Wakes formed by dust grains in supersonically flowing plasmas

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Interesting wake effects are found in simulations of dust grains in supersonically flowing plasma. A Mach cone is formed at an angle to the flow determined by the ratio of flow to Bohm speed. The latter is well approximated by $[k(T_e + \gamma T_i)/m_i]^{1/2}$ with $\gamma = 3$. For ion temperatures significantly lower than the electron temperature, a second (inner) cone forms due to flow convergence. An "ion vacuum" and stagnation point occur downstream. These latter effects cannot be described by conventional (cold-ion) gas dynamics. Critically, none of the cones observed are shocks but are more akin to weak discontinuities.

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I. INTRODUCTION

Wake effects are encountered in many areas of physics with varying length scales and complexities. The effect of an object in a stream of neutral gas moving with supersonic flow velocity is one of the most important and well-studied problems of compressible hydrodynamics. As is well known, there are two distinct regions in the flow: Upstream there is a "zone of silence" which is unaffected by the presence of the object, and downstream is a roughly conical region in which the flow is considerably modified. This latter region is usually termed the "Mach cone," although it only forms a true cone with a sharp vertex for a pointed object aligned with the flow. In the corresponding dusty plasma situation, complex wake effects have been observed in dust crystal lattices [1] and investigated theoretically [2]. In the case of a single small object immersed in a supersonically flowing plasma, simulations have suggested interesting and complicated wake behavior [3]. It has become customary to use the term "Mach cone" rather indiscriminately to describe this phenomenon. In this paper, we report detailed simulations of such structure and show that the Mach cones formed in neutral gas and plasma have significant differences. These differences reflect underlying differences in the basic physics of the interaction between the object and the high-velocity medium. The study of this problem is important for many practical reasons, ranging from industrial application to fusion. The problem of dust charging is nontrivial; plasma temperature and constituent components, dust grain size, temperature, and velocity all play critical roles. The motion of the dust relative to the plasma may be supersonic in a number of situations ranging from astrophysical dust and spacecraft charging to dust present in fusion devices such as tokamaks and stellarators.

The problem investigated is that of a single dust grain with radius (a) much larger than the electron Debye length (λ_{De}). Due to recombination at the dust grain surface, the grain is effectively absorbing. The ions are warm and collisionless, and the electrons are isothermal. We define the parameters $\beta = T_i/T_e$, where T_i is the ion temperature in the unperturbed plasma upstream of the dust grain. We also define the following normalized variables: $\rho = a/\lambda_{De}$, v is flow velocity normalized by $\sqrt{kT_e/m_i}$, and V is potential normalized by kT_e/e. In the following figures, r and z are normalized by a.

The cones are investigated via the particle-in-cell (PIC) code SCEPTIC, which was developed by Hutchinson [4–8] and is freely available. The ions have finite temperature and are advanced individually, and the electrons are governed by the Boltzmann relation. The fields are calculated on a semicircular mesh, and we use cylindrical coordinates (r,z) with the z direction defined as the flow direction. The origin is at the center of the grain. The problem is two dimensional with cylindrical symmetry; that is, the fields are assumed to be independent of azimuthal angle θ. As we are interested in large grains, resolving the electron Debye length over a sufficient distance requires a large number of cells and hence a large number of PIC particles. The potential of the grain is determined by setting the ion current equal to the electron current and averaging over a few tens of time steps. For a detailed outline of the SCEPTIC code, see [4–8].

The major differences between plasma-immersed dust and an object in a neutral gas can be summarized as follows. In the neutral gas case, a shock is present at the Mach cone, but in the plasma case, this boundary is not a shock. In the neutral gas case, there is a standoff distance in front of a blunt object, like a sphere, and the bow shock starts here. For large objects in the plasma case, the observed cones are truncated and start on the grain surface. In the neutral case, the flow must move around the object, whereas in the plasma case, the flow impinging on the grain is effectively absorbed as recombination takes place. These differences are illustrated schematically in Fig. 1. In addition, there are two further important differences. First, in the neutral case, a second shock may be seen downstream due to converging flow. In the plasma case, a single cone is seen at high ion temperature but for $\beta \lessapprox 0.5$ two "nested" cones are always observed. Finally, in the neutral case, the shock extends over large distances, but the plasma cones extend for a few grain radii before being damped away.

These differences are due to the following key factors. In the neutral gas case, the interaction is mediated by gas pressure, whereas in the plasma the interaction is mediated by electrostatic fields. As mentioned above, in the plasma case, the object is perfectly absorbing and hence no density buildup is possible in front of the object. In the neutral case, the gas does impinge on the object, after the shock wave, whereas in the plasma case the upstream, quasineutral plasma is separated from the object by a thin sheath of positive space charge. Finally, in the neutral case, the characteristic velocity is the sound speed. For the plasma, the characteristic velocity is the Bohm speed (ion acoustic speed), although it is unclear a

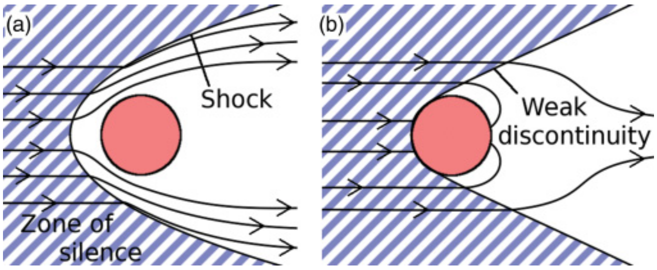

FIG. 1. (Color online) Schematic representation of the flow pattern in (a) the conventional case of an object in supersonic flow and (b) an absorbing dust grain in supersonic flow.

priori what value this has for the case of warm ions, considered here.

II. SIMULATION

We now examine these results in more detail. For a stationary plasma, the sheath and presheath around a dust grain are spherically symmetric. As flow is introduced, the spherical symmetry of the presheath is lost as the presheath is deformed. For supersonic flows, a well-defined, truncated Mach cone is seen, and the form of this cone is dependent on the ion temperature. For $\beta \gtrapprox 0.5$, a single Mach cone forms, and for lower ion temperatures a double cone is seen (Fig. 2). Wake structures and rarefaction cones have previously been observed with SCEPTIC [4] but not explored in detail. We are primarily concerned with low ion temperatures and double cones. The inner cone can be seen to begin at some point behind the grain. As already noted, the outer cone is not a conventional shock wave as the density and velocity vary smoothly across the cone (i.e., there are no jumps at the cone boundary). Figure 3 shows the ion density and z component of velocity at $z = 2$. These quantities are seen to vary smoothly everywhere. This is a critical point; both cones are more akin to weak discontinuities [9] but with fuzzy boundaries (i.e., the derivatives change quickly over a short distance but are not discontinuous). Ion density decreases across the outer cone to some minimum and then increases toward the axis. Experimental work by Merlino and D'Angelo [10] found a rarefaction wave in the wake of a negatively charged object (a conducting disk). Ion deflection in their case was assumed to be due to the sheath, whereas we are concerned with deflection in the presheath but similarities can be seen between their

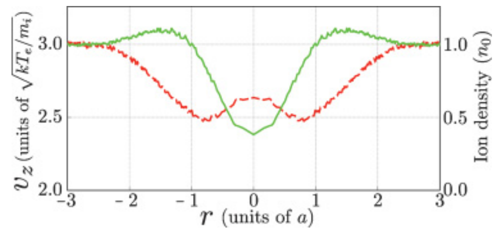

FIG. 3. (Color online) Ion density contour (dashed) and v_z (solid): $z = 2.0$, $\beta = 0.3$, $v = 3.0$, $\rho = 40$.

Fig. 6 and our Fig. 3. The z velocity increases on entering the outer cone and reaches a peak, due to the form of the potential. The z velocity then falls below the flow velocity to some minimum on axis. As these go further downstream, the density and velocity perturbations become smaller. For flow velocities up to approximately twice the sound speed, a spherically symmetric sheath of positive space charge is seen in a thin sheath ($\sim 5\lambda_{De}$) around the grain. We define the sheath edge as the point at which quasineutrality breaks down (specifically, when the ion and electron densities differ by more than 3%). For larger flows, the positive space charge region behind the grain is reduced due to depletion of ion orbits. This ion depletion has been observed in numerical work concerning spacecraft wakes in the ionosphere [11]; this is an example of the direct application of large ρ grains in a supersonic plasma. As the flow velocity increases further, a negative space charge region develops downstream. This initially enhances the negative potential immediately downstream before being screened. This is a particularly interesting point, as the Bohm criterion does not have to be satisfied for negative space charge. The position of the sheath edge is shown in Fig. 4 as a function of flow velocity. For flow around the sound speed, a stagnation point (the point on the downstream axis where the average z velocity is zero) is observed. The stagnation point is in the plasma (not in the sheath), and there is a significant ion density at the stagnation point. The position of and density at the stagnation point are shown in Fig. 4. As v is increased, the stagnation point moves toward the grain. The sheath boundary also moves toward the grain due to the increased depletion of ion orbits; that is, the width of the positive space charge sheath downstream is reduced. In the case shown, at $v \approx 3.0$ the sheath boundary downstream reaches the grain surface (i.e., there is no sheath downstream). Increasing the flow further causes the sheath boundary to move away from the surface, but the space charge in the sheath region is now negative.

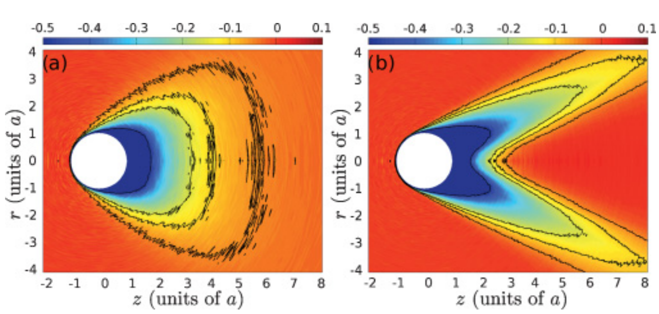

FIG. 2. (Color online) Potential distribution V (units of kT_e/e). (a) Single cone: $\beta = 1.0$, $\rho = 80$, $v = 2.5$. (b) Double cone: $\beta = 0.2$, $\rho = 80$, $v = 2.5$.

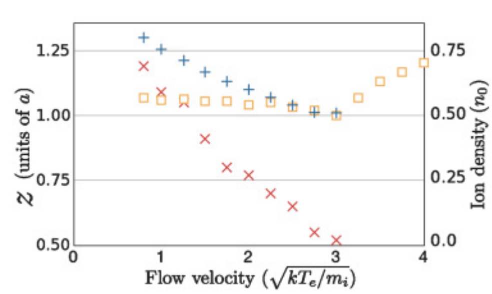

FIG. 4. (Color online) $\beta = 0.2$, $\rho = 80$. Position of the stagnation point (+), position of the downstream sheath boundary ($\square$), and density at the stagnation point ($\times$).

As can be seen in Fig. 4, the position of the sheath boundary changes faster with increasing flow when the sheath is negative than when positive.

The inner cone, seen in Fig. 2(b), is due to converging flows and does not start at the stagnation point. The stagnation point, when it exists, is always closer to the probe than the apex of the inner cone. The inner cone is lost for $\beta \gtrapprox 0.5$. This is probably due to the increasing thermal energy of the ions causing them to "wash out" potential structure. Interestingly, for a given (normalized) flow speed, the position of the stagnation point is independent of β.

In trying to understand theoretically a problem involving supersonic, compressible flows, the obvious choice of model is gas dynamics. Assuming cold ions (isothermal) there is a well-defined sound speed, $c = 1$ (normalized by $(kT_e/m_i)^{1/2}$), and the Bohm criterion [12] is $v \geqslant c$. That is, the velocity of the ions at the sheath edge must be greater than or equal to the sound speed. For a nonabsorbing spherical object in supersonic flow, a shock wave is expected to form upstream at some standoff distance with the half angle of the cone being the Mach angle, $\sin\alpha = c/v$. For the isothermal case, no vacuum region is expected [13]. If there is a stagnation point, the flow must have transitioned from supersonic to subsonic, a shock would be expected somewhere in the neighboring region, and the density at the stagnation point should be large.

III. INTERPRETATION

We compare our results with an analytic model. Historically we would have chosen a cold-ion model; however, this does not adequately describe the situation due to the ion pressure. The Vlasov equation, for finite values of T_i, does not yield the cold-ion case as T_i tends to zero; the latter case is singular in nature. Unfortunately, as we do not have a warm-ion model, we are limited to a comparison with cold-ion gas dynamics.

Work has been carried out by Stangeby and Allen [14] for objects in a flowing plasma of cold ions. Ion and electron density in the plasma are given by the Boltzmann relation, and the ion motion is described by the equations of fluid flow. In the cold-ion case, the motion of a fluid element is the same as the motion of an individual particle; hence the ion trajectories follow or are the streamlines. In a region of supersonic flow, a Mach surface is defined such that the fluid velocity component perpendicular to the "Mach surface" is equal to the sound speed. A plasma-sheath boundary, at a positive-space charge sheath, is a Mach surface [14], but this is no longer the case for a negative-space charge sheath, which may form over part of the plasma boundary. For sufficiently high flow velocities, as seen in Fig. 4, the ion depletion downstream causes a region of negative space charge to form downstream. As such, a Mach surface around the grain will close on the grain surface rather than extending around the back of the grain, and ions cannot be accelerated sufficiently by the presheath to satisfy the Bohm criterion and form a positive sheath downstream.

Now we contrast our results with the gas dynamic theory. The primary cone may be approximated but very little of the downstream behavior is predicted. In the case of an absorbing dust grain, the shock is replaced by something resembling a weak discontinuity beginning approximately on the grain surface. A thin sheath still exists around the grain, though it is deformed downstream for larger flows. At no point does the plasma move to avoid the grain. Ions are also collected downstream (depending on the flow velocity), and this may lead to a stagnation point somewhere behind the dust. The outer cone is truncated upstream and "fades away" downstream, whereas the inner cone has an apex. The fact that the Mach cone is truncated upstream is due to the absorbing nature of the grain; the plasma upstream does not need to "know" about the grain unless the ion velocity perpendicular to the plasma-sheath transition is less than the hot-ion Bohm (sound) speed. The flow is accelerated and deflected upon entering the outer cone and then decelerated as it approaches the inner cone. Here one might expect a shock wave; however, no jumps in the potential, density, or velocity are observed. This is unpredicted, considering that at some supersonic flow velocities we have a stagnation point and would expect a shockwave, from a compressible fluid point of view, at the transition from supersonic to subsonic flow. The density increases on the axis as the flow converges with the effects on the velocity, potential, and density becoming less pronounced the further downstream we look.

The presheath around a dust grain in a stationary plasma has the function of accelerating the ions to a velocity perpendicular to the sheath, which satisfies $v \geqslant c$. If the ions are streaming toward an object at a velocity already satisfying the Bohm criterion, then there will be no presheath. If our object were a cone at the Mach angle [i.e., the same shape as the outer potential contour in Fig. 2(b) for that specific case], the plasma could flow onto it freely as the Bohm criterion would be already satisfied, and hence there would be no presheath upstream. For the same conditions, a cone with a smaller half-angle would require a presheath in order to satisfy the Bohm criterion at its surface. The outer cone is a two-dimensional (2D) collisionless presheath required to deflect the ions so they satisfy the Bohm criterion at the sheath edge.

The speed of sound is no longer a simple quantity; it depends on the ion temperature but is not described by any simple expression. The wave is also damped (Landau damping), but less so when T_i is small compared with T_e [15]. In the collision-free case, with finite ion temperature initially, one cannot deduce a polytropic relation between pressure and density. The warm-ion Bohm speed may be estimated by observing the flow velocity at which the upstream presheath width falls to zero (that is, the velocity at which ions approaching from directly upstream are unperturbed right up to the sheath edge; see Fig. 5). Alternatively, we may use the

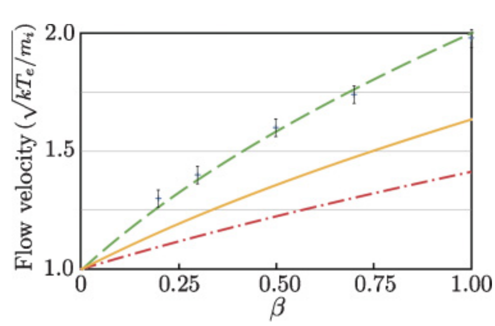

FIG. 5. (Color online) Flow velocity at which upstream presheath width goes to zero. Fits are $\sqrt{1+\gamma\beta}$ with $\gamma = 1$ (dot-dashed line, red), $\gamma = 5/3$ (solid line, yellow), and $\gamma = 3$ (dashed line, green).

half-angle of the outer cone to determine the speed at which information propagates perpendicular to the flow. Both methods result in a hot-ion Bohm (sound) speed well approximated by the semiempirical formula $c_{\text{hot}} = \sqrt{k(T_e + \gamma T_i)/m_i}$, with $\gamma \approx 3$. The Mach angle is then $\sin\theta = c_{\text{hot}}/v$.

IV. CONCLUSIONS

In conclusion, the isothermal ($\gamma = 1$) gas dynamic model is not appropriate for describing the processes seen in warm, supersonic ion flow past an absorbing object. An ion vacuum is observed downstream for significant flow velocities, and $\gamma = 3$ is found to be an excellent approximation for the Bohm velocity. For large flows, the presheath cannot sustain a large enough radial electric field, and maintain quasineutrality, to accelerate the ions sufficiently to satisfy the Bohm condition.

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