

Critical short-time dynamics in a system with interacting static and diffusive populations

C. Argolo and Yan Quintino

Instituto Federal de Ciência e Tecnologia do Estado de Alagoas, 57020-510 Maceió, Alagoas, Brazil

Iram Gleria* and M. L. Lyra

Instituto de Física, Universidade Federal de Alagoas, 57072-970 Maceió, Alagoas, Brazil

(Received 4 September 2011; published 5 January 2012)

We study the critical short-time dynamical behavior of a one-dimensional model where diffusive individuals can infect a static population upon contact. The model presents an absorbing phase transition from an active to an inactive state. Previous calculations of the critical exponents based on quasistationary quantities have indicated an unusual crossover from the directed percolation to the diffusive contact process universality classes. Here we show that the critical exponents governing the slow short-time dynamic evolution of several relevant quantities, including the order parameter, its relative fluctuations, and correlation function, reinforce the lack of universality in this model. Accurate estimates show that the critical exponents are distinct in the regimes of low and high recovery rates.

DOI: [10.1103/PhysRevE.85.011111](https://doi.org/10.1103/PhysRevE.85.011111)

PACS number(s): 05.70.Jk, 64.60.Ht, 05.70.Ln

I. INTRODUCTION

Classical many-particle systems, stochastically defined by a set of allowed configurations, are good examples of complex systems with a master equation governing the transition rates [1]. Nonequilibrium phase transitions from an active to an absorbing state have been a topic of great interest in this field [2–4]. In this class of systems, there is no detailed balance and the fluctuation-dissipation theorem does not hold, features that are usually explored in studies of equilibrium phase transitions. The basic model possessing a dynamic transition from an active state into an absorbing state is the contact process (CP), a prototype model for the directed percolation (DP) universality class [5,6]. If diffusion is allowed, the critical behavior of the absorbing state phase transition is heavily influenced and deviations from the DP class have been reported [7–19]. In a CP in which the active and inactive particles can diffuse, it has been demonstrated, both by field theoretical renormalization group arguments [8] and numerical simulations [9], that three distinct regimes associated with the absorbing state phase transition emerge depending on the relative diffusivity of the active and inactive particles.

The critical behavior of one-dimensional diffusive epidemic processes (DEP) with coupled static and diffusive populations was recently analyzed and the scaling behavior of quasistationary quantities was explored [11,15,18]. It was obtained from finite-size scaling arguments that the exponent β/ν_{\perp} governing the size dependence of the order parameter at the critical point is weakly dependent on the model parameters, particularly the recovery rate at which active individuals become inactive. By contrast, the correlation length exponent ν_{\perp} varies from the diffusive CP universality class value at low recovery rates to the DP value at large recovery rates. This result points to distinct critical regimes in these two limits, even though active and inactive particles have the same diffusivity. However, simulations based on quasistationary properties are usually limited to relatively small system sizes and the slow convergence to

the asymptotic scaling behavior may compromise the accuracy of the evaluated critical parameters. Therefore independent estimates of the critical exponents derived from an alternative procedure are in order to further probe such conjectured lack of universality.

Here we address the above question by exploring the critical short-time dynamics. Due to the critical slowing down, the physical properties evolve in time as power laws. Besides being able to provide new dynamical critical exponents, scaling relations can be used to extract the static ones which can be compared with the estimates based on quasistationary calculations. In particular, we follow the time evolution of the order parameter, its logarithmic derivative, and the relative order-parameter fluctuations starting from the full active state. These can be used to estimate the order-parameter exponent β , the correlation length exponent ν_{\perp} , and the dynamical exponent z . Further, starting from a state with an infinitesimal order, the critical initial slip exponent θ is estimated. All results are consistent with a nonuniversal behavior with a distinct set of critical exponents governing the transition at low and high recovery rates.

II. MODEL, SIMULATIONS, AND SCALING

We consider a one-dimensional lattice populated with two distinct species [18]. One of them is composed of static individuals that can be in either an active (infected) state P_a or an inactive (noninfected) state P_i . With no diffusion, the infected individuals cannot contaminate their neighbors. In order to spread the infection we consider a second population (vectors) which also can be found in either an active (infective) state V_a or an inactive (noninfective) state V_i . The vector population diffuses performing a random walk on the lattice. Active states are transmitted whenever an inactive individual of one population occupies the same site of an active individual of the other population.

We consider chains of size L with periodic boundary conditions. The diffusion rates of infective and noninfective vectors are assumed to be equal. The inactive vector becomes

*iram@fis.ufal.br; iramgleria@hotmail.com

active whenever its site is occupied by an active individual P_a . However, it may recover to the inactive state with probability ϕ . Within the same lattice sweep, each inactive individual becomes active if its site is occupied by at least one active vector. Active individuals become inactive with probability λ .

In Ref. [18] we reported numerical data for some quasistationary quantities (density of infected individuals and its relative fluctuations) from simulations for $\phi = \lambda = 0.1$ (low recovery rates) and $\phi = \lambda = 0.7$ (high recovery rates). For high densities of vectors, the system always remains in an active phase with a fluctuating finite density of infected individuals. In contrast, at low density of vectors, the system evolves toward the absorbing inactive state. Therefore, the density of vectors is a natural parameter to control the transition between the absorbing and active states. For the simulation at a large recovery rate, $\lambda = \phi = 0.7$, the critical density of vectors is very high and the diffusive particles do not need to walk to spread the infection because practically all sites of the lattice are occupied by a vector. In this case, the usual exponents of directed percolation were obtained from the quasistationary calculations. In the limit of low recovery rates, the critical density of vectors is small. Under this condition of scarce vectors, the diffusive particles need to walk long distances to spread the infection. In this regime, the results are consistent with the renormalization group prediction of $\nu_{\perp} = 2/d = 2$ for the diffusive epidemic process with equally diffusing active and inactive particles [8]. However, as mentioned in the previous section, the slow convergence toward the quasistationary state due to the critical slowing down phenomena restricted the accuracy of the estimated critical exponents.

In what follows we take an alternative approach that actually explores the critical slowing down. The slow time evolution of the physical quantities at criticality can also be described by power laws with characteristic critical exponents [20,21]. We use such an alternative method of short-time dynamics and its related finite-size scaling relations to find the critical exponents characterizing the nonequilibrium phase transition displayed by the present model. At a critical point, static as well as dynamic properties of a physical system show power-law behavior. While the static properties exhibit a slow power-law dependence on the system size at criticality, dynamic properties computed at very large system sizes evolve in time as power laws. Related to the dynamical properties, the relaxation process is of great importance and may show distinct trends in the short-time and long-time regimes. Right after the start of the relaxation, the short-time behavior is governed by nonuniversal processes. There is an intermediate period which appears after the initial steps of the relaxation and before the long-time behavior. This is the critical initial slip described in Ref. [20]. According to the dynamic scaling approach, the order parameter (density of infected individuals) $\psi(t)$, at the critical point, shall obey the scaling form given by

$$\psi(t) = \psi(0)t^{\theta} f_{\psi}[t^{\theta+\beta/(v_{\perp}z)}\psi(0)], \quad (1)$$

where we consider

$$\begin{aligned} f_{\psi}(x) &= 1 & \text{if } x \rightarrow 0, \\ &= 1/x & \text{if } x \rightarrow \infty. \end{aligned} \quad (2)$$

At the critical point of a continuous phase transition the average stationary value of the order parameter is zero. The critical initial increase is a phenomenon that at the critical point an initially nonzero order parameter will first grow with time as a power law t^{θ} . The above equations contain the crossover from the short-time $\psi(t) = \psi(0)t^{\theta}$ behavior to the long-time $\psi(t) = \psi(0)t^{-\beta/(v_{\perp}z)}$ behavior. The exponent θ is known as the critical initial increase exponent (or growth exponent) [21]. At a typical crossover time $\psi(0) = t_c^{-[\theta+\beta/(v_{\perp}z)]}$ the order parameter stops increasing and starts decreasing. Therefore, for an infinitesimal initial order parameter, the short-time dynamical regime is quite long. On the other hand, only the long-time relaxation regime can be probed when the system starts from its fully ordered state.

In our time-dependent simulation we perform calculations at the critical vector densities (average number of vector individuals per site) which are $\rho_c = 0.116(1)$ for $\lambda = \phi = 0.1$ and $\rho_c = 25.89(2)$ for $\lambda = \phi = 0.7$ [18]. These critical densities were evaluated exploring the finite-size scaling behavior of the statistically stationary order-parameter moment ratio $m_L(\rho)$ (see detailed definition in the next section). In order to achieve the statistically stationary regime, we disregard the first $10L^2$ lattice sweeps for all chain sizes and vector densities. After that, 2×10^5 distinct configurations, separated by L lattice sweeps, are used to compute the order parameter and its second moment. Reflecting boundaries are used to avoid the system to become trapped in the absorbing state. According to finite-size scaling arguments, the moment ratio becomes size independent at the critical point, with small corrections to scaling due to finite-size corrections. The above critical densities were obtained as the scale invariant point of the order-parameter moment ratio using lattice sizes up to 2000 sites (for further detail see Ref. [18]). Here, we tune the system at criticality and let it to evolve in time, averaging the measured quantities over a large number n_c of copies. Each copy has a different initialization of the random number generator, thus leading to distinct evolutions of the stochastic process. We calculate the average density of infected individuals $\psi(t)$ at time t over all copies. We do not employed reflecting boundaries in the present dynamical analysis. The runs in which the system reached the absorbing state, with all individuals in the healthy state, are also included in the configurational averages.

III. RESULTS

A. Long-time dynamics

We start by letting the system evolve from a totally ordered state [$\psi(0) = 1$] where all individuals are in an infected state. In this case, only the long-time critical relaxation regime can be probed. The following scaling relations hold at the critical point [20,21]:

$$\begin{aligned} \psi(\rho_c, t) &= \left\langle \frac{1}{N} \sum_{n=1}^N P_n(t) \right\rangle \propto t^{-\beta/(v_{\perp}z)}, \\ \frac{d}{d\rho} \ln \psi(\rho_c, t) &\propto t^{1/v_{\perp}z}, \\ m(\rho_c, t) &= \frac{\left(\left\langle \frac{1}{N} \sum_{n=1}^N P_n(t) \right\rangle^2 \right)}{\left(\left\langle \frac{1}{N} \sum_{n=1}^N P_n(t) \right\rangle \right)^2} - 1 \propto t^{d/z}, \end{aligned} \quad (3)$$

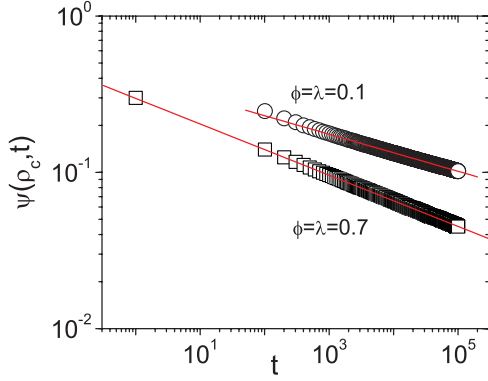


FIG. 1. (Color online) Long-time relaxation of the order parameter at criticality. We considered chains with $L = 10000$ sites and $n_c = 2000$ for the case of a low recovery rate ($\phi = \lambda = 0.1$) and $n_c = 1000$ for the case of a high recovery rate ($\phi = \lambda = 0.7$). The relaxation exponents are distinct in these two regimes: $\beta/(v_{\perp}z) = 0.12(1)$ for $\phi = \lambda = 0.1$ and $\beta/(v_{\perp}z) = 0.161(4)$ for $\phi = \lambda = 0.7$.

where, in the present one-dimensional simulations, $d = 1$. P_n represents the state of the static individual located at the n th site which is taken to be $P_n = 0$ for an inactive individual and $P_n = 1$ for an active one. From the above scaling relations (3) we can extract the exponents z , v_{\perp} , and β . In Fig. 1 we show the time evolution of the density of sick individuals $\psi(t)$. For the case $\phi = \lambda = 0.1$ we considered a chain with $L = 10000$ sites and averaged over $n_c = 2000$ copies. The best fit to a power law of the asymptotic decay provides $\beta/(v_{\perp}z) = 0.12(1)$. Notice that the actual asymptotic regime only sets up after a short transient of the order of 10^2 lattice sweeps. The decay exponent was estimated from the power law fitting in the interval between 10^3 and 10^5 lattice sweeps. For the case $\phi = \lambda = 0.7$ we considered the same system size with $L = 10000$ sites but averaged over a smaller number of copies $n_c = 1000$ because the fluctuations are rather smaller in this case (large critical vector density) as compared with the previous one (small critical density). For such a large recovery rate the asymptotic regime sets up rather fast. The dynamical exponent governing the long-time relaxation of the order parameter is estimated to be $\beta/(v_{\perp}z) = 0.161(4)$. Although the difference between these exponents is larger than the estimated error bars, which points to distinct universality classes in these two regimes, they are still close to each other. At this point, one cannot exclude the possibility of such a difference being due to small corrections to scaling that would require simulations on much larger system sizes to become irrelevant.

In Fig. 2 we present our results for the time evolution of the slope of the derivative of the logarithm of the order parameter at criticality. Its power-law growth gives an estimate of the critical exponent ratio $1/v_{\perp}z$. The best fit of the data provides $1/(v_{\perp}z) = 0.264(8)$ for $\phi = \lambda = 0.1$ and $1/(v_{\perp}z) = 0.54(3)$ for $\phi = \lambda = 0.7$. These two exponents are quite distinct (a factor of 2), thus giving solid numerical support that these two regimes indeed belong to distinct universality classes.

In Fig. 3 we present the data for the long-time growth of the order-parameter fluctuations at the critical point. According to its scaling behavior, the dynamical exponent $1/z$ can be extracted from these data. For the case of a low recovery

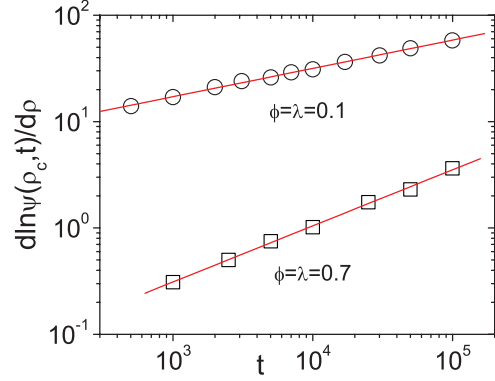


FIG. 2. (Color online) Long-time relaxation of $\frac{d \ln \psi(\rho_c, t)}{d \rho}$ at criticality. Its power-law growth allows us to estimate the critical exponent ratio $1/(v_{\perp}z)$. For $\phi = \lambda = 0.1$ we obtain $1/(v_{\perp}z) = 0.264(8)$, while for $\phi = \lambda = 0.7$ we obtain $1/(v_{\perp}z) = 0.54(3)$. Lattice size and sampling were the same as those used in Fig. 1.

$\phi = \lambda = 0.1$ we get $z = 2.00(1)$. On the other hand, we estimate $z = 1.59(3)$ for $\phi = \lambda = 0.7$. All three exponent ratios are distinct for the cases of low and high recovery rates. The results for a low recovery rate are consistent with the renormalization group prediction of $z = v_{\perp} = 2$ for the DEP with equally diffusing active and inactive particles [7,8]. On the other hand, the estimated exponents for the case of a high recovery rate are similar to those of the usual directed percolation universality class [6].

B. Short-time dynamics

In order to probe the short-time critical dynamics we let the system evolve after starting with only one infected individual [$\psi(0) = 1/N$]. In this case, the critical initial slip persists during a period long enough to allow us to compute the critical exponent related to the initial growth of the order parameter. Besides following the time evolution of the order parameter, we also compute the temporal evolution of the autocorrelation

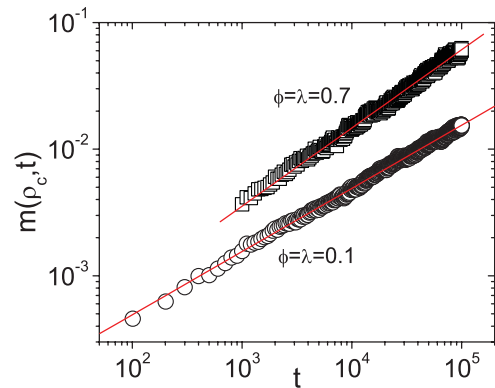


FIG. 3. (Color online) Long-time relaxation of the order-parameter relative fluctuation at criticality. From its power-law growth the dynamical exponent $1/z$ can be estimated. For $\phi = \lambda = 0.1$ one obtains $1/z = 0.500(3)$, which implies $z = 2.00(1)$. For $\phi = \lambda = 0.7$ one gets $1/z = 0.63(1)$, thus giving $z = 1.59(3)$. Lattice size and sampling were the same as those used in Fig. 1.

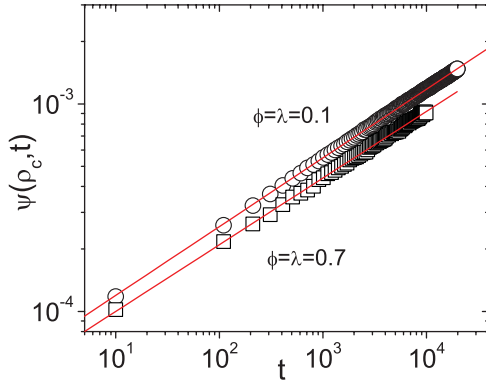


FIG. 4. (Color online) Initial increase of the density of infected individuals starting from a configuration with a single infected individual. For $\phi = \lambda = 0.1$ we considered $L = 10\,000$ and $n_c = 20\,000$. The straight line represents the power law with exponent $\theta = 0.33(1)$. For $\phi = \lambda = 0.7$ we considered $L = 10\,000$ and $n_c = 5\,000$. The power-law fitting led to $\theta = 0.32(1)$.

function at criticality $A(\rho_c, t)$ defined as

$$A(\rho_c, t) \equiv \frac{1}{N} \sum_{n=1}^N \langle P_n(t) P_n(0) \rangle - \langle P_n(t) \rangle \langle P_n(0) \rangle. \quad (4)$$

In the short-time critical regime, these quantities satisfy the following scaling relations:

$$\psi(\rho_c, t) \propto t^\theta, \quad (5)$$

$$A(\rho_c, t) \propto t^{\theta-d/z}, \quad (6)$$

from which we estimate the exponents θ and z . We followed the initial power-law growth of the order parameter over 3 decades, as shown in Fig. 4. Calculations of the initial growth regime exhibit stronger fluctuations than those displayed in the long-time relaxation regime. In order to account for such stronger fluctuations, we considered a larger number of copies in order to achieve good statistics. However, the error bars in the estimated exponents are usually larger than those derived from the long-time regime.

In Fig. 4 we show the initial increase of the density of infected individuals starting from $[\psi(0) = 1/N]$. For the case $\phi = \lambda = 0.1$ (low recovery rate) we considered a chain size with $L = 10\,000$ sites and averaged over $n_c = 20\,000$ copies at the critical density $\rho_c = 0.116(1)$. The power-law fitting provides the growth exponent $\theta = 0.33(1)$. For the case of a high recovery rate, $\phi = \lambda = 0.7$, a smaller number of $n_c = 5\,000$ copies was required to achieve similar statistics. In this case, for which the critical density is $\rho_c = 25.89(2)$, we obtained $\theta = 0.32(1)$. These two estimates are quite closer, differing only 3% from each other. The present accuracy is not high enough to exclude the possibility of these two exponents being the same.

Finally, in Fig. 5 we present our results for the evolution of the autocorrelation function, from which we can give a second independent estimate of the dynamical exponent z using the previous values of θ . Contrary to the previous quantities, the autocorrelation function displays larger fluctuations in the regime of high recovery rates. Its calculation is also more time-consuming which has restricted its use in past

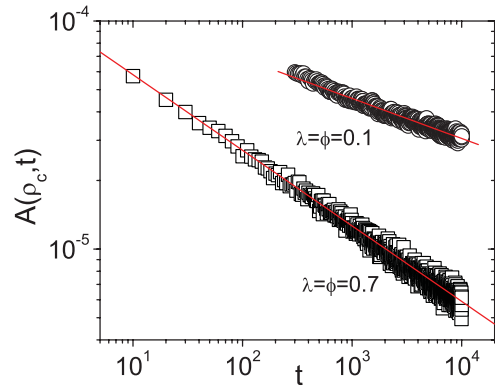


FIG. 5. (Color online) Time evolution of the autocorrelation function started from $\psi(0) = 1/N$. For $\phi = \lambda = 0.1$ we considered chains with $L = 10\,000$ sites and averaged over $n_c = 2\,500$ copies. The best-fit power law provides $\theta - 1/z = -0.18(1)$ from which we obtain $z = 1.96(8)$. For $\phi = \lambda = 0.7$ we averaged over $n_c = 16\,000$ copies. In this case we obtained $\theta - 1/z = -0.33(1)$, which gives $z = 1.54(5)$.

studies of absorbing state second-order phase transitions. For $\phi = \lambda = 0.1$ we averaged over $n_c = 2\,500$ copies, while $n_c = 16\,000$ copies were required to obtain reasonably good statistics for $\phi = \lambda = 0.7$. For the low recovery rate we obtained $\theta - 1/z = -0.18(1)$, from which we get $z = 1.96(8)$. For the case of a high recovery rate the autocorrelation function exponent is estimated to be $\theta - 1/z = -0.33(1)$, thus providing $z = 1.54(5)$. Although less accurate, these two estimates of the dynamical exponent z from the short-time dynamics scaling are consistent with the values found from the long-time relaxation dynamics.

IV. SUMMARY AND CONCLUSIONS

In summary, we have investigated the dynamical critical behavior of an interacting two-species diffusion-limited reaction one-dimensional model which mimics the propagation of a disease in a static population mediated by a population of diffusive vectors. The model presents a transition from an absorbing to an active state at a critical density of vectors which depends on the typical recovery rate. At low recovery rates the critical vector density is small and diffusivity is a relevant mechanism to spread the infection. On the other hand, the critical vector density is large at high recovery rates and the process is mainly activated by contact. We used the short-time dynamics scaling relations to obtain accurate estimates of the stationary and dynamical critical exponents. Our results give further support that the absorbing state transition exhibited by this model belongs to distinct universality classes in the regimes of low and high recovery rates. The long-time dynamical critical exponent ratios $\beta/(v_\perp z)$, $1/(v_\perp z)$, and $1/z$ are distinct in these two regimes. In the case of a low recovery rate they are compatible with the diffusive contact process that, according to field-theoretical renormalization group calculations, shall have $v_\perp = z = 2$ in one-dimensional systems. In the high recovery rate case, the exponents are compatible with the usual directed percolation universality class. Further, we computed the critical exponent θ governing the initial growth of the order parameter in simulations starting

from a state with an infinitesimal order parameter. The data give quite close estimates for the growth exponent θ in the two regimes of recovery rate. Within our numerical accuracy, we are not able to decide if these two exponents are indeed distinct in the thermodynamic limit. However, the present results bring additional evidence that absorbing state phase transitions occurring in systems with diffusive populations may display a multitude of universality classes and that models with coupled diffusive and nondiffusive populations appear

to be good candidates to investigate the crossover between distinct nonequilibrium critical points.

ACKNOWLEDGMENTS

We would like to thank CAPES, CNPq, and FINEP (Brazilian Research Agencies) as well as FAPEAL (Alagoas State Research Agency) for partial financial support.

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