_{\rm ext}$ is to be added to the charging equation and will affect the equilibrium dust charge $q_{\rm eq}$, by reducing its negative value or even making it positive. This, however, will not alter the expressions for the dust charge fluctuations and consequently the spectral densities, since $I_{\rm ext}$ can be considered constant in the temporal and spatial scales of the fluctuations [16,18]. In addition, one should also include relevant source terms in the plasma Klimontovich equations, these will also be nonfluctuating and hence have no effect on the spectral densities.

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APPENDIX: A CONDITION FOR THE OMISSION OF NATURAL PLASMA DENSITY FLUCTUATIONS

One of the basic assumptions of the kinetic model [16] is the treatment of electrons and ions as continuous Vlasov fluids in the low-frequency regime of dust dynamics (roughly defined so far as $\omega \ll k v_{Ti}$). Neglecting plasma discreteness also implies that the electron and ion binary collisions can be neglected compared to collisions with dust.

Due to the random character of the natural plasma fluctuations, the range of validity of such an assumption can only be estimated in light of its effect in the deliverables of the theory, i.e., the permittivity, the collision integrals, and the spectral densities of fluctuations. The permittivity cannot provide any insight since it depends only on the induced fluctuations, while estimates based on the characteristic frequencies of the collisional processes have yielded the condition $P Z_d > 1$ for natural plasma fluctuations to be negligible [16,28]. Here, we find a condition on neglecting plasma discreteness based on the spectral densities of fluctuations.

The effect of plasma discreteness on the spectral densities will initially be evaluated through the *multicomponent model of fully ionized plasmas*. Also taking $\delta f_{\boldsymbol{p}}^{e,(0)}(\boldsymbol{r},t)$ and $\delta f_{\boldsymbol{p}}^{i,(0)}(\boldsymbol{r},t)$ into account will yield the spectral densities [15,17]

$$S^{\alpha}_{\boldsymbol{k},\omega} = S^{\alpha,(0)}_{\boldsymbol{k},\omega}\left(1 - 2\mathrm{Re}\left\{\frac{\chi^{\alpha}_{\boldsymbol{k},\omega}}{\epsilon_{\boldsymbol{k},\omega}}\right\}\right) + \left|\frac{\chi^{\alpha}_{\boldsymbol{k},\omega}}{\epsilon_{\boldsymbol{k},\omega}}\right|^2 \times \left(S^{\alpha,(0)}_{\boldsymbol{k},\omega} + S^{\beta,(0)}_{\boldsymbol{k},\omega} + Z_d^2 S^{d,(0)}_{\boldsymbol{k},\omega}\right), \tag{A1}$$

$$S^{\phi}_{\boldsymbol{k},\omega} = \frac{16\pi^2 e^2}{k^4|\epsilon_{\boldsymbol{k},\omega}|^2}\left(S^{\alpha,(0)}_{\boldsymbol{k},\omega} + S^{\beta,(0)}_{\boldsymbol{k},\omega} + Z_d^2 S^{d,(0)}_{\boldsymbol{k},\omega}\right), \tag{A2}$$

where $\alpha,\beta = \{i,e\}$ and $\alpha \neq \beta$. Therefore, a meaningful comparison is between the spectral density of natural plasma charge density fluctuations $e^2 S^{\alpha,(0)}_{\boldsymbol{k},\omega}$ and the spectral density of natural dust charge density fluctuations $Z_d^2 e^2 S^{d,(0)}_{\boldsymbol{k},\omega}$. Imposing $Z_d^2 e^2 S^{d,(0)}_{\boldsymbol{k},\omega} > e^2 S^{\alpha,(0)}_{\boldsymbol{k},\omega}$ and using the Maxwellian expressions for the natural correlators, we can obtain a simple condition for neglecting natural plasma fluctuations (also using $v_{Td} \ll v_{T\alpha}$ or equivalently keeping only the zero-order terms in the small parameter $\frac{\omega}{k v_{T\alpha}}$): $\frac{\omega}{k v_{Td}} < \Lambda_\alpha$ with the dimensionless parameter defined by $\Lambda_\alpha = \sqrt{2}\{\ln(\frac{Z_d^2 n_d}{n_\alpha}\frac{v_{T\alpha}}{v_{Td}})\}^{1/2}$. Note that for typical plasma and dust parameters and for grains of radii ranging from 20 nm to 10 μm, we have $\Lambda_i = 5$–7 and $\Lambda_e = 6$–8.

In the multicomponent model the effects of charging in the system's kinetics are not considered, hence the only discrete collisional process are Coulomb collisions with the resulting spectral functions $S^{\gamma,(0)}_{\boldsymbol{k},\omega}$, $\gamma = \{i,e,d\}$. In the *full kinetic model of fully ionized plasmas* due to the effect of the plasma discreteness in inelastic charging collisions and the velocity dependence of the capture cross sections new spectral functions will appear for electrons and ions. Yet, when compared to $S^{e,(0)}_{\boldsymbol{k},\omega}$ and $S^{i,(0)}_{\boldsymbol{k},\omega}$ respectively in the region $\frac{\omega}{k v_{T\alpha}} \ll 1$ they are much smaller and thus the condition $\frac{\omega}{k v_{Td}} < \Lambda_\alpha$ still holds. The new spectral functions will be of the form [13]

$$T^{\alpha}_{\boldsymbol{k},\omega} = \frac{2\pi}{\nu_{\rm ch}} \int \nu_{d,\alpha}(v)\Phi^{\alpha}_{\boldsymbol{p}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\frac{d^3p}{(2\pi)^3}, \quad \text{(A3)}$$

$$\widetilde{T}^{\alpha}_{\boldsymbol{k},\omega} = \frac{2\pi}{\nu^2_{\rm ch}} \int \nu^2_{d,\alpha}(v)\Phi^{\alpha}_{\boldsymbol{p}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\frac{d^3p}{(2\pi)^3}, \quad \text{(A4)}$$

where $\nu_{d,\alpha}(v) = n_d v \sigma_\alpha(q,v)$ is the instantaneous frequency of inelastic charging collisions with dust and $\nu_{\rm ch}$ is the charging frequency.

New ion spectral functions. For $T^i_{\boldsymbol{k},\omega}$, we acquire

$$T^i_{\boldsymbol{k},\omega} = \frac{(2\pi)^2 n_d}{\nu_{\rm ch}} \int_{\omega/k}^{\infty} \frac{v^2}{k}\sigma_i(q,v)\Phi^i(v)dv,$$

which for Maxwellian distributions, orbit motion limited (OML) collisionless cross sections $\sigma_i(q_{\rm eq},v)$ [29] and $\zeta_i = \frac{\omega}{\sqrt{2}kv_{Ti}}$ will lead to

$$T^i_{\boldsymbol{k},\omega} = \frac{\sqrt{2\pi}n_i}{kv_{Ti}} \frac{\sqrt{2}n_d v_{Ti}\pi a^2}{\nu_{\rm ch}} \times \left[\zeta_i e^{-\zeta_i^2} + \left(\frac{1}{2} + \frac{z}{\tau}\right)\sqrt{\pi}[1 - \mathrm{erf}(\zeta_i)]\right],$$

where erf(.) is the error function. Expansion in terms of ζ_i gives the zero-order term

$$T^i_{\boldsymbol{k},\omega} \sim S^{i,(0)}_{\boldsymbol{k},\omega} \frac{\sqrt{2}n_d v_{Ti}\pi a^2}{\nu_{\rm ch}}\left(\frac{1}{2} + \frac{z}{\tau}\right)\sqrt{\pi}.$$

Use of $\nu_{\rm ch} = \frac{a\omega_{pi}}{\sqrt{2\pi}\lambda_{Di}}(1 + z + \tau)$ and typical parameters leads to the estimate $T^i_{\boldsymbol{k},\omega}/S^{i,(0)}_{\boldsymbol{k},\omega} \sim 10^{-3}$.

Similarly for $\widetilde{T}^i_{\boldsymbol{k},\omega}$ we get

$$\widetilde{T}^i_{\boldsymbol{k},\omega} = \frac{\sqrt{2\pi}n_i}{kv_{Ti}} \frac{2n_d^2\pi^2 a^4}{\nu^2_{\rm ch}} v^2_{Ti} \times \left[\left(1 + \frac{2z}{\tau} + \zeta_i^2\right)e^{-\zeta_i^2} + \frac{z^2}{\tau^2}\mathrm{Ei}(\zeta_i^2)\right],$$

where Ei(.) is the exponential integral, this leads to the low-frequency expansion

$$\widetilde{T}^i_{\boldsymbol{k},\omega} \sim S^{i,(0)}_{\boldsymbol{k},\omega} \frac{2n_d^2\pi^2 a^4 v^2_{Ti}}{\nu^2_{\rm ch}} \frac{z^2}{\tau^2}\mathrm{Ei}(\zeta_i^2),$$

and the estimate $\widetilde{T}^i_{\boldsymbol{k},\omega}/S^{i,(0)}_{\boldsymbol{k},\omega} \sim 10^{-4}$.

New electron spectral functions. The main difference is that only electrons with $v > v^* = \sqrt{\frac{2Z_d e^2}{am_e}}$ can cross the repulsive barrier set by the negatively charged dust and be captured by the grain, hence the lower integration limit of the integrals will be $\max\{v^*, \frac{\omega}{k}\} = v^*$, since typically $v^* \simeq 10^7$–10^8 cm/s while $v_{Ti} \simeq 10^4$ cm/s. Therefore, the electron spectral functions will be nearly of the form of the ion ones, with z instead of ζ_e^2, i.e.,

$$T^e_{\boldsymbol{k},\omega} = \frac{\sqrt{2\pi}n_e}{kv_{Te}} \frac{\sqrt{2}\pi a^2 n_d}{\nu_{\rm ch}} v_{Te} \times \left\{\sqrt{z}e^{-z} + \left(\frac{1}{2} - z\right)\sqrt{\pi}[1 - \mathrm{erf}(\sqrt{z})]\right\},$$

$$\widetilde{T}^e_{\boldsymbol{k},\omega} = \frac{\sqrt{2\pi}n_e}{kv_{Te}} \frac{2n_d^2\pi^2 a^4}{\nu^2_{\rm ch}} v^2_{Te}\{(1 - z)e^{-z} + z^2\mathrm{Ei}(z)\},$$

which yield the estimates $T^e_{\boldsymbol{k},\omega}/S^{e,(0)}_{\boldsymbol{k},\omega} \sim 10^{-2}$ and $\widetilde{T}^e_{\boldsymbol{k},\omega}/S^{e,(0)}_{\boldsymbol{k},\omega} \sim 10^{-3}$, respectively.

Finally, to complete the proof, in the *full kinetic model of partially ionized plasmas*, due to the effect of electron discreetness in the electron impact ionization of neutrals and the velocity dependence of the ionization cross sections $\sigma_{iz}(v)$, additional spectral functions will arise. They will be of the form

$$Z^e_{\boldsymbol{k},\omega} = \frac{2\pi}{\nu_e} \int \nu^I(v)\Phi^e_{\boldsymbol{p}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\frac{d^3p}{(2\pi)^3}, \quad \text{(A5)}$$

$$\widetilde{Z}^e_{\boldsymbol{k},\omega} = \frac{2\pi}{\nu^2_e} \int [\nu^I(v)]^2\Phi^e_{\boldsymbol{p}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\frac{d^3p}{(2\pi)^3}, \quad \text{(A6)}$$

$$Z^{e,\rm ch}_{\boldsymbol{k},\omega} = \frac{2\pi}{\nu_e\nu_{\rm ch}} \int \nu^I(v)\nu_{d,e}(v)\Phi^e_{\boldsymbol{p}}\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\frac{d^3p}{(2\pi)^3}, \quad \text{(A7)}$$

with $\nu^I(v)$ the instantaneous ionization frequency and ν_e the average ionization frequency over the electron distribution function. They can always be neglected for $\frac{\omega}{kv_{Te}} \ll 1$ compared to $S^{e,(0)}_{\boldsymbol{k},\omega}$. Moreover, due to the effect of ion-neutral collisions in ion absorption on dust grains, the inelastic charging collision cross sections for ions will not be given by the OML model but by a collision enhanced collection model [19,30] and will be significantly larger on average. However, the charging frequency $\nu_{\rm ch}$ will also be significantly larger, hence $T^i_{\boldsymbol{k},\omega}$, $\widetilde{T}^i_{\boldsymbol{k},\omega}$ can still be neglected compared to $S^{i,(0)}_{\boldsymbol{k},\omega}$. Thus we conclude that the condition $\frac{\omega}{kv_{Td}} < \Lambda_\alpha$ is a sufficient criterion for neglecting the discreetness of plasma particles.

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