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Noise can induce explosions for dissipative solitons

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We study the influence of noise on the spatially localized, temporally regular states (stationary, one frequency, two frequencies) in the regime of anomalous dispersion for the cubic-quintic complex Ginzburg-Landau equation as a function of the bifurcation parameter. We find that noise of a fairly small strength η is sufficient to reach a chaotic state with exploding dissipative solitons. That means that noise can induce explosions over a fairly large range of values of the bifurcation parameter μ. Three different routes to chaos with exploding dissipative solitons are found as a function of μ. As diagnostic tools we use the separation to characterize chaotic behavior and the energy to detect spatially localized explosive behavior as a function of time.

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One of the most fascinating types of spatially localized solutions are exploding dissipative solitons found by Akhmediev *et al.* for anomalous linear dispersion in the cubic-quintic complex Ginzburg-Landau equation [1]. While exploding dissipative solitons are characterized by spatiotemporally irregular large-amplitude variations, they show deterministically an average repeat time with a fairly narrow temporal distribution of their disappearance and reappearance as a function of time. These remarkable objects have been further characterized with respect to their deterministic behavior experimentally [2] and theoretically [3–5]. In addition, it has been shown [6,7] that a number of temporally regular localized solutions emerge as the bifurcation parameter μ, the distance from linear onset, is increased toward the regime for which explosive dissipative solitons are found with the system following an analog of the Ruelle-Takens route for spatially localized solutions [7]. Notice that our system is not excitable. The stable localized solutions under study exist in a narrow range where the zero solution coexists with an homogeneous solution (bistable system). Therefore there exists a saddle function separating the stable zero solution from the stable localized solution. An explosion happens when perturbations in the wings overcome this saddle function.

There appear to be very few systematic studies of the influence of noise on systems showing stable pulse-type localized solutions in nonlinear dissipative spatially extended nonequilibrium systems, although in 2010 it was found that a reaction-diffusion system driven by noise can lead to localized objects of finite lifetime [8]. Stimulated by the observation of the partial annihilation of pulses with fixed shape near the onset of binary fluid convection in an annulus [9,10] and for the catalytic oxidation of CO on Pt(110) [11] we investigated the influence of additive noise on colliding pulses of fixed shape. We found a partial annihilation of pulses for small noise strength [12] and clarified the mechanism of partial annihilation.

Motivated by our recent studies of the nature of the transition to exploding dissipative solitons as the bifurcation parameter is varied [7], we investigate here the influence of noise on the three states preceding exploding dissipative solitons as a function of μ. The expectation is that at least the state with two frequencies deterministically should be rather sensitive to noisy perturbations, since it directly precedes the transition to exploding dissipative solitons.

In this Rapid Communication three different routes to chaos with exploding dissipative solitons are found as a function of μ. For μ values for which localized solutions are stationary we obtain the following sequence: noisy without explosions, noisy with explosions, chaos with explosions. In the range of μ where we have localized solutions with one frequency we obtain either the previous sequence or noisy states followed by nonexplosive chaotic solutions before the transition to explosive chaos. For μ values for which localized solutions have two frequencies we obtain only the last mentioned sequence. As diagnostic tools we use the separation [13,14], the asymptotic limit of the time evolution of the distance between two initially infinitesimally close trajectories, a quantity closely related to the largest Liapunov exponent, for the detection of chaotic behavior and the energy to characterize spatially localized explosive behavior. This technique has been used for the analysis of convectively unstable states [13] as well for chaotically breathing localized states [14].

To study noise-induced exploding dissipative solitons we use the stochastic cubic-quintic complex Ginzburg-Landau equation

$$\begin{aligned}\partial_t A = \ & \mu A + (\beta_r + \mathrm{i}\beta_i)|A|^2 A + (\gamma_r + \mathrm{i}\gamma_i)|A|^4 A \\ & + (D_r + \mathrm{i}D_i)\partial_{xx} A + \eta\,\xi,\end{aligned} \tag{1}$$

where $A(x,t)$ is a complex field, β_r is positive, and γ_r is negative in order to guarantee that the bifurcation is subcritical, but saturates to quintic order. The stochastic force $\xi(x,t)$ denotes white noise with the properties $\langle\xi\rangle = 0$, $\langle\xi(x,t)\ \xi(x',t')\rangle = 0$ and $\langle\xi(x,t)\ \xi^*(x',t')\rangle = 2\delta(x-x')\delta(t-t')$, where ξ^* denotes the complex conjugate of ξ.

In our numerical simulations we keep all parameters fixed except for μ, the distance from linear onset, and η, the noise strength. The parameter values are $\beta_r = 1$, $\beta_i = 0.8$, $\gamma_r = -0.1$, $\gamma_i = -0.6$, $D_r = 0.125$, and $D_i = 0.5$

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(positive) corresponding to an anomalous dispersion regime. The parameter μ is varied from -0.265 until -0.215. This range includes three different behaviors for $\eta = 0$, namely, stationary localized solutions ($-1.23 < \mu < -0.254$), oscillatory localized solutions with one frequency ($-0.254 < \mu < -0.232$), and oscillatory localized solutions with two frequencies ($-0.232 < \mu < -0.213$).

In the discretized problem the stochastic force $\xi(x,t)$ is replaced by $(\chi_r + i\chi_i)/\sqrt{dx\,dt}$, where χ_r and χ_i are uncorrelated random numbers obeying a standard normal distribution.

To characterize the influence of the noise on the solutions of the Eq. (1) for a given value of μ and noise strength η we study the dynamical evolution of two nearby states by means of

$$\begin{aligned}\partial_t(\delta A) = \; & \mu\delta A + (\beta_r + i\beta_i)(2|A|^2\delta A + A^2\delta A^*) \\ & + (\gamma_r + i\gamma_i)(3|A|^4\delta A + 2A^2|A|^2\delta A^*) \\ & + (D_r + iD_i)\partial_{xx}(\delta A). \end{aligned} \tag{2}$$

We define the separation of these states as

$$\zeta(t) = \left(\int_0^L |\delta A(x,t)|^2 dx\right)^{1/2}, \tag{3}$$

whose slope gives us a measure of the largest Lyapunov exponent for the whole extended system.

Time integration of equation (1) was performed in all regions using a time-splitting pseudospectral scheme, with a box size $L = 50$ and $N = 1024$, so we get a grid spacing of $dx \sim 0.05$. Initial conditions are solutions of the equation in the asymptotic time regime for $\eta = 0$. In order to perform temporal integration of δA in Eq. (2) we use the same time-splitting scheme with a time step $dt = 0.005$ in regions where stationary localized solutions and oscillatory localized solutions with one frequency are expected for $\eta = 0$. For the region where oscillatory localized solutions with two frequencies are expected for $\eta = 0$ we use a second-order pseudospectral Adams-Bashforth scheme with a time step $dt = 0.001$. This scheme is used in order to avoid numerical instabilities. Initial conditions for δA are shaped like Gaussian functions with a height ~ 0.02 and width ~ 8. Equation (1) is, in fact, integrated in parallel with Eq. (2) for each time step. We used a different temporal scheme to integrate Eq. (2) in the region where oscillatory localized solutions with two frequencies are expected in the limit zero noise. The reason is that this equation becomes unstable in that region of parameters, even in the absence of any chaotic behavior by Eq. (1). To overcome that instability we used a smaller dt and a more stable, but resource consuming, temporal scheme in the integration of Eq. (2). We also remark that the reduction in dt does not affect, in any measurable way, the behavior of Eq. (1).

In Fig. 1 we summarize our qualitative main results for the effects of small noise of a strength up to 10^{-2} on the stable localized solutions that are stationary or oscillatory. We have plotted the patterns arising as a function of the bifurcation parameter μ for varying noise strength, which is plotted on the ordinate on a logarithmic scale. We read off immediately from Fig. 1 that for sufficiently large noise amplitude a transition

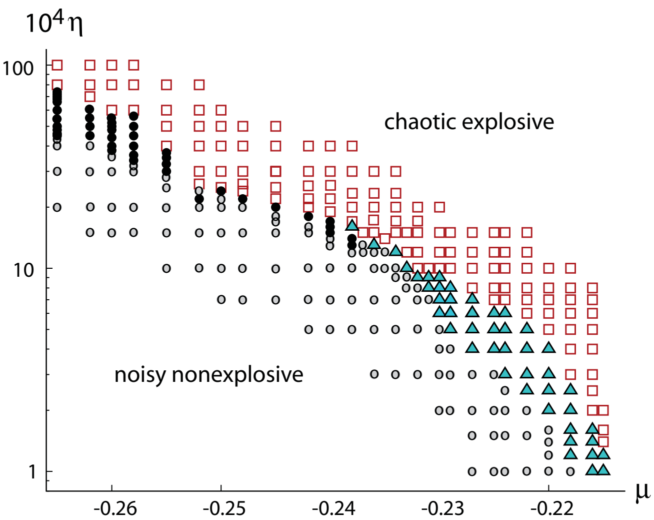

FIG. 1. (Color online) Phase diagram showing the observed patterns as a function of noise strength η on the ordinate (plotted on a logarithmic scale) vs the bifurcation parameter μ. We show the parameter range μ from -0.265 to -0.215. It includes for $\eta = 0$: (a) stationary localized solutions ($-1.23 < \mu < -0.254$), (b) oscillatory localized solutions with one frequency ($-0.254 < \mu < -0.232$), and (c) oscillatory localized solutions with two frequencies ($-0.232 < \mu < -0.213$). Right after localized solutions with two frequencies and before the onset of explosive solutions, there exists a narrow range ($\Delta\mu \sim 10^{-4}$) where solutions are localized, chaotic, and nonexplosive. Open circles (○) correspond to noisy states without any explosion. Squares (□) represent chaotic states with explosions. Black solid circles (•) correspond to noisy states with explosions. Triangles (△) represent chaotic states without any explosions. All data points shown correspond to numerical runs with a duration $T = 2 \times 10^4$.

to a chaotic state with explosions arises independent of the deterministic starting state.

Inspection of Fig. 1 shows that there are three qualitatively different routes from the deterministic states to a chaotic localized state with explosions. Starting with a stationary localized state one has a first transition to a noisy state with explosions followed by a chaotic state with explosions. Starting with an oscillatory localized state with one frequency the first transition is either to a noisy state with explosions or to a chaotic state without explosions and then a second transition to explosions. Starting with an oscillatory localized state with two frequencies the first transition is to a chaotic localized state without explosions, which then shows for increasing noise amplitude a transition to a chaotic localized state with explosions. For large noise ($\eta \sim 0.15$) not shown in Fig. 1 the chaotic state with explosions is replaced by a spatiotemporal pattern filling the entire system.

The borders in the phase diagram also have a stochastic aspect. This means that borders are not strictly fixed: Small variations occur when the numerical runs are repeated, as is to be expected for a system with a certain amount of noise. Since the amount of noise studied here is quite small, the borders are almost fixed.

To characterize chaotic versus nonchaotic quantitatively for these spatially localized states we use the tool of separation

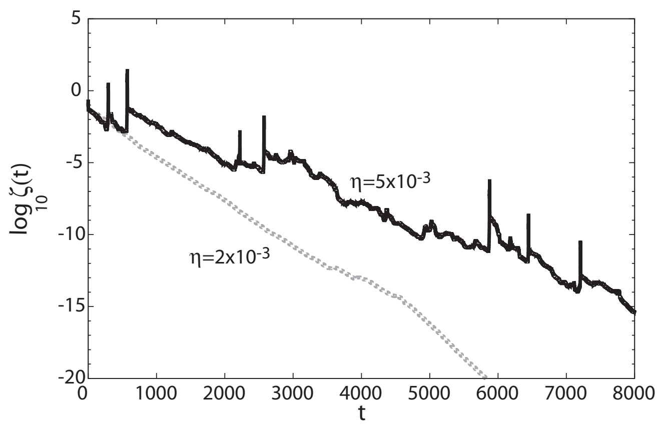

FIG. 2. The separation ζ is plotted on a logarithmic scale as a function of time for two values of the noise strength η for $\mu = -0.26$: for $\eta = 2 \times 10^{-3}$ (dotted line) noisy, nonchaotic, as well as nonexplosive behavior is obtained, while for $\eta = 5 \times 10^{-3}$ (solid line) noisy, nonchaotic, but explosive behavior arises. For this value of μ one finds deterministically stationary localized solutions.

discussed above. In Fig. 2 we have plotted the logarithm of the separation for a value of the bifurcation parameter, for which deterministically a stationary localized state appears, for two values of the superposed noise strength η. For $\eta = 2 \times 10^{-3}$ the separation is decaying exponentially and monotonically signaling for the localized state a nonchaotic noisy behavior without any explosions. As the noise strength is increased to $\eta = 5 \times 10^{-3}$, one obtains noisy behavior, but now with intermittent explosions visible as spikes in the plot of the logarithm of the separation.

In Fig. 3 we have plotted as a solid line the logarithm of the separation for $\mu = -0.25$, for which one has deterministically an oscillatory localized state with one frequency and a noise

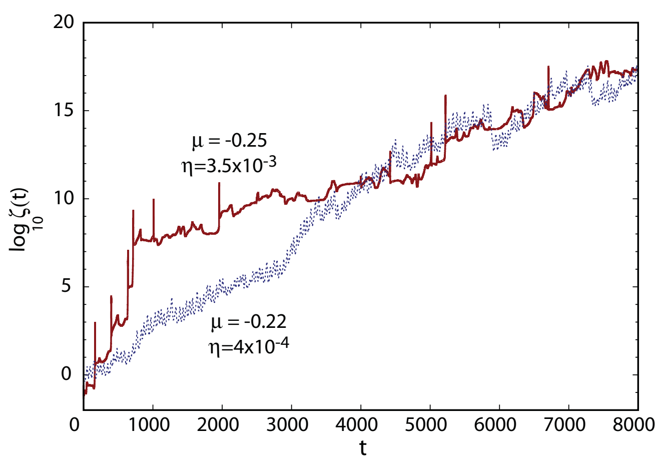

FIG. 3. (Color online) The separation ζ is plotted as a solid line on a logarithmic scale as a function of time for $\mu = -0.25$ and for $\eta = 3.5 \times 10^{-3}$; chaotic as well as explosive behavior is observed. For this value of μ one finds deterministically oscillatory localized solutions with one frequency. In addition, the separation ζ is plotted as dashed line on a logarithmic scale as a function of time for $\mu = -0.22$ and for $\eta = 4 \times 10^{-4}$: chaotic, nonexplosive behavior is obtained. For this value of μ one finds deterministically oscillatory localized solutions with two frequencies.

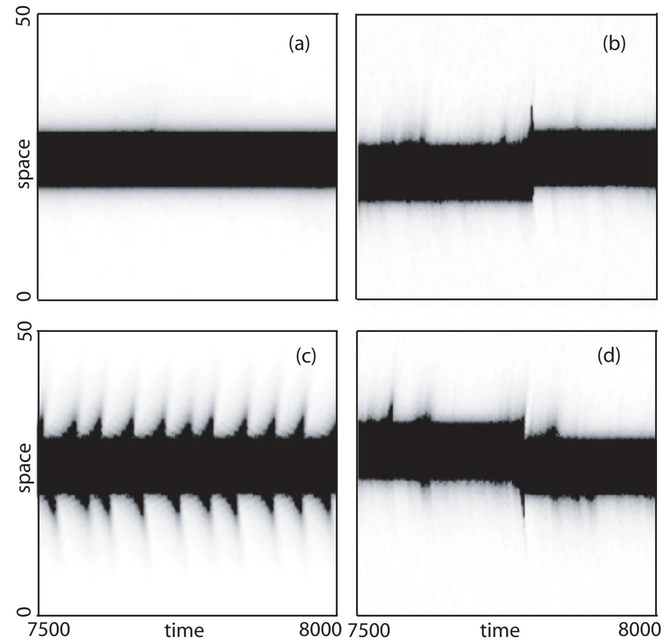

FIG. 4. x-t plots are shown in the asymptotic time regime for four different types of states (a) nonchaotic, nonexplosive $\mu = -0.26$, $\eta = 2 \times 10^{-3}$, (b) nonchaotic with explosions $\mu = -0.26$, $\eta = 5 \times 10^{-3}$ (c) chaotic without explosions $\mu = -0.22$, $\eta = 4 \times 10^{-4}$, and (d) chaotic with explosions $\mu = -0.25$, $\eta = 3.5 \times 10^{-3}$.

strength of $\eta = 3.5 \times 10^{-3}$ has been applied. The separation is seen to grow exponentially with intermittent spikes corresponding to explosions. Also in Fig. 3 the logarithm of the separation is plotted as a dashed line for $\mu = -0.22$, for which deterministically an oscillatory localized state with two frequencies is stable. A noise strength of $\eta = 4 \times 10^{-4}$ is implemented. The separation grows exponentially on the average, and there are no spikes that would be associated with intermittent explosions. From the solution of Eq. (2) and the definition of the separation ζ, it is clear that there will be no saturation for localized chaotic states in the asymptotic time limit (see also Refs. [13,14]).

That the spikes shown in the plots for the separation as a function of time (Figs. 2 and 3) correspond to explosions can be concluded from the plot energy ($\equiv \int |A|^2\, dx$) as a function of time. In addition, it can be checked using time series as well as snapshots of the corresponding states. Therefore we present in Fig. 4 space-time (x-t) plots in the asymptotic time regime for four types of states: (a) nonchaotic, nonexplosive, (b) nonchaotic with explosions (c) chaotic without explosions, and (d) chaotic with explosions. These plots, corresponding to the parameter values of the plots shown in Figs. 2 and 3, demonstrate that the objects presented stay localized.

In previous studies qualitative changes as a function of noise strength for space-filling patterns have been investigated experimentally mainly for two types of systems. For multiplicative noise applied to electroconvection in nematic liquid crystals it has been shown [15] that not only the onset of electroconvection could be suppressed, but that also several types of regular spatial patterns do not occur anymore for sufficiently large applied noise strength. The second type of systems for which one has seen qualitative changes in the spatiotemporal patterns due to external noise experimentally [16–18] and theoretically [16,17] is the catalytic oxidation of CO on surfaces such as Ir(111) under UHV conditions. In this case the noise is superposed on the flow of CO, thus leading to combination of applied additive and multiplicative noise [16]. As a consequence one obtains, for example, a replacement of spatially periodic patterns by spatiotemporal intermittency.

Here we have demonstrated that a small amount of additive noise has a profound effect on stable spatially localized solutions of a prototype envelope equation: the cubic-quintic complex Ginzburg-Landau equation characteristic for a weakly inverted bifurcation to traveling waves in the regime of anomalous linear dispersion. We have shown that with increasing noise strength a chaotic state with explosions is reached via three qualitatively different routes. As intermediate states for smaller noise strength we find either a noisy state with explosions or a chaotic state without explosions. These results on the influence of a small amount of noise (of up to ∼1% of the amplitude of the localized solutions) reveal a qualitatively new perspective for the phenomena associated with the explosive dissipative solitons found and characterized deterministically in modeling and experiment [1–7].

To test our results we suggest the performance of two types of experiments in the presence of additive noise. One is along the lines of those reported for a solid-state passively mode-locked laser [2]. The suggestion is to add noise to the complex electric field and to check which sequence of transitions to exploding dissipative solitons can be induced by noise starting from stationary or temporally periodic spatially localized solutions. The other is sheared annular electroconvection [19], a system for which it has been shown [19] by simulating the underlying full hydrodynamic equations that the analog of the Ruelle-Takens-Newhouse route is expected to occur experimentally for localized solutions.

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