# Why the water bridge does not collapse

Artem A. Aerov

*Physics Department, Moscow State University, Moscow RU-119991, Russia*

(Received 7 December 2010; revised manuscript received 26 May 2011; published 20 September 2011)

In 2007 an interesting phenomenon was discovered [J. Phys. D **40**, 6112 (2007)]: a horizontal thread of water, the so-called water bridge, hangs in a horizontal electrostatic field. A different explanation of the water bridge stability is proposed herein: the force supporting it is the surface tension of water, while the role of the electric field is to not allow the water bridge to reduce its surface energy by breaking into separate drops. It is proven that electrostatic field is not the origin of the tension holding the bridge.

DOI: 10.1103/PhysRevE.84.036314 PACS number(s): 47.55.nk, 47.20.Dr, 47.65.−d, 77.22.−d

## I. INTRODUCTION

After the water bridge (see Fig. 1) was rediscovered in 2007 [1] (it was first observed in 1893 [2]), it immediately captured attention [3] and even entered some television shows because the experiment is easy to reproduce and because it can be regarded as evidence of some unique properties of water. What keeps the bridge stable against gravity? The first thing one can suppose is that the water in the bridge has properties similar to those of a polymer melt; namely, in the electrostatic field, water molecules are arranged in quasi polymer chains that play the role of the bridge load-carrying structure [4]. It has been also supposed that hydrogen bonds induce the formation of the water bridge [5], but in the computer simulation of Ref. [5] the bridge consisted of $10^3$ molecules and could be formed only if the electrostatic field was at least $\approx 10^3$ times stronger than the ones in real experiments with macroscopic water bridges [1,6–12]. Some attempts have been made to reveal a specific structure of the bridge by means of neutron scattering [6,7] and Raman scattering [11], but still no exhaustive explanation of the stability has been found. Two interesting features of the water bridge are the complicated water flow and the electric current inside it [8,9], but it has not been proven that this dynamics is related to the stability. It has even been supposed that the bridge stability in gravity is a quantum effect [13]. In this context, experiments with the water bridge in reduced gravity are particularly interesting [14], but the parabolic flight lasts only a few seconds, which is hardly sufficient for investigating the equilibrium. A detailed review of water bridge investigations has already appeared [15].

However, the best explanations of newly discovered phenomena are often the simplest ones. Indeed, it has been stated [10,16] that not the specific properties of water but just its high dielectric permittivity is likely to be the reason for the phenomenon. Formation of a bridge (dielectric liquid bridge) by another small-molecule polar liquid [12,15] (e.g., methanol or glycerine) proves this. How can the high dielectric permittivity of a dielectric liquid give rise to the bridge stability? It is straightforward to assume that the bridge is kept stable in gravity by tension as a hanging rope, with the tension being somehow produced by the electric field [10,12,16]. In this paper, we prove that the electric field is, however, not the origin of the tension; instead, we show that it is likely the surface tension that holds the bridge.

## II. ELECTROSTATIC TENSION OF A DIELECTRIC LIQUID BRIDGE

In a typical experiment the bridge hangs between two beakers, in which electrodes are immersed (see, e.g., [6,8,10]). The electric field between the beakers is evidently nonuniform. However, the beakers' diameter is 5–10 cm, the electrodes are immersed at the points farthermost from the bridge, and the bridge is shorter than 2.5 cm. Therefore, the field around the bridge can be split into the uniform and the nonuniform parts, the latter being not dominant. Moreover, such bridges have also been produced between two large flat metallic plates [12] (see Fig. 1). The field around the central part of the bridge is evidently almost uniform in the latter configuration. This proves that nonuniformities of the field are not the basic reason for the phenomenon. Hence, it is relevant to consider the bridge roughly as a dielectric liquid cylinder in a uniform electrostatic field parallel to its axis. Let us use this approximation to analyze the bridge tension.

A dielectric liquid cylinder is in a uniform field $E$ parallel to its axis if, for example, the cylinder bases touch two infinite conducting planes to which a voltage $\Delta\varphi = LE$ is applied (see Fig. 2), where $L$ is the cylinder's length. To simplify the reasoning, let us imagine that there are two infinitely thin gaps between the cylinder bases and the planes. It follows from the electrostatic field boundary conditions [17] that the field is equal to $E$ everywhere between the planes (inside and outside the cylinder), except in the gaps, where it is equal to $\varepsilon E$, where $\varepsilon$ is the liquid's dielectric constant; the surface densities of charges induced on the cylinder bases are $\mp\sigma_{\text{ind}} = \mp E(\varepsilon - 1)/(4\pi)$, while the densities on the corresponding adjacent areas of the planes are $\pm\sigma_0 + \sigma_{\text{ind}} = \pm\varepsilon E/(4\pi)$, and the densities on the corresponding remaining parts of the planes are $\pm\sigma_0$ (see Fig. 2).

To derive the tension, let us suppose that the planes are isothermally moved apart by $dL$ at constant $\Delta\varphi$, and the cylinder is respectively elongated by $dL$. The field $E$ between the planes is then decreased by $E\,dL/L$, and the cylinder becomes thinner. This leads to the change $dU$ of the total energy $\frac{1}{8\pi}\int \varepsilon E^2 d^3\mathbf{r}$ of electrostatic field in the system. This leads also to the redistribution of charges induced on the planes (i.e., the voltage does the work when carrying charges from one plane to the other). For the part of the system external to the cylinder, the difference between the electrostatic field energy increase and the voltage work is then the work against the planes' mutual Coulomb attraction force existing

 

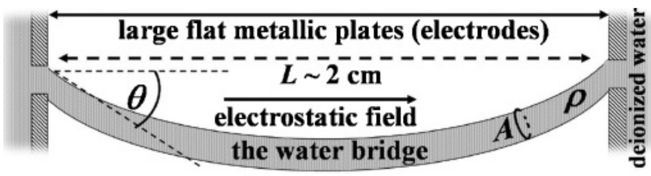


FIG. 1. Schematic of water bridge in the setup [12] that produces a uniform electrostatic field.

independently of the cylinder. For the cylinder part, the field energy is increased by

$$dU = E^2 [\varepsilon A dL + (\varepsilon - 1) L dA - 2\varepsilon A dL]/(8\pi), \quad (1)$$

where $A$ is the cylinder's cross-sectional area. The first term of Eq. (1) is due to the elongation of the domain occupied by the dielectric with electrostatic field. The second term is due to the thinning of the domain occupied by the dielectric. The last term is due to the decrease of the field in the whole volume of the cylinder.

The work of the voltage for the cylinder part is:

$$dW = E [\sigma_{\text{ind}} L dA - (\sigma_0 + \sigma_{\text{ind}}) A dL], \quad (2)$$

where the first term is due to the change of the area of the cylinder bases and the second term is due to the decrease of the field. The field $\varepsilon E$ in a gap is the field $\varepsilon E/2$ of the adjacent plane plus the field $\varepsilon E/2$ external with respect to the plane. $dU - dW$ is the work against the cylinder tension $\tau$ and the Coulomb force exerted by the external field $\varepsilon E/2$ on a plane circle adjacent to a cylinder base:

$$dU - dW = [\tau + (\sigma_0 + \sigma_{\text{ind}}) \varepsilon E/2] A dL. \quad (3)$$

To first approximation, the liquid is incompressible (i.e., $LdA + AdL = 0$). Therefore, it follows from Eqs. (1)–(3), that $\tau = -(\varepsilon - 1)^2 E^2/(8\pi)$. The negative tension corresponds to the stretching of a dielectric liquid drop along uniform external electrostatic field [18]. In Ref. [16], positive tension was obtained because the pressures produced by the electrostatic field on the liquid-air interfaces were assumed therein to be equal to the corresponding Maxwell stresses in the liquid. Maxwell stresses on both sides of the interface must be subtracted instead from each other to obtain the pressures [17].

Are the planes pushed apart by the cylinder with pressure $-\tau > 0$? Let us analyze the Coulomb interaction between the

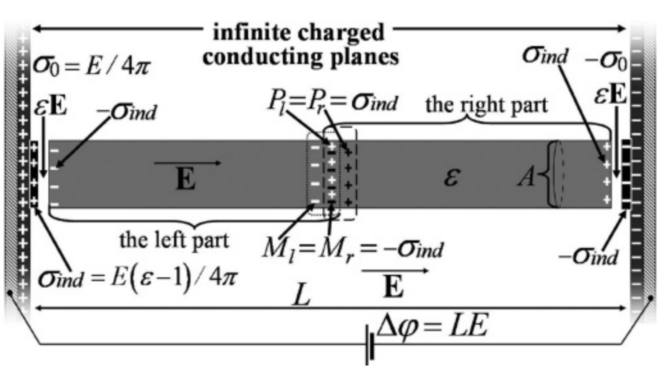


FIG. 2. Schematic of a dielectric liquid cylinder in a uniform electrostatic field between two infinite conducting planes. This is a simple model of a dielectric liquid bridge.

cylinder and, say, the left plane. There are two charges on the plane. The uniform charge density $\sigma_0$ produces the uniform field, which does not exert force on the cylinder since the total charge of the cylinder is zero. The other charge is the circle with the density $\sigma_{\text{ind}}$ induced additionally in front of the cylinder base. If $L \ll \sqrt{A}$, the fields of the charges of the cylinder bases cancel outside of the cylinder, because these opposite charges are very close to each other. Hence, the dielectric liquid "pancake" does not exert a force on the plane by the electrostatic field but exerts on it, really, only the pressure derived above of $-\tau > 0$. But in the case of the bridge with $L \gg \sqrt{A}$, the right cylinder base is far from the left plane so the field produced by the charge of the base is zero on the left plane. Therefore, the attraction between the $\sigma_{\text{ind}}$ circle on the left plane and the long cylinder is equal to the attraction $2\pi\sigma_{\text{ind}}^2 A$ between the circle and the charge opposite to it on the left cylinder base, which is adjacent. Hence, the bridge attracts the left (right, as well) plane by an electrostatic field with the force $2\pi\sigma_{\text{ind}}^2 A$, which cancels together with the pressure $-\tau$ that is exerted by the bridge on the plane.

The same cancellation applies to the interaction between two adjacent parts of the bridge. (They have to attract each other if the bridge is supported by its tension as a hanging rope.) A dielectric cylinder in a uniform electrostatic field parallel to its axis is a stack of identical and equally oriented one-dipolar-molecule-thick double electrostatic layers. Neighboring layers penetrate each other: the area of their overlap is neutral since the positive charge of one layer and the negative charge of the other are intermixed there. Thus, each point inside the cylinder is neutral. The positive charge of the last layer at one cylinder base and the negative charge of the last layer at the other base are not neutralized. The surface densities of these charges are the charges induced on the bases: $\sigma_{\text{ind}}$ and $-\sigma_{\text{ind}}$. This means that each of the double layers is composed of $\pm\sigma_{\text{ind}}$ charges. Probably, the anisotropy of the water bridge detected by means of neutron scattering [6] is related to this ordering of molecules along the electrostatic field. The left and the right parts of the long cylinder, each consisting of an integer number of double layers, interact as follows: (Dividing the cylinder by a plane into two nonoverlapping parts would make no sense because dipole molecules of one layer would be cut into pieces belonging to different parts). The rightmost layer $M_l/P_l$ of the left part (schematically presented in Fig. 2 as a dotted rectangle with white pluses and minuses inside) and the leftmost layer $M_r/P_r$ (dashed rectangle with black pluses and minuses) of the right part overlap. ($M_l$, $P_l$, and $M_r$, $P_r$ symbolize the negative and positive charges of the two layers, respectively.) The left part taken without the $M_l/P_l$ layer has the uncompensated charge density $\sigma_{\text{ind}}$ on its right base, which attracts the right part with the force $2\pi\sigma_{\text{ind}}^2 A$, like the left plane attracts the whole cylinder. At the same time, the overlapping double layers $M_l/P_l$ and $M_r/P_r$ repel each other with the same force because $M_l$ repels $M_r$ and $P_l$ repels $P_r$ with the force $2\pi\sigma_{\text{ind}}^2 A$, $M_l$ attracts $P_r$ with the same force, and there is no interaction parallel to the cylinder axis between $P_l$ and $M_r$ since they overlap. By the way, the same forces expulse from the cylinder its last layers at the bases, which is the origin of the pressure $-\tau$ exerted by the cylinder bases on the planes. Regarding the interaction of the $M_l/P_l$

layer with the right part taken without the $M_r/P_r$ layer, it is negligibly weak. So, the total tension of a dielectric liquid bridge produced by the electrostatic field is zero. It should be noted also that the electrostatic field hypothesis of the bridge tension ($\tau \sim E^2$, [10,12,16]) is not really consistent with experiments, because it allows the existence of bridges longer than 4 cm in stronger fields, which seems to be not the case.

## III. THE REASON FOR THE BRIDGE STABILITY

What tension holds the bridge then? Let us just estimate the tension of the bridge produced by surface tension. If a liquid cylinder is elongated by $dL$ without changing its volume, the cylinder's lateral surface area is increased by $l dL/2$, where $l$ is the bridge cross-section perimeter. Hence, if the surface tension coefficient of the liquid is $\gamma$, the total tension of the bridge produced by surface tension is $l\gamma/2$. In a hanging bridge, horizontal projection of the tension at an end is equal to the tension in the center, while the doubled vertical projection of the tension at an end is the weight of the bridge. Hence, the tension in the center is $\simeq \rho g A L/(2 t g \Theta)$, where $\rho$ is the density of the liquid and $\Theta$ is the angle between an end of the bridge and the horizontal (see Fig. 1). (It is supposed here when calculating the weight that the bridge is straight and its cross section is uniform). Therefore, surface tension can hold the bridge if

$$\rho g \cot(\Theta) L A / l \simeq \gamma. \tag{4}$$

One can find evidence that Eq. (4) is approximately valid by analyzing the photos of dielectric liquid bridges presented in Refs. [6,8,10,12]. Enlarged photos from the electronic versions of the papers should be used. For example, let us consider the central part of Fig. 7 in Ref. [12], which presents a glycerine ($\rho = 1250$ kg/m$^3$ [12]) bridge, in the setup with the configuration producing a uniform electrostatic field. Let us suppose that the cross section of the bridge is roughly a circle of radius $r$. (In fact, horizontal and vertical projections of water bridges presented in one of the figures of Ref. [12] show that the heights of the bridges are about 1.5 times larger than their widths.) Then $A/l = r/2$ and, having measured $\cot\Theta \simeq 1.6$, $r \simeq 1.1$ mm, and $L \simeq 7.6$ mm, we obtain from Eq. (4) $\gamma \simeq 82$ mN/m, which is close to the actual value of 64 mN/m [12] for glycerine. For example, such an analysis of the schematic of the water bridge in Fig. 1 gives $\cot\Theta \simeq 1.6$, $r \simeq 0.63$ mm, and $L \simeq 21$ mm, which with $\rho = 980$ kg/m$^3$ gives $\gamma \simeq 102$ mN/m. The real value for water is $\gamma = 72$ mN/m [12] (i.e., the bridge in the illustration should be, say, thinner). The accuracy of such "measurements" is very low of course. The precisely determined shape of a bridge must be analyzed to prove exhaustively that the bridge is supported by surface tension. Collapse of a water bridge caused by the addition of surfactant reported in Ref. [1] also corroborates the surface tension origin of the bridge stability.

Why is a dielectric liquid bridge not possible without an electric field? The reason is that surface tension plays, actually, an ambivalent role. On the one hand, it does not allow gravity to tear the bridge. But, on the other hand, as has been mentioned in Refs. [12,15], it "wants" a sufficiently long and thin bridge to turn into separate round drops, because then the surface energy would decrease (i.e., the bridge would be in a labile equilibrium without the outer longitudinal electric field). The latter provides stable equilibrium: it does not allow distortion of the bridge shape to start, because the field energy is the lowest if the shape is nonperturbed. This phenomenon has been extensively studied long ago [19–22]. To complete the basic explanation of the bridge stability let us report briefly the main relevant conclusions of Refs. [20–22] without repeating the derivations presented therein. In Ref. [20], the energy change caused by small sinusoidal distortions of an infinite cylindrical jet of dielectric liquid (an infinitely long dielectric liquid bridge in zero gravity, in other words) was analyzed. It was proven that the longitudinal electrostatic field $E_{cr}$ necessary for the stability is $\sim\sqrt{\gamma}$, and it is lower for larger $A$ or $\varepsilon$. In Ref. [21], the equilibrium shape of a bridge of one dielectric liquid surrounded by another one of the same density was studied. (It should be noted that the system considered in the present paper and the system described in Ref. [21] are, of course, not the same since the gravity effect is canceled out in the latter one.) The possibility of an equilibrium shape very close to the cylindrical one was used as the instability criterion, and the same results were obtained: $E_{cr} \sim \sqrt{\gamma_i} L/\sqrt{A}$, where $\gamma_i$ is the surface tension coefficient of the interface between the two liquids; and $E_{cr}$ is lower the larger is the ratio of the two dielectric constants of the liquids. This explains why it is hard to make a long dielectric liquid bridge: the bridge must be thin to withstand gravity [see Eq. (4)], but a thinner and longer bridge needs a much stronger field to keep the shape. The consideration of Ref. [21] was generalized in Ref. [22] for the case when the bridge is vertical, and there is a small difference in the two liquid densities. It was shown that even the small effect of axial gravity strongly destabilizes the equilibrium. This explains the lower stability of vertical water bridges as compared to horizontal ones [11].

## IV. CONCLUSIONS

Our reasoning describes the basic roles of surface tension and electric field in providing the dielectric liquid bridge stability. It does not explain why the increase of voltage leads to the thickening of the horizontal water bridge between the beakers [11] and to the deformation of the horizontal glycerine bridge [12] in the setup producing uniform electrostatic field. The bridges are thicker and slacker or thinner and straighter, which somehow depends on the voltage. However, our approximate Eq. (4) is valid for all cases. How does the bridge shape depend on the parameters? We believe that this is a secondary question, the answer to which would only supplement the basic explanation proposed by us of the bridge stability. To determine the equilibrium shape one has to minimize the energy of the system, taking into account the electrostatic field energy, the surface energy, and the potential energy in gravity. In other words, the liquid surface must be found, along which the equilibrium is kept between the pressures produced by the electrostatic field [17], the surface tension, and the liquid head. This seems to be a very complicated problem. Probably, it cannot be solved if the electrostatic field nonuniformities are neglected. Probably, free charges are present in the bridge and are also to be taken into account.

Lastly, let us propose three small hints for experiment. (1) It has been reported that a water bridge is possible in an oscillating electric field [11]. At the same time it is known that the water must be deionized, evidently because free charges relocate, thus screening the field. But if the field oscillates frequently enough, the ions do not have time to relocate [21]. Therefore, one might avoid deionizing the water if a high-frequency oscillating voltage is used. It would also be possible then to measure the nonelectrostatic tension of the dielectric bridge separately from the Coulomb attraction of the electrodes. (The mutual attraction of the beakers between which water bridge hangs has already been measured [8]). (2) It would be interesting to make a bridge from a liquid with a dielectric permittivity higher than that of water. A longer bridge might then be possible. The dielectric constant of *N*-methylformamide is around 200 [23,24]. The challenge is to make sure that the liquid is free of ion-producing contaminants—the first of which is water. Otherwise, the conductivity would be too high [24]. (3) If one uses electrodes covered with glass, one can probably make a dielectric liquid bridge without electric current and liquid flow inside, which would prove that the dynamics of the bridge is not related to its stability.

## ACKNOWLEDGMENTS

We thank E. Fuchs and K. Gatterer for introducing the phenomenon and providing their pioneering works on the subject; V. I. Denisov, O. E. Philippova, A. M. Lotonov, I. A. Malyshkina, S. V. Venev, A. V. Gorshkov, and A. F. Aerov for discussions. This work was supported by the Ministry of Science and Education of the Russian Federation (state contract П1365, signed 02.09.2009) in the framework of the realization of the federal task program "Scientific and scientific-educational personnel of innovational Russia" in the years 2009–2013.

[1] E. C. Fuchs *et al.*, J. Phys. D **40**, 6112 (2007).
[2] W. G. Armstrong, *The Electric Engineer* (The Newcastle Literary and Phylosophical Society, New Castle, 1893), p. 154.
[3] E. Hand, Nature (London) **449**, 517 (2007).
[4] H. Nishimi and H. Honda, Res. Lett. Phys. Chem. 371650 (2009).
[5] T. Cramer, F. Zarbetto, and R. Garcia, Langmuir **24**, 6116 (2008).
[6] E. C. Fuchs *et al.*, J. Phys. D **43**, 105502 (2010).
[7] E. C. Fuchs *et al.*, J. Phys. D **42**, 065502 (2009).
[8] J. Woisetschlager, K. Gatterer, and E. C. Fuchs, Exp. Fluids **48**, 121 (2010).
[9] E. C. Fuchs *et al.*, Proc. SPIE **7376**, 73761E (2010); J. Phys. D **41**, 185502 (2008); M. Eisenhut *et al.*, Cent. Eur. J. Chem. **9(3)**, 391 (2011).
[10] F. Saija *et al.*, J. Chem. Phys. **133**, 081104 (2011).
[11] R. C. Ponterio *et al.*, J. Phys. D **43**, 175405 (2010).
[12] A. G. Marin and D. Lohse, Phys. Fluids **22**, 122104 (2010).
[13] E. D. Giudice and G. Vitiello, Water **2**, 133 (2011); E. D. Giudice, E. C. Fuchs, and G. Vitiello, *ibid.* **2**, 69 (2010).
[14] E. C. Fuchs *et al.*, J. Phys. D **44**, 025501 (2011).
[15] E. C. Fuchs, Water (MDPI) **2**, 381 (2010).
[16] A. Widom, J. Swain, J. Silverberg, S. Sivasubramanian, and Y. N. Srivastava, Phys. Rev. E **80**, 016301 (2009).
[17] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1960), Chapter II.
[18] N. K. Nayyar and G. S. Murty, Proc. NISI (Phys. Sci.) **25**, 373 (1959).
[19] D. A. Saville, Annu. Rev. Fluid. Mech. **29**, 27 (1997).
[20] N. K. Nayyar and G. S. Murty, Proc. Phys. Soc. **75**, 369 (1960).
[21] H. Gonzalez *et al.*, J. Fluid Mech. **206**, 545 (1989).
[22] H. Gonzalez and A. Castellanos, J. Fluid. Mech. **249**, 185 (1993).
[23] G. R. Leader and J. F. Gormley, JACS **73**, 5731 (1951); J. Richardi *et al.*, Chem. Phys. Lett. **273**, 115 (1997).
[24] J. Swiergiel and J. Jadzyn, JPC B **113**, 14225 (2009).