Phase controlling of collisions between solitons in the two-dimensional complex Ginzburg-Landau equation without viscosity

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We present a systematic analysis of the outcome of soliton collisions upon variation of the relative phase ϕ of the solitons, in the two-dimensional cubic-quintic complex Ginzburg-Landau equation in the absence of viscosity. Three generic outcomes are identified: merger of the solitons into a single one, creation of an extra soliton, and quasielastic interaction. The velocities of the merger soliton and the extra soliton can be effectively controlled by ϕ. In addition, the range of the outcome of creating an extra soliton decreases to zero with the reduction of gain or the increasing of loss. The above features have potential applications in optical switching and logic gates based on interaction of optical solitons.

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I. INTRODUCTION

The complex Ginzburg-Landau (CGL) equation is an important model which occurs in many areas, such as in superconductivity and superfluidity, fluid dynamics, reaction-diffusion phenomena, nonlinear optics, Bose-Einstein condensates, and quantum field theories [1,2]. The CGL equation with the cubic-quintic (CQ) nonlinearity was first proposed in two-dimensional form by Petviashvili and Sergeev [3] as a model generating stable localized modes. Subsequently, numerous complex stable patterns in this model have been investigated including, among others, stable vortices [4], stable soliton clusters [5,6], fusions of necklace-ring patterns [7,8], and bound states [9,10].

Recently, the application of spatial solitons in all-optical devices [11–14] has been discussed extensively in conservative systems, given their particlelike properties in collision and interaction. Recently, the collisions between solitons or vortices have been reported both in conservative systems [15,16] and dissipative systems [17–25]. In the CQ CGL model, three alternative outcomes of collisions have been studied between dissipative solitons or vortices. However, the influence of relative phase of participating solitons on the collisional outcomes has not been studied in detail. By modulating relative phase, the variation of the interaction should have an important impact on the collisional outcome.

In this work, we study the impact of the relative phase on the three generic outcomes of collisions in two-dimensional (2D) CQ CGL model in the absence of viscosity. Depending on the collisional velocity P, three generic outcomes are identified: at small P, merger of the solitons into a single one whose velocity can be effectively modulated with ϕ; at large P, quasielastic interactions arise; and in the intermediate region, an extra soliton may be created whose velocity also significantly varies with ϕ. Additionally, we also investigated the influence on the three outcomes with the variety of gain and loss coefficients.

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II. THE MODEL

We consider the 2D CQ CGL equation in a general form [4,12]:

$$iu_z - i\delta \cdot u + (1/2 - i\beta)(u_{xx} + u_{yy}) + (1 - i\varepsilon)|u|^2 u - (\nu - i\mu)|u|^4 u = 0, \tag{1}$$

where ν is the quintic self-defocusing coefficient, δ is the linear loss coefficient, μ is the quintic-loss parameter, $\varepsilon > 0$ is the cubic-gain coefficient, and β is the diffusivity term (viscosity). The latter appears in a model of laser cavities, where it is generated by the interplay of the dephasing of the local polarization in the dielectric medium, cavity loss, and detuning between the cavity's frequency and atomic frequencies [26]. We set $\beta = 0$ in order to enable the free motion of the solitons. The dependence of the collisional outcomes on the relative phase may be adequately represented at the following values of parameters: $\mu = 1, \nu = 0.1, \delta = 0.4$, and $\varepsilon = 1.85$, which corresponds to a physically realistic situation and, simultaneously, makes the evolution relatively fast, thus helping to elucidate its salient features [10].

We have solved Eq. (1) using a split-step Fourier method with typical transverse and longitudinal step sizes $\Delta x = \Delta y = 0.2$ and $\Delta z = 0.1$ in all cases below. The second-order derivative terms in x and y are solved in Fourier space with the periodic boundary conditions. Other linear and nonlinear terms in the equation are solved in real space using a fourth-order Runge-Kutta method. For this case, the stable soliton solution was obtained by the evolution of an input pulse $u = A\exp[-(x^2 + y^2)/2w^2]$ based on the split-step Fourier method; see Fig. 1(a).

Thus, we consider the outcomes of collisions between two solitons by varying their relative phase. The two solitons are set in motion in the transverse direction, separated by large distance R, i.e., multiplying each soliton by $\exp(\pm iPr)$, which is taken as

$$u(x,y,z) = u_1(x + R/2,y,z)\exp(iPx) + u_2(x - R/2,y,z)\exp(-iPx)\exp(\pm i\phi), \tag{2}$$

where u_1 and u_2 are two stable soliton solutions in Eq. (1). ϕ is the relative phase ($0 < \phi \leqslant \pi$). With $\phi = 0$, three generic

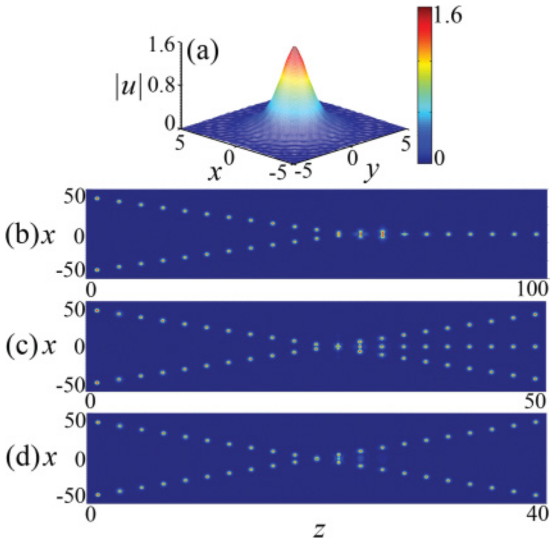

FIG. 1. (Color online) Three outcomes of collision between two in-phase 2D dissipative solitons. (a) Stable soliton solution in Eq. (1); (b) merger of the two solitons at $P = 0.7$, $R = 80$; (c) adding an extra soliton at $P = 1.5$, $R = 80$; (d) quasielastic passage through each other at $P = 2$, $R = 80$.

outcomes are identified. For $P < 0.95$, two solitons fuse into a single one [Fig. 1(b)]. For $0.95 \leqslant P \leqslant 1.79$, an extra soliton is created after the collision in the middle [Fig. 1(c)]. And for $P > 1.79$, the two solitons pass through each other [Fig. 1(d)]. By analyzing the evolution of collision, three second pulses are generated after collision. Furthermore, the energy of the middle pulse increases with collision velocity P, but the two bilateral pulses are opposite. The pulses with enough

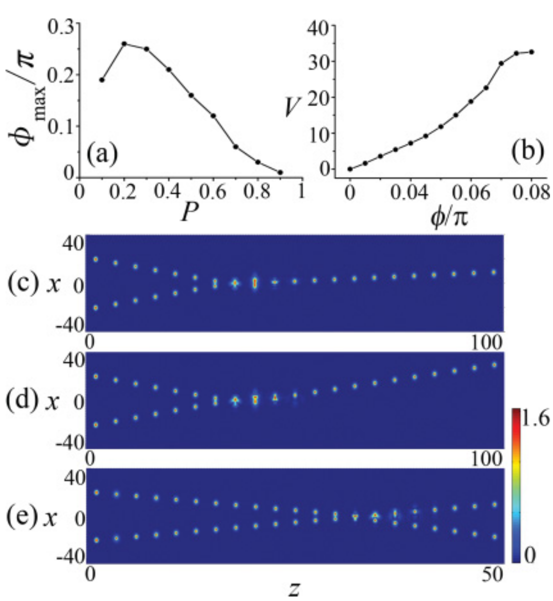

FIG. 2. (Color online) Relative phase controls momentum of merger soliton. (a) Relationship between $\phi_{\max}$ and P; (b) relationship between velocity of merger soliton and relative phase ϕ upon propagation, $z = 100$ for $P = 0.6$ and $R = 40$; (c) and (d) merger into one with transverse momentum for $\phi = 0.05\pi$ and 0.1π at $P = 0.6$, $R = 40$; (e) bounce off each other for $\phi = 0.2\pi$ at $P = 0.4$, $R = 20$.

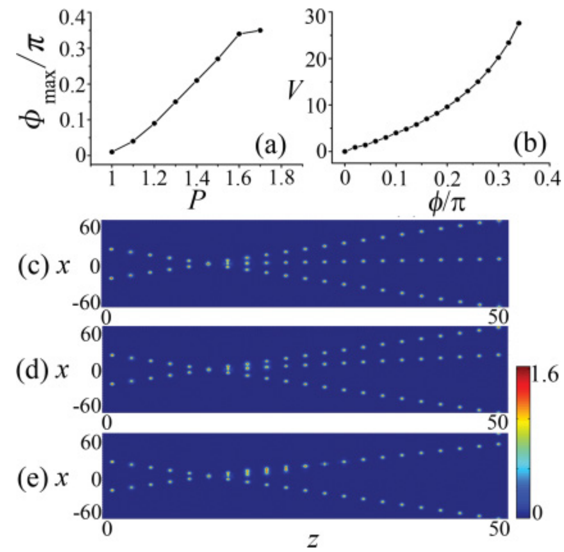

FIG. 3. (Color online) Phase control of the momentum of extra soliton. (a) Relationship between $\phi_{\max}$ and P; (b) relationship between velocity and ϕ on propagation, $z = 50$ for $P = 1.6$ and $R = 40$; (c) and (d) extra soliton with transverse momentum for $\phi = 0.15\pi$ and 0.3π, at $P = 1.6$, $R = 40$; (e) the extra soliton interacts with the original soliton, and they fuse into one for $\phi = 0.4\pi > \phi_{\max}$ at $P = 1.6$, $R = 40$.

energy can self-trap into stable solitons; otherwise they rapidly dissipate. So, for a low collision velocity P, the two bilateral pulses without enough energy dissipate; for a high P, the middle pulse without enough energy dissipates; and for the intermediate region, all of three pulses self-trap into solitons. Next, we are chiefly interested in the impact of the variation of relative phase ϕ on the above results.

III. RESULTS AND ANALYSIS

For the collisions with $P < 0.95$, there exists a critical value $\phi \leqslant \phi_{\max}$ that the two solitons still fuse into one, but the soliton resulting from the merger acquires transverse momentum at $\phi \leqslant \phi_{\max}$. The relationship between $\phi_{\max}$ and P is shown in Fig. 2(a). Clearly, there is a significant range of ϕ which can be used to modulate the transverse momentum of the merger-resulting soliton. Figures 2(c) and 2(d) clearly show the evolutions of the merger of the two solitons with a momentum for $\phi = 0.05\pi$ and 0.1π (at $P = 0.6$). The velocity of merger-resulting soliton for $\phi = 0.1\pi$ is obviously larger than $\phi = 0.05\pi$. Thus, by performing a longer distance numerical simulation, the relationship between the velocity of

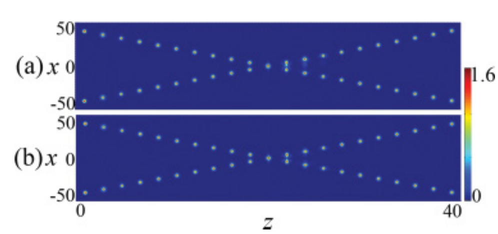

FIG. 4. (Color online) Quasielastic interaction. (a) and (b) Evolution of collisions for $\phi = \pi/2$ and π at $P = 2$ and $R = 80$.

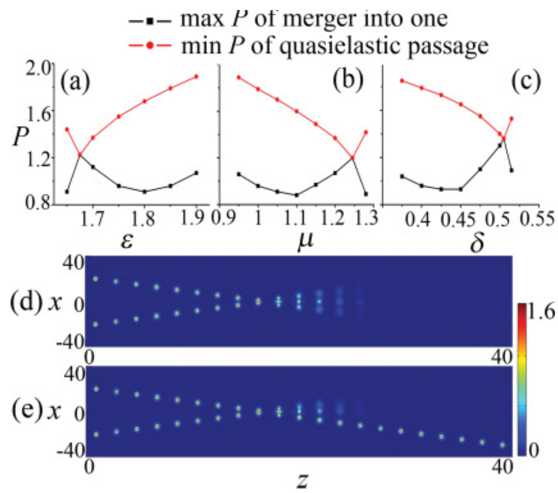

FIG. 5. (Color online) (a)–(c) Region of the three types of collisions by the variety of ε, μ, and δ, respectively. (d) and (e) different types of collisions at $\varepsilon = 1.65$: (d) Both solitons dissipate for in phase at $P = 1.2$, $R = 80$, and (e) one of them dissipates for $\phi = 0.2\pi$ at $P = 1.2$, $R = 80$.

merger-resulting soliton and ϕ at $P = 0.6$ is shown in Fig. 2(b). These exists an effective range of relative phase ϕ in the curve where the velocity of the resulting soliton increases with ϕ. The nature of the soliton interaction changes as a function of ϕ from attractive for in-phase ($\phi = 0$) to repulsive for out-of-phase ($\phi = \pm\pi$) [12,14,21]. As a result, for $\phi > \phi_{\max}$, the two solitons bounce off each other with strong repulsive force [shown in Fig. 2(e)].

At $0.95 \leqslant P \leqslant 1.79$, the collisions of two in-phase solitons will create an extra soliton. There also exists a critical value $\phi_{\max}$ in this case. For $\phi \leqslant \phi_{\max}$, the two solitons, upon collision, also create an additional one, and the transverse velocity of the offspring can again be modulated by ϕ. However, the original solitons maintain their original velocities. We show two typical examples in Figs. 3(c) and 3(d); the extra solitons transversely move with different velocities for $\phi = 0.15\pi$ and for $\phi = 0.3\pi$ at $P = 1.6$. The relationship between the velocity of extra solitons after collision and ϕ for $P = 1.6$, $R = 40$ is shown in Fig. 3(b). Obviously, the velocity of the extra soliton significantly increases with the growth of ϕ. Through this increase upon increase of ϕ, the velocity of the extra soliton gradually approaches that of the original solitons. If the critical value $\phi_{\max}$ is exceeded, the extra soliton with large velocity interacts with one of the original solitons [shown in Fig. 3(e)].

On the other hand, for $P > 1.79$, the two solitons pass through each other in a quasielastic collision. In the case of such quasielastic events, the outcome of the collisions does not change upon variation of ϕ. Figures 4(a) and 4(b) show the evolutions of collisions with $\phi = \pi/2$ and $\phi = \pi$.

In addition, we also study the influence on the collisional outcomes by the variety of gain (ε) and loss (μ and δ). The variety of the region is shown in Fig. 5(a) by only changing ε; when $1.675 \leqslant \varepsilon \leqslant 1.9$, the region of creating an extra soliton (between the red circle and black square dotted line) gradually decreases to zero with reducing of ε (gain). At $1.65 \leqslant \varepsilon < 1.675$, a different type of collision reported in Ref. [24] appears instead of the creation of an extra soliton. For the in-phase collisons, both solitons dissipate [shown in Fig. 5(d)]; but for collisons with a little ϕ, the phase lag soliton dissipates, while the phase advance soliton maintains its original velocity [shown in Fig. 5(e)]. The same results also can be obtained in Figs. 5(b) and 5(c) by inceasing μ and δ (loss). To sum up, the region of creating an extra soliton gradually decreases to zero by reducing gain or increasing loss, and if continue reducing gain or increasing loss continues, a different type of collision appears.

IV. CONCLUSIONS

In summary, we have studied phase modulation of the three possible types of collisional outcomes in the context of the general 2D CQ CGL model in the absence of viscosity. For low enough initial soliton momenta, the collision results in a merger. In turn, the velocity of the resulting merger soliton can be effectively modulated by ϕ when $\phi \leqslant \phi_{\max}$. For intermediate velocity regimes, the collision leads to the production of an extra soliton. The velocities of such extra solitons can be significantly modulated by relative phase in an appropriate range $\phi \leqslant \phi_{\max}$. For high enough initial momenta, the solitons interact quasielastically, an outcome which is not affected by the variation of ϕ. The properties of phase modulation on collision have the potential application of enabling the design of optical switches and logic gates based on collision of solitons, depending on the regime of parametric operation of models such as the CQ CGL above. In addition, upon reduction of gain coefficients (ε) or enhancing of loss coefficients (δ and μ), the intermediate velocity regime for creation of an extra soliton gradually decreases to zero, and a different type of collision appears instead.

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[1] S. Aranson and L. Kramer, Rev. Mod. Phys. **74**, 99 (2002).
[2] B. A. Malomed, in *Encyclopedia of Nonlinear Science*, edited by A. Scott (Routledge, New York, 2005), pp. 157–160.
[3] V. I. Petviashvili and A. M. Sergeev, Dokl. Akad. Nauk SSSR **276**, 1380 (1984).
[4] D. Mihalache, D. Mazilu, F. Lederer, Y. V. Kartashov, L.-C. Crasovan, L. Torner, and B. A. Malomed, Phys. Rev. Lett. **97**, 073904 (2006).
[5] A. G. Vladimirov, J. M. McSloy, D. V. Skryabin, and W. J. Firth, Phys. Rev. E **65**, 046606 (2002).
[6] D. V. Skryabin and A. G. Vladimirov, Phys. Rev. Lett. **89**, 044101 (2002).
[7] Y. J. He, B. A. Malomed, and H. Z. Wang, Opt. Express **15**, 17502 (2007).
[8] B. Liu, Y. J. He, Z. R. Qiu, and H. Z. Wang, Opt. Express **17**, 12203 (2009).

[9] V. V. Afanasjev and N. Akhmediev, Opt. Lett. **20**, 1970 (1995).
[10] Y. J. He, B. A. Malomed, D. Mihalache, B. Liu, H. C. Huang, H. Yang, and H. Z. Wang, Opt. Lett. **34**, 2976 (2009).
[11] Y. D. Wu, Opt. Express **12**, 4172 (2004).
[12] Y. D. Wu, Opt. Express **14**, 4005 (2004).
[13] Y. D. Wu, Appl. Opt. **44**, 4144 (2005).
[14] S. Blair and K. Wagner, Appl. Opt. **38**, 6749 (1999).
[15] D. Mihalache, D. Mazilu, F. Lederer, and Y. S. Kivshar, Phys. Rev. A **79**, 013811 (2009).
[16] D. Mihalache, D. Mazilu, F. Lederer, and Y. S. Kivshar, Opt. Commun. **282**, 3000 (2009).
[17] N. Akhmediev, J. M. Soto-Crespo, and P. Grelu, Chaos **17**, 037112 (2007).
[18] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, and B. A. Malomed, Phys. Rev. A **77**, 033817 (2008).
[19] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, and B. A. Malomed, Phys. Rev. E **78**, 056601 (2008).
[20] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, and B. A. Malomed, Eur. Phys. J. Special Topics **173**, 245 (2009).
[21] G. Wainblat and B. A. Malomed, Physica D **238**, 1143 (2009).
[22] S. V. Dmitriev and T. Shigenari, Chaos **12**, 324 (2002).
[23] S. V. Dmitriev, P. G. Kevrekidis, B. A. Malomed, and D. J. Frantzeskakis, Phys. Rev. E **68**, 056603 (2003).
[24] D. Mihalache, D. Mazilu, and F. Lederer, Cent. Eur. J. Phys. **8**, 77 (2010).
[25] D. Mihalache, Rom. Rep. Phys. **62**, 697 (2010).
[26] J. Lega, J. V. Moloney, and A. C. Newell, Phys. Rev. Lett. **73**, 2978 (1994).