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Angular momentum flux of nonparaxial acoustic vortex beams and torques on axisymmetric objects

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An acoustic vortex in an inviscid fluid and its radiation torque on an axisymmetric absorbing object are analyzed *beyond* the paraxial approximation to clarify an analogy with an optical vortex. The angular momentum flux density tensor from the conservation of angular momentum is used as an efficient description of the transport of angular momentum. Analysis of a monochromatic nonparaxial acoustic vortex beam indicates that the *local* ratio of the axial (or radial) flux density of axial angular momentum to the axial (or radial) flux density of energy is *exactly* equal to the ratio of the beam's topological charge l to the acoustic frequency ω. The axial radiation torque exerted by the beam on an axisymmetric object centered on the beam's axis due to the transfer of angular momentum is proportional to the power absorbed by the object with a factor l/ω, which can be understood as a result of phonon absorption from the beam. Depending on the vortex's helicity, the torque is parallel or antiparallel to the beam's axis.

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I. INTRODUCTION

Vortex beams, characterized by a screw phase dislocation of the wave field around its propagation axis with a magnitude null at its core [1], have been generated in both acoustical [2,3] and quantum (optical [4,5] and electron [6]) fields. The wave field is known to carry orbital angular momentum (AM) associated with its phase dependence on the azimuthal angle ϕ in the form of $\exp(il\phi)$, where the integer l is called the topological charge or the order of the vortex whose sign defines the direction of helicity. The orbital AM carried by the optical vortex is known to be in units of $\hbar l$ ($\hbar$ is Planck's constant) per photon with an energy $\hbar\omega$ (ω is the beam's angular frequency) [7], being partially analogous to the spin AM value of $\pm\hbar$ for circularly polarized light. As the classical counterpart, the acoustic vortex was studied more recently and identified to be analogous to the optical vortex within the paraxial approximation [2,8,9]. By considering the angular momentum flux density in the form of a tensor, this paper extends the analogy beyond the paraxial approximation by analyzing the angular momentum transport of a monochromatic acoustic vortex in a homogeneous inviscid fluid. The connection of the paraxial approximation with the description of angular momentum is also addressed.

When interacting with an object, the vortex field can transfer AM to the object, and hence exert torques on the object, as demonstrated by several experiments in both optics [10] and acoustics [11]. Our emphasis is on angular momentum associated with axisymmetric wave fields, and it differs from the situation associated with the average torque on a Rayleigh disk [12,13]. A superposition of the wave field considered here approximates the standing wave that has been used to generate a radiation torque on axisymmetric objects such as spheres and spheroids [14]. Our approach begins with the simpler case of the transfer of the angular momentum from a *progressive* vortex beam directed along the object's symmetry axis. The absorption of energy by the object is accompanied by the transfer of angular momentum. Based on the angular momentum flux density tensor, we shall analyze the axial torque of a nonparaxial acoustic vortex on axisymmetric objects immersed in an ideal surrounding fluid. The result indicates that the torque is proportional to the absorbed power and the ratio l/ω. Our method of integrating the angular momentum flux crossing a spherical surface enclosing the object is partially analogous to a method that has been used to show that the radiation torque of circularly polarized light on a sphere is directly proportional to the absorbed optical power [15].

The transport of momentum in fluids is sometimes presented using phonon concepts [16]. While the inclusion of phonon concepts is not central to our presentation, understanding acoustic vortices beyond the paraxial approximation should have potential importance in fundamental investigations [17] and in applications (such as acoustic alignment [2], wave computation [18], imaging technique [19], acousto-optic interaction [20], acoustic tweezers [21], acoustic spanners for noncontact rotational manipulation [11], etc.). The indicated analogy between an acoustical vortex and a quantum vortex may assist in the understanding of acoustical vortices because of the better explored optical vortex and the concept of twisted photons [22]. The angular momentum flux of optical vortices is sometimes introduced to separate angular momentum into spin and orbital contributions beyond the paraxial approximation [23]. Even though the analysis in this study concerns scalar fields in the fluid, acoustical transverse waves in a solid may be circularly polarized and thus also carry spin angular momentum [24]. It is plausible that an extension of our approach to angular momentum may aid the investigation of circularly polarized acoustic vortices in solids.

II. ANGULAR MOMENTUM FLUX OF NONPARAXIAL ACOUSTIC VORTEX BEAMS

Assuming the beam propagates along the z axis (refer to Fig. 1), as characterized by an azimuthal phase dependency,

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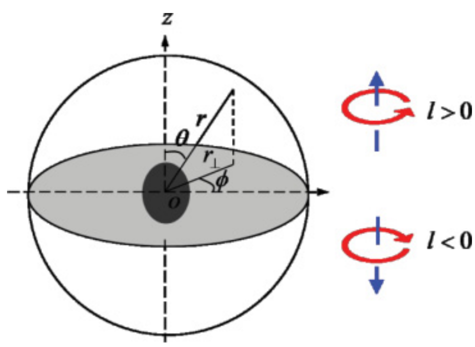

FIG. 1. (Color online) A vortex beam propagating along the z axis exerts an axial torque on the absorbing axisymmetric object (dark gray). Depending on the vortex's helicity (curved arrows pointing to the direction of increasing phase), the torque is parallel or antiparallel to the beam's axis (vertical arrows).

the complex velocity potential of a monochromatic vortex may be expressed generally in cylindrical coordinates as [9]

$$\psi(r_\perp,z,\phi;\omega,t) = \psi_0(r_\perp,z)\exp\{i[k\varphi(r_\perp,z) + l\phi - \omega t]\}, \quad (1)$$

where ω is the angular frequency, $k = \omega/c_0$ is the wave number, c_0 is the phase speed, and $\psi_0(r_\perp,z)$ and $\varphi(r_\perp,z)$ are the amplitude and phase dependencies on the axial z and radial $r_\perp$ coordinates, respectively. The amplitude has a null at the core $r_\perp = 0$ of the phase singularity. This general vortex beam is a solution of the wave equation or its paraxial approximation. Some typical beams (acoustical and optical) are described as Laguerre-Gaussian modes, helicoidal Bessel beams, r vortex, etc. [9,25]. For an acoustic vortex in a nonviscous fluid, the first-order acoustic velocity $\mathbf{u}$, pressure p, and density ρ are given in terms of velocity potential ψ as $\mathbf{u} = \mathrm{Re}[\nabla\psi]$ and $p = c_0^2\rho = \mathrm{Re}\,[i\omega\rho_0\psi]$, where Re denotes the real part and ρ_0 is the unperturbed density of the medium, the total density being $\rho_0 + \rho$.

Prior studies [8,9,26,27] used the angular momentum density $\mathbf{j} = \mathbf{r}\times\mathbf{g}$, where $\mathbf{g} = \rho_0\mathbf{u} + \rho\mathbf{u}$ is the density of linear momentum [28], to discuss the angular momentum of acoustic vortices. The corresponding density of sound energy is $E = \rho_0 u^2/2 + p^2/2\rho_0 c_0^2$. The analysis in this paper concerns time averages (denoted by $\langle\rangle$), and the first-order terms in all of these quantities vanish while averaging over an acoustic time cycle. The momentum density $\mathbf{g}$ is related to the energy flux density $\mathbf{S} = p\mathbf{u}$ (the Kirchhoff-Poynting vector for acoustic waves [28]) by $\langle\mathbf{S}\rangle = c_0^2\langle\mathbf{g}\rangle$. Because of the azimuthal phase dependence, the Poynting vector $\mathbf{S}$ and momentum $\mathbf{g}$ of the vortex beam (1) have an azimuthal component, and hence the vortex carries axial angular momentum. The axial angular momentum density of the beam is Eq. (19) of Ref. [27]

$$\langle j_z\rangle = \langle\rho(\mathbf{r}\times\mathbf{u})_z\rangle = lf(\psi), \quad f(\psi) \equiv \tfrac{1}{2}\omega\rho_0 c_0^{-2}|\psi|^2. \quad (2)$$

The value is identical to the optical quantities, with $\rho_0 c_0^{-2}$ replaced by the permittivity ϵ_0 and $|\psi|$ by the modulus of the complex vector potential associated with optical vortices [4,8]. Under the paraxial approximation and the assumption of a quasiplane wave front, the energy density and axial linear momentum density approximate to $E \simeq \omega f(\psi)$ and $g_z \simeq kf(\psi)$. Hence the beam is analogous to phonons carrying energy, momentum, and angular momentum, with these quantities having the ratios $\langle j_z\rangle/\langle E\rangle \simeq l/\omega$ and $\langle j_z\rangle/\langle g_z\rangle \simeq l/k$ [8,9]. However, these *local* ratios do not hold beyond the paraxial approximation. The ratio l/ω applies to a superposition of Bessel beams when evaluating the total amount per unit length of these physical parameters along the propagating axis [27].

When considering nonparaxial beams, it is natural to use the *acoustic* angular momentum flux density tensor $\mathbf{M}$ to describe the transport of angular momentum. In the absence of dissipation, from the conservation of angular momentum [13,29,30], $\partial_t\mathbf{j} + \nabla\cdot\mathbf{M} = \mathbf{0}$, where $\mathbf{M} = \mathbf{r}\times\Pi$, with Π being the momentum flux density tensor of the sound field [28,31]. For a sound field propagating in a homogeneous inviscid medium, this gives

$$\mathbf{M} = (p + P)\mathbf{r}\times\mathbf{I} + (\rho_0\mathbf{r}\times\mathbf{u})\mathbf{u}, \quad (3)$$

where $\Pi = (p + P)\mathbf{I} + \rho_0\mathbf{uu}$, $P = -\rho_0 u^2/2 + p^2/2\rho_0 c_0^2$ is the second-order acoustic radiation pressure, and $\mathbf{I}$ denotes the unit tensor [32]. For time-harmonic wave fields, $\langle\partial_t\mathbf{j}\rangle$ vanishes so that in regions free of dissipation $\langle\mathbf{M}\rangle$ becomes a solenoidal tensor field: $\nabla\cdot\langle\mathbf{M}\rangle = \mathbf{0}$. Introducing the permutation tensor, the time-averaged components of $\mathbf{M}$ are expressed as $\langle M_{ji}\rangle = \epsilon_{ipq}r_p(\langle P\rangle\delta_{qj} + \rho_0\langle u_q u_j\rangle)$, where $\delta_{qj} = 0$ for $q \neq j$ and $\delta_{qj} = 1$ for $q = j$. The component $\langle M_{ji}\rangle$ denotes the flux density of the i component of angular momentum through a surface oriented in the direction j. The first term of Eq. (3) associated with radiation pressure P does not contribute to the diagonal components of $\mathbf{M}$. The second term is related to the *first-order* angular momentum density $\rho_0\mathbf{r}\times\mathbf{u}$.

Since the radiation pressure only contributes to off-diagonal terms, it follows from (3) that the flux density of the axial angular momentum crossing a plane normal to the propagating direction becomes

$$\langle M_{zz}\rangle = \langle(\rho_0\mathbf{r}\times\mathbf{u})_z u_z\rangle = lc_0 h(\psi),$$
$$h(\psi) \equiv \frac{1}{2}\rho_0 c_0^{-1}\mathrm{Im}[\psi^*\partial_z\psi], \quad (4)$$

where Im denotes imaginary part. Integration of Eq. (4) over a transverse plane agrees with Eq. (38) of Ref. [29]. The corresponding axial energy flux density is $\langle S_z\rangle = c_0^2\langle g_z\rangle = \langle pu_z\rangle$. Since $(\rho_0\mathbf{r}\times\mathbf{u})_z = \rho_0 r_\perp u_\phi = \mathrm{Re}[\rho_0\partial\psi/\partial\phi] = \mathrm{Re}[il\rho_0\psi]$, such that $(\rho_0 r\times\mathbf{u})_z/p = l/\omega$, it follows that

$$\frac{\langle M_{zz}\rangle}{\langle S_z\rangle} = \frac{l}{\omega}. \quad (5)$$

This expression states that, for a monochromatic acoustic vortex with helicoidal phase dislocation $\exp(il\phi)$, the *local* ratio of the axial flux density of the axial angular momentum to the energy flux density through transverse planes is equal to the ratio of the beam's topological charge to acoustic frequency. The sign of l indicates the direction of the carried axial AM being parallel or antiparallel to the beam's axis. This local ratio is *exact* without any approximations or restrictions on the beam's profile, and hence holds for any vortex beam with the phase dislocation $\exp(il\phi)$. An analogous relation exists for an optical vortex for evaluating the total angular momentum flux crossing the whole transverse plane, where there is an extra term corresponding to spin angular momentum for the case of circular polarization [23]. Our expression (5) implies a corresponding ratio for the total flux crossing the whole transverse plane associated with an acoustic beam. This agrees with Eq. (57) of Ref. [29] giving the total flux of a Bessel beam superposition.

The angular momentum flux, within the paraxial limit, has been approximated by multiplying the angular momentum density with the sound speed [2], rather than from the angular momentum flux density introduced here. It is then valuable to examine the connection between the angular momentum density and the paraxial approximation of angular momentum flux density. Under the paraxial approximation $\partial_z \psi \simeq ik\psi$, it follows from Eqs. (2) and (4) that $h(\psi) \simeq f(\psi)$ and hence $\langle M_{zz}\rangle \simeq c_0\langle j_z\rangle$. [Note that together with Eq. (5), this approximation implies the relation $\langle j_z\rangle/\langle g_z\rangle \simeq l/k$ in the paraxial approximation.] Hence the quantity $c_0\langle j_z\rangle$ does give the same result as the quantity $\langle M_{zz}\rangle$ for paraxial beams. However, this is not true for nonparaxial beams. Take a helicoidal Bessel beam as an example: $\psi(r_\perp,z,\phi) = \psi_0 J_l(\mu r_\perp)\exp[i(\kappa z + l\phi - \omega t)]$, where μ and κ are radial and axial wave numbers with $k = \sqrt{\kappa^2+\mu^2}$. It is a nondiffracting exact solution of the wave equation in cylindrical coordinates [25,33,34]. The Bessel beam is equivalent to the superposition of plane-wave components whose wave vectors have a titled conic angle $\beta = \tan^{-1}(\kappa/\mu)$ relative to the z axis [25,33,34]. Substituting the velocity potential of the helicoidal Bessel beam into Eqs. (2) and (4), it follows that $\langle M_{zz}\rangle = \cos\beta\,(c_0\langle j_z\rangle)$. The extra factor $\cos\beta$ is the axial projection of the wave components. It implies that the strictly correct expression for the angular momentum flux density is the tensor introduced by the conservation of angular momentum, instead of the product of angular momentum density multiplied by sound speed. The latter approaches the former only under the paraxial approximation.

Considering also a sound field in spherical coordinates (refer to Fig. 1) having the azimuthal phase dependency $\exp(il\phi)$,

$$\psi(r,\theta,\phi;\omega,t) = \psi_0(r,\theta)\exp\{i[k\varphi(r,\theta) + l\phi - \omega t]\}. \quad (6)$$

Here, the phase singularity and intensity null are at $\theta = 0$ and π (the z axis). This vortex in spherical coordinates (called a *spherical wave vortex* for convenience) could be used to recast the vortex beam Eq. (1) into spherical coordinates, or describe its scattered field by an axisymmetric object placed on the beam's axis (where the unchanged azimuthal phase dependence of the scattered field is due to the symmetry of the object). For scattering of a helicoidal Bessel beam by a sphere, see, for example [34]. This spherical wave vortex could also be used to describe the wave modes of the sound fields emitted by a spherical source, analogous to the distribution of the electromagnetic field emitted by multipoles. Again, this vortex carries axial angular momentum due to the same azimuthal phase dependency, but with the radially outgoing radiation one may evaluate the cycle-averaged flux density of the z component of angular momentum through a radially oriented spherical surface with a radius r: $\langle M_{rz}\rangle = \langle \rho_0(\mathbf{r}\times\mathbf{u})_z u_r\rangle$, which follows from $\mathbf{M}$ in Eq. (3) by recognizing the disappearance of the term associated with radiation pressure [30]. The corresponding radial energy flux is $\langle S_r\rangle = c_0^2\langle g_r\rangle = \langle p u_r\rangle$. Again, it follows from $(\rho_0\mathbf{r}\times\mathbf{u})_z/p = l/\omega$ that

$$\frac{\langle M_{rz}\rangle}{\langle S_r\rangle} = \frac{l}{\omega}, \quad (7)$$

that is, the ratio holds for the radial flux density of axial angular momentum to the radial energy flux density. Only in the far-field region $kr \gg 1$, where the vortex approximates to a locally progressive plane wave such that $u_r \simeq \mathrm{Re}[ik\psi]$, does the wave have $\langle M_{rz}\rangle \simeq c_0\langle j_z\rangle$. Both (5) and (7) apply to a nonparaxial acoustic vortex with a phase dislocation $\exp(il\phi)$. It should also be noticed that the flux of axial angular momentum depends not only on the azimuthal component of velocity but also on the velocity component along the flux direction being evaluated.

III. AXIAL TORQUES ON AXISYMMETRIC OBJECTS

The acoustic vortex exerts an axial torque on an axisymmetric object when accompanied by the absorption of energy by the object. The acoustic radiation torque on an object immersed in an ideal fluid could be evaluated as the integral of the time-averaged flux of the angular momentum density dyadic of the *total* field (incident vortex beam + scattered field) over any spherical surface enclosing the object with its center at the scatterer's centroid (refer to Fig. 1) [30]: $\mathbf{T} = -\int_S\langle\mathbf{M}\rangle\cdot d\mathbf{A} = -\int_S\langle(\rho_0\mathbf{r}\times\mathbf{u})\,\mathbf{u}\rangle\cdot d\mathbf{A}$, where $d\mathbf{A} = \mathbf{n}dA$ is in the direction of the radially outward normal $\mathbf{n}$. The axial torque of an acoustic vortex on an axisymmetric object becomes

$$T_z = -\int_S\langle(\rho_0\mathbf{r}\times\mathbf{u})_z\,\mathbf{u}\rangle\cdot d\mathbf{A} = -\int_S\langle M_{rz}\rangle dA. \quad (8)$$

Likewise, the acoustic power absorbed by the object could be evaluated as the integral of the acoustic energy flux of the total field over the (same) spherical surface bounding the object:

$$P_{\mathrm{abs}} = -\int_S\langle\mathbf{S}\rangle\cdot d\mathbf{A} = -\int_S\langle p\mathbf{u}\rangle\cdot d\mathbf{A} = -\int_S\langle S_r\rangle dA. \quad (9)$$

These evaluations using the integrals over a surface enclosing the object are based on the conservation of angular momentum and energy. In the absence of dissipation in the fluid surrounding the object, $\nabla\cdot\langle\mathbf{M}\rangle = \mathbf{0}$ and $\nabla\cdot\langle\mathbf{S}\rangle = \mathbf{0}$ so that the integrations in Eqs. (8) and (9) may be moved away from the object. The changes of the mean angular momentum and energy in a unit time in the region enclosed by the surface are transferred to or absorbed by the object. The evaluation should still be a useful approximation provided that the dissipation associated with thermal-viscous effects in the surrounding fluid occurs close to the object and the dissipation is included in the analysis of the scattering [30].

The total field consisting of the incident vortex beam and the scattered field has the same phase dependency $\exp(il\phi)$ according to the axisymmetry, such that (7) holds for the integrands in Eqs. (8) and (9), giving

$$T_z = \frac{l}{\omega}P_{\mathrm{abs}}. \quad (10)$$

This expression indicates that the radiation torque vanishes for ordinary acoustic beams ($l = 0$) for axisymmetric objects. It also vanishes for an ideal nonabsorptive object in a nonabsorbing fluid because either the beam does not transfer angular momentum or the radiation pressure does not cause a torque because of the axisymmetry. Otherwise, the axial torque is proportional to the absorbed power with a factor l/ω. This can be understood as a result of phonon absorption from the beam whose ratio of carried angular momentum to energy is l/ω. The torque is parallel or antiparallel to the beam's axis, depending on the direction of carried axial angular momentum (that is, the vortex's helicity, namely, the sign of l), as sketched in Fig. 1.

Though we have emphasized the case of incident waves having the form of a vortex beam (1) in our evaluation of the torque (10), notice that the result (10) applies to other situations in which the total field is described by Eq. (6). For example, when radiation torques are generated on centered axisymmetric objects by standing waves having an appropriate phase dependence [14], (6) should be approximately applicable, provided the rescattering of energy back onto the object by the surrounding walls can be neglected. In that case, (10) is also applicable.

IV. DISCUSSIONS AND SUMMARY

In summary, we have indicated the analogy and connection of acoustic vortices with optical vortices beyond the paraxial approximation by analyzing the transport of angular momentum by acoustic vortices and its transfer to axisymmetric objects. The angular momentum flux density from the conservation of angular momentum provides a description of the transport of angular momentum. Only for a limited class of vortex beams that can be paraxially approximated in the region of interest (such as near the center of a Bessel beam having a small cone angle) is the axial angular momentum flux density approximated as $\langle M_{zz}\rangle \simeq c_0\langle j_z\rangle$, where $\langle j_z\rangle$ is the angular momentum density from Eq. (2). Furthermore, in the analysis of the angular momentum carried by a scattered wave, only in the far field is the radial measure of the axial angular momentum flux density approximated as $\langle M_{rz}\rangle \simeq c_0\langle j_r\rangle$. The more general relationships useful for the analysis of the flux of angular momentum and energy are given in Eqs. (5) and (7).

The axial torque (10) is proportional to the power absorbed by the object and the factor l/ω. Torque is parallel or antiparallel to the vortex's axis depending on the vortex's helicity. In the case of a helicoidal acoustic Bessel beam with a sphere placed on the beam's axis, an expression for the absorbed power, needed in the evaluation of Eq. (10), has been derived [34]. That expression, Eq. (18) of Ref. [34], contains a summation of terms proportional to $(1-|s_n|^2)$, where the partial wave scattering coefficients of the corresponding plane-wave case are proportional to (s_n-1), where n is the partial wave index and s_n depends on material properties and the frequency. In the case of an absorbing sphere in a nonviscous fluid, the expression for P_{abs} is directly applicable, where $|s_n|<1$ as a consequence of the material properties of the sphere. The series for P_{abs} also contains factors depending on l and the conic angle of the beam β [34]. In the case in which the sphere is in a slightly viscous fluid, the expression (10) should be approximately applicable provided the $|s_n|$ are reduced as a consequence of dissipation in the fluid close to the sphere. (For an approximation to the scattering for a solid sphere from which the s_n may be found in that case, see, for example [35].) While the derivation given here is not directly applicable in that case, when the oscillating thermal viscous boundary layers are thin in comparison to the radius of the sphere, it is anticipated that (10) is approximately true for the case of a steady torque associated with an unmodulated acoustic wave. It is noteworthy, however, that for some applications it may be desirable to modulate the helicity of the beam, which may be accompanied with a modification [36] of the usual form of a helicoidal beam transducer [2]. The magnitude of the modulated torque in that case will depend on the time scale for diffusion of momentum in the fluid near the sphere in comparison to the modulation period.

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