Inwardly rotating spirals in nonuniform excitable media

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(Received 19 August 2011; revised manuscript received 28 November 2011; published 24 January 2012)

Inwardly rotating spirals (IRSs) have attracted great attention since their observation in an oscillatory reaction-diffusion system. However, IRSs have not yet been reported in planar excitable media. In the present work we investigate rotating waves in a nonuniform excitable medium, consisting of an inner disk part surrounded by an outer ring part with different excitabilities, by numerical simulations of a simple FitzHugh-Nagumo model. Depending on the excitability of the medium as well as the inhomogeneity, we find the occurrence of IRSs, of which the excitation propagates inwardly to the geometrical spiral tip.

DOI: 10.1103/PhysRevE.85.016213 PACS number(s): 89.75.Kd, 82.40.Ck, 47.54.−r

I. INTRODUCTION

Inwardly propagating waves, e.g., inwardly propagating concentric waves and inwardly rotating spirals (IRSs), were first observed in the oscillatory Belousov-Zhabotinsky (BZ) reaction that dispersed in water droplets of a water-in-oil aerosol OT (AOT) microemulsion (BZ-AOT system) [1,2]. Subsequently, inwardly propagating waves were discovered in other systems, such as the chlorite-iodide-malonic acid (CIMA) reaction [3,4], the oscillatory CO oxidation on Pt (110) [5], an artificial tissue of oscillatory cells [6], and glycolysis [7]. Numerous theoretical and numerical studies have also been carried out to investigate the behaviors of these inwardly propagating waves [8–17].

However, in excitable systems which are typical in the heart tissue and other biological systems, the existence of IRSs is still an unsettled issue. Although pairs of inwardly and outwardly rotating spirals have been observed on spherical surfaces of excitable media [18,19], the appearances of IRSs are highly topologically constrained. In Refs. [20,21], the authors conclude that IRSs cannot exist in homogeneous planar excitable media.

As is known, real systems are commonly spatially inhomogeneous. But, theoretical [22–26] and experimental [27] investigations on the effects of inhomogeneity in excitable and oscillatory systems are mainly focusing on the outwardly rotating spiral waves. In Ref. [28], Lázár *et al.* study the chemical wave propagation in an annular membrane with a slow inner and a fast outer zone, and observe inwardly rotating waves in the slow inner zone. However, compared to annular domains, rotating waves in holeless domains would be more scientifically interesting [29], because most experimental examples of spiral waves have no hole in the center. Recently, we reported the sink spiral in holeless oscillatory medium with a disk-shaped inhomogeneity [30]. Unlike the outward group velocities of inwardly and outwardly rotating spirals in homogeneous oscillatory media, the group velocity of the sink spiral point inward. The followings are two examples of the holeless excitable medium with a disk-shaped inhomogeneity: In the genesis of ischemic arrhythmias, the ischemic area with reduced excitability and conduction velocity is surrounded by the normal tissue [31,32]; in *Dictyostelium* amoebae, high excitable PST cells aggregate to the center of the mound, and leave the periphery of the mound in a relatively low excitable state [33]. Besides the two above examples in the real world, the light sensitive BZ reaction [34] and the CO oxidation on Pt(110) with different metal components on the Pt(110) surface [35] can also build the holeless medium with a disk-shaped inhomogeneity.

In this paper, for the sake of simplicity, we simplify the outer zone to be a thin ring, and examine the wave propagation in a nonuniform excitable medium consisting of an inner disk part surrounded by this outer ring part with different excitabilities. We show that IRSs could arise and their formation greatly depends on the properties of the medium as well as the inhomogeneity.

II. NUMERICAL MODEL AND RESULTS

The following two-variable reaction-diffusion model of the simple FitzHugh-Nagumo type [36,37] is used:

$$\frac{\partial u}{\partial t} = D\Delta u + (3u - u^3 - v), \quad \frac{\partial v}{\partial t} = \varepsilon(u - \delta).$$

In Fig. 1(a), we divide the medium into two parts: the inner disk part and the outer ring part, fix $\varepsilon = 0.013$ and $D = 1$, and change parameters δ_{ring} and δ_{disk} for different excitabilities. We apply the phase field method (or so-called smoothed boundary method) [38] to add a no-flux boundary condition outside the ring part: We set phase field ϕ to be 1 inside the ring part and 0 outside, and use the relaxation method to smooth its value in the interface; the reaction-diffusion model is then changed to be

$$\frac{\partial u}{\partial t} = \nabla(\log \phi) \cdot (D\nabla u) + \nabla(D\nabla u) + (3u - u^3 - v),$$
$$\frac{\partial v}{\partial t} = \varepsilon(u - \delta).$$

When the width of the interface become just a few space units (su), in our case 6 su, as proved by Ref. [38], the no-flux boundary condition is implicitly added in the above revised reaction-diffusion model. The explicit Euler integration method is applied with a discrete step in space $\Delta x = 0.3$ and in time $\Delta t = 0.02$. The system size is 1024×1024 grid

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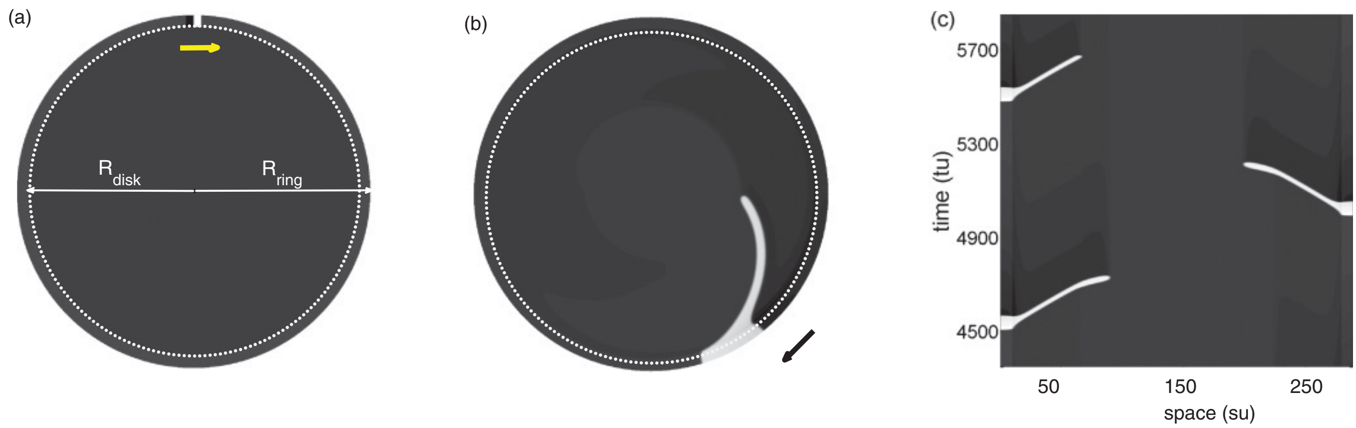

FIG. 1. (Color online) (a) Initial stimulus. The arrow indicates the initial stimulus rotates clockwise; the dotted line is the border between the inner disk part and the outer ring part. $R_{\rm disk} = 134.4$ su and $R_{\rm ring} = 144$ su. (b) The IRS at 4500 time units (tu). (c) Space time plot along the horizontal cut through the medium center. The parameters are $\delta_{\rm ring} = -1.42$ and $\delta_{\rm disk} = -1.63$.

points. The initial stimulus is added on the top of the ring part, and circulates clockwise.

IRSs are obtained in the medium with a weakly excitable (or even subexcitable) disk part and a relatively higher excitable ring part. As shown in Fig. 1(b), the initial stimulus propagates into the inner disk part. Due to the higher excitability, the excitation in the ring part travels faster than the one in the disk part. Thus the wave in the ring part drags the whole wave rotating clockwise, and acts as a source which sustains to excite the neighbor unexcited medium in the disk part. And the free geometrical tip acts as a sink where the excitation propagating from the source is quenched. The space time plot in Fig. 1(c) demonstrates that the excitation propagates towards the center of the medium.

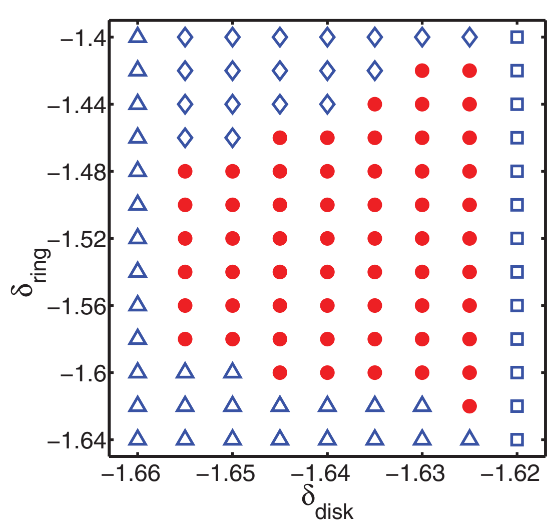

FIG. 2. (Color online) Phase diagram of patterns for $\delta_{\rm disk}$ vs $\delta_{\rm ring}$. The circles denote the IRS patterns, the diamonds the unsteady patterns, the squares the intermediate patterns, and the triangles the no-excitation propagation patterns. Note that the IRS in Fig. 1(b) is the longest wave in our simulation, and longer waves with several wavelengths are not observed, even at a larger medium (with system size 4096×4096 grid points).

In addition to IRSs, we also observe unsteady patterns, intermediate patterns, and no-excitation propagation patterns. In Fig. 2, we give the phase diagram of the types of patterns in the $\delta_{\rm disk}$ vs $\delta_{\rm ring}$ plane. To notify, the rotor boundary (∂R) lies at $\delta = -1.628$, and the propagation boundary (∂P) at $\delta = -1.667$, by the use of the method in Ref. [39]. Both boundaries do not coincide with the border of IRSs.

The intermediate pattern between inwardly and outwardly rotating spirals lies at the right of Fig. 2, i.e., rising $\delta_{\rm disk}$ to some critical point. In our simulation, it is $\delta_{\rm disk} = -1.62$, which is still weakly excitable, not subexcitable. When $\delta_{\rm disk} = -1.62$, as shown in Fig. 3(a), the central and outer parts in such a spiral are wound in opposite directions, as the twisted spiral in periodically forced oscillatory media [40]. The central and outer parts compete with each other; the central part acts as an outwardly rotating spiral and the outer part as an IRS. The wave finally drifts out of the medium. Further enhancing $\delta_{\rm disk}$, the central part suppresses the outer part, and the outwardly rotating spiral dominates the whole medium.

The no-excitation propagation pattern is located at the left and bottom of Fig. 2. As shown in Fig. 3(b), no excitation propagates into the disk part. However, for this pattern, $\delta_{\rm disk}$ is larger than the propagation boundary ($\delta = -1.667$), so the disk part is still excitable.

The IRS gets unsteady as increasing $\delta_{\rm ring}$, plotted at the top of Fig. 2. This is because large $\delta_{\rm ring}$ makes the period of rotation

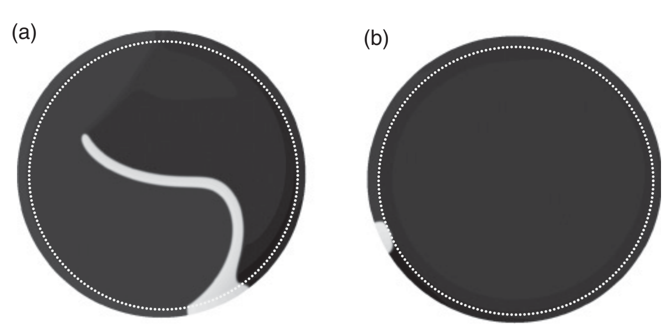

FIG. 3. (a) The intermediate pattern between inwardly and outwardly rotating spirals. $\delta_{\rm disk} = -1.62$ and $\delta_{\rm ring} = -1.55$. (b) The no-excitation propagation pattern. $\delta_{\rm disk} = -1.66$ and $\delta_{\rm ring} = -1.5$.

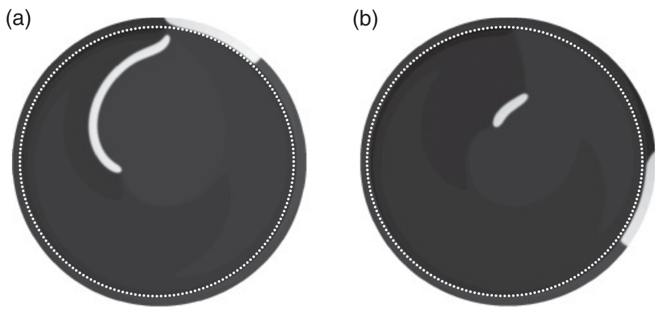

FIG. 4. The unsteady pattern. The parameters are $\delta_{\text{disk}} = -1.63$ and $\delta_{\text{ring}} = -1.2$.

shorter than the refractory time in the disk part. As shown in Fig. 4, after the IRS forms, it breaks up at the interface between disk and ring parts [Fig. 4(a)]. The wave segment in the disk part shrinks as it rotates, whereas the excitation in the ring part continues rotating [Fig. 4(b)]. Unless the disk part is restored to the excitable state, the excitation does not propagate into it and grow into an IRS again. These breakups and growths of the IRS repeat.

The space time plot in Fig. 1(c) implies that the trajectory of a spiral tip is not a rigid circle. Therefore we study the tip path and illustrate it in Fig. 5. At the right part of the IRS region in Fig. 2, the IRS has a long arm [Fig. 5(a)] and the spiral tip meanders. However, in the left part of the IRS region in Fig. 2, the tip moves along a circle [Fig. 5(b)] and the core radius is relatively large.

We study the dependence of the core radius of the IRS on δ. As mentioned above, when the spiral arm is long enough, the core is not a circle. Therefore, we use two variables to measure the core radius: One is the minimal core radius, which is defined as the nearest tip location to the center of the medium; the other is the mean core radius, which is obtained by averaging the distance from tip locations to the center of the medium. In Fig. 6, we present the dependence of the minimal and mean core radii on δ_{disk} and δ_{ring} separately; both the minimal and mean core radii decrease with the increase of δ_{disk} and δ_{ring}. Figure 6 also shows that the more excitable the disk and/or ring part is, the more different the minimal and mean core radii become, and therefore, the more irregular the tip path becomes.

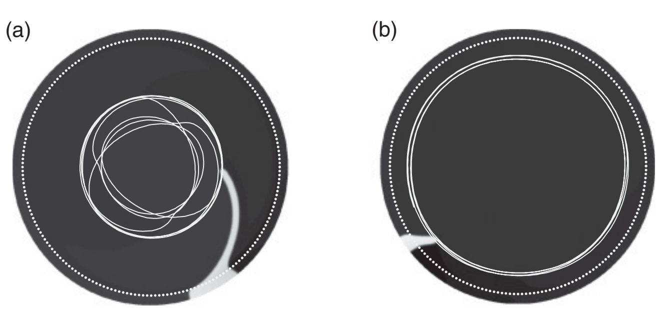

FIG. 5. Tip path. The locations of the tip are obtained by calculating the maximum of $|\nabla u \times \nabla v|$ [40]. The solid line stands for the tip path through several rotations after the transient (about initial three rotations). (a) $\delta_{\text{disk}} = -1.63$ and $\delta_{\text{ring}} = -1.42$; (b) $\delta_{\text{disk}} = -1.635$ and $\delta_{\text{ring}} = -1.58$.

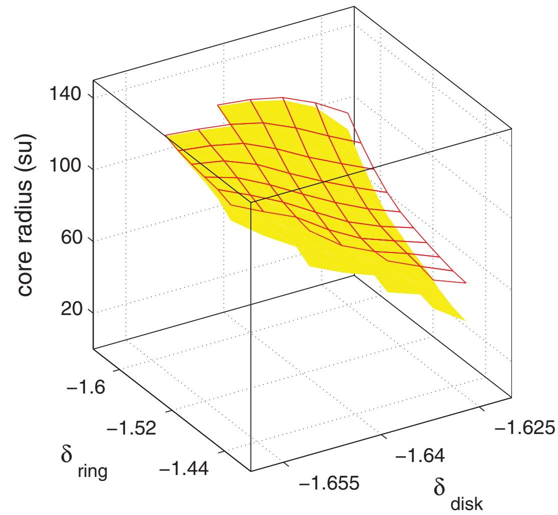

FIG. 6. (Color online) Minimal (yellow solid surface) and mean (red mesh surface) core radii of IRSs.

III. DISCUSSION

As shown in Fig. 1(b), the curvature of the IRS is concave. Generally, in homogeneous media, the concave-shaped waves are unstable, but the stability in our case is established on the properties of the medium and inhomogeneity: In order to match the higher rotation speed in the ring part, the wave in the disk part must be accelerated. In two-dimensional excitable media, the velocity-curvature relationship of waves fronts can be written in the form of an eikonal equation [29,41]:

$$c_n = c_p - Dk,$$

where c_n is the normal velocity, c_p is the plane wave velocity, D is the diffusion coefficient, and k is the wave curvature. To accelerate the wave in the disk part, k should be negative according to the eikonal relation, i.e., the shape of the wave in the disk part should be concave. This situation is similar with the concave wave stabilized in a sandwichlike medium consisting of a lower excitable stripe amid two higher excitable ones [41,42].

Besides the eikonal equation, k, c_n, and the tangential velocity c_τ of a rigidly IRS also obey the following system of differential equations [41]:

$$\frac{dc_n}{ds} = \omega + kc_\tau, \quad \frac{dc_\tau}{ds} = -kc_n,$$

where ω is the angular velocity and s is the arc length along the wave front. The kinematic model above has succeeded in many aspects of spiral waves in homogeneous media [37,43], but needs further study in nonuniform media, such as the medium with a disk-shaped inhomogeneity [44]. Although we lack a rigorous analysis, we can still give an illuminating discussion about IRSs using the dispersion relation for periodic excitation waves in one-dimensional (1D) excitable media.

It is known that the dispersion relation plays a key role in the occurrence of IRSs in oscillatory systems: As Refs. [2,20] show, one of the essential conditions for IRSs in oscillation media is that the dispersion $d\omega/dk$ is negative at the characteristic wave number k_0. From this point of view, we intend to study the dispersion relation in 1D excitable media,

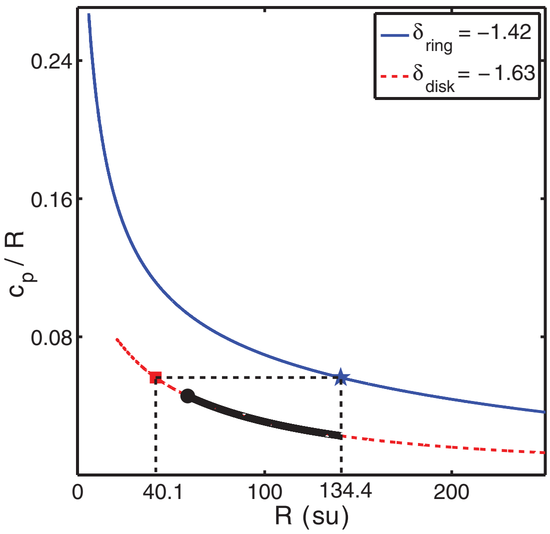

FIG. 7. (Color online) Scheme of the dispersion core radius. The parameters are $\delta_{\mathrm{disk}} = -1.63$ and $\delta_{\mathrm{ring}} = -1.42$, which are the same as in Fig. 1(b).

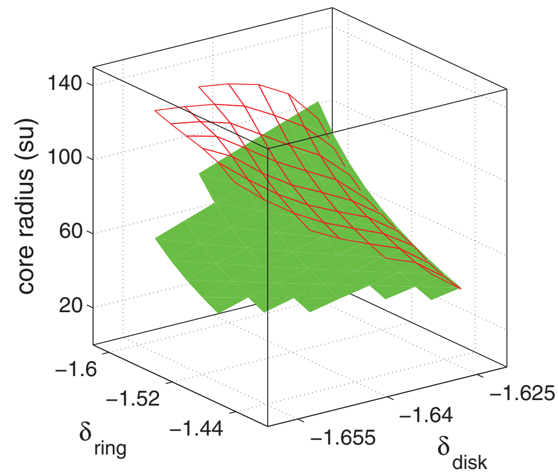

FIG. 9. (Color online) The radii of dispersion (green solid surface) and minimal (red mesh surface) cores.

to grasp some of the criteria for the occurrence of IRSs in nonuniform excitable media.

The dispersion curves are obtained by the method in Ref. [40], in which an excitation is circulating along a 1D ring medium with given δ, whose perimeter ($2\pi R$) is gradually shortened until the excitation is collapsed at some minimal perimeter. In the case of our disk-shaped inhomogeneous medium, we plot the dispersion relations of both disk and ring parts, as shown in Fig. 7, for instance $\delta_{\mathrm{ring}} = -1.42$ (thin, blue solid line) and $\delta_{\mathrm{disk}} = -1.63$ (red dashed line), the same parameters as in Fig. 1(b).

For a rigid IRS, each of the points at the wave has its own concentric circle orbit, shown as dashed lines in Fig. 8. Now, we can investigate the dispersion relation of an excitation in a 1D ring with the same perimeter of this concentric circle orbit. The radius of the outer ring part in this paper is fixed at $R = 134.4$. Therefore, c_p/R in the ring part is 0.054 (blue star point in Fig. 7). We also superimpose each of the points' c_p/R in the inner ring part at its corresponding radius of concentric circle orbit, as a thick, black solid line (the tip is especially plotted as a black circle point) in Fig. 7.

In Fig. 7, we plot a horizontal dashed line from the star point to cross the dispersion relation curve of the disk part at the square point. The abscissa value of the red square point is defined as the dispersion core radius. Figure 7 shows that all the c_p/R in the dispersion curve corresponding to the disk part are less than the one corresponding to the ring part. In other words, for a steady IRS, all radii of the concentric circle orbits in the disk part (thick, black solid line), including the minimal core radius (black circle point), are larger than the dispersion core radius (red square point). This probably is an essential condition for the emergence of the IRS.

We guess that the essential condition above is satisfied through the whole region of the IRS in Fig. 2. Figure 9 supports our guess: The radii of minimal cores are larger than those of the dispersion cores for all steady IRSs except at $\delta_{\mathrm{disk}} = -1.625$. This is just the border between the IRSs and the intermediate patterns in Fig. 2; at this border, the radii of dispersion cores are nearly equal to those of the minimal cores (for details, see Fig. 10).

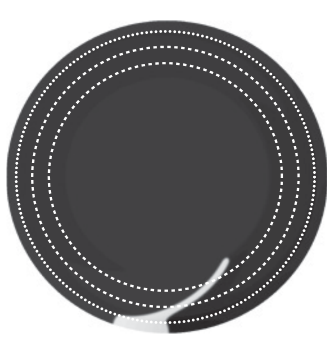

FIG. 8. Points of the wave in their concentric circle orbits (dashed lines).

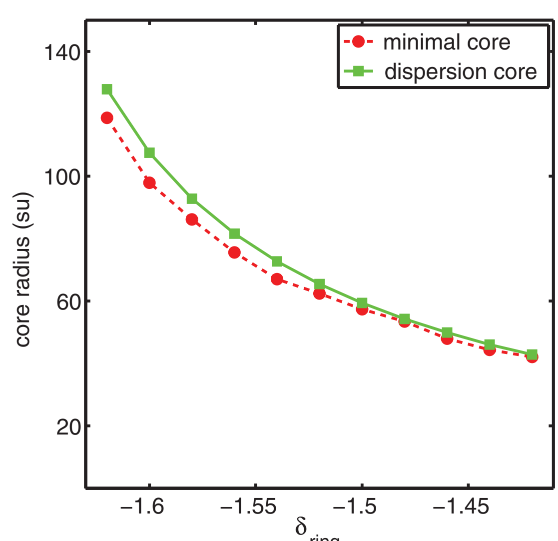

FIG. 10. (Color online) The radii of dispersion and minimal cores at $\delta_{\mathrm{disk}} = -1.625$.

IV. CONCLUSION

In this work, we have found the existence of IRSs in an excitable system with a disk-shaped inhomogeneity. Depending strongly on the properties of the medium and inhomogeneity, IRSs and other patterns emerge. Using the dispersion relation, we also propose an essential condition for the occurrence of the IRSs: The dispersion core radius is less than the minimal core radius; the intermediate pattern between inwardly and outwardly rotating spirals occurs when the essential condition is violated. To check whether our results are sensitively model independent, we also study rotating waves in the Barkley model [45] with a disk-shaped inhomogeneity and observe IRSs in this excitable system, which suggests our findings are robust. Note that our inhomogeneous medium is similar to the leading-circle model [46] in cardiac arrhythmias: The outer ring part serves as the leading circle; the inner disk part is like the excitable tissue excited by the impulse shedding inwardly from the leading circle to the core. Moreover, by contrast to the excited and refractory core predicted in the leading-circle model, which is contradicted by the evidence that the core of functional reentry is excitable, yet remains nonactivated [47], the centripetal propagation wave in the IRS circulates around an excitable but nonexcited core. Considering the inhomogeneity in the heart tissue [31,32], we expect the existence of IRSs in the cardiac system. Finally, we also hope our results could be reproduced in the experiments such as the BZ reaction [34] and the CO oxidation on Pt(110) [35], which will make our work more interesting.

ACKNOWLEDGMENTS

This work is supported by the National Nature Science Foundation of China (Grants No. 10975117 and No. 10947021), the Program for New Century Excellent Talents in University, the Fundamental Research Funds for the Central Universities, and the Soft Matter Research Center of Zhejiang University.

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