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Percolation current in a periodic segregation of a binary granular mixture

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A periodic segregation in a binary granular mixture of alumina and copper spheres has been found when the mixture is subjected to vertical vibrations, in which a stable percolation process has been experimentally studied. Measurements reveal that the percolation time of alumina particles through a permeable layer of copper particles increases linearly as the number of alumina particles increases. While the percolation current of alumina particles proves to be independent of the number of alumina particles, it shows a monotonically decreasing relationship with the number of copper particles in the mixture. Finally, we briefly discuss possible candidates to affect the percolation current.

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I. INTRODUCTION

Vibrating segregation in a binary granular mixture, which is a great theoretical challenge with wide applications in practice, has attracted intensive interest from science and engineering communities in past years [1,2]. With energy input from the external environment, segregations on the size or density of grains produce various patterns, such as the Brazil nut effect (BNE) [3,4], the reverse Brazil nut effect (RBNE) [5,6], the sandwich pattern (SP) [7], the lighter mixed state (LMS) [8], etc. It is found that the behavior of the seemingly simple granular system under external vibrations proves to be extremely complicated. Many factors, such as vibrating conditions, media among grains, the properties of grains, etc., can impact the segregation. As a result, several typical theoretical models which include convective motion [9], air dissipation [10], geometrical reorganization [11], inertia [12], kinetic theory [13], and competition between percolation and condensation [14] were proposed to explain each type of segregation observed in experiments. Theorists also made efforts to unify a large variety of mechanisms to understand the underlying physics further. Recently some progress was achieved. In the work of Huerta *et al.*, they considered inertia, convection, and fluidization (void filling) as three predominant mechanisms for segregation under vertical vibration [15]. Afterwards, Matthias Schröter *et al.* in their review presented three mechanisms as well: void filling (also called geometrical mechanism or percolation; to avoid confusion, we use percolation in our paper), convection, and thermal diffusion predicted by kinetic theory. Almost all their results can be explained by any one or a combination of the three mechanisms [4]. Although a consensus on these theories mentioned above is still absent, most theorists agree that percolation is one of the most important mechanisms [4,5,7,15–20].

Generally more than one mechanism most likely governs the segregation behavior in a granular system, which may favor or disfavor segregation simultaneously. The complexity increases the difficulties of investigation. To deeply understand the segregation, isolating every mechanism experimentally seems promising for future solutions. Finding a granular system where percolation is the dominant effect is desirable to investigate the effect of percolation on segregation. Therefore, we initially take efforts to find such a segregation process where percolation plays a central role, and then we research the mechanism in detail.

In most past reports, segregation induced by vertical vibrations showed phenomena with time independence, which indicated that once segregations were formed, whether BNE, RBNE, or SP, they would be steady. However, a new segregation behavior found recently shows time dependence, that is, periodicity in the segregation is observed [7,21]. The segregation exhibits a cyclic process in which the status of the granular mixture evolves from BNE to RBNE and SP in sequence, and returns next to BNE. The complete period includes a percolation process in which a RBNE evolves to a BNE. Since the mechanism of percolation fundamentally dominates the process, it is suitable for investigating the effects of percolation on the segregation.

In the present study, we report a novel segregation behavior referred to as periodic segregation in a binary granular mixture subjected to vertical vibration, and describe a complete cycle by successive snapshots. In the complete period, we focus on the percolation process since a single mechanism which is called void filling or percolation dominates here. We then measure how the percolation time depends on the number of each type of particle in the binary mixture. The percolation current is defined and measured, and its dependence on particle number is shown. Finally, arguments for the observed linear dependence of a percolation current are presented.

II. EXPERIMENTS

The binary mixture used in our experiment consisted of copper spheres of density 8.38 $\mathrm{g/cm^3}$ and of diameter 320 ± 30 μm, and alumina spheres of density 1.65 $\mathrm{g/cm^3}$ and of diameter 170 ± 10 μm. The mixture is confined in a 50 long by 10 wide rectangular glass box with a height of 100 mm. The container is attached to an electromechanical transducer that is used to excite vertical sinusoidal motion, with the ratio of the

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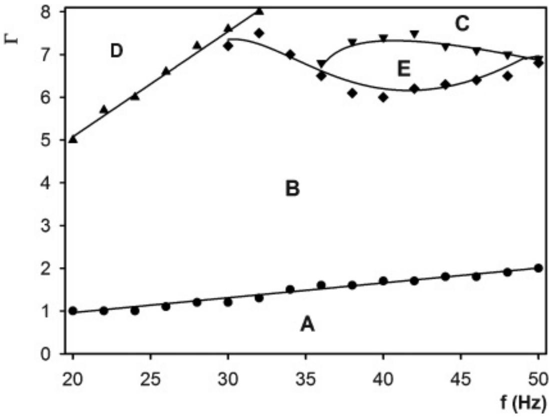

FIG. 1. Schematic frequency f vs amplitude phase diagram for a binary mixture of copper-alumina particles, showing the regions of behavior described in the text. Solid lines are drawn to guide the eye.

horizontal vibration amplitude to that of the vertical being less than 3%. Additionally, the frequency f of the transducer can be varied from 20 to 100 Hz and its acceleration amplitude Γ from 1.0 to 80. Amplitude A and frequency of vibration f applied to the particles are computer controlled. In general, frequency f and acceleration amplitude $4A\pi^2f^2/g$ are used as control parameters, where g is the acceleration of gravity. Before the experiment, both types of particle are mixed intimately and handled by an antielectrostatic process. We grounded the container, and sprayed antielectrostatic surfactant on the inner wall of the container, which forms a transparent film to slow the buildup of static charges. In addition, every hour we replaced the mixture with freshly prepared particles to avoid possible static buildup. To eliminate other undesirable factors which disturb the resulting segregation as much as possible, the temperature and humidity are maintained at 25 °C and in the range 40%–50%, respectively. After all experimental conditions are well established, we control the transducer to vibrate at various frequencies and amplitudes, and record each stable status of the mixture under vibrations.

A phase diagram to represent the vibrating behavior is shown as Fig. 1 withranges of f and Γ. Region A denotes the weak vibration state in which particles are well mixed in every local area, and both kinds of particles show the same convection and heaping behavior. Region B is the well-known BNE, and region C is defined as mixed state (MS) because percolation and migration exist simultaneously. Although there is a tendency that groups of the same kind of particles condense and form clusters, these clusters do not compose a uniform layer, and the structure of these clusters varies rapidly with time. Region D describes a throwing status where the copper spheres are visibly thrown free out of the upper surface, forming a surface wave. Below the surface wave there is a mixture of copper and alumina particles. Region E is the periodic segregation mentioned before. It is worthy of note that all stable segregations above are defined as a pattern which are maintained 5 min or longer at given f and Γ except for region E.

Unless specified otherwise, all measurements are performed in region E. To show the percolation process we are interested in, Fig. 2 uses a series of successive snapshots to illustrate a complete period of the segregation starting from a BNE [22]. Initially the mixture is vibrated for a while, turning out to be a BNE, in which dark particles (copper) constitute the upper layer and white particles (aluminum oxide) form the lower layer, and a sharp segregation boundary is clearly shown [see Fig. 2(a)]. However, the segregation is unstable, and the boundary tilts gradually. The copper layer heaps along one side of the container wall, and the alumina particles are extruded upwards by tilting copper particles, as Fig. 2(b) shows. Along one side of the container wall alumina particles sometimes burst out of the thinnest part of the copper layer, surging on the top surface as shown in Figs. 2(c) and 2(d). From the onset of alumina particles immigrating into the uppermost layer to the complete immigration over the copper ones to form a RBNE, the process is fast, taking 2 s or less. We refer to the moment of forming a RBNE as the starting time of the periodic segregation, as Fig. 2(e) shows. However, the configuration of a RBNE is unstable; immediately smaller alumina particles on the top begin to seep through the copper layer to reach the bottom of the container. Strictly speaking, the RBNE is a quasisteady or a transitional state. As more time elapses, the deposited alumina particles constitute a new layer below the copper one; the RBNE evolves to be a SP, as shown in Fig. 2(f). The amount of alumina particles on the top layer continuously decreases as they percolate through the copper layer and end up on the bottom layer [see Figs. 2(g) and 2(h)], until the return to a new BNE in which the copper particles occupy the upper layer again [Fig. 2(i)]. The procedure of BNE-RBNE-SP-BNE circulates stably under constant conditions, although some fluctuations are observed in the period of the segregation. Naturally, the time between Figs. 2(e) and 2(i) is considered as percolation time, which is directly used to characterize how quickly the percolation occurs.

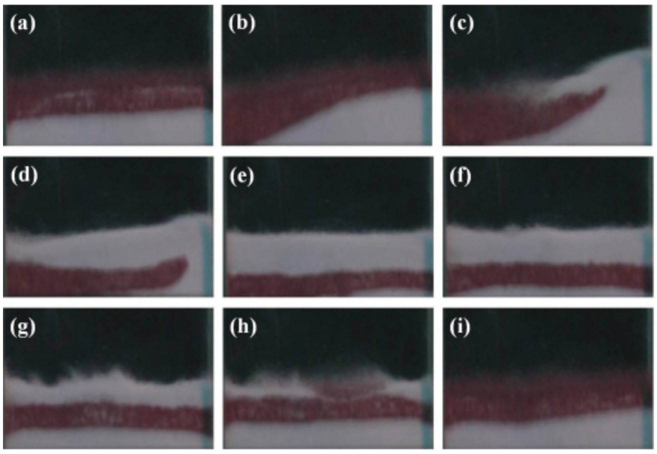

FIG. 2. (Color online) Series of consecutive snapshots (a) to (i) extracted from a video which records a typical periodic segregation represent a complete cycle. Here vibration frequency $f = 40$ Hz and acceleration amplitude $\Gamma = 6.6$. White particles are alumina spheres, and dark particles are copper spheres in these pictures. The pictures form a time sequence from upper left to lower right and correspond to 0, 6, 8, 9, 10, 85, 160, 235, and 314 s of vibration.

To further study the percolation process, we measure the percolation time for different amounts of alumina particles, fixing the amount of copper particles which act as the

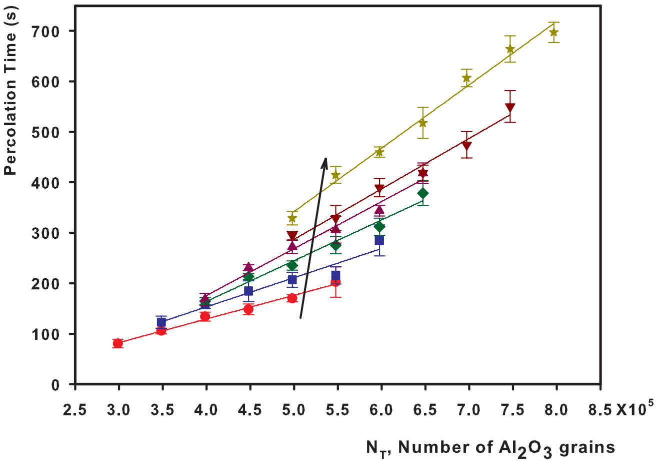

FIG. 3. (Color online) Percolation time measured as a function of the number of alumina particles at different amounts of copper particles: (●), 5.22×10^4 copper particles; (■), 5.97×10^4; (◆), 6.71×10^4; (▲), 7.46×10^4; (▼), 8.21×10^4; and (★), 8.95×10^4. The arrow indicates the trend of results for increasing numbers of copper particles. Straight solid lines which are fitting curves manifest the linear dependence of the percolation time on the numbers of alumina particles. All data points are averaged 10 times.

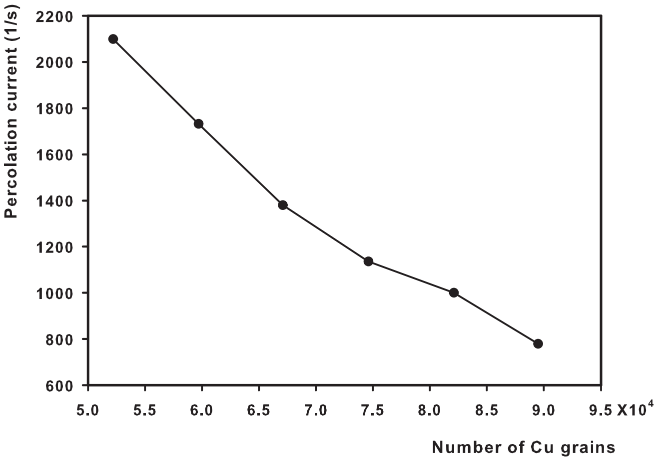

FIG. 4. Percolation current extracted from Fig. 3 as a monotonic decreasing function of the number of copper particles. The solid line is plotted to guide the eye.

permeable layer. Figure 3 demonstrates the relationship between the percolation time and the amount of alumina particles. Due to fluctuations in the measurement, multiple measuring is conducted to suppress the error of the percolation time. Every data point is measured ten times and averaged, and error bars are shown as well. It is worthy of note that an inappropriate ratio or large total volume of both particles can destroy the periodic segregation, since we observed that the cycle was interrupted by insufficient or excessive copper particles.

III. RESULTS AND DISCUSSION

Data points in Fig. 3 that have the same symbol represent the percolation time as a function of the numbers of alumina particles in the container, as the numbers of copper particles are fixed; straight solid lines correspond to fitting curves. It is worth pointing out that the parameter directly measured in our experiment is the volume of particles. However, we convert the volume to the numbers of particles in Figs. 3 and 4, though the thickness or the volume of particles appears more intuitive than numbers. The reason is that the percolation current proposed later is defined by the percolation time and the numbers of particles, and thus it is more convenient to use numbers instead of volume or thickness. We estimate the amount of particles by a simple equation of $N = \frac{V_{\text{fill}}}{V_{\text{sphere}}}\gamma$, where N is the amount of particles, V_{fill} and V_{sphere} are the volume of particles filled in the container and of an individual particle, respectively, and γ, the static packing fraction, is regarded as a constant due to the same filling method each time. By using $\gamma = \frac{M}{\rho V_{\text{fill}}}$, where M is the mass of particles, ρ is the particle density, and the γ is measured to be 0.60.

It is readily shown that the percolation time increases with the numbers of alumina particles in a nearly linear fashion in Fig. 3. The reciprocal of the slope of the fitting straight line, $1/k$, is naturally regarded as the percolation current of alumina particles through the copper layer. Thus, the percolation current used to characterize the strength of percolation is defined by $I = \frac{dN_T}{dt}$, where N_T is the percolating amount of alumina particles through the copper layer. According to the definition of the percolation current, the linear behavior of the fitting lines indicates that the percolation current is a constant for a given number of copper particles. When the number of copper particles is changed from 5.22×10^4 to 8.95×10^4, the percolation current apparently drops because the slopes of fitting curves increase correspondingly. To determine the percolation current accurately, we measure the percolation time, from which the percolation current is extracted by a fitting equation as follows:

$$T(N_T, N_b) = \frac{1}{I(N_b)} N_T + a, \qquad (1)$$

where $T(N_T, N_b)$ is the percolation time, and $I(N_b)$ is the percolation current, which is the function of N_b. Equation (1) is used to fit the experimental data in Fig. 3, and then the percolation current is extracted. Consequently Fig. 3 illustrates that the percolation current is a function of the amount of the copper particles and is insensitive to that of alumina particles. With the result extracted from Figs. 3 and 4 clearly exhibits a dependence of percolation current on the number of copper particles. With increasing copper particle number, the percolation current drops in an approximately linear fashion within the range of measurement. The relation between the percolation current of alumina particles and the numbers of copper particles suggests a way to compare them with fluid flow through porous media. When the porosity of a porous medium is decreased, the flux of fluids through the porous medium will be reduced correspondingly [23]. In this sense, there should be a close connection between the numbers of copper particles and the porosity of porous media. If we can consider the copper particle bed as a dynamic porous medium, the increased copper particles might reduce the dynamic void volume within the vibrating bed.

We therefore suspect the dynamic porosity of the copper layer as the most likely candidate for influencing the percolation current. However, it is very challenging to directly measure dynamic porosity throughout the vibration. We attempted to identify and compare the thickness of copper layers of different particle numbers when vibrated, but found that large fluctuations prevented the validity of measurement. In the future, if the bridge between the amount and the porosity of copper particles, or other relevant parameters, can be established, more light will be shed on the further understanding of percolation current.

IV. CONCLUSION

In conclusion, we observe periodic segregation in a mixture of binary granular particles, which is a suitable system for research on the effect of percolation. We establish a phase diagram that delineates the different segregation phases and the boundary of the periodic segregation region. Also, we describe a complete process of periodic segregation by sequential snapshots. By studying the stage where percolation dominates, we find the dependence of the percolation time on the number of alumina and copper particles in the mixture. The percolation time of alumina particles through a permeable layer is linearly proportional to the number of alumina particles if vibrating conditions and the amount of copper particles maintain constant. The percolation current defined later shows a monotonically decreasing relationship with the number of copper particles, and is nearly independent of the number of alumina particles. Finally we briefly discussed some factors that could influence the percolation current.

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[1] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).

[2] J. M. Ottino and D. V. Khakhar, Annu. Rev. Fluid Mech. **32**, 55 (2000).

[3] M. E. Möbius, B. E. Lauderdale, S. R. Nagel, and H. M. Jaeger, Nature (London) **414**, 270 (2001).

[4] M. Schröter, S. Ulrich, J. Kreft, J. B. Swift, and H. L. Swinney, Phys. Rev. E **74**, 011307 (2006).

[5] A. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, Phys. Rev. Lett. **58**, 1038 (1987).

[6] T. Shinbrot and F. J. Muzzio, Phys. Rev. Lett. **81**, 4365 (1998).

[7] N. Burtally, P. J. King, Michael R. Swift, and M. Leaper, Gran. Matt. **5**, 57 (2003).

[8] Q. Shi, G. Sun, M. Hou, and K. Lu, Phys. Rev. E **75**, 061302 (2007).

[9] J. B. Knight, H. M. Jaeger, and S. R. Nagel, Phys. Rev. Lett. **70**, 3728 (1993).

[10] X. Yan, Q. Shi, M. Hou, K. Lu, and C. K. Chan, Phys. Rev. Lett. **91**, 014302 (2003).

[11] J. Duran, J. Rajchenbach, and E. Clement, Phys. Rev. Lett. **70**, 2431 (1993).

[12] Y. Nahmad-Molinari, G. Canul-Chay, and J. C. Ruiz-Suarez, Phys. Rev. E **68**, 041301 (2003).

[13] S. S. Hsiau and M. L. Hunt, Acta Mech. **114**, 121 (1996).

[14] P. V. Quinn, D. C. Hong, and S. Luding, Phys. Rev. Lett. **89**, 189604 (2002).

[15] D. A. Huerta and J. C. Ruiz-Suarez, Phys. Rev. Lett. **92**, 114301 (2004).

[16] N. Shishodia and C. R. Wassgren, Phys. Rev. Lett. **87**, 084302 (2001).

[17] L. Vanel, A. D. Rosato, and R. N. Dave, Phys. Rev. Lett. **78**, 1255 (1997).

[18] J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, Phys. Rev. E **50**, 5138 (1994).

[19] K. Liffman, K. Muniandy, M. Rhodes, D. Gutteridge, and G. Metcalfe, Gran. Matt. **3**, 205 (2001).

[20] R. Jullien, P. Meakin, and A. Pavlovitch, Phys. Rev. Lett. **69**, 640 (1992).

[21] Wu Yu-Hang, Zheng Ning, Wen Ping-Ping, Li Liang-Sheng, Shi Qing-Fan, and Sun Gang, Acta Physica Sinica **60**, 024501 (2011).

[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.84.041307 for a movie demonstrating a complete periodic segregation at the vibrational conditions of $f = 40$ Hz and $\Gamma = 6.6$.

[23] A. Koponen, M. Kataja, and J. Timonen, Phys. Rev. E **56**, 3319 (1997).