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Performance analysis of a two-state quantum heat engine working with a single-mode radiation field in a cavity

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We present a performance analysis of a two-state heat engine model working with a single-mode radiation field in a cavity. The heat engine cycle consists of two adiabatic and two isoenergetic processes. Assuming the wall of the potential moves at a very slow speed, we determine the optimization region and the positive work condition of the heat engine model. Furthermore, we generalize the results to the performance optimization for a two-state heat engine with a one-dimensional power-law potential. Based on the generalized model with an arbitrary one-dimensional potential, we obtain the expression of efficiency as $\eta = 1 - \frac{E_C}{E_H}$, with E_H (E_C) denoting the expectation value of the system Hamiltonian along the isoenergetic process at high (low) energy. This expression is an analog of the classical thermodynamical result of Carnot, $\eta_c = 1 - \frac{T_C}{T_H}$, with T_H (T_C) being the temperature along the isothermal process at high (low) temperature. We prove that under the same conditions, the efficiency $\eta = 1 - \frac{E_C}{E_H}$ is bounded from above the Carnot efficiency, $\eta_c = 1 - \frac{T_C}{T_H}$, and even quantum dynamics is reversible.

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I. INTRODUCTION

The concept of a quantum heat engine was introduced by Scovil and Schultz-Dubois [1] and subsequently extended in many meaningful publications [2–17]. The working substance of a quantum heat engine includes various quantum systems, such as spin systems [8–12], harmonic-oscillator systems [2], two-level or multilevel systems [5,13], cavity quantum electrodynamics systems [7,14], coupled two-level systems [17], etc. Bender *et al.* [5] set up a two-level quantum engine model of a particle confined in a one-dimensional (1D) box trap. Unlike in the quantum Carnot engine where the working substance couples to the heat bath during an isothermal process, this quantum heat engine, in which heat fluxes and thus temperature are not introduced, produces work changing the potential width at a very slow speed.

It is of significance that the optimization of heat engines would have to proceed under constraints that determine the very path of the engine evolution. Based on the assumption that the minimum value of the potential width is fixed and the potential wall moves at a low but finite speed, the efficiency at the maximum power output has been found to be a constant for the engine model proposed by Bender *et al.* [15]. However, a question that naturally arises is whether the efficiency depends on the form of the potential. Moreover, so far there has been no comprehensive discussion of such a quantum heat engine model in the literature [5,15,16], and thus the properties of the heat engine model have not been addressed adequately and clearly.

In this paper, we present a performance analysis of a quantum heat engine model proposed first by Bender *et al.* [5], changing the two-state particle in a box into a two-state radiation field in a cavity. Instead of heat baths, the heat engine model includes energy baths. The expectation value of the Hamiltonian (instead of the temperature variable in classical thermodynamics) will be used, and thus "temperature" means the expectation value of the Hamiltonian. Assuming that the potential width moves at a very slow but fixed speed in one cycle, we obtain the expressions for some important parameters and determine the optimization region. We find that the value of efficiency at the maximum power output is dependent of the form of the potential, though it is not dependent of the parameter for a given potential. Furthermore, we discuss the general case of a heat engine model that uses a particle confined in an arbitrary 1D potential as the working substance. In the general case, the expression of the efficiency is found to be

$$\eta = 1 - E_C/E_H, \tag{1}$$

with E_C (E_H) denoting the expectation value of the system Hamiltonian along the isoenergetic process at low (high) energy. The expression of the efficiency, as expressed in Eq. (1), is independent of the form of the potential and analogous to the expression of the classical Carnot efficiency. Finally, based on our generalized quantum heat engine model, we demonstrate that the efficiency is bounded from above the Carnot value.

The plan of the paper is as follows. In Sec. II, we briefly review the first law of thermodynamics in quantum systems. We capitulate the structure of a reversible two-level engine model of a single-mode radiation field and obtain the expression of the efficiency in Sec. III. We study the optimization on power output, and we analyze the optimal ranges of the efficiency and of the engine structure in Sec. IV. In Sec. V, we generalize the results to the case in which a two-state engine model works with one 1D power-law potential. In Sec. VI, we discuss the relationship between the efficiency of the engine model and that of the corresponding quantum Carnot cycle. Section VII presents our conclusions.

II. THE FIRST LAW OF THERMODYNAMICS

Like the classical thermodynamics, the first law of thermodynamics in quantum-mechanical systems can be expressed

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as a function of eigenenergies ε_n and probability distributions p_n [6],

$$dE = đQ + đW = \sum_n \varepsilon_n dp_n + \sum_n p_n d\varepsilon_n, \tag{2}$$

where

$$đQ = \sum_n \varepsilon_n dp_n \tag{3}$$

and

$$đW = \sum_n p_n d\varepsilon_n \tag{4}$$

depict the heat exchange and work done, respectively, during a thermodynamic process. As in a classical system in which the generalized force Y_n, conjugate to the generalized coordinate y_n, is given by $Y_n = -\frac{đW}{dy_n}$, for a quantum system one defines the force as

$$F = -\frac{đW}{dL} = -\sum_n P_n \frac{d\varepsilon_n}{dL}, \tag{5}$$

where L is the generalized coordinate corresponding to the force F.

III. A HEAT ENGINE MODEL OF A SINGLE-MODE RADIATION FIELD IN A CAVITY

We now consider a single-mode radiation field in a cavity. Schrödinger's equation of the radiation field can be given by $\hat{H}|u_n\rangle = \varepsilon_n|u_n\rangle$, where $|u_n\rangle$ and ε_n stand for the eigenstates and eigenenergies, respectively. An arbitrary state $|\psi\rangle$ can be expanded in terms of the eigenstates as $|\psi\rangle = \sum_n a_n|u_n\rangle$, where the expansion coefficients satisfy the constraint that

$$\sum_n |a_n|^2 = 1. \tag{6}$$

The expectation value of the Hamiltonian of the system is given by

$$E = \langle\psi|\hat{H}|\psi\rangle = \sum_n \varepsilon_n |a_n|^2. \tag{7}$$

Considering the fact that the von Neumann entropy $S^{\rm VN} = k_B {\rm Tr}(\rho \ln \rho)$, with k_B being Boltzmann constant, identically vanishes when the density matrix ρ is of a pure state [15,16], we will use the Shannon entropy instead of the von Neumann entropy. The Shannon entropy can be expressed in terms of the expansion coefficients as

$$S = -k_B \sum_n |a_n|^2 \ln |a_n|^2. \tag{8}$$

According to the quantum adiabatic theorem [18], an isolated system would remain in its initial state during an adiabat. It is clear from Eq. (8) that the entropy in an adiabat remains invariant because the absolute values $|a_n|$ of the expansion coefficients do not change.

For a single-mode radiation field in a cavity, the eigenstate energies are given by [6]

$$\varepsilon_n = (n + 1/2)\hbar\omega \tag{9}$$

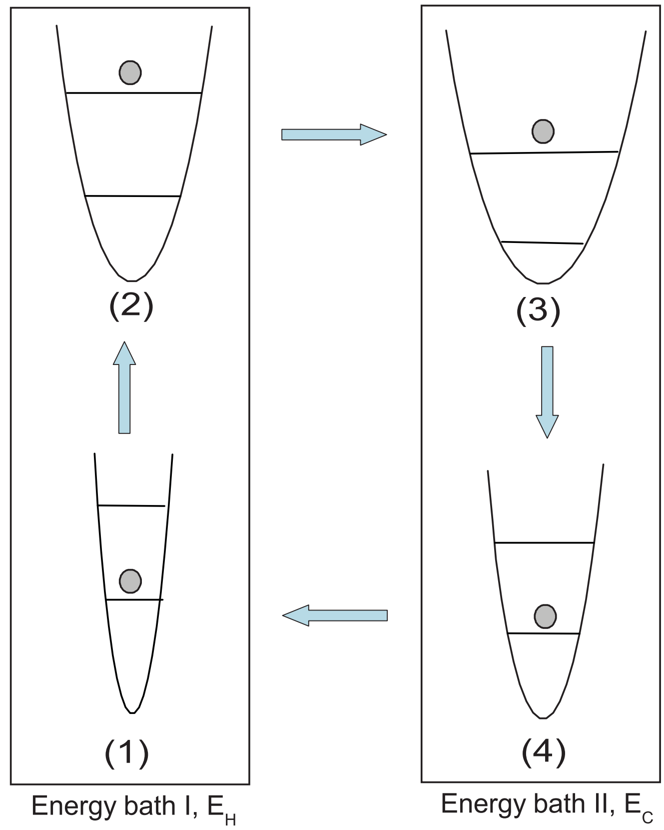

FIG. 1. (Color online) The graphic sketch of a quantum heat model based on a single-mode radiation in a cavity. At instants 1 and 4, the system stays at the ground energy level, while at instants 2 and 3, the system stays at the excited energy level. In the process $1 \rightarrow 2$ ($3 \rightarrow 4$), the system absorbs (releases) energy from (to) energy bath I (II) and the energy of the system is kept unchanged at constant E_H (E_C). In the adiabatic processes $2 \rightarrow 3$ and $4 \rightarrow 1$, the system is decoupled from the energy bath and stays in a fixed state.

or

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar \frac{s\pi c}{L} (n = 0,1,2,\ldots), \tag{10}$$

where L is the width of cavity and $\omega = \frac{s\pi c}{L}$ is the frequency of the radiation field, with s being an integer and c being the speed of light. Like the piston in a 1D cylinder for a classical thermodynamic system, the cavity width L can be changed when the external trap parameters are varied. In what follows, we will assume that there are only two energy levels $n = 0$ and 1 employed by the quantum heat engine.

A quantum heat engine, using a single-mode radiation field in a cavity as the working substance and consisting of two quantum adiabatic and two isoenergetic processes, is illustrated in Fig. 1. During an isoenergetic process, the cavity width L changes as the cavity wall moves, but the expectation value of the Hamiltonian, $E(L)$, remains constant when the system is coupled to an energy bath. During the adiabatic process, there does not exist any heat exchange between the system and the surroundings. The system remains a fixed state and the expansion coefficients $|a_n|^2$ do not change in an adiabat, as mentioned earlier.

In the first process $1 \to 2$, the energy level of the system is excited, while the expectation value of the Hamiltonian is kept fixed. The total energy of the system reads $E_{12} = E_H = \frac{\hbar s \pi c}{2L_1}$. The state of the system in this expansion is a linear combination of the lowest two energy eigenstates. Using the condition $|a_1|^2 + |a_2|^2 = 1$, the total energy as a function of cavity width L can be expressed as

$$E_H = \frac{\hbar s \pi c}{2L}(3 - 2|a_1|^2) = \frac{\hbar s \pi c}{2L_1}. \tag{11}$$

Accordingly, we have $L = L_1(3 - 2|a_1|^2)$. When $a_1 = 0$, the possible maximum value of L in the isoenergetic expansion is obtained with $L_2 = 3L_1$. At the point of $L = L_2$, the system is purely in the second energy eigenstate since $a_1 = 0$. From Eq. (11), along this isoenergetic expansion the force is determined by

$$F_{12}(L) = \frac{\hbar s \pi c}{2L^2}(3 - 2|a_1|^2) = \frac{\hbar s \pi c}{2L_1 L}. \tag{12}$$

In the process $2 \to 3$ the system is expanded adiabatically from $L = L_2$ until $L = L_3$. During this expansion, the system remains in the excited state and the expectation value of the Hamiltonian is $E_{23} = \frac{3\hbar s \pi c}{2L}$, which yields the force as a function of L: $F_{23} = \frac{3\hbar s \pi c}{2L^2}$.

In the third process $3 \to 4$, the system is compressed isoenergetically from $L = L_3$ until $L = L_4$ with $L_4 = \frac{1}{3}L_3$. During this compression, energy is extracted to keep the expectation value of the Hamiltonian constant. At instant 3, the system is in the excited state $|u_2\rangle$, whereas the system is back in the ground state $|u_1\rangle$ at instant 4. During this isoenergetic compression, the expectation value of the Hamiltonian is kept constant as

$$E_{34} = E_C = \frac{3\hbar s \pi c}{2L_3}. \tag{13}$$

Similarly to the first process $1 \to 2$, the radiation force as a function of L can be obtained as

$$F_{34}(L) = \frac{3\hbar s \pi c}{2L_3 L}. \tag{14}$$

During the fourth process $4 \to 1$, the system is compressed adiabatically from $L = L_4$ until it returns to the starting point $L = L_1$. During this compression, the system remains in the ground state $|u_1\rangle$ and the expectation value of the Hamiltonian is $E_{41} = \frac{\hbar s \pi c}{2L}$. Then the force applied to the cavity wall is $F_{41}(L) = \frac{\hbar s \pi c}{2L^2}$. The reversible quantum cycle that we have just described is constructed as in Fig. 2.

In the two constant-energy processes, the system is coupled to an energy bath at constant energy E_H and an energy bath at constant energy E_C, respectively. The heat quantity Q_H absorbed from the energy bath I and the heat quantity Q_C released to the energy bath II are, respectively,

$$Q_H = \int_{L_1}^{L_2} F_{12}(L)dL = \frac{\ln 3}{2L_1}\hbar s \pi c, \tag{15}$$

$$Q_C = \left| \int_{L_3}^{L_4} F_{34}(L)dL \right| = 3\frac{\ln 3}{2L_3}\hbar s \pi c. \tag{16}$$

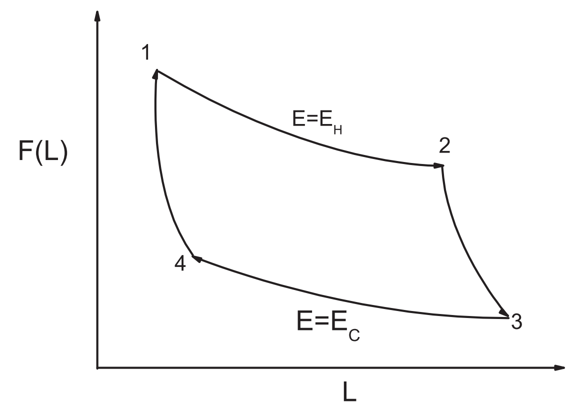

FIG. 2. Schematic diagram of a quantum heat engine cycle in the plane of the width L and force $F(L)$. The quantum states of the radiation field and the values of the potential width at the four special instants are as follows: $|u_1\rangle$ and L_1 at instant 1, $|u_2\rangle$ and $L_2 = 3L_1$ at instant 2, $|u_2\rangle$ and L_3 at instant 3, and $|u_1\rangle$ and $L_4 = L_3/3$ at instant 4.

Since there is no heat exchange between the system and the surroundings in any adiabat, the mechanical work W per cycle can be directly according to

$$W = Q_H - Q_C = \frac{\ln 3}{2}\hbar s \pi c\left(\frac{1}{L_1} - \frac{3}{L_3}\right). \tag{17}$$

Therefore, the efficiency η of our two-state quantum heat engine is given by

$$\eta = \frac{W}{Q_H} = 1 - \frac{3L_1}{L_3} = 1 - \frac{E_C}{E_H}. \tag{18}$$

The expression of efficiency for this quantum heat engine is identical to that of efficiency for the heat engine working with a single particle confined in a box. Let $r \equiv L_3/L_1$; the positive work ($W > 0$) condition becomes

$$r > 3, \tag{19}$$

which is different from that obtained in the model with a single particle in a box [15]. Only when this positive work condition is satisfied can the positive work be extracted. This result indicates that the heat engine of a single-mode radiation field in a cavity would produce positive work if adiabatic expansion ($2 \to 3$) and compression ($4 \to 1$) are satisfied with the relations $L_3 > \frac{3}{2}L_2$ and $L_4 > L_1$, respectively.

IV. OPTIMIZATION ON THE PERFORMANCE OF THE HEAT ENGINE

Now, we are in a position to discuss the power output of the quantum heat engine. Upon realizing the finite power, we are not able to introduce heat fluxes into the system because of the absence of heat baths. It is noteworthy that in order for the adiabatic theorem to apply, the time scale of the change of the state must be much larger than that of the dynamical one, $\sim\hbar/E$ [15,16,18]. Assuming that the potential wall moves at a small but finite speed, we let $\bar{v}(t)$ and τ be the average speed of the change of L and the total cycle time, respectively. The speed $\bar{v}(t)$ should be slow enough so that the variation of L is much slower compared with the dynamical time scale, $\sim\hbar/E$, as pointed out above. The total amount of movement during a single cycle, L_0, is given by

$$L_0 = (L_2 - L_1) + (L_3 - L_2) + (L_3 - L_4) + (L_4 - L_1) = 2(L_3 - L_1). \quad (20)$$

The total cycle time τ can be given by

$$\tau = L_0/\bar{v} = 2(L_3 - L_1)/\bar{v}. \quad (21)$$

The power output, after a single cycle, is

$$P = \frac{W}{\tau} = \frac{\ln 3}{4L_1^2}\bar{v}\hbar s\pi c\frac{r-3}{r^2-r}. \quad (22)$$

Assuming that L_1 and $\bar{v}$ are fixed, one can control r so as to maximize power output P. The maximization condition $(\frac{\partial P^*}{\partial r})_{r=r_m} = 0$, with the dimensionless power output $P^* \equiv P/(\frac{\ln 3}{4L_1^2}\bar{v}\hbar s\pi c)$, leads to the following equation:

$$r_m^2 - 6r_m + 3 = 0, \quad (23)$$

which has two real solutions: $r_m^1 = 3 - \sqrt{6}$ and $r_m^2 = 3 + \sqrt{6}$. Thus, the value of efficiency η_m at the maximum power output is

$$\eta_m = 1 - \frac{3}{3+\sqrt{6}} \simeq 0.44949, \quad (24)$$

which is different from that obtained in the engine working with a particle in a 1D box trap [15]. It is shown that the value of the efficiency depends on the form of the potential. Combing Eqs. (18) and (22), we obtain the dimensionless power output P^* as a function of the efficiency η,

$$P^* = \frac{(1-\eta)\eta}{2+\eta}. \quad (25)$$

We plot the characteristic curve of the dimensionless power output P^* as a function of efficiency η, as shown in Fig. 3.

There exists a maximum dimensionless power output and the corresponding efficiency η_m for given parameter L_1 and average speed $\bar{v}$. Figure 3 shows that, when $P^* < P^*_{\max}$, there are two different efficiencies for a given power output, where one is smaller than η_m and another is larger than η_m. When $\eta < \eta_m$, the dimensionless power output P^* decreases as the efficiency decreases, such that the efficiencies, smaller than η_m, are not the optimal values for the heat engine. The optimal region of the efficiency is thus given by

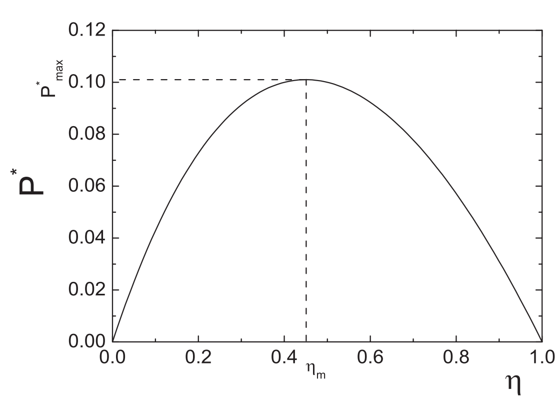

FIG. 3. The dimensionless power output P^* vs the efficiency η.

$$\eta_m \leqslant \eta < 1. \quad (26)$$

When the heat engine is operated in this region, the dimensionless power output will increase as the efficiency decreases, and vice versa. The value of η_m is of primary importance, since it determines the allowable value of the lower bound of the optimal efficiency. It is found that the value of ratio r, determining the structure of the heat engine model, should be

$$r \geqslant r_m \equiv (3 + \sqrt{6}). \quad (27)$$

When determining the structure of the heat engine cycle, the condition $L_3 = r_m L_1$ must be satisfied so that the engine is operated in the optimal region.

V. GENERAL CASE

To address the general case, we shall consider 1D power-law potentials that are parametrized by a single-particle energy spectrum of the form [19–21]

$$\varepsilon_n = \varepsilon_g(L)n^\sigma, \quad (28)$$

where $\varepsilon_g(L) \equiv \varepsilon_1(L)$ is the energy of the ground state, n is a positive integer quantum number, and σ is an index of the single-particle energy spectrum. The ground-state energy can be assumed to be proportional to $L^{-\alpha}$, i.e., $\varepsilon_g(L) = BL^{-\alpha}$, where B is a constant for a given potential, and the index α is positive and depends on the form of the trapping potential. For instance, there are several special cases: (i) $\sigma = \alpha = 2$ for a box and a harmonic potential, $B = \frac{\pi^2\hbar^2}{2m}$ for a box potential, while $B = \frac{\hbar^2}{m}$ for a harmonic potential [22], with m being the particle mass. (ii) $\sigma = \alpha = 1$ and $B = \pi\hbar c$ for extremely relative particles in a box potential [23]. (iii) $\sigma = \frac{4}{3}$, $\alpha = 2$, and $B = \frac{\hbar^2}{m}$ for a quartic potential [24].

Using a similar method adopted in Sec. III, the fixed energy in the process $1 \to 2$ is calculated to be

$$E_H = \varepsilon_g(L_1) = \varepsilon_g(L)|a_1|^2 + 2^\sigma\varepsilon_g(L)(L)|a_2|^2. \quad (29)$$

At the instant 2, the maximum value of L is achieved when $|a_1| = 0$, with $L_2 = 2^{\sigma/\alpha}L_1$. According to Eq. (29), the force is $F_{12} = \alpha\varepsilon_g(L_1)/L$. Therefore, the heat quantity absorbed from the energy bath is

$$Q_H = \int_{L_1}^{L_2}\frac{\alpha\varepsilon_g(L_1)}{L}dL = \sigma\varepsilon_g(L_1)\ln 2. \quad (30)$$

During the process $3 \to 4$, the force is given by $F_{34} = 2^\sigma\alpha\varepsilon_g(L_3)/L$. This leads to the heat quantity released from the system,

$$Q_C = \left|\int_{L_3}^{L_4}\frac{2^\sigma\alpha\varepsilon_g(L_3)}{L}dL\right| = 2^\sigma\sigma\varepsilon_g(L_3)\ln 2. \quad (31)$$

During the adiabat $2 \to 3$ $(4 \to 1)$, the system remains in the state $|u_2\rangle$ $(|u_1\rangle)$ and thus there is no heat exchange. The total amount of the mechanical work W, with $W = Q_H - Q_C$, can be written as

$$W = B\sigma\left(\frac{1}{L_1^\alpha} - \frac{2^\sigma}{L_3^\alpha}\right)\ln 2, \quad (32)$$

which yields the following positive work condition:

$$r > 2^{\sigma/\alpha}, \tag{33}$$

where r was defined above Eq. (19). We can readily obtain the efficiency as

$$\eta = 1 - 2^{\sigma}\frac{L_1^{\alpha}}{L_3^{\alpha}} = 1 - \frac{E_C}{E_H}. \tag{34}$$

Here, the expression of the efficiency is analogous to that of Carnot efficiency and independent of the form of the trapping potential. The power output $P = W/\tau$, with τ being given by Eq. (21), is given by

$$P = \frac{B\bar{v}}{2}\frac{1}{L_1^{\alpha+1}}\frac{r^{\alpha} - 2^{\sigma}}{r^{\alpha+1} - r}. \tag{35}$$

The maximization condition $(\frac{\partial P^*}{\partial r})_{r=r_m} = 0$, with the dimensionless $P^* = P/(\frac{B\bar{v}}{2}\frac{1}{L_1^{\alpha+1}})$, gives rise to the following relation:

$$r_m^{1+\alpha} - 2^{\sigma}(\alpha + 1)r_m + 2^{\sigma}\alpha = 0. \tag{36}$$

Under the positive work condition that was given by Eq. (33), we can obtain the optimal value $r = r_m$ and thus determine the corresponding efficiency η_m of the model based on a given potential. It is clear that the value of r_m determining the efficiency η_m depends on the index α of the potential. That is, the value of the efficiency varies from system to system, as emphasized below Eq. (24). For the quantum heat engine working with a particle in a 1D box potential, $\sigma = \alpha = 2$ and the relation Eq. (36) becomes $r_m^3 - 12r_m + 8 = 0$, confirming the result obtained in Ref. [15].

VI. RELATIONSHIP BETWEEN THE EFFICIENCY OF THE QUANTUM CYCLE WITH ENERGY BATHS AND THAT OF THE CORRESPONDING QUANTUM CARNOT CYCLE WITH HEAT BATHS

In this section, we discuss the relationship between the efficiency of the quantum heat engine cycle mentioned above and that of the quantum Carnot cycle. The quantum Carnot cycle consists of two quantum adiabatic and isothermal processes. In the quantum isothermal process, the working substance, such as a particle confined in a trapping potential, is coupled to a heat bath at a constant temperature. For a Carnot cycle $1 \to 2 \to 3 \to 4 \to 1$, in the quantum isothermal processes $1 \to 2$ and $3 \to 4$, the system is coupled with heat baths at constant temperatures, T_H and T_C, respectively, whereas $2 \to 3$ and $4 \to 1$ are adiabats with fixed expansion coefficients $|a_n|^2$, as shown in Fig. 2. (Instead of two constant energies $E = E_H$ and $E = E_C$, the system couples to heat baths at constant temperatures $T = T_H$ and T_C in the processes $1 \to 2$ and $3 \to 4$, respectively.)

Let $\xi = |a_1|^2$ (and thus $|a_2|^2 = 1 - \xi$) for a two-level system with the ground state and the excited state. The ratio $\gamma \equiv (1 - \xi)/\xi$ of the two occupation probabilities should satisfy the Boltzmann distribution $\gamma = \frac{1-\xi}{\xi} = e^{-\Delta(L)/k_BT}$, where $\Delta(L) \equiv \varepsilon_e - \varepsilon_g$ is the energy gap between the ground state and the first excited state. The occupation probability ξ of the ground state can thus be given in terms of trap size L and temperature T:

$$\xi(L,T) = \frac{1}{e^{-\frac{\Delta(L)}{k_BT}} + 1}. \tag{37}$$

Note that, for a particle confined in a 1D box trap, $\Delta(L) = \frac{3\pi^2\hbar^2}{2mL^2k_BT}$, Eq. (37) becomes identical to that obtained in a different way [16]. Again, we would like to emphasize that the time scale associated with the change of L should be much larger than the dynamical one, $\sim\hbar/E$, and the change of the state can thus be represented by the change of ξ [16].

For simplicity, instead of using Eq. (3), we apply $đQ = TdS$ directly to the calculation of the heat exchange $đQ$ in any quantum isothermal process. The heat exchanges Q_H and Q_C can be determined by $Q_H = T_H[S(2) - S(1)]$ and $Q_C = T_C|[S(4) - S(3)]|$. According to the first law of the thermodynamics, the work W per cycle can be calculated as

$$W = Q_H - Q_C = (T_H - T_C)[S(2) - S(1)]. \tag{38}$$

In obtaining Eqs. (38), the relations $S(3) = S(2)$ and $S(4) = S(1)$ have been used, since the occupation probabilities are not varied and the entropies remain fixed during any quantum adiabat. So the efficiency of the quantum Carnot engine, $\eta_c = \frac{W}{Q_H}$, becomes

$$\eta_c = 1 - \frac{T_C}{T_H} = 1 - \left(\frac{L_2}{L_3}\right)^{\alpha}, \tag{39}$$

where the relation $\frac{T_C}{T_H} = (\frac{L_2}{L_3})^{\alpha}$ has been used due to the fact that the occupations probabilities are fixed in the adiabatic process.

For the two-level engine model working with heat baths at different temperatures, the expectation value of the Hamiltonian is given by $E = \frac{B}{L^{\alpha}}\xi + 2^{\sigma}(1 - \xi)\frac{B}{L^{\alpha}}$. In the quantum isothermal process $1 \to 2$, the energy of the system must not decrease in this process, yielding $\frac{E(L_1)}{E(L_2)} = (\frac{L_2}{L_1})^{\alpha}\frac{2^{\sigma}+(1-2^{\sigma})\xi(L_1,T_H)}{2^{\sigma}+(1-2^{\sigma})\xi(L_2,T_H)} \leqslant 1$, i.e., $(\frac{L_2}{L_1})^{\alpha} \leqslant \frac{2^{\sigma}+(1-2^{\sigma})\xi(L_2,T_H)}{2^{\sigma}+(1-2^{\sigma})\xi(L_1,T_H)}$. We thus have the relation $(\frac{L_2}{L_1})^{\alpha} < 2^{\sigma}$. Combing Eqs. (34) and (39), we have the inequality

$$\eta = 1 - \frac{E_C}{E_H} < 1 - \frac{T_C}{T_H} = \eta_c. \tag{40}$$

This implies that under the same conditions, the quantum engine with heat baths at constant temperatures runs more efficiently than the quantum engine with energy baths at constant energies.

VII. CONCLUSIONS

We have considered the optimization region and the positive work condition for the quantum heat engine, using a single-mode radiation field in a cavity as the working substance, under the approximation that only two levels are included. Our performance analysis has been generalized to a heat engine working at an arbitrary power-law trapping potential. The expression of the efficiency for such a heat engine model has been shown to be analogous to that of Carnot efficiency and independent of the form of the potential. It has been found that the value of the efficiency depends on the form of the potential and is bounded from above the Carnot value.

A natural extension of this work would be to consider the engine models consisting of other physical systems, such as interacting particles confined in a potential under conditions in which occupation probabilities correspond to either microcanonical [25] or canonical forms [26]. Such an issue deserves a deeper study in view of the substantial recent progress in theoretical considerations and the ability to handle particles in a potential.

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