Dust-acoustic rogue waves in a nonextensive plasma

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We present an investigation for the generation of a dust-acoustic rogue wave in a dusty plasma composed of negatively charged dust grains, as well as nonextensive electrons and ions. For this purpose, the reductive perturbation technique is used to obtain a nonlinear Schrödinger equation. The critical wave-number threshold k_c, which indicates where the modulational instability sets in, has been determined precisely for various regimes. Two different behaviors of k_c against the nonextensive parameter q are found. For small k_c, it is found that increasing q would lead to an increase of k_c until q approaches a certain value q_c, then further increase of q beyond q_c decreases the value of k_c. For large k_c, the critical wave-number threshold k_c is always increasing with q. Within the modulational instability region, a random perturbation of the amplitude grows and thus creates dust-acoustic rogue waves. In order to show that the characteristics of the rogue waves are influenced by the plasma parameters, the relevant numerical analysis of the appropriate nonlinear solution is presented. The nonlinear structure, as reported here, could be useful for controlling and maximizing highly energetic pulses in dusty plasmas.

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I. INTRODUCTION

The field of dusty plasma physics is rapidly growing due to its potential applications in processing plasmas, solar system, cosmic plasmas, magnetically confined fusion plasmas, as well as in laboratory plasmas. During the past two decades, tremendous progress has been made in the area of collective dust-plasma interactions [1–3] involving waves, instabilities, coherent nonlinear structures, etc. The starting point of the theoretical studies in dusty plasmas was the theoretical prediction of the existence of the dust-acoustic wave (DAW) by Rao, Shukla, and Yu [4], which was later confirmed experimentally by several experimental groups [5–7]. It is well known that the linear and nonlinear properties of the plasma depend on the velocity distribution functions of the plasma particles. In a collisionless plasma, the most commonly used distribution is the Maxwellian velocity distribution, which is a distribution that is in the thermal equilibrium. However, various observations of fast ions and electrons in space environments indicate that these particles can have velocity distributions that deviate from the Maxwellian behavior. On the other hand, a number of observations clearly indicate the presence of fast particles in space described by non-Maxwellian distributions. For example, nonthermal and superthermal electrons have been observed in the Earth's bow-shock [8], in the upper ionosphere of Mars [9], in the vicinity of the Moon [10], and in the magnetospheres of Jupiter and Saturn [11,12]. Motivated by these observations, many efforts have been made to understand the properties of non-Maxwellian distributions in plasmas; see, e.g., Refs. [13–18]. Based on the above findings, important modification to the Maxwellian distribution is recognized by Renyi [19], and subsequently proposed by Tsallis [20]. In this context, the nonextensive statistical mechanics (based on the deviations of the Maxwellian distribution, and namely the q distribution) was successfully applied to a number of applications, including dissipative optical lattices [21], thermoluminescence dosimetry and dating of archaeological and geological minerals [22], earthquake dynamics [23], solar neutrino problem [24], and plasma wave propagation [25–31].

Rogue waves are a singular, rare, high-energy event with very high amplitude that carries dramatic impact. It appears in seemingly unconnected systems in the form of oceanic rogue waves, stock market crashes, communication systems, superfluid helium, Bose-Einstein condensates, opposing currents flows, propagation of acoustic-gravity waves in the atmosphere, atmospheric physics, and plasma physics [32–53]. Indeed, understanding the origin of the rogue waves appearing in systems characterized by many waves is currently a matter of debate [54]. Clearly, we cannot do much if we leave the creation of rogue waves to chance. So, preparing special initial conditions and understanding the features of this wave could be useful either to avoid rogue waves or to generate highly energetic pulses. For example, in optics the rogue waves can be created systematically and obtain high-energy pulses each time when one wants them. A possible mechanism for the formation of rogue waves is the modulation instability, which is a universal phenomenon that occurs in many physical systems, such as in optics [55], in hydrodynamics [56], and even in biology [57].

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In this paper, we study the dust-acoustic (DA) rogue wave in a dusty plasma composed of negatively charged dust grains, as well as the electrons and ions obeying the q-nonextensive distribution functions. In Sec. II, we present the basic equations describing the dynamics of the nonlinear DA rogue waves. We use the reductive perturbation method to derive the nonlinear Schrödinger (NLS) equation. The latter predicts the nonlinear evolution of modulationally unstable DA wave packets. Numerical results and discussions are presented in Sec. III. Finally, the results are summarized in Sec. IV.

II. BASIC EQUATIONS AND DERIVATION OF THE NLS EQUATION

Let us consider a one-dimensional, collisionless, unmagnetized dusty plasma composed of negative dust grains, nonextensive electrons, and ions. Thus, at equilibrium, we have $Z_d n_{d0} + n_{e0} = n_{i0}$, where n_{d0}, n_{e0}, and n_{i0} are the unperturbed dust, electron, and ion number densities, respectively, and Z_d is the number of electrons residing on the dust grain surface. We are interested in examining the nonlinear propagation of the low phase velocity (in comparison with the electron and ion thermal speeds) electrostatic mode on the time scale of the DA wave period. The nonlinear dynamics of the low-frequency DA waves in our dusty plasma is governed by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \quad (1)$$

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x}\right) u_d + \frac{\sigma}{n_d}\frac{\partial n_d}{\partial x} - \frac{\partial \varphi}{\partial x} = 0, \quad (2)$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_d + \mu_e [1 + (q-1)\beta\varphi]^{\frac{q+1}{2(q-1)}} - \mu_i [1 - (q-1)\varphi]^{\frac{q+1}{2(q-1)}}, \quad (3)$$

where n_d is the dust number density normalized by its equilibrium value n_{d0}, u_d the dust fluid velocity normalized by the dust-acoustic speed $C_{sd} = (Z_d k_B T_i/m_d)^{1/2}$, φ the electrostatic wave potential normalized by $k_B T_i/e$, x the space variable normalized by $\lambda_{Dd} = (k_B T_i/4\pi e^2 Z_d n_{d0})^{1/2}$, and t the time variable normalized by the dust-plasma period $\omega_{pd}^{-1} = (m_d/4\pi e^2 Z_d^2 n_{d0})^{1/2}$. We assumed that $\mu_e = n_{e0}/Z_d n_{d0}, \mu_i = n_{i0}/Z_d n_{d0}$, $\beta = T_i/T_e$, and $\sigma = 3T_d/Z_d T_i$. Here T_i, T_e, and T_d are the ion, electron, and dust temperatures, respectively, and k_B is the Boltzmann constant.

To examine the hydrodynamics of one-dimensional modulated DAWs propagating in our dusty plasma, we analyze the outgoing solutions of Eqs. (1)–(3) by introducing the stretched coordinates [58–61],

$$\xi = \epsilon(x - V_g t) \quad \text{and} \quad \tau = \epsilon^2 t, \quad (4)$$

where ϵ is a small (real) parameter and V_g the envelope group velocity to be determined later. The dependent variables are expanded as

$$\mathbf{A}(x,t) = \mathbf{A}_0 + \sum_{m=1}^{\infty} \epsilon^m \sum_{L=-\infty}^{\infty} \mathbf{A}_L^{(m)}(\xi,\tau) \exp(iL\Theta), \quad (5)$$

where

$$\mathbf{A}_L^{(m)} = \left[n_{d_L}^{(m)} \ u_{d_L}^{(m)} \ \ \varphi_L^{(m)}\right]^T,$$

$$\mathbf{A}_L^{(0)} = [1 \ 0 \ 0]^T, \quad \text{and} \quad \Theta = kx - \omega t.$$

Here k and ω are real variables representing the fundamental (carrier) wave number and frequency, respectively. Since $\mathbf{A}_L^{(m)}$ must be real, the coefficients in Eq. (5) have to satisfy the condition $\mathbf{A}_{-L}^{(m)} = \mathbf{A}_L^{*(m)}$, where the asterisk indicates the complex conjugate.

Substituting Eqs. (4) and (5) into Eqs. (1)–(3) and collecting terms of the same powers of ϵ, the first-order ($m = 1$) equations with $L = 1$, gives

$$n_{d_1}^{(1)} = \frac{k^2}{k^2\sigma - \omega^2}\varphi_1^{(1)}, \quad u_{d_1}^{(1)} = \frac{\omega k}{k^2\sigma - \omega^2}\varphi_1^{(1)}, \quad (6)$$

with the neutrality condition $\mu_i = 1 + \mu_e$ and ω satisfies the relation

$$D(\omega,k) \equiv 2k^2(1 + k^2\sigma - \omega^2) + (1+q)(k^2\sigma - \omega^2)(\beta\mu_e + \mu_i) = 0. \quad (7)$$

The second-order ($m = 2$) reduced equations with $L = 1$ are

$$n_{d_1}^{(2)} = \frac{k}{(\omega^2 - k^2\sigma)^2}\left[-k(\omega^2 - k^2\sigma)\varphi_1^{(2)} + 2i\omega(\omega - kV_g)\frac{\partial \varphi_1^{(1)}}{\partial \xi}\right], \quad (8)$$

$$u_{d_1}^{(2)} = \frac{1}{(\omega^2 - k^2\sigma)^2}\left[-k\omega(\omega^2 - k^2\sigma)\varphi_1^{(2)} + i(\omega^2 + k^2\sigma)(\omega - kV_g)\frac{\partial \varphi_1^{(1)}}{\partial \xi}\right], \quad (9)$$

with the compatibility condition

$$V_g = \frac{\partial \omega}{\partial k} = \frac{1}{k\omega}[\omega^2 - (\omega^2 - k^2\sigma)^2]. \quad (10)$$

We recall that the compatibility condition (10) is the group velocity of the envelope DA rogue wave.

The second harmonic modes ($m = L = 2$) arising from nonlinear self-interactions of the carrier waves are obtained in terms of $(\varphi_1^{(1)})^2$ as

$$n_{d_2}^{(2)} = \frac{k^2}{2(\omega^2 - k^2\sigma)^3}[k^2(3\omega^2 - k^2\sigma) - 2(\omega^2 - k^2\sigma)^2\Delta_1]\big(\varphi_1^{(1)}\big)^2, \quad (11)$$

$$u_{d_2}^{(2)} = \frac{k\omega}{2(\omega^2 - k^2\sigma)^3}[k^2(\omega^2 + k^2\sigma) - 2(\omega^2 - k^2\sigma)^2\Delta_1]\big(\varphi_1^{(1)}\big)^2, \quad (12)$$

and

$$\varphi_2^{(2)} = \Delta_1\big(\varphi_1^{(1)}\big)^2, \quad (13)$$

where Δ_1 is given in the Appendix.

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth-order harmonic. Its strength is analytically determined by taking the $L = 0$ component of the third-order reduced equations, which can be expressed as

$$n_{d_0}^{(2)} = \Delta_2|\varphi_1^{(1)}|^2, \tag{14}$$

$$u_{d_0}^{(2)} = \Delta_3|\varphi_1^{(1)}|^2, \tag{15}$$

and

$$\varphi_0^{(2)} = \Delta_4|\varphi_1^{(1)}|^2, \tag{16}$$

where Δ_2, Δ_3, and Δ_4 are given in the Appendix.

Finally, the third harmonic modes ($m = 3$ and $L = 1$), with the aid of Eqs. (11)–(16), give a set of equations. The compatibility condition for these equations yields the NLS equation

$$i\frac{\partial\phi}{\partial\tau} + \frac{1}{2}P\frac{\partial^2\phi}{\partial\xi^2} + Q|\phi|^2\phi = 0. \tag{17}$$

For simplicity, we have denoted $\varphi_1^{(1)} \equiv \phi$. The dispersion coefficient P and the nonlinear coefficient Q are, respectively,

$$P = \frac{1}{2k^2\omega(k^2\sigma - \omega^2)} \times \left[\begin{array}{c} -k^6\sigma^3 + 3k^2\sigma(k^2\sigma - 1)\omega^2 - (3k^2\sigma + 1)\omega^4 \\ +\omega^6 + kV_g[4(k^2\sigma\omega + \omega^3) - (k^3\sigma + 3k\omega^2)V_g] \end{array}\right] \tag{18}$$

and

$$Q = -\frac{1}{8k^2\omega(k^2\sigma - \omega^2)^3} \times \left[\begin{array}{c} 2k^6(k^4\sigma^2 - 5\omega^4) \\ +(-k^2\sigma + \omega^2)^2[\Delta_1 L_1 + (k^2\sigma - \omega^2)L_2] \end{array}\right], \tag{19}$$

where L_1 and L_2 are given in the Appendix.

The NLS equation (17) has a rational solution that is located on a nonzero background and localized both in τ and ξ directions [45] as

$$\phi = \sqrt{\frac{P}{Q}}\left[\frac{4(1 + 2iP\tau)}{1 + 4P^2\tau^2 + 4\xi^2} - 1\right]\exp(iP\tau). \tag{20}$$

The solution (20) predicts the concentration of the DA wave energy into a small region due to the nonlinear properties of the medium. Such a solution is able to concentrate a significant amount of the DA wave energy into a relatively small area in space. This property of the nonlinear system under consideration may serve as the basis for the explanation of the DA rogue waves in dusty plasmas. The rogue waves are usually an envelope of a carrier wave with a wavelength smaller than the central region of the envelope.

III. RESULTS AND DISCUSSION

It is known (see, e.g., Refs. [62–64]) that the evolution of a wave whose amplitude obeys Eq. (17) depends on the product PQ, which may be investigated in terms of the physical parameters involved. To see this, we first ensure that Eq. (17) supports the plane wave solution $\phi = \phi_0 \exp(iQ|\phi_0|^2\tau)$; the standard linear analysis consists in perturbing the amplitude by setting $\widehat{\phi} = \widehat{\phi}_0 + \varepsilon\widehat{\phi}_{1,0}\cos(\widetilde{k}\xi - \widetilde{\omega}\tau)$ (the perturbation wave number $\widetilde{k}$ and the frequency $\widetilde{\omega}$ should be distinguished from their carrier wave homologue quantities, denoted by k and ω). One thus obtains the (perturbation) dispersion relation:

$$\widetilde{\omega}^2 = \tfrac{1}{2}P\widetilde{k}^2\left(\tfrac{1}{2}P\widetilde{k}^2 - 2Q|\widehat{\phi}_{1,0}|^2\right). \tag{21}$$

One immediately sees that, if $PQ > 0$, the amplitude modulated envelope is "unstable" for $\widetilde{k} < \sqrt{4Q/P}|\widehat{\phi}_{1,0}|$; i.e., for perturbation wavelengths larger than a critical value. If $PQ < 0$, the amplitude modulated envelope will be "stable" against external perturbations. In other words, for "positive" PQ, the carrier wave is modulationally "unstable;" it may either "collapse," due to (possibly random) external perturbations, or lead to the formation of "bright" envelope modulated wave packets, i.e., localized envelope "pulses" confining the carrier wave [15,65,66]. The instability saturates usually by the formation of a train of envelope pulses, the so-called bright solitons. The latter can be stationary in time, but the system can also oscillate periodically back and forth between the soliton state and an almost homogeneous state, usually referred to as the Fermi-Pasta-Ulam oscillation. For $PQ < 0$, the carrier wave is modulationally "stable" and may propagate in the form of a "dark" ("black" or "gray") envelope wave packet, i.e., a propagating localized "hole" (a "void") amidst a uniform wave energy region [15,65,66]. Here, the wave train is stable to the modulational instability and the wave train will not fall apart into a train of solitons. Thus there exist dark solitons, which are local depletions of the amplitude, while the amplitude of the wave train remains stable on both sides of the solitons. We here investigate a special modulational unstable solution for $PQ > 0$, which is local both in space and time. This is typical for rogue waves, which appear suddenly and then disappear without trace.

It is noticed that the solution (20) is slightly different from those in Refs. [49–52]. Recalling that the NLS equations in Refs. [51,52] are limited to the range of low wave frequency, i.e., when the frequency of the carrier wave is much smaller than the ion plasma frequency. Furthermore, the nonlinear coefficient P and the dispersion coefficient Q in Refs. [49–52] are always positive. In the present case, the derivation of the NLS equation has been carried out for an arbitrary frequency of the carrier wave, and the coefficients P and Q could be negative. For negative P, the solution in Refs. [49–52] is not appropriate to obtain the rogue wave profile. Therefore, we slightly modify this solution to be suitable for describing the rogue waves for either positive or negative P [53].

Based upon the above finding, we determine in various regimes the critical wave number threshold k_c for which $PQ = 0$, which indicates where the modulational instability sets in. The variation of the critical wave number k_c (represented by the red curve) with the nonextensive parameter q is depicted in Fig. 1. It is obvious that for low wave number, $k < 0.2$, increasing q would lead to an increase of the critical wave number k_c until it approaches a certain value $q \equiv q_c \approx -0.8$; then a further increase in q beyond q_c decreases the value of k_c. For $q > -0.1$, the critical wave number k_c increases with the increase of q, which is the same behavior for $k_c > 1$. The yellow (white) color corresponds to

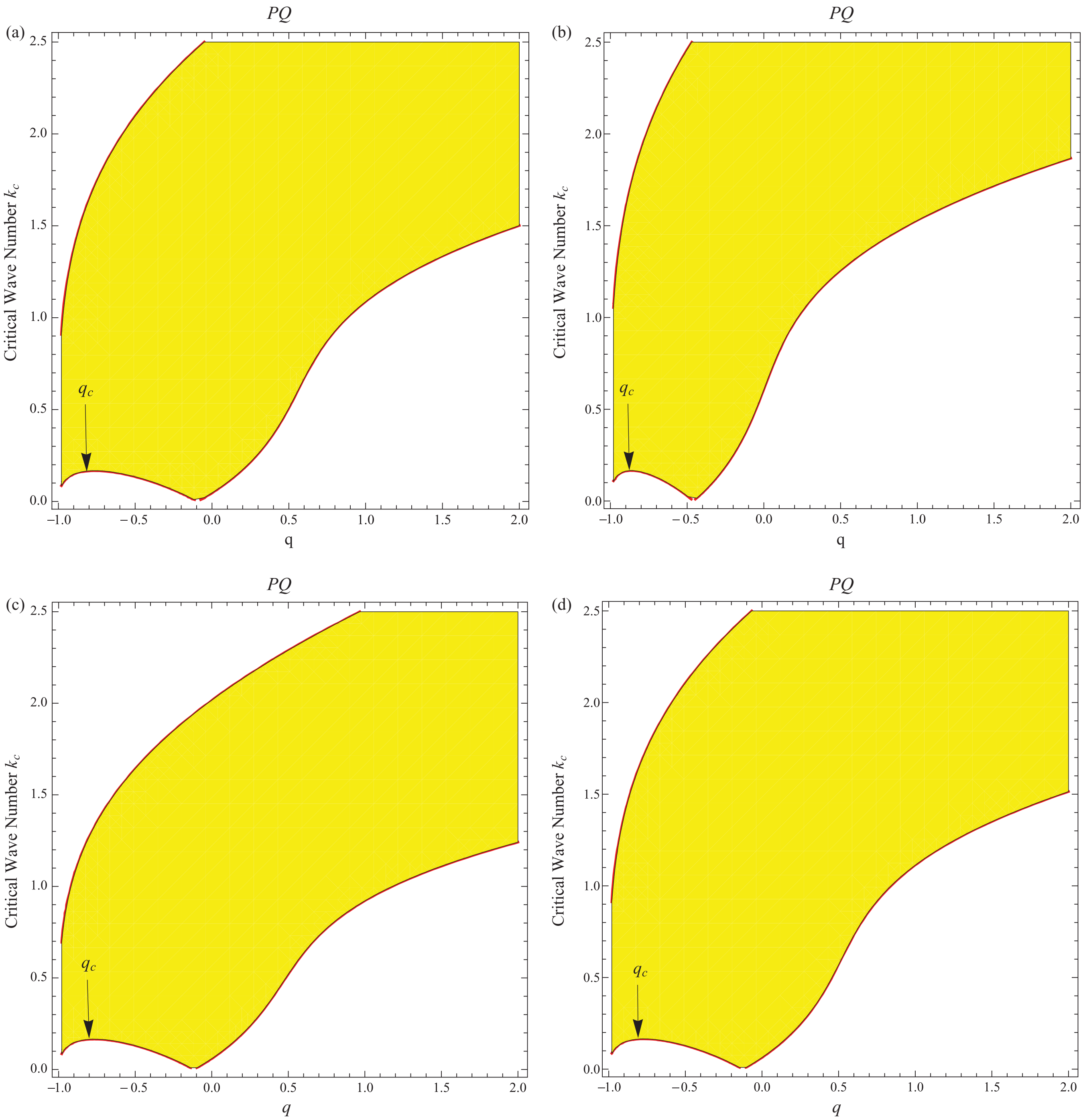

FIG. 1. (Color online) Product PQ contour is depicted against the critical wave number k_c and the nonextensive parameter q; the yellow (white) color represents the region where the unstable (stable) pulses set in. (a) $\mu_e = 0.1, \sigma = 0.05$, $\beta = 0.1$; (b) $\mu_e = 0.9$, $\sigma = 0.05$, $\beta = 0.1$; (c) $\mu_e = 0.1$, $\sigma = 0.15$, $\beta = 0.1$; (d) $\mu_e = 0.1$, $\sigma = 0.05$, and $\beta = 0.3$.

$PQ > 0(PQ < 0)$; i.e., the region of modulational unstable (stable) pulses. Therefore, the rogue waves may propagate for plasma parameters within the yellow or unstable region, as depicted in Fig. 2. Our numerical results exhibit that the effects of both the electron-to-dust number density ratio μ_e and the dust-to-ion temperature ratio are salient to reduce the rogue waves existence region, but the ion-to-electron temperature ratio β is inapparent. Furthermore, it is found that increasing μ_e (we used here $\mu_e = 0.9$) would lead to reducing $q_c \approx -0.9$. Therefore, the increase of μ_e shrinks the unstable region and allows for the rogue wave to propagate in a narrower region. In a qualitative manner, addition of non-Maxwellian electrons seems to favor stability to the envelope excitation against external perturbations.

Now, we numerically analyze the maximum rogue wave envelope amplitude $|\phi_M|$ and investigate how the nonextensive parameter q, the electron-to-dust number density ratio μ_e, the dust-to-ion temperature ratio σ, and the ion-to-electron

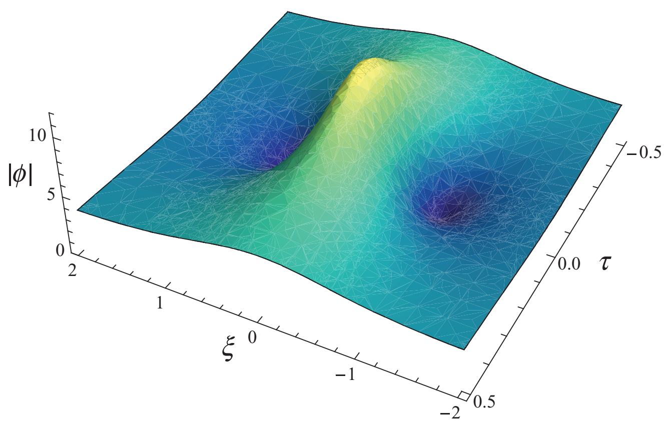

FIG. 2. (Color online) Absolute of the electrostatic potential envelope $|\phi|$ is depicted vs ξ and τ directions. Here, $\mu_e = 0.1$, $\sigma = 0.05$, $q = -0.8$, $k = 0.3$, and $\beta = 0.1$.

temperature ratio β change its profile. Figure 3 depicts that for $q < 0$ the amplitudes of the rogue waves decrease with the increase of q, but, for positive q, the amplitudes increase with the increase of q. It should be emphasized that the physical state described by the q distribution is not the thermodynamic equilibrium. The nonextensive parameter q was proved to be related with the temperature gradient and the potential energy of the system in terms of the formula $k_B \nabla T + (1 - q) Q \nabla \phi = 0$. Thus the deviation of q from unity qualifies the degree of the inhomogeneity of the temperature or the deviation from the equilibrium state [28]. Therefore, the properties of the DA rogue waves derived here are actually those of a nonequilibrium stationary plasma state. Our numerical results exhibit that the nonextensive effect is salient for both positive and negative q. We speculate that this behavior could be explained as follows: for certain values of q the deviation from the equilibrium is strong enough to enhance the nonlinearity and concentrate a significant amount of energy, which makes the pulses taller; however, for other values of q the deviation from the equilibrium is not strong enough to increase the nonlinearity and then the pulses become shorter. Indeed, increasing q causes the rogue wave amplitudes to be enhanced. Furthermore, increasing the wave number k would lead to shrinking the amplitude of the DA rogue waves; i.e., increasing k reduces the nonlinearity and disperses the energy, which makes the pulses shorter. It is interesting to trace the influence of the electron-to-dust density ratio μ_e on the profile of the DA rogue waves. It is clear that an increase of μ_e yields the same qualitative behavior as in Fig. 3(a), but the enhancement of the rogue wave amplitude could happen for a negative value of q ($q > -0.55$). Furthermore, the increase of μ_e (we used here $\mu_e = 0.9$) leads to an enhancement of the rogue wave amplitude. On the other hand, an increase of μ_e would lead to an enhancement of the nonlinearity and then concentrate a significant amount of energy, which makes the pulses taller. Increasing σ and β has the same qualitative behavior on the DA rogue waves profile as Fig. 3(a), so we did not include these figures here. Thus our results indicate that μ_e is the most important parameter for concentration of a significant amount of the DA rogue wave energy, more than the other plasma parameters.

Finally, it is interesting to note that the DA rogue waves can propagate in the absence of nonextensive distributions. For example, the nonlinear coupling between the small, but finite, amplitude Langmuir waves and quasistationary density

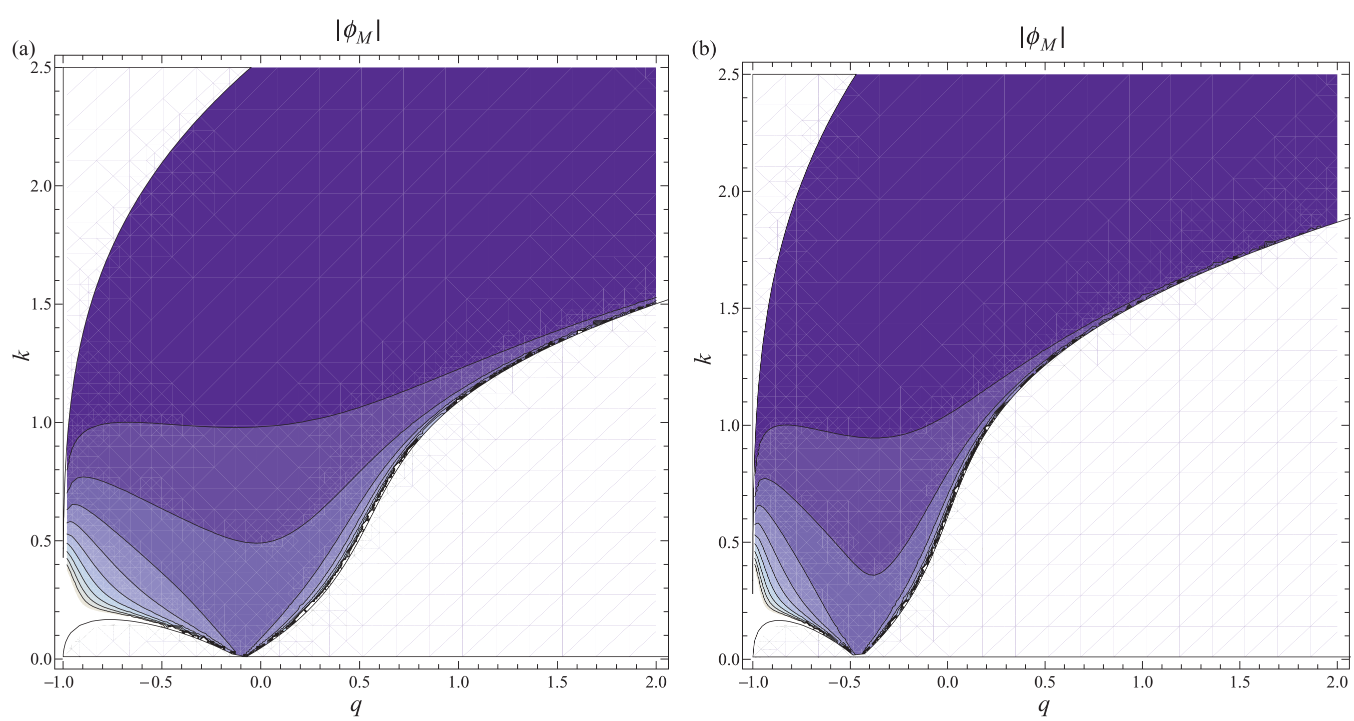

FIG. 3. (Color online) Maximum rogue wave amplitude ϕ_M contour is depicted against the wave number k and the nonextensive parameter q. (a) $\mu_e = 0.1$ and (b) $\mu_e = 0.9$. Here, $\sigma = 0.05$ and $\beta = 0.1$. Light-colored regions correspond to high values of ϕ_M.

perturbations in an electron-positron plasma gives rise to propagating the rogue waves [49]. Also, the electrostatic surface plasma rogue waves can be excited and propagate along a plasma-vacuum interface due to the nonlinear coupling between the high-frequency surface plasmons and the low-frequency ion oscillations [53].

IV. SUMMARY

To summarize, we have investigated the modulational instability of the envelope dust-acoustic waves in a three-component dusty plasma composed of warm dust grains, nonextensive positive ions, and electrons. The critical wave number threshold k_c, which indicates where the modulational instability sets in, has been determined for various regimes. The present study shows that our dusty plasma would introduce unique features for the nonlinear wave modulation, which do not exist in an ordinary dusty plasma. For low wave number, increasing the nonextensive parameter q would lead to increasing k_c until q approaches a certain value q_c, then further increase of q beyond q_c would decrease the value of k_c. For large wave numbers, the increase of q would lead to an increase of k_c. Within the modulational unstable envelope pulse region, it is possible for a random perturbation of the amplitude to grow and may thus lead to the creation of DA rogue waves. The dependence of the latter on the presence of non-Maxwellian electron and ion distribution parameters is numerically examined. It is found that the nonextensive parameter q plays a significant role in maximizing or minimizing the energy of the DA rogue waves. On the other hand, negative q cannot concentrate the energy in the rogue pulses, while positive q causes the rogue waves to obtain much energy. The increase of the electron-to-dust number density ratio μ_e yields the same qualitative behavior as q, but the enhancement of the rogue wave amplitudes could occur for negative values of q. The present results may be useful in understanding the basic features of the DA rogue waves that may be of some relevance to space physics, for example, in Saturn's rings, and they are certainly a topic that may arise in laboratory dusty plasmas.

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APPENDIX: COEFFICIENTS OF EQS. (11)–(19)

The coefficients of Eqs. (11)–(19) are as follows:

$$\Delta_1 = \frac{[3+(2-q)q - 4(k^6\sigma - 3k^4\omega^2)(k^2\sigma-\omega^2)^{-3} + (q-3)(1+q)(\beta^2-1)\mu_e]}{4\{1+q+2k^2[4+(k^2\sigma-\omega^2)^{-1}] + (1+q)(1+\beta)\mu_e\}},$$

$$\Delta_2 = \frac{(1+q)}{\Omega_1}\begin{pmatrix} -4k^3\omega V_g(1+(1+\beta)\mu_e) \\ +k^2(k^2\sigma-\omega^2)(2+3\sigma-q\sigma)+\omega^2(k^2\sigma-\omega^2)(-3+q) \\ +\mu_e(1+\beta)(k^2\sigma-\omega^2)\{k^2[2+(-3+q)(-1+\beta)\sigma]-(-3+q)(-1+\beta)\omega^2\}\end{pmatrix},$$

$$\Delta_3 = -\frac{2k^3\omega}{(-k^2\sigma+\omega^2)^2} + \frac{(1+q)V_g\Omega_2}{\Omega_1},$$

$$\Delta_4 = \frac{1}{\Omega_1}\begin{pmatrix} (-k^2\sigma+\omega^2)\{k^2[4+(-3+q)(1+q)\sigma^2]-(-3+q)(1+q)\sigma\omega^2\} \\ +V_g[8k^3\omega+(-3+q)(1+q)(-k^2\sigma+\omega^2)^2V_g] \\ +\mu_e(-3+q)(1+q)(-1+\beta^2)(-k^2\sigma+\omega^2)^2\left(\sigma-V_g^2\right)\end{pmatrix},$$

$$\Omega_1 = 2(-k^2\sigma+\omega^2)^2\left\{2+\sigma+q\sigma+(1+q)(1+\beta)\sigma\mu_e-(1+q)V_g^2[1+(1+\beta)\mu_e]\right\},$$

$$\Omega_2 = -4k^3\omega V_g[1+(1+\beta)\mu_e]+k^2(k^2\sigma-\omega^2)(2+3\sigma-q\sigma)$$
$$+(k^2\sigma-\omega^2)[(-3+q)\omega^2+(1+\beta)\{k^2[2+(-3+q)(-1+\beta)\sigma]-(-3+q)(-1+\beta)\omega^2\}\mu_e],$$

$$L_1 = k^4[4+(q-3)(q+1)\sigma^2](3\omega^2-k^2\sigma)+(q-3)(q+1)[\omega^6-3k^2\sigma\omega^4+(\beta^2-1)(k^2\sigma-\omega^2)^3\mu_e],$$

and

$$L_2 = (\omega^2-k^2\sigma)\{4k^2\Delta_2+(q-3)(q+1)(\omega^2-k^2\sigma)[(\beta^2-1)\mu_e-1]\Delta_4\}+8k^3\omega\Delta_3.$$

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