Resonant acceleration of charged particles in the presence of random fluctuations

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We present a quantitative theory of the nonlinear dynamics and surfatron resonant acceleration of charged particles in the presence of random fluctuations of magnetic field. We demonstrate that the surfatron mechanism of acceleration is sufficiently stable versus the influence of fluctuations and particle accelerate even in the presence of a random noise. We estimate the maximum energy which particles could gain in the course of the surfatron acceleration.

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I. INTRODUCTION

Transport and acceleration of charged particles in turbulent electromagnetic fields is one of the challenging problems of plasma physics. In collisionless plasma (e.g., interplanetary plasma, interstellar medium, regions of planetary magnetospheres) neither the redistribution of energy between various particle populations nor the energy dissipation (transformation of magnetic field energy into kinetic energy of particles) can be achieved by particle collisions. One of the alternative mechanisms is the interaction of charged particles with the electromagnetic turbulence (EMT) appearing due to the growth and eventual saturation of plasma instabilities.

Various physical conditions in different systems lead to quite diverse models of the acceleration of charged particles by EMT. Fluid models are used to describe the acceleration of particles by large-scale turbulence in the Solar corona [1,2]. For many systems, the turbulence can be approximated as an ensemble of localized electromagnetic structures (so-called "magnetic clouds"). The interaction of particles with such clouds was first discussed in Ref. [3], and similar models are still in use (see, e.g., Ref. [4]). An ensemble of plane electromagnetic waves is one of the most straightforward models of EMT [5–8]. It was shown in Refs. [6] and [8] that in such a model the acceleration of particles can be almost free, with the average energy of the particle ensemble growing as $\sim t^2$. The presence of this regime indicates that at the resonance particles interact not with the waves' ensemble as a whole, but rather with a particular wave. The free acceleration of a particle by a single electromagnetic (or electrostatic) wave can be explained by the so-called "surfatron" mechanism (see, e.g., Refs. [9–12]). However, the presence of other harmonic(s) in the turbulent spectrum puts the upper bound on the time a particle spends in the resonance with a particular wave (see Ref. [13]).

The surfatron mechanism is traditionally used to describe the particle acceleration by shock waves [14–16]. In this mechanism particles can accelerate along the nonlinear wave front in the presence of a background magnetic field in the Solar corona [17], in the interplanetary medium and in planetary magnetospheres (see Refs. [18,19] and references therein), and in a magnetospheres of stars [20]. Such a mechanism also is realized in the vicinity of the X line appearing during magnetic reconnection [21]. However, the stability of surfatron acceleration in the presence of fluctuations of the magnetic field near shock-wave fronts and in the vicinity of the X line remained an open question. In the current paper we demonstrate that fluctuations of the magnetic field in a relatively broad range of frequencies cannot completely destroy the surfatron resonance.

II. MAIN EQUATIONS

We study the resonant interaction of a charged particle with a single electromagnetic wave in the presence of surrounding random magnetic fluctuations that model the rest of EMT. We consider the following geometry of the system (as in Ref. [12]): The background magnetic field B_0 and the random magnetic field $B_\Gamma(t)$ are directed along the $\hat{z}$ axis, the plane electromagnetic wave (with frequency $\widehat{\omega}$ and wave vector $\widehat{k}$) propagates along the $\hat{y}$ axis, and particles move in the $(\hat{x},\hat{y})$ plane (a schematic view is presented in Fig. 1). To make the calculations more straightforward, we consider only fluctuations that are parallel to B_0. The more general case of three-dimensional (3D) fluctuations will be considered in subsequent publication(s).

The Equations of motion of a nonrelativistic particle with charge q and mass m can be written as

$$\begin{aligned}\dot{\widehat{v}}_x &= (q/mc)[\widehat{v}_\phi B_w \sin\phi + \widehat{v}_y(B_0 - B_w\sin\phi + B_\Gamma)],\\ \dot{\widehat{v}}_y &= -(q/mc)\widehat{v}_x(B_0 - B_w\sin\phi + B_\Gamma),\\ \dot{\widehat{x}} &= \widehat{v}_x,\ \dot{\widehat{y}} = \widehat{v}_y.\end{aligned}\tag{1}$$

In (1), $\widehat{\mathbf{v}}$ is the particle velocity, $\phi = \widehat{k}\widehat{y} - \widehat{\omega}\widehat{t}$, B_w is the amplitude of the wave, $\widehat{v}_\phi = \widehat{\omega}/\widehat{k}$, and c is speed of light. Introduce dimensionless quantities $t = \widehat{t}\omega_0$, $\mathbf{v} = \widehat{\mathbf{v}}/\widehat{v}_0$, $v_\phi = \widehat{v}_\phi/\widehat{v}_0$, $k = \widehat{k}\widehat{v}_0/\omega_0$, $\omega = \widehat{\omega}/\omega_0$, $\mathbf{r} = \widehat{\mathbf{r}}\omega_0/\widehat{v}_0$, $\beta = B_w/B_0$, and $\Gamma(t) = B_\Gamma(t)/B_0$. Here $\widehat{v}_0$ is a typical initial particle velocity, and $\omega_0 = qB_0/mc$ is the cyclotron frequency. Throughout the text we use $\beta = 2\pi$. With the different variables (1) takes the form

$$\begin{aligned}\dot{v}_x &= v_\phi\beta\sin\phi + v_y(1+\Gamma(t) - \beta\sin\phi),\\ \dot{v}_y &= -v_x(1+\Gamma(t)-\beta\sin\phi).\end{aligned}\tag{2}$$

We assume that $\Gamma(t)$ is a random process with zero mean (a nonzero mean can be included in B_0). $\Gamma(t)$ is constant during the fixed time interval $\widehat{\tau} = \tau/\omega_0$ and when $t = t_i = \tau i$

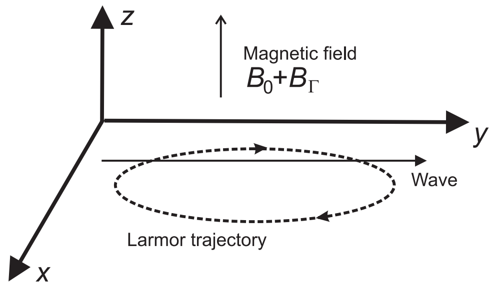

FIG. 1. Scheme of particle motion.

($i = 1,2,3,\ldots$) the value of $\Gamma(t)$ changes randomly according to its probability distribution.

In the present paper we assume that $k \gg 1$ and the high-frequency EMT corresponds to $\tau \ll 1$. This means that during one cyclotron gyration the particle experiences many $\Gamma(t)$ jumps. The averaging of (2) over the fast-oscillating random field corresponds to omitting the terms with $\Gamma(t)$. In such a system the majority of particles move around the Larmor circles (the dashed line in Fig. 1). It was shown in Refs. [9–13] that the most interesting phenomena are associated with the resonance wave-particle interaction, occurring when the y component of the particle velocity matches the phase speed of the wave $v_y \approx v_\phi$. Over one period of Larmor rotation, most of the particles pass through resonance with just small changes in energy (the process called scattering on resonance [12,13]). However, some particles can be captured into resonance with the wave (over a sufficiently long time almost all the particles would be captured, see, e.g., Ref. [12] and references therein). When a particle is captured, it ceases moving along a Larmor circle and starts moving together with the wave, while accelerating along its front. In the absence of the random field $\Gamma(t)$, the equations of motion in the captured state are

$$\dot{v}_x = v_\phi, \quad \ddot{\phi} = -kv_x(1-\beta\sin\phi). \tag{3}$$

As $k \gg 1$, ϕ changes much faster than v_x. The Hamiltonian for the second equation in (3) is

$$H_\phi = \dot{\phi}^2/2 + kv_x(\phi + \beta\cos\phi).$$

A schematic view of the phase portrait of this system (for a fixed value of $v_x > 0$) is presented in Fig. 2. A characteristic frequency of ϕ oscillations is $\Omega = (v_x\beta k)^{1/2}$ [12].

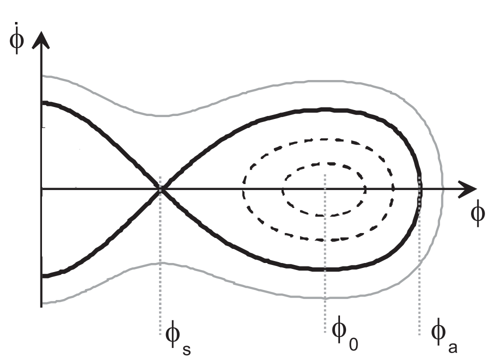

FIG. 2. A schematic phase portrait of the system without fluctuations for $\beta > 1$. The gray curve is a "scattered" trajectory, the dashed curves are captured orbits, and the solid black curve is a separatrix.

The captured motion has an adiabatic invariant $I = \oint \dot{\phi}\, d\phi$, which is the area under the unperturbed trajectory (the integral is taken along the route passing though a current location of the particle). The once-captured particle stays captured as long as the value of I is smaller than the area S under the separatrix on the resonant phase plane. The value of S is

$$S = \oint_{\text{sep}} \dot{\phi}\, d\phi = 2^{3/2}\sqrt{k|v_x|} \times \int_{\phi_s}^{\phi_a} \sqrt{|(\phi_s - \phi) + \beta(\cos\phi_s - \cos\phi)|}\, d\phi, \tag{4}$$

where ϕ_s is the value of ϕ at the hyperbolic point and ϕ_a is shown in Fig. 2. As the particle accelerates in the resonance $v_x \sim v_\phi t$ [see (3)], the value of S grows. Together with the conservation of I (which is the case in the absence of fluctuations of the magnetic field) the growth of S ensures the permanent capture (see Ref. [12]).

III. INFLUENCE OF RANDOM FLUCTUATIONS OF MAGNETIC FIELD

The particle motion is qualitatively different for different values of $\Omega\tau$. We consider the case $\Omega\tau \ll 1$. The presence of fast oscillations of the magnetic field does not significantly change the dynamics of particles during the Larmor rotation, but, as we show below, it is important for resonance acceleration. Equations (3) become

$$\begin{aligned}\dot{v}_x &= v_\phi(1+\Gamma(t)),\\ \ddot{\phi} &= -kv_x(1-\beta\sin\phi) - kv_x\Gamma(t).\end{aligned} \tag{5}$$

Due to the presence of random fluctuations, the value of I changes. As $\Omega = 2\pi dH_\phi/dI$ (see Ref. [22]), we have $\Delta I \approx 2\pi\Delta H_\phi/\Omega$ for small ΔI. Approximating the action of $\Gamma(t)$ as a series of pulses with impulses $\Gamma(t)\tau$ and uniform periodicity τ, we obtain the change $(\Delta I)_i$ during the ith pulse:

$$(\Delta I)_i \approx \frac{2\pi}{\Omega}\dot{\phi}_i(\Delta\dot{\phi})_i = \frac{2\pi}{\Omega}\dot{\phi}_i k v_x \Gamma(t_i)\tau. \tag{6}$$

A typical dynamics of a single particle is presented in Fig. 3. The particle is captured at the rightmost point of the small circle in the center of Fig. 3(a). At the moment of capture $I = S$. Then S starts growing as $\sim(v_\phi\beta kt)^{1/2}$ and I changes according to (6). If τ is sufficiently small, at the initial stage S grows faster than I and the particle "falls" toward the bottom of the potential well during the first few rotations on the $(\phi,\dot{\phi})$ plane. While the particle is deep inside the well, its dynamics is quite similar to the dynamics without $\Gamma(t)$ discussed above. The particle accelerates (the horizontal strip, whose width grows as the particle accelerates, as shown in Fig. 3(a); see also Ref. [12]) and Ω grows. In Fig. 3(b), the dashed line is $\Omega = (v_\phi\beta kt)^{1/2}$ and the solid line is the numerical value, obtained as the inverse of twice the time between two consecutive crossings of the $\dot{\phi} = 0$ line (times 2π). The deviations of the numerical value from the theoretical line are due to the (weak) dependence of Ω on $I(t)$, in other words, on the position of the particle inside the separatrix loop. If the particle comes closer to the separatrix, the ratio $I(t)/S(t)$ increases and $\Omega(I)$ decreases, which is most pronounced near $t \approx 68$ and at the very right, where the particle comes close to the separatrix and

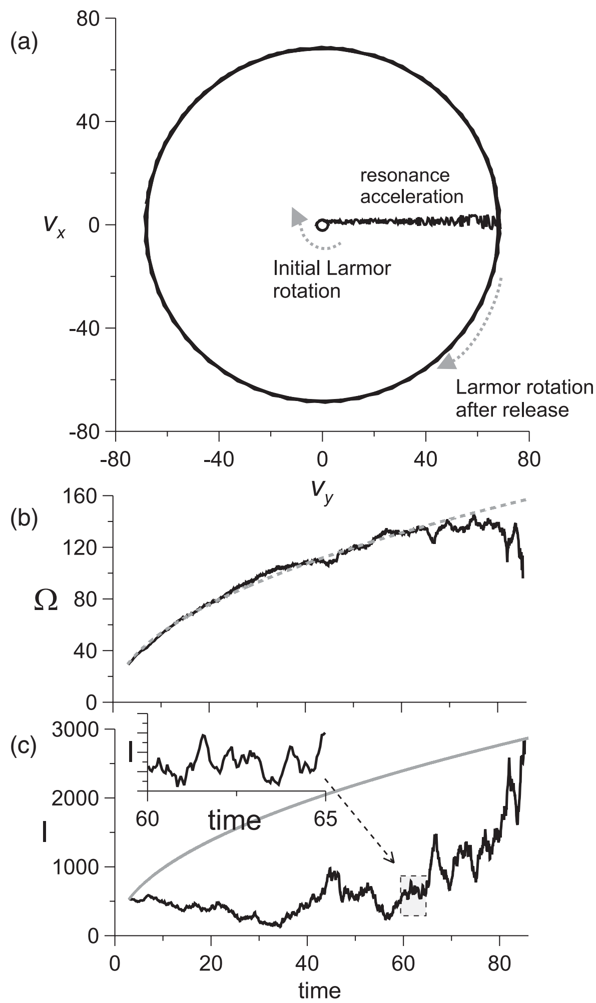

FIG. 3. Capture and release in the presence of fluctuations; $k = 100$, $\mathrm{Var}[\Gamma_i(t)] = 1$, $\tau = 10^{-3}$. Right after the capture $\Omega\tau \approx 3\times 10^{-2}$ and just before the release $\Omega\tau \approx 10^{-1}$.

eventually crosses it. However, the dependence of Ω on $I(t)$ is indeed quite weak: Deviations of numerical $\Omega(t)$ from the theoretical curve are much smaller than variations of $I(t)$.

The evolution of I can be viewed as random walk (6) with time-dependent statistics of the steps—see Fig. 3(c). The inset shows the magnification of a typical interval containing several periods $\sim 2\pi/\Omega$. If in the process of the random walk the value of I exceeds the current value of S, the particle is released from resonance and starts moving along a larger Larmor circle.

We describe the behavior of a ensemble of particles in terms of the probability distribution function $\Psi(I,t)$: the probability for a particle to have the value of adiabatic invariant in the interval $(I-\delta I/2, I+\delta I/2)$ at the time t is given by $\Psi(I,t)\delta I$. We get

$$\frac{\partial\Psi}{\partial K} = \tau\frac{\partial\Psi}{\partial t} = \frac{\partial}{\partial I}\left(D(I,t)\frac{\partial\Psi}{\partial I}\right), \qquad (7)$$

where K is the number of realizations of the random field $\Gamma(t)$. The diffusion coefficient $D(I,t)$ is given by the variance of the right-hand side in (6):

$$D(I,t) = \left(\frac{2\pi}{\Omega}kv_x\tau\right)^2 \mathrm{Var}[\dot{\phi}_i\Gamma_i(t)]. \qquad (8)$$

Writing (8) we took into account that random walk (6) has two distinct time scales. The first one is the period of a single rotation in resonance ($\sim 2\pi/\Omega$). The other is the time at which the resonance orbit evolves. The results of numerical simulations presented in Fig. 3 indicate that a typical particle makes many turns before being released from the capture, and for a one period of captured motion the values of Ω and v_x can be considered to be constant. Assuming that $\dot{\phi}_i$ and $\Gamma_i(t)$ are uncorrelated and have zero mean, we get

$$D(I,t) = (2\pi kv_x\tau/\Omega)^2\,\mathrm{Var}(\dot{\phi}_i)\mathrm{Var}[\Gamma_i(t)].$$

For $\mathrm{Var}(\dot{\phi}_i)$ we have

$$\mathrm{Var}(\dot{\phi}_i) \approx \frac{\Omega}{2\pi}\oint \dot{\phi}_i^2\,dt = \frac{\Omega}{2\pi}I.$$

Therefore,

$$D(I,t) = \left(\frac{2\pi}{\Omega}kv_x\tau\right)^2 \frac{\Omega}{2\pi}I\Upsilon^2,$$

where we denoted $\Upsilon^2 = \mathrm{Var}[\Gamma_i(t)]$. As it was noted above, we can neglect the dependence of Ω on I (except in the immediate vicinity of the separatrix) and assume $\Omega \sim (v_x\beta k)^{1/2}$. As $v_x \sim v_\phi t$, we get

$$\frac{\partial\Psi}{\partial t} = D_0 t^{3/2}\frac{\partial}{\partial I}\left(I\frac{\partial\Psi}{\partial I}\right), \qquad (9)$$

where $D_0 = 2\pi\tau\Upsilon^2 k^{3/2}v_\phi^{3/2}/\beta^{1/2}$. Introducing a unique time $t' = (2/5)t^{5/2}$, we can reduce (9) to a standard diffusion equation

$$\frac{\partial\Psi}{\partial t'} = D_0\frac{\partial}{\partial I}\left(I\frac{\partial\Psi}{\partial I}\right). \qquad (10)$$

The long-time dynamics of an ensemble of particles can be described as follows: The initial distribution $\Psi(I,0)$ is a δ function at a given value of $I = I_0$. The value of S at that moment is $S_0 = I_0$. After that the value of S starts growing as $S \sim \sqrt{k\beta v_\phi t} \sim (t')^{1/5}$. Meanwhile the evolution of I can be described as a set of walks (6) and $\Psi(I,t)$ starts drifting and spreading according to (10). One obtains from (10) that on a given trajectory the expected value of I grows as $\sim D_0 t' \sim D_0 t^{5/2}$. In the asymptotical regime (when $t \gg 1$) S grows slower than the expected value of I. At a certain moment $t = t_*$ [defined as $I(t_*) = S(t_*)$] the particle is released from resonance. At that moment $\sqrt{k\beta v_\phi t_*} \sim D_0 t_*^{5/2}$, and the velocity of a particle at the moment of release is

$$v_{\max} = v_\phi t_* \sim \Upsilon^{-1}\sqrt{\beta v_\phi/\tau k}. \qquad (11)$$

IV. NUMERICAL SIMULATIONS

We performed a set of numerical simulations for different values of k, τ, and Υ. For each set of parameters, we computed the average value of the maximum velocities achievable in the process of capture by integrating (2) for an ensemble of 10^3 particles all starting with the initial energy $\varepsilon_0 = (v_x^2 + v_y^2)/2 = 2$ [for each particle, the value of the initial angle $\arctan(v_y/v_x)$ was chosen randomly]. For each trajectory we used an individual realization of $\Gamma(t)$. We computed the final velocity and averaged it over the ensemble. The results are

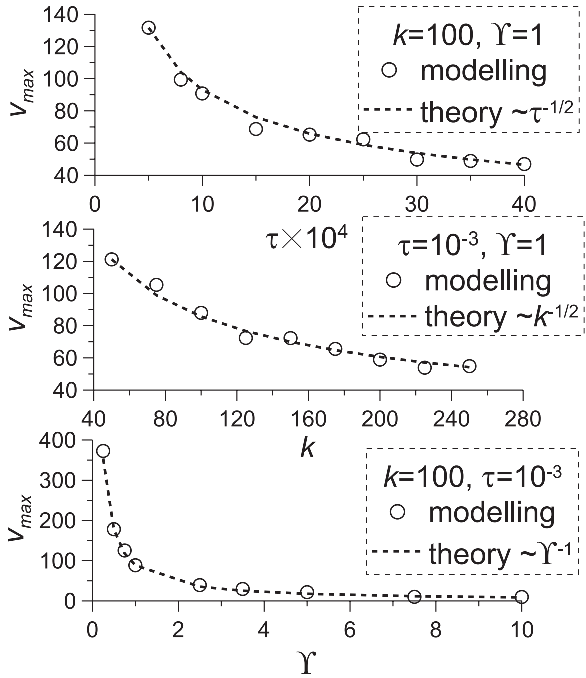

FIG. 4. Ensemble average of $v_{\max}$ as a function of k, τ, and Υ.

presented in Fig. 4. The symbols are the results of numerical simulations and the curves are obtained from the analytic description (11). One can see that the estimates of scaling of $v_{\max}$ are in a good agreement with the results of the direct numerical modeling.

To verify the probability density function (PDF)-based description (10) and the evolution as random walk (6), we performed numerical simulations in two ways. We integrated exact system (2) with $k = 100$, $\Upsilon = 1$, $\tau = 10^{-3}$ for 10^4 particles with initial energy $\varepsilon_0 = 2$ until the release from resonance and computed the distribution of $v_{\max}$. We also computed 10^6 trajectories as a random walk of I using (6) as long as $I(t) < S(t)$. For each trajectory we obtained its value t_* such that $I(t_*) = S(t_*)$ and computed $v_{\max} = v_\phi t_*$. Two distributions of $v_{\max}$ are presented in Fig. 5: Their similarity shows that the random-walk approximation can adequately describe the dynamics of (2).

V. DISCUSSION AND CONCLUSIONS

In the present paper we considered the fast-noise approximation for the magnetic field fluctuations ($\Omega\tau \ll 1$). However,

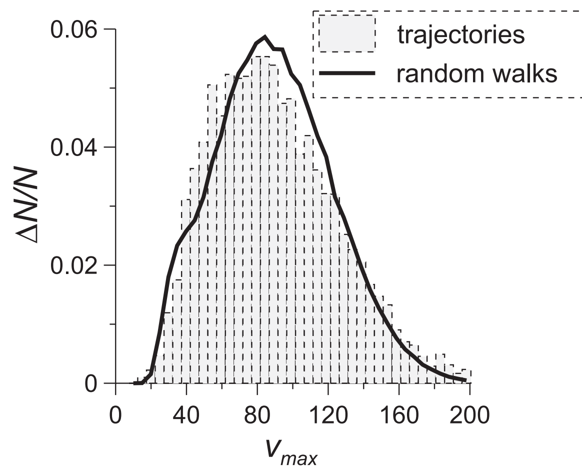

FIG. 5. The distributions of $v_{\max}$. ΔN is the number of particles with $v_{\max}$ (the width of histogram column is $\delta v = 5$) and N is the total number of particles.

the regimes with $\Omega\tau \sim 1$ and $\Omega\tau \gg 1$ are also quite important for the description of the interaction of charged particles with EMT. The regime $\Omega\tau \sim 1$ could be investigated only numerically due to the absence of the separation of time scales. The regime $\Omega\tau \gg 1$ could be described analytically by the method of averaging. Both problems will be subjects of our further investigation.

Summing up, we considered the resonance wave-particle interactions in the presence of random fluctuations of the background magnetic field. Random fluctuations are responsible for the jumps of the adiabatic invariant of the captured motion I, thus limiting the duration of particle resonance acceleration.

We demonstrated that the system can be modeled as a random walk or as a diffusion equation in the (t,I) space. We estimated the maximum value of energy $v_{\max}^2/2 \sim (\beta v_\phi)/(\Upsilon^2 \tau k)$ that could be gained by captured particles.

ACKNOWLEDGMENTS

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