Energy gain of an electron by a laser pulse in the presence of radiation reaction

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A well-known no-energy-gain theorem states that an electron cannot gain energy when being overrun by a plane (transverse) laser pulse of finite length. The theorem is based on symmetries which are broken when radiation reaction (RR) is included. It is shown here that an electron, e.g., being initially at rest, will gain a positive velocity component in the laser propagation direction after being overrun by an intense laser pulse (of finite duration and with intensity of order 5×10^{22} W/cm^2 or larger). The velocity increment is due to RR effects. The latter are incorporated in the Landau-Lifshitz form. Both linear as well as circular polarization of the laser pulse are considered. It is demonstrated that the velocity gain is proportional to the pulse length and the square of the peak amplitude of the laser pulse. The results of numerical simulations are supported by analytical estimates.

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I. INTRODUCTION

Research in ultrarelativistic laser-material interaction [1,2] is progressing significantly. Intensities of 10^{19} W/cm^2 are now available in terawatt tabletop laser systems based on chirped-pulse amplification. In that intensity regime, the relativistic γ factor for electrons becomes already much larger than 1, and clearly relativistic effects are dominating the nonlinear particle dynamics. Large laser systems may deliver intensities up to $I \approx 10^{23}$ W/cm^2 at the focal spot. In the near future, extreme laser intensities $I \geqslant 10^{23}$ W/cm^2, when the electrons become ultrarelativistic within a fraction of a wave period, are expected to become available. New effects, like the radiation reaction (RR) force, enter the scene. Because of the huge acceleration, electrons will emit quite large amounts of electromagnetic radiation. The emitted fields may influence the motion of the electron itself, an effect which is covered by the RR force [3,4]. Numerical simulations suggest that the RR force becomes important at intensities exceeding 5×10^{22} W/cm^2 [5–7].

Neglecting the RR force, relativistic particle and wave motion during laser-plasma interaction was analyzed in the classical paper by Akhiezer and Polovin [8], and later by Dawson and Kaw [9,10]. It was shown that the laser can generate longitudinal plasma waves due to the Lorentz force. Relativistic coupling between transverse and longitudinal oscillations occurs. An interesting phenomenon in the new field of relativistic optics [2] is the completely different particle motion in relativistic waves compared to the well-known quiver motion in nonrelativistic oscillations. In linearly polarized laser light "figure-eight" orbits [11] appear as new solutions of the single particle orbits in the average rest frame (where the average momentum is zero). During past years, the systematic understanding of periodic plasma motion in relativistic waves persistently grew. In most considerations, the RR force was absent. One should note that even in the absence of the RR force the analysis is by far not trivial because of the nonlinear nature of the problem. Nonintegrability complicates the understanding of the complex dynamics. Besides regular also chaotic solutions occur. The basic equations are in general not integrable, and only in some limiting cases analytical work could be done. Nevertheless, besides purely numerical solutions, also approximate analytical methods have been applied to discuss the various forms of the solutions in highly relativistic regimes [12–19] without taking into account the RR.

As has been mentioned already, at intensities exceeding 5×10^{22} W/cm^2, RR is expected to become significant. The inclusion of RR effects into a classical equation of motion goes back to Lorentz, Abraham, and Dirac (LAD); see, e.g., Ref. [3]. Several models have been introduced afterward (see, e.g., the discussions in Refs. [20,21]). Most recently, Skolov *et al.* [7,22,23] expressed the radiation backreaction on the electron motion in terms of the emission probability. They succeeded in deriving the RR force experienced by an electron from QED principles, extending the applicability to QED-strong fields. For relatively weak fields, the Landau-Lifshitz (LL) equation is widely accepted. It does not produce inconsistencies (such as, e.g., exponentially diverging acceleration even without an external field) and it is equivalent to the LAD equation up to the first order in the coupling parameter [21]. By perturbation theory, the LL form was derived from basic considerations of energy and momentum conservation [21,24].

Inclusion of effects of radiation emission on the particle dynamics have been discussed already a long time ago, e.g., in Refs. [11,25]. Only recently, the full ultrarelativistic case has been considered for plane waves [21,26,27] and other time-varying electromagnetic fields [28].

In this paper, we investigate RR effects during the interaction of an electron (which, e.g., initially is at rest) with a transverse laser pulse. Compared to a longitudinally infinite plane wave solution, the finite duration of the interaction should be noted. Short laser pulses are envisaged as realistic jtitles for laser-based particle (electrons or ions) acceleration in vacuum. Several acceleration processes have been proposed, e.g., electron acceleration in wake-fields [29] as well as ponderomotive pushing [30]. It is well known that ponderomotive laser acceleration only leads to a net energy gain when the three-dimensional geometry of the laser pulse is taken into account. Profile tailoring of the laser pulse may lead to very effective acceleration and focusing [31]. This has to be contrasted to the opposite case of an infinite transverse extend, i.e., the plane wave approximation of the longitudinally localized laser pulse. In the latter case, although an electron may reach relativistic velocities within the electromagnetic pulse, the electron comes to rest after the wave has overtaken the electron. We want to test, in the presence of the RR force, whether that classical prediction, namely that "an electron cannot gain net energy in a plane light pulse," remains true. According to this classical statement for the no energy gain after a plane transverse pulse has overrun an electron, the electron needs breaking of planar symmetry for a net energy gain. As mentioned already, the breaking typically occurs in experimental configuration due to a finite beam radius [31,32].

Gunn and Ostriker [25] argued already that, in the field of electromagnetic radiation, radiative losses will ultimately lead to an increase in particle energy. Since the drag causes a phase lag between velocity and field, an electron may keep the energy increase after it was overrun by a pulse of finite duration. This is exactly what we want to investigate here in detail. The proof-of-principle study of energy gain in plane laser pulses, being exemplified on several quantitative examples, is at the focal point of the considerations. However, to bring the RR effect on energy gain into the context of experimental significance, we have no doubt that in most experiments the energy gain of an electron due to finite beam radii will dominate over the energy gain caused by RR.

The paper is organized as follows. In the next section, Sec. II, we briefly present the model which will be used for the motion of an electron in an electromagnetic field in the presence of the RR force. The no-energy-gain theorem in the absence of RR is discussed in Sec. III. In Sec. IV it is shown, both numerically and via analytical estimates, that the RR force causes a velocity increase in the laser propagation direction after an electron is overrun by a finite laser pulse. The weakly relativistic case is discussed first, since in that case analytical estimates are simple. The manuscript is concluded by a short summary.

II. MODEL

Let us start with the covariant formulation of the motion of a relativistic electron in the presence of an electromagnetic wave. The four components $i = 1,2,3,4$ of the four-velocity $u^i = (\gamma,\gamma\mathbf{v}/c)$ and the proper time element ds are

$$u^i = \frac{dx^i}{ds}, \quad ds = c\,dt\sqrt{1-\frac{v^2}{c^2}} \equiv \frac{c}{\gamma}dt. \tag{1}$$

The relativistic four-velocity is related to the four-momentum by $p^i = mcu^i$, where m is the electron rest mass. We use the notation of Landau and Lifshitz [4] with the metric tensor $g^{ik} = g_{ik}$, whose diagonal elements are $1,-1,-1,-1$. Taking into account the electron charge $-e$, the Lorentz force, and the RR force, we start from [4]

$$mc\frac{du^i}{ds} = -\frac{e}{c}F^{ik}u_k + g^i. \tag{2}$$

F^{ik} is the electromagnetic tensor and g^i the RR force. The latter may be formulated in the Lorentz-Abraham-Dirac form [3]

$$g^i = \frac{2}{3}\frac{e^2}{c}\left(\frac{d^2u^i}{ds^2} - u^i u^k \frac{d^2u_k}{ds^2}\right). \tag{3}$$

However, we follow Landau and Lifshitz (LL) and assume that g^i is small in the rest frame of the particle. This allows us to express the derivatives in Eq. (3) in terms of the derivatives of the external fields [using Eq. (2) with $\frac{dg^i}{ds} \approx 0$],

$$g^i = \frac{2}{3}\frac{e^2}{c}\left[-\frac{e}{mc^2}\frac{\partial F^{ik}}{\partial x^l}u_k u^l - \frac{e^2}{m^2c^4}F^{ik}F_{lk}u^l + \frac{e^2}{m^2c^4}(F_{lk}u^k)(F^{lm}u_m)u^i\right], \tag{4}$$

where we used the antisymmetric nature of the field tensor ($F^{ik} = -F^{ki}$). Expression (4) is given in [4] and further discussed in [6]. Considering the various terms of Eq. (4), it is physically consistent to neglect the first term. The argument is that a particle with a spin degree of freedom is also subject to a force in an external field and that this force (the Frenkel force), in the case of a plane wave, is much larger than the first term in Eq. (4). Since we shall work with a classical equation of motion neglecting the spin, it is consistent to drop the first term, leading to

$$g^i \approx -\frac{2}{3}\frac{e^4}{m^2c^5}F^{ik}F_{lk}u^l + \frac{2}{3}\frac{e^4}{m^2c^5}(F_{lk}u^k)(F^{lm}u_m)u^i. \tag{5}$$

The equation of motion becomes

$$mc\frac{du^i}{ds} = -\frac{e}{c}F^{ik}u_k - \frac{2}{3}\frac{e^4}{m^2c^5}F^{ik}F_{lk}u^l + \frac{2}{3}\frac{e^4}{m^2c^5}(F_{lk}u^k)(F^{lm}u_m)u^i. \tag{6}$$

Next we normalize time with inverse wave-frequency, i.e., $t\omega \to t$, velocity components of $\mathbf{v}$ by c, components of momentum $\mathbf{p}$ by mc, and change $\frac{eF_{ik}}{\omega mc} \to F_{ik}$. In terms of the fields $\mathbf{E}$ and $\mathbf{H}\hat{=}\mathbf{B}$ that means that the dimensionless forms of the electric field $\frac{e\mathbf{E}}{mc\omega} \to \mathbf{E}$ and the magnetic field $\frac{e\mathbf{H}}{mc\omega} \to \mathbf{H}$, corresponding to a dimensionless vector potential $\frac{e\mathbf{A}}{mc^2} \to \mathbf{A}$, appear. In dimensionless form, the equation of motion becomes

$$\frac{d\mathbf{p}}{dt} = -(\mathbf{E}+\mathbf{v}\times\mathbf{H}) + \mathcal{F}\{[\mathbf{E}(\mathbf{v}\cdot\mathbf{E}) + (\mathbf{E}\times\mathbf{H}) + \mathbf{H}\times(\mathbf{H}\times\mathbf{v})] - \gamma^2\mathbf{v}[(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2]\}. \tag{7}$$

The (dimensionless) prefactor $\mathcal{F}$ is

$$\mathcal{F} = \frac{2}{3}\frac{e^2\omega}{mc^3} \approx 1.474 \times 10^{-8} \tag{8}$$

for a $\lambda = 800$ nm wavelength ($\omega = 2\pi c/\lambda$).

III. NO-ENERGY-GAIN THEOREM IN THE ABSENCE OF RADIATION REACTION

In the absence of RR, the solution of the (dimensionless) equation of motion

$$\frac{d\mathbf{p}}{dt} = -(\mathbf{E}+\mathbf{v}\times\mathbf{H}) \tag{9}$$

in an infinitely long plane electromagnetic wave is known for both circular and linear polarization. Let us summarize the result for linear polarization $\mathbf{A}_\perp(x,t) = A_y(x-t)\hat{e}_y = A_0\cos(x-t)\hat{e}_y$. In the eigentime $\mathcal{T}$, which can be determined from time t via the implicit equation $\mathcal{T} = t - x[t(\mathcal{T})]$ where $x[t]$ is the x position of the electron at time t, we find $y(\mathcal{T}) = \sin(\mathcal{T})$ and $x(\mathcal{T}) \approx x_d + \frac{1}{2}\frac{A_0^2}{4+A_0^2}\sin(2\mathcal{T})$. Because of

$x_d = x_d(t) = \frac{A_0^2}{4+A_0^2}t$, the electron obtains a time-independent drift velocity,

$$v_x^{\rm drift} = \frac{A_0^2}{4+A_0^2}, \tag{10}$$

within an infinitely long plane electromagnetic wave. Superimposed is a figure-eight motion described by $x(\mathcal{T}) - x_d$ and $y(\mathcal{T})$ in the comoving x,y plane. Summarizing, the electron drifts in the positive x direction (when the electromagnetic wave propagates in positive x direction) and shows a figure-eight motion in the drift frame. However, that solution does not imply that an electron will be left behind a finite-length electromagnetic wave with a finite velocity, e.g., the drift velocity.

To determine the electron velocity after having experienced the action of a finite-length electromagnetic pulse, we start from the (dimensionless) Lagrangian

$$L = -\sqrt{1-v^2} - \mathbf{v}\cdot\mathbf{A}, \tag{11}$$

which leads to the generalized momentum $\mathbf{P} = \mathbf{p} - \mathbf{A}$. When a plane electromagnetic pulse does not depend on the perpendicular direction $\mathbf{r}_\perp = (0,y,z)$, the Lagrange equations lead to the constant of motion

$$\mathbf{P}_\perp = \mathbf{p}_\perp - \mathbf{A}_\perp = 0, \tag{12}$$

where its value has been set to zero for an electron initially (before the pulse arrives) not performing transverse oscillations.

Next, the form $\mathbf{A} = \mathbf{A}(t-x)$ implies for the Lagrangian the form $L = L(x,\mathbf{v},t)$ and subsequently for the Hamiltonian $H = H(x,\mathbf{P},t) \equiv E(t)$. From

$$\frac{dH}{dt} = \frac{dE}{dt} = -\frac{\partial L}{\partial t} = \frac{\partial L}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial v_x} = \frac{dP_x}{dt} = \frac{dp_x}{dt}, \tag{13}$$

we find the constant of motion

$$E - p_x = \gamma_0 - p_{x0}, \quad \text{with} \quad \gamma_0 = \sqrt{1+p_{x0}^2}. \tag{14}$$

Here, $E = \gamma \equiv \sqrt{1+\mathbf{p}^2}$ (in nondimensional form) and p_{x0} is the initial momentum of the electron in propagation direction x of the pulse. We have $p_{x0} = 0$ when the electron is initially at rest.

From Eqs. (12) and (14), we find

$$\gamma\gamma_0 = 1 + \frac{\mathbf{A}_\perp^2}{2} + p_x p_{x0}, \quad \gamma = \sqrt{1+p_x^2+\mathbf{A}_\perp^2}. \tag{15}$$

Looking for solutions $p_x = p_x(\mathbf{A}_\perp = 0)$ outside the pulse, i.e., initially as well as for times when the pulse has passed the electron such that again $\mathbf{A}_\perp = 0$ at the position of the electron, a short calculation leads to

$$[p_x(\mathbf{A}_\perp = 0) - p_{x0}]^2 = 0, \tag{16}$$

i.e., after the pulse has passed the electron the latter has the same longitudinal as well as transverse momentum as initially (before the pulse arrived). An electron does not get kinetic energy when being overrun by a finite (transverse) laser pulse. This "no-energy-gain theorem" can be abbreviated as $\gamma = \gamma_0$ for the electron before and after the pulse. We have solved numerically the equation of motion for an electron under the action of an electromagnetic pulse. Without the RR force, the broken line in Fig. 1 demonstrates the expected "no energy-gain behavior" for linear polarization. An electron with initial momentum $p_{x0} = -100$ is overrun by a linearly polarized pulse,

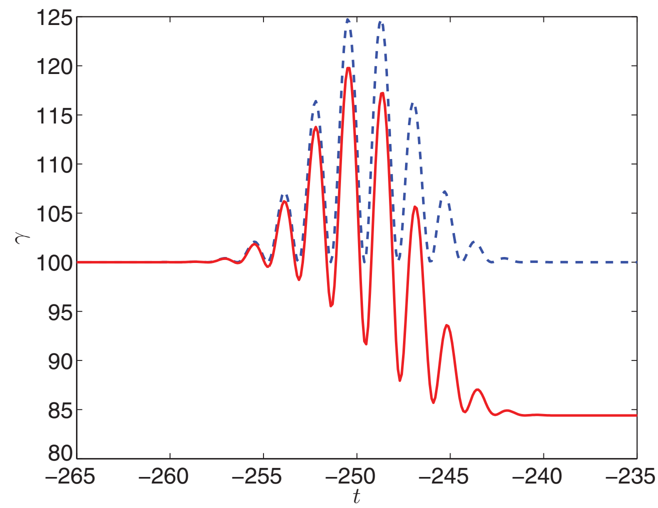

FIG. 1. (Color online) Broken (blue) line shows the normalized energy of an electron with initial momentum $p_{x0} = -100$ when being overrun by a linearly polarized, transverse pulse of the form (17) for $E_{\max} = 100$ and $\tau = 10$ when no RR is present. The solid (red) line shows the same scenario in the presence of RR.

$$E_y = E_{\max}\exp\left\{-\frac{(x-t)^2}{\tau^2}\right\}\cos(x-t), \tag{17}$$

propagating in positive x direction with maximum amplitude $E_{\max} = 100$ and width $\tau = 10$. The energy of the electron falls back to $\gamma = \gamma_0$ after the pulse has passed. Note that, for large widths τ, inside the pulse the electron obtains an additional drift velocity $v_x^{\rm drift} = \frac{E_{\max}^2}{4+E_{\max}^2}$, causing a corresponding shift in position. However, no velocity change is observed after the pulse has passed. The same occurs for circular polarization in the absence of RR as shown by the broken line in Fig. 2. After being overrun by a circularly polarized pulse of the form

$$\vec{E} = \frac{1}{\sqrt{2}}(\hat{e}_y + i\hat{e}_z)E_{\max}\exp\left\{-\frac{(x-t)^2}{\tau^2}\right\}\cos(x-t), \tag{18}$$

the electron has not gained any additional kinetic energy in the absence of RR. Comparing the broken lines shown in Figs. 1 and 2, we recognize the typically different dynamical behaviors of the electron while interacting with a linearly or circularly polarized pulse, respectively.

IV. ENERGY GAIN IN THE PRESENCE OF RADIATION REACTION

Now we turn to the numerical simulations of the full equation of motion (7) (including the RR force). First, Fig. 1 shows, for linear polarization, the typical effect of the RR force. Compared to the broken (blue) line, which depicts the time dependence of the relativistic γ factor when no RR force is present, the solid (red) line shows the same scenario

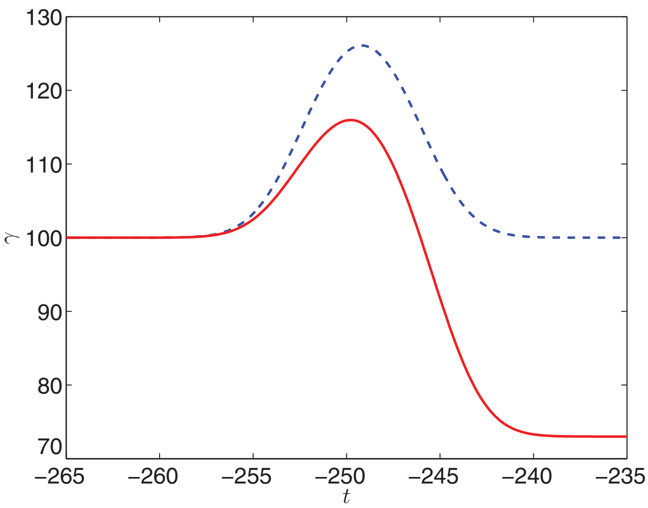

FIG. 2. (Color online) Same as Fig. 1, but for circular polarization.

in the presence of the RR force. The electron has gained momentum $\Delta p_x = p_x - p_{x0}$ in the positive x direction due to the interaction with the pulse. Note that, in this example, the initial momentum component $p_{x0} = -100$ was negative, such that a momentum increase in positive direction reduces the γ factor. Comparing pulses with the same intensity, in the case of circular polarization, the same momentum gain occurs as for linear polarization. This can be recognized from a comparison of the solid lines in Figs. 1 and 2, respectively. Independence of polarization is a generic behavior for the energy gain in the presence of RR. Therefore, in the following we present only results for one type of polarization, i.e., linear polarization.

Next we show results for the velocity change of an electron in dependence of the parameters pulse width τ and maximum amplitude $E_{\max}$ of the linearly polarized pulse (17). The wavelength λ is 800 nm. Because of linear polarization, the magnetic field has only a z component with $H_z = E_y$. We measured the velocity increase Δv_x, which the electron gained after the pulse passed. Because of space limitations, we only present cases with electrons being initially at rest and for linearly polarized laser pulses. However, similar results occur for other initial conditions as well as for circular polarization.

We found the generic result $\Delta v_x \equiv \gamma^{-1}\Delta p_x \sim E_{\max}^2 \tau$. The electron always gets a push in the direction of the laser pulse propagation. Its strength is proportional to the pulse length and the square of the maximum pulse amplitude. To better understand the behavior, we split the discussion into two parts.

A. Weakly relativistic case

First, when $E_{\max}$ is small, the electron will not gain large velocities, and also the velocity increases will be small. The situation is only weakly relativistic over the whole time. Typical results are shown in Figs. 3 and 4.

The velocity increase is always in the positive x direction. We clearly recognize a linear dependence on the pulse width τ and a quadratic increase with the pulse maximum $E_{\max}$.

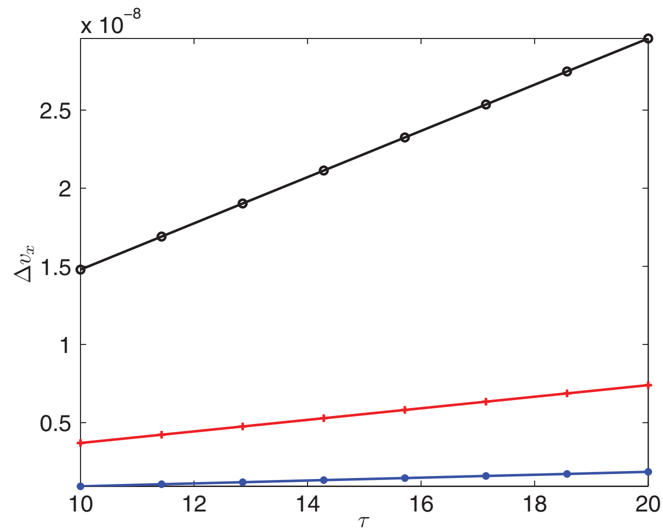

FIG. 3. (Color online) Results for the velocity increase Δv_x (in unit c) for an electron which was initially at rest, after the pulse (17) has passed, as a function of pulse width τ (in unit ω^{-1}). Solutions are from Eq. (7). Three small values have been chosen for $E_{\max}$ (in unit $mc\omega/e$): 0.4 (black, circles), 0.2 (red, crosses), and 0.1 (blue, diamonds), respectively.

To better understand this generic behavior, we perform a perturbation analysis of Eq.(7) in the weakly relativistic limit. For small velocities, the RR force may be approximated by

$$\mathbf{f}_{\mathrm{RR}} = \mathcal{F}\{[\mathbf{E}(\mathbf{v}\cdot\mathbf{E}) + (\mathbf{E}\times\mathbf{H}) + \mathbf{H}\times(\mathbf{H}\times\mathbf{v})] - \gamma^2\mathbf{v}\big[(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2\big]\} \approx \mathbf{f}_{\mathrm{RR}}^{\mathrm{weak}}, \quad (19)$$

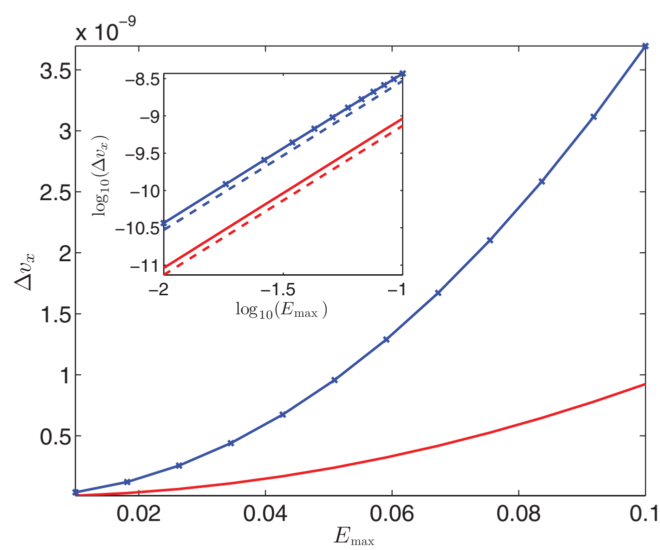

FIG. 4. (Color online) Results for the velocity increase Δv_x (in unit c) for an electron which was initially at rest, after the pulse (17) has passed, as a function of pulse maximum $E_{\max}$ (in unit $mc\omega/e$). Solutions are from Eq. (7). Two values have been chosen for τ (in unit ω^{-1}): 40 (blue, solid line with squares) and 10 (red, solid line), respectively. The inlet shows a logarithmic plot, including a comparison with the theoretical estimate depicted by the dashed lines.

where

$$\mathbf{f}_{\rm RR}^{\rm weak} = \mathcal{F}(\mathbf{E}\times\mathbf{H}). \tag{20}$$

For the discussion of the weakly relativistic case, we thus may start from

$$\frac{d\mathbf{p}}{dt} \approx -(\mathbf{E}+\mathbf{v}\times\mathbf{H}) + \mathbf{f}_{\rm RR}^{\rm weak}\,, \tag{21}$$

instead of the full equation (7). Note that $\mathbf{E}\times\mathbf{H}$ is proportional to the Poynting vector $\mathbf{S} = c\frac{H^2}{4\pi}\hat{\mathbf{n}}$ (in dimensional form). Thus the RR pushes the electron in the propagation direction of the pulse, i.e., x direction. This explains why we always get a positive velocity increase after the pulse has passed the electron. To estimate the order of magnitude, we write for the (time) average of the x component of the RR force

$$\left\langle \mathbf{f}_{\rm RR}^{\rm weak}\right\rangle_x \approx \mathcal{F}\langle E_y H_z\rangle \approx \tfrac{1}{2}\mathcal{F}E_{\max}^2. \tag{22}$$

This force acts on the electron approximately during the pulse duration, leading to a velocity increase of the order

$$\Delta v_x \approx \tfrac{1}{2}\mathcal{F}E_{\max}^2\tau \approx 7.4\times 10^{-9}E_{\max}^2\tau \quad \text{for} \quad \lambda = 800\ \text{nm}. \tag{23}$$

This formula is expressed in the same nondimensional quantities as being used in Figs. 3 and 4. It is straightforward to check that the agreement between the analytical estimate and the numerical result is very good.

B. Ultrarelativistic case

When the maximum amplitude $E_{\max}$ of the pulse is large, an electron can reach large velocities. In case the electron reaches large velocities very close to c, the γ factor becomes very large, and the RR force can be approximated by the terms containing the largest powers in the velocities and field components, i.e.,

$$\begin{aligned}\mathbf{f}_{\rm RR} &= \mathcal{F}\{[\mathbf{E}(\mathbf{v}\cdot\mathbf{E}) + (\mathbf{E}\times\mathbf{H}) + \mathbf{H}\times(\mathbf{H}\times\mathbf{v})]\\ &\quad -\gamma^2\mathbf{v}[(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2]\} \approx \mathbf{f}_{\rm RR}^{\rm ultra},\end{aligned} \tag{24}$$

with

$$\mathbf{f}_{\rm RR}^{\rm ultra} = -\mathcal{F}\gamma^2\mathbf{v}[(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2]. \tag{25}$$

Landau and Lifshitz [4] mentioned already that this part of the force is in opposite direction to the velocity $\mathbf{v}$. Let us present a simple argument for that behavior.

Since the RR force should be small compared to the Lorentz force, we may approximate in the first term on the right-hand side of Eq. (25)

$$\mathbf{E}+\mathbf{v}\times\mathbf{H} \approx -\frac{d\mathbf{p}}{dt}. \tag{26}$$

In addition, the second term on the right-hand side of Eq. (25) is related to the change of kinetic energy,

$$\mathbf{E}\cdot\mathbf{v} = \frac{dE_{\rm kin}}{dt} = \frac{d\gamma}{dt} = \frac{1}{\gamma}\mathbf{p}\cdot\frac{d\mathbf{p}}{dt}. \tag{27}$$

Thus, when evaluating the right-hand side of Eq. (25), we find

$$(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2 \approx \left(\frac{d\mathbf{p}}{dt}\right)^2 - \frac{1}{\gamma^2}\left(\mathbf{p}\cdot\frac{d\mathbf{p}}{dt}\right)^2. \tag{28}$$

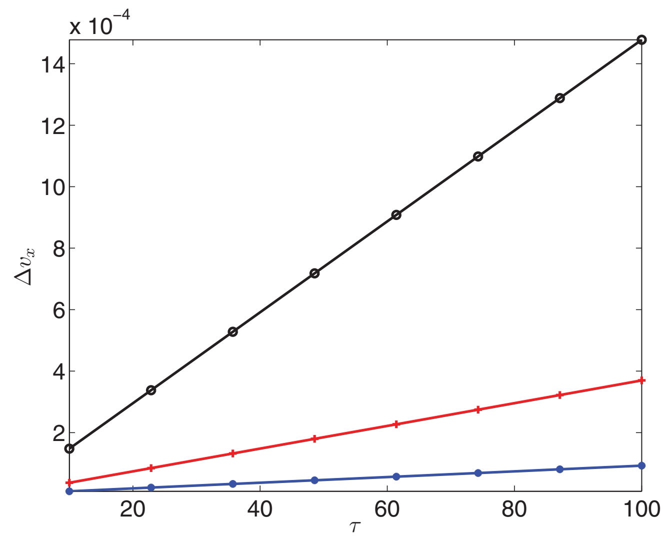

FIG. 5. (Color online) Same as Fig. 3, but now for large values of $E_{\max}$: 40 (black, circles), 20 (red, crosses), and 10 (blue, diamonds), respectively.

Making use of Schwarz inequality,

$$\left(\mathbf{p}\cdot\frac{d\mathbf{p}}{dt}\right)^2 \leqslant \mathbf{p}^2\left(\frac{d\mathbf{p}}{dt}\right)^2, \tag{29}$$

we may estimate

$$\begin{aligned}(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2 &\geqslant \left[1 - \frac{\mathbf{p}^2}{1+\mathbf{p}^2}\right]\left(\frac{d\mathbf{p}}{dt}\right)^2\\ &= \frac{1}{1+\mathbf{p}^2}\left(\frac{d\mathbf{p}}{dt}\right)^2 \geqslant 0\,.\end{aligned} \tag{30}$$

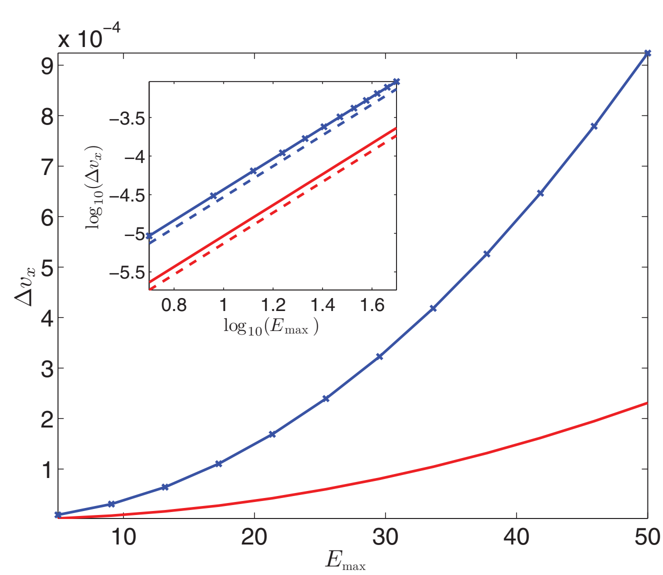

FIG. 6. (Color online) Same as Fig. 4, but now for larger values of $E_{\max}$.

This leads to the already mentioned behavior that the ultrarelativistic RR force is directed opposite to the velocity of the particle,

$$\mathbf{f}_{\mathrm{RR}}^{\mathrm{ultra}} \sim -\mathcal{F}\gamma^2 \mathbf{v} \underbrace{[\cdots]}_{\geqslant 0}. \tag{31}$$

That prediction was first checked with our code in case of fast particle motion in simple (constant) electromagnetic fields including the RR force. In the ultrarelativistic velocity regime, the approximation (24) is very good, and the direction (31) was confirmed.

Now it becomes interesting to see what happens when we turn to the motion of an electron in the electromagnetic pulse at large pulse amplitudes such that the initially resting electron can reach large velocities. Results are shown in Figs. 5 and 6. We get similar behaviors to those reported in the weakly relativistic case. The velocity change is also positive (in pulse propagation direction) and proportional to the pulse duration as well as the square of the maximum amplitude. The results clearly show that the estimate (23) also applies in the so-called ultrarelativistic regime (when $E_{\max}$ is large and the electron velocity can go up to ultrarelativistic velocities).

The velocity increase in the direction of the laser pulse propagation is not in contradiction to the estimate (31). During the whole interaction phase of the electron with the electromagnetic fields of the pulse, the quiver velocity is not always ultrarelativistic even if $E_{\max}$ is large. Furthermore, the common scaling of the velocity increment with the square of the mximum amplitude in the whole energy range can be understood as follows. Let us go back to the full form of the RR as written on the right-hand side of Eq. (7). In the frame of reference comoving with the electron (where $\vec{v} = 0$ and of course the Lorentz-transformed fields appear), the averaged, velocity-independent term $\mathcal{F}(\mathbf{E} \times \mathbf{H})$ leads to the velocity push. The result is Eq. (23), which therefore also applies in the ultrarelativistic region.

V. SUMMARY

In the present paper, we have investigated the interaction of an electron with a plane transverse laser pulse of finite duration. When the laser intensity is large (e.g., of order 5×10^{22} W/cm^2 or larger), the RR force has to be taken into account. A well-known theorem states that an electron cannot gain energy when being overrun by a plane (transverse) laser pulse of finite length. The no-energy-gain theorem results from symmetries which are broken when RR is included. It is shown here that an electron, e.g., being initially at rest, will gain a positive velocity after being overrun by an intense laser pulse (with intensity of order 5×10^{22} W/cm^2 or larger) due to RR effects. Both linear as well as circular polarization of the laser pulse have been considered. It was demonstrated that the velocity gain is proportional to the pulse length and the square of the peak amplitude of the laser pulse. The results of numerical simulations were supported by analytical estimates.

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